Implicit Monte Carlo Radiative Transfer in DRACO*

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Abstract

Implicit Monte Carlo method is used to solve the coupled photon transport equation and plasma energy conservation equation with improved stability for large time steps. The essence is that a forward plasma temperature is estimated at the beginning of the time step in the transport equation using the so-called Fleck factor. We are developing an Implicit Monte Carlo code module in DRACO for parallel, two-dimensional, multi-group frequency radiation transfer. A simple equilibrium solution in an infinite medium is used as an analytical benchmark. In other test cases, analytical opacities, as in the Fleck and Cummings' paper, are applied to compare with their one-dimensional results using our two-dimensional code.

Outline

1. Introduction of the Implicit Monte Carlo (IMC) method for radiation transport.
2. Simulation of radiation Marshak wave problem in 1D and 2D.
3. Comparison with diffusion results.
4. Summary and future work.

Implicit Monte Carlo Radiation Transport

Photon transport and plasma energy equations:

$$\frac{1}{c}\frac{\partial I(r,\Omega,\nu,t)}{\partial t} + \Omega\cdot\nabla I(r,\Omega,\nu,t) + \mu_t(\nu) I(r,\Omega,\nu,t) = \mu_a(\nu) B(\nu)$$

$$\frac{\partial u_m(r,t)}{\partial t} = \iint \mu_t(\nu) I(r,\Omega,\nu,t)\, d\nu d\Omega - 4\pi\int \mu_a(\nu) B(\nu)\, d\nu$$

Change variable by $\frac{\partial u_m(r,t)}{\partial u_r(r,t)} = \frac{1}{\beta(r,t)}$ $\quad u_r(r,t) \equiv$ equilibrium radiation energy

IMC transport and plasma energy conservation equations

$$\frac{1}{c}\frac{\partial I(r,\Omega,\nu,t)}{\partial t} + \Omega\cdot\nabla I(r,\Omega,\nu,t) + \mu_t(\nu) I(r,\Omega,\nu,t) = \frac{c}{4\pi}\mu_a(\nu) b_\nu u_r(r,t)$$

$$\frac{1}{\beta(r,t)}\frac{\partial u_r(r,t)}{\partial t} = \iint \mu_t(\nu) I(r,\Omega,\nu,t)\, d\nu d\Omega - c\sigma_p(r,t) u_r(r,t)$$

Fleck and Cummings IMC Method

In the integration of the plasma energy equation from t^n to t^{n+1}, the integrands are approximated by the mean-value theorem and the centered value $u_r(r,t)$ is approximated as a linear combination of t^n and t^{n+1} timestep values:

$$u_r(r,t) = \alpha\, u_r(r,t_{n+1}) + (1-\alpha) u_r(r,t_n)$$

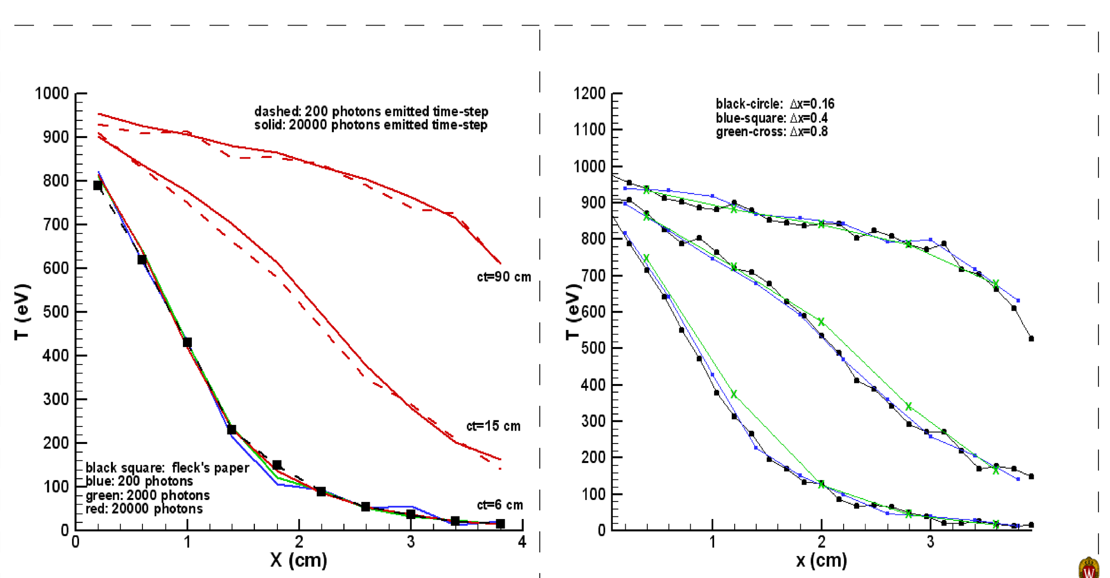
Define: $$f = \frac{1}{1+\alpha c \beta \sigma_p \Delta t}$$

$$\frac{1}{c}\frac{\partial I(r,\Omega,\nu,t)}{\partial t} + \Omega\cdot\nabla I(r,\Omega,\nu,t) + \mu_t(\nu) I(r,\Omega,\nu,t) = f\frac{c}{4\pi}\mu_a(\nu) b_\nu u_r(r,t) + (1-f)\mu_a(\nu) b_\nu \frac{1}{4\pi\sigma_p}\iint \mu_a(\nu) I(r,\Omega,\nu,t)\, d\nu d\Omega$$

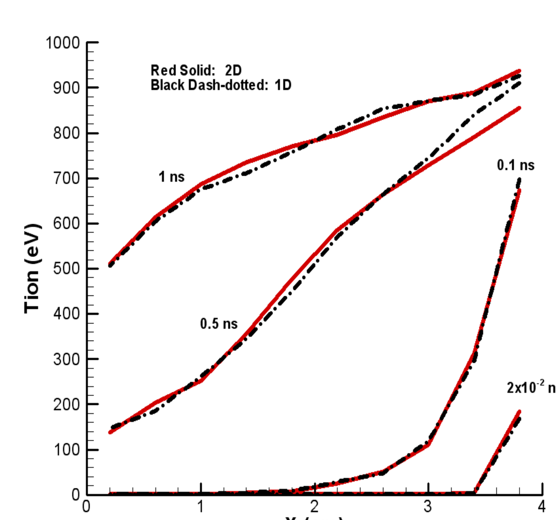
$$\frac{1}{\beta(r,t)}\frac{\partial u_r(r,t)}{\partial t} = f\iint \mu_t(\nu) I(r,\Omega,\nu,t)\, d\nu d\Omega - fc\sigma_p(r,t) u_r(r,t)$$

Simulation of Radiation Transport in 1D with IMC

1) A slab 4cm in thickness, heated by a 1 keV blackbody source at x=0
2) IMC results by varying the number of sampled photons and the mesh size
3) The agreement with Fleck's paper is very good



Comparison of 2D Simulation with the 1D Result



A slab heated by a 1 keV blackbody source from the right boundary

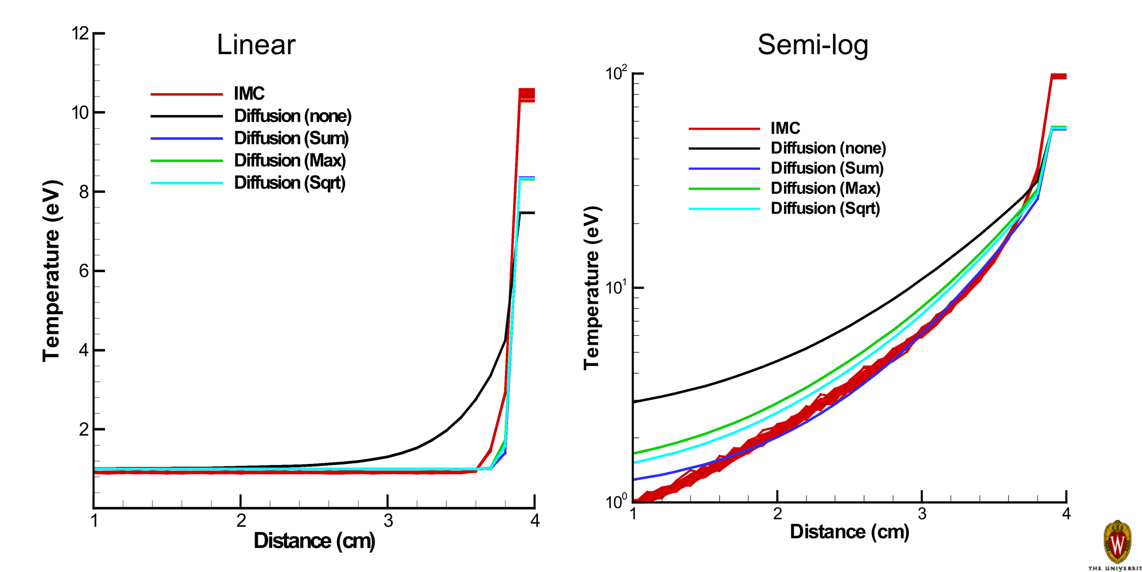
The results from 2D IMC are averaged for I-lines to compare with the 1D results

The 2D simulations agree with Fleck and Cummings results.

2D Comparison of IMC with the Diffusion Results

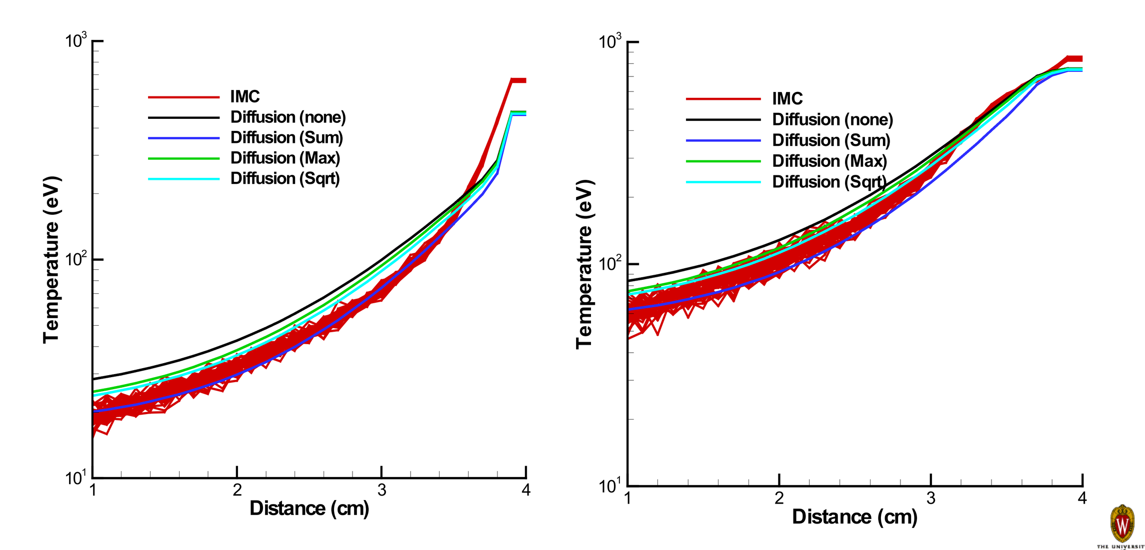
Electron thermal conductivity is set to a very small value so that the IMC and diffusion are solving the same problem.

None: no flux limiter is used
Sum: summation form of flux limiter
Max: maximum form of flux limiter
Sqrt: square root form of flux limiter

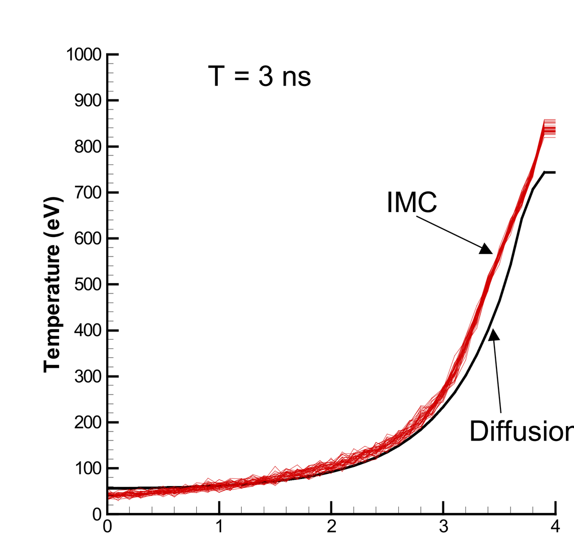


2D Comparison of IMC with the Diffusion Results

1) Diffusion with the summation form of flux limiter agrees with IMC better than other forms of flux limiter far from the source.
2) Close to the source, IMC agrees with diffusion with the forms of flux limiter other than the summation form.



Operator Splitting in Energy Source Coupling



1) Full thermal transport with real thermal conductivity.

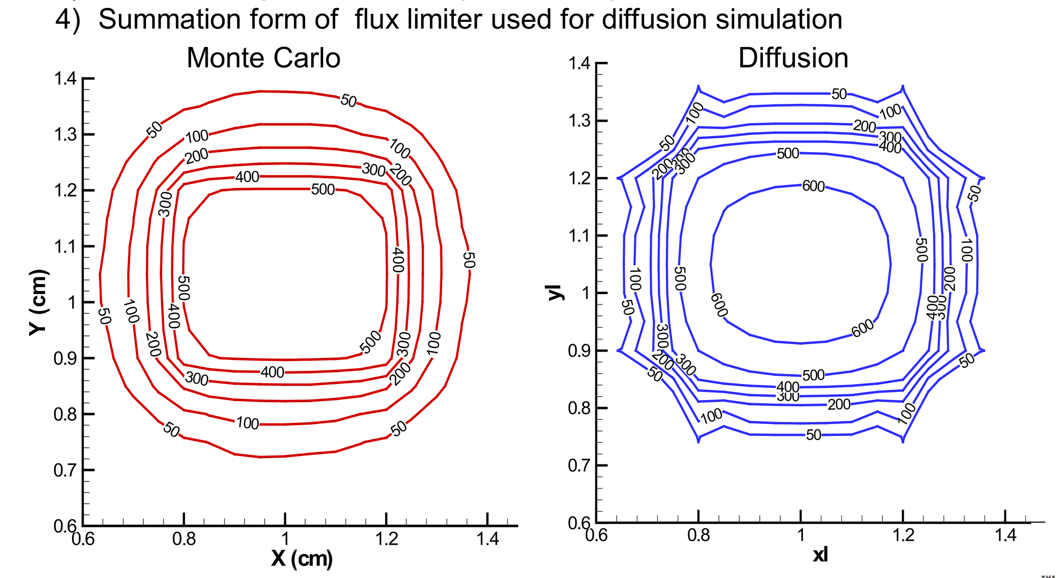
2) Temperature is updated before plasma electron conduction in IMC splitting method.

3) In diffusion method, the radiation energy is a source term in the electron thermal transport equation.

4) The operator splitting method in IMC agrees with diffusion results.

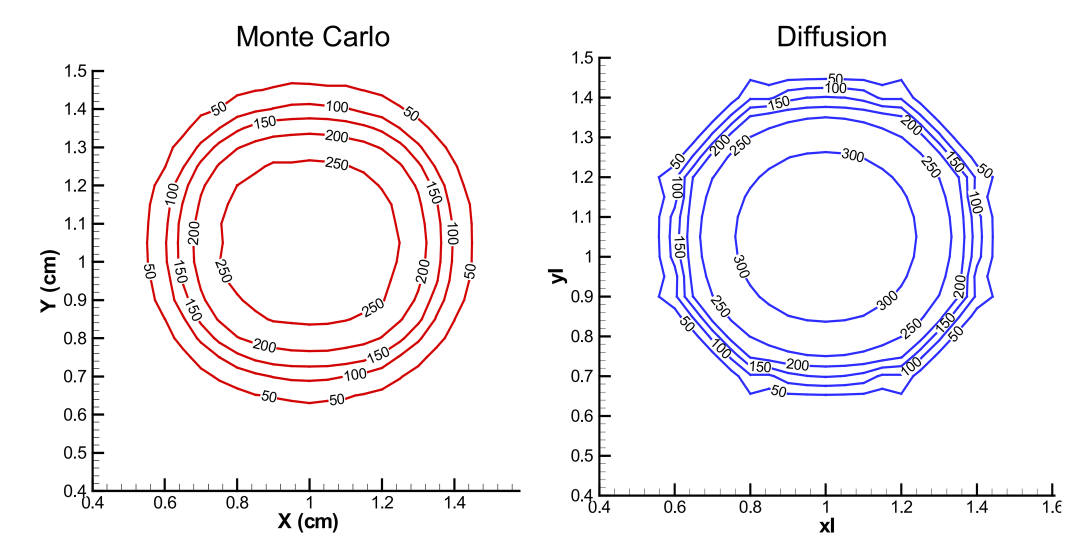
2D Contour Comparison of IMC with Diffusion

1) A square source (0.5cm by 0.5cm) with a temperature of 1 keV at the center of a region (2cm by 2cm).
2) Mesh size: 40 by 40
3) Monte Carlo photon number per time step: 100,000
4) Summation form of flux limiter used for diffusion simulation



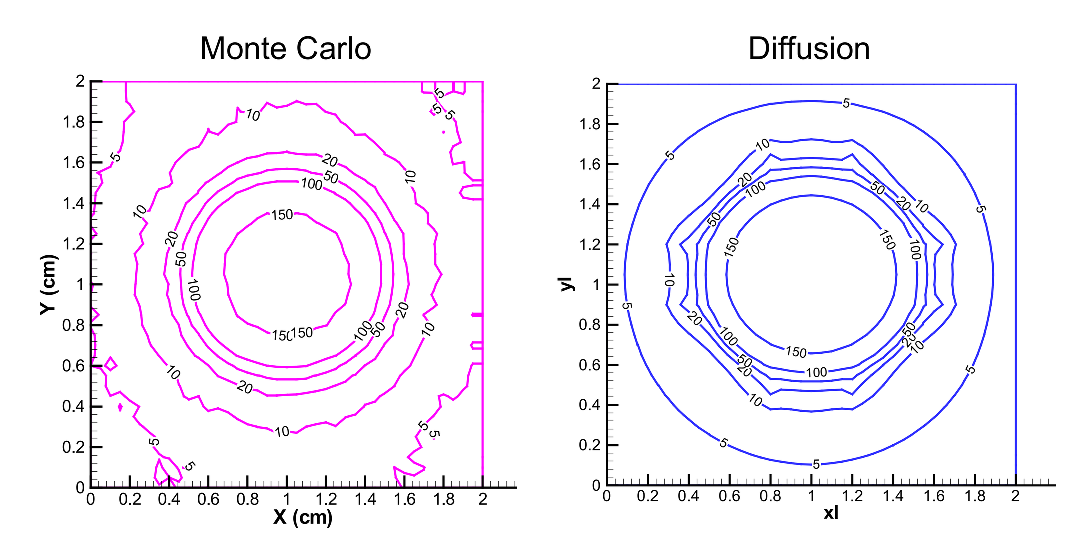
2D Contour Comparison of IMC with Diffusion

1) Diffusion algorithm shows non-smoothness at the corners of the hot square; the results from IMC are much smoother.
2) The size of heated area is similar for both methods, however, the temperature at the inner circle for the diffusion method is higher than predicted by IMC



2D Contour Comparison of IMC with Diffusion

1) The statistical error in IMC is evident for the regions with lower temperature since many fewer photons reach those areas.
2) The thermal radiation wave at the center propagates faster in the diffusion method compared with the IMC.



Summary and Future Work

We are implementing an Implicit Monte Carlo method for radiation transport into the multi-dimensional hydrodynamic code DRACO. Operator splitting techniques are used to couple with electron thermal transport. Simulations of radiation Marshak wave on a diffusive plasma show agreement with the diffusion results. In the future, continued testing on full target simulations will be performed on a massively parallel computer. Variance reduction methods will be introduced.

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