Three-Dimensional Neutron Source Models for Toroidal Fusion Energy Systems

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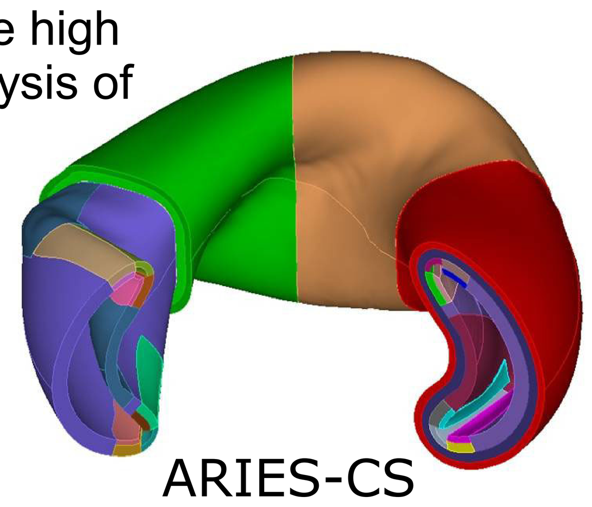
Introduction and Motivation

- New developments enable high fidelity 3D neutronics analysis of complex geometries.
- *Goal:* couple results to other types of engineering analysis.
- CAD-based, continuous energy Monte Carlo calculations are combined with high res output.



- *Result:* increased importance of neutron source distribution.
- *Cylindrical Mesh* - toroidally symmetric machines
- Uses poloidal magnetic flux represented on a 2D quadrilateral R-Z grid in the community standard *geqdsk* format.
- *Conformal Hexahedral Mesh* - complex sources
- Uses plasma physics data defined on a uniform hexahedral grid in an idealized plasma coordinate system which is transformed to real space through a Jacobian transformation.
- Closer coupling of plasma physics simulations with neutronics analysis enabling easier design iteration.
- Each method was applied to a representative fusion machine.
- The neutron wall loading distribution, or NWL, is used as the basis for analyzing the results.
- NWL is more strongly affected by the source than other parameters.

Cylindrical Mesh Method

Method

- A source density, S, is assigned to each vertex of the cylindrical mesh:

$$S(R,Z) = S\{T_i[\psi(R,Z)], n_i[\psi(R,Z)]\}$$

- The relative probability of each cell is:

$$D_{i,j} = 2\pi \int_Z dZ \int_r RdRS(R,Z) \approx 2\pi\Delta Z \left[\int_r RdR \left\{\frac{S(R,Z_{j+1}) + S(R,Z_j)}{2}\right\}\right]$$

$$= 2\pi\Delta Z \frac{\bar{S}_{z_{i+1}} - \bar{S}_{z_i}}{R_{i+1} - R_i}\left[\left(\frac{R_{i+1}^2}{2} - R_i \cdot R_{i+1}\right) - \left(\frac{R_i^2}{2} - R_i \cdot R_i\right)\right] + \bar{S}_{z_i}(R_{i+1} - R_i)$$

- A linear search is carried out through Z then R where:

$$PDF_{i,j} = \frac{D_{i,j}}{N} \leftarrow N = \sum_{i,j} D_{i,j}$$

- The CDF is the cumulative sum of the PDF
- The location within the cell is sampled uniformly in Z and the toroidal angle, and linearly in R

ARIES-RS

- 1881.5 MW tokamak
- 5.12 m major radius
- 1.28 m minor radius
- 55 cm magnetic shift

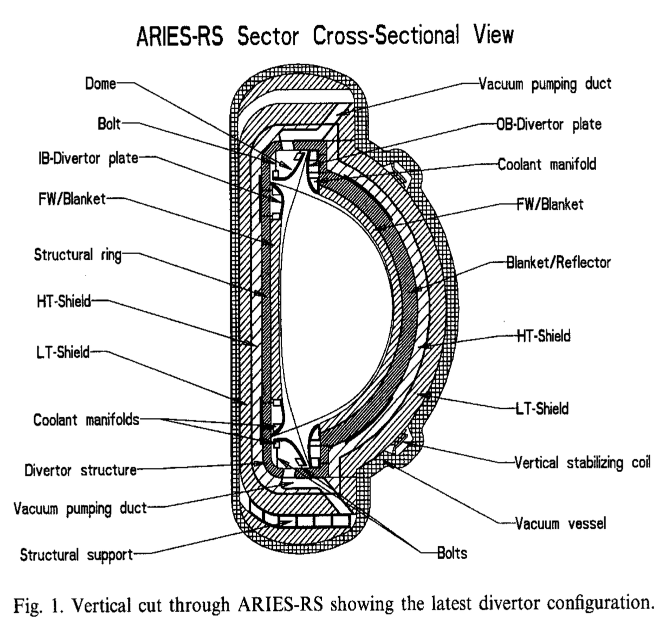
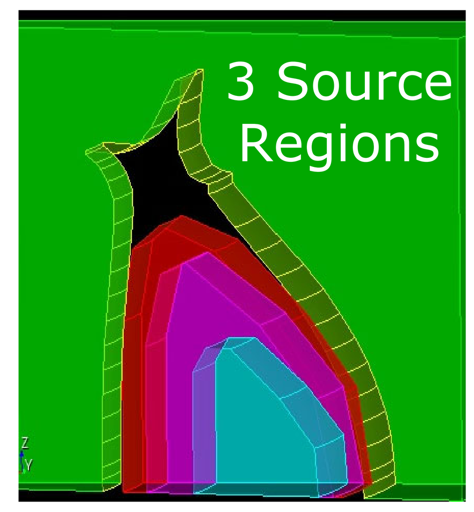
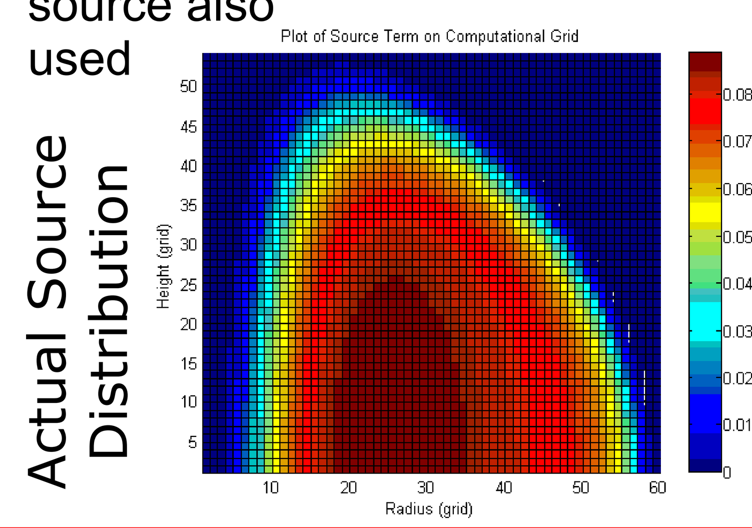


Fig. 1. Vertical cut through ARIES-RS showing the latest divertor configuration.

Symmetric about the midplane

- Analyzed in 1996[1] with 3-region source weighted to represent real source distribution
- 1-region source also used

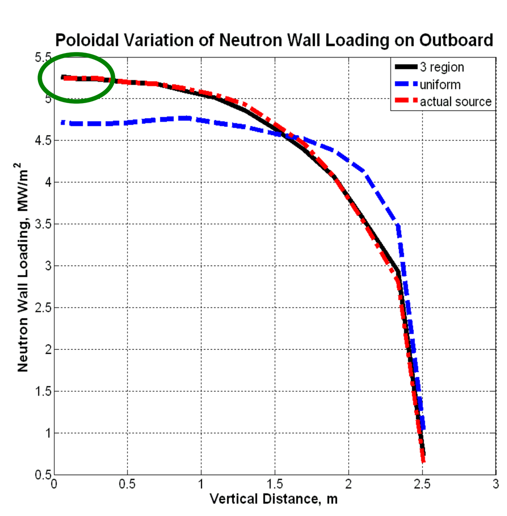


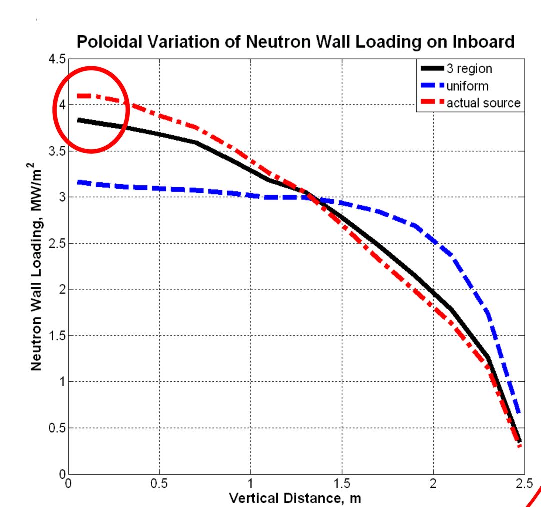


Results

- Actual source should be used for accurate analysis

NWL	1-uniform	3-uniform	actual
Peak Inboard	3.2 MW/m²	3.8 MW/m²	**4.1 MW/m²**
Peak Outboard	4.8 MW/m²	5.3 MW/m²	5.3 MW/m²
Average	3.1 MW/m²	3.1 MW/m²	3.1 MW/m²





Conformal Hexahedral Mesh Method

Method

- Each vertex has a source strength
- Each hex is mapped to the natural coordinate system
- Average source at each vertex is found by integration with Gaussian quadrature

1) Convert from real space to natural space:

$$x(\vec{\xi}) = \sum_{a=1}^{8} N_a(\vec{\xi}) x_a^e$$

ξ η ζ: Any point

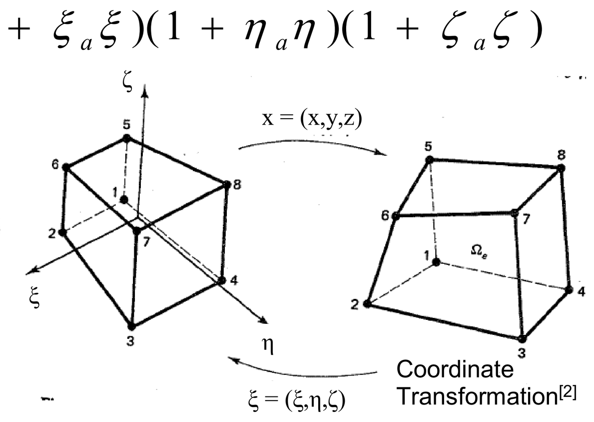
ξ_a η_a ζ_a: Vertices

ξ_l η_l ζ_l: Integration points

$$x_a^e = x(\vec{\xi}_a)$$

$$N_a(\vec{\xi}) = \frac{1}{8}(1+\xi_a\xi)(1+\eta_a\eta)(1+\zeta_a\zeta)$$

- Done for source values and coordinates at vertices and quadrature points



2) Numerically integrate to get the relative probability for each vertex in real space:

$$D_{i,j,k}(\vec{\xi}) \equiv \int_{-1}^{1} g(\vec{\xi})d\vec{\xi} \cong \sum_{l=1}^{n_{int}} g(\tilde{\xi}_l, \tilde{\eta}_l, \tilde{\zeta}_l)W_l \qquad P_{i,j,k} = \frac{D_{i,j,k}}{N}$$

$$g(\vec{\xi}_l) = S(\vec{\xi}_l) j(\vec{\xi}_l)$$

Define:

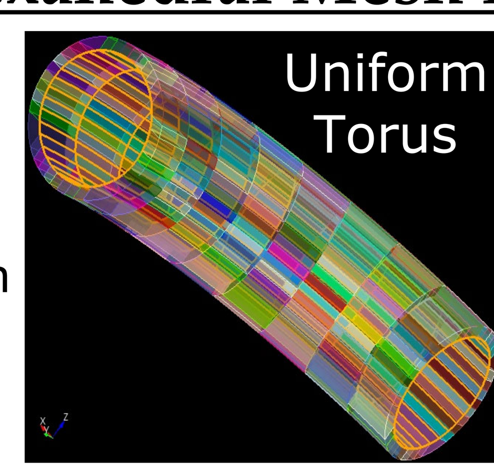
$$j(\vec{\xi}_l) = \begin{vmatrix} x_\xi x_\eta x_\zeta \\ y_\xi y_\eta y_\zeta \\ z_\xi z_\eta z_\zeta \end{vmatrix} \quad \leftarrow x_\xi(\vec{\xi}_l) = \sum_{a=1}^{8} N_{a,\xi}(\vec{\xi}_l) x_a^e \quad \leftarrow N_{a,\xi}(\vec{\xi}) = \frac{1}{8}(\xi_a)(1+\eta_a\eta)(1+\zeta_a\zeta)$$

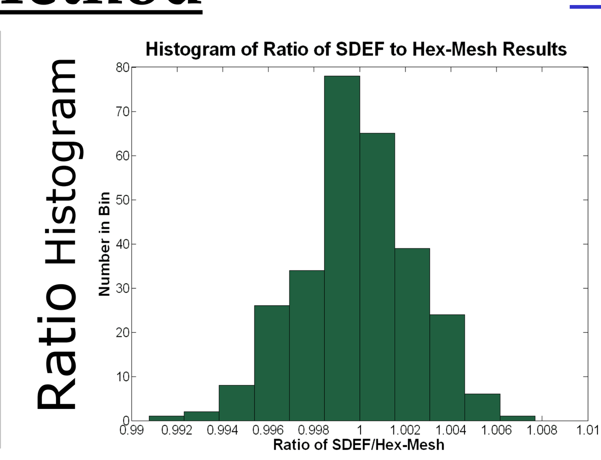
3) The CDF is the cumulative sum of the PDF

4) The CDF is sampled in MCNP:
- The cell to be sampled is determined using a hierarchical (linear) search
  - Location of particle within the hex is found by: $[(Src)(j)](\vec{x}) \le Src_{max} j_{max}$

Uniform Torus

- Comparison of this method with MCNPX's built in general source
- Test geometry: ¼ torus with 5 m major radius and 1 m minor radius
- Source: uniform in volume

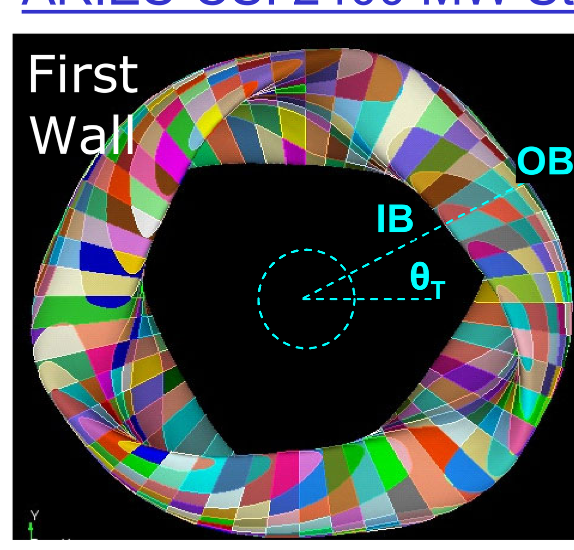


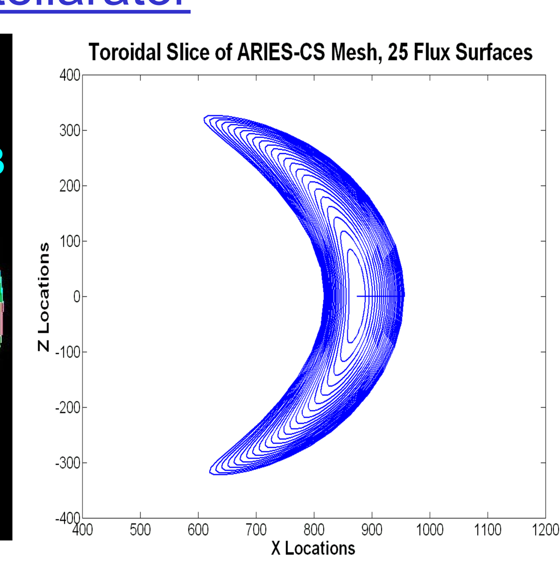


Results

- Discrepancies dominated by statistical error
- Method validated
- Average ratio: 0.9999
- Standard deviation: 2.5×10^{-3}
- Max relative difference: 7.7×10^{-3}
- Max statistical error: 2.9×10^{-3}
- Average statistical error: 2.4×10^{-3}

ARIES-CS: 2400 MW Stellarator





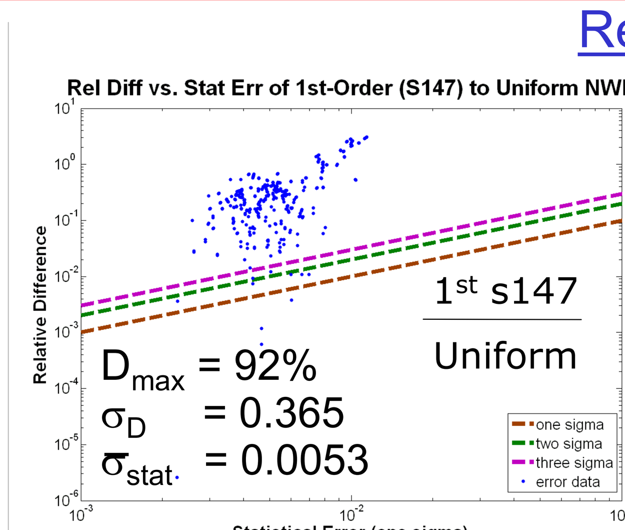
Toroidal slice of ARIES-CS with 25 flux surfaces

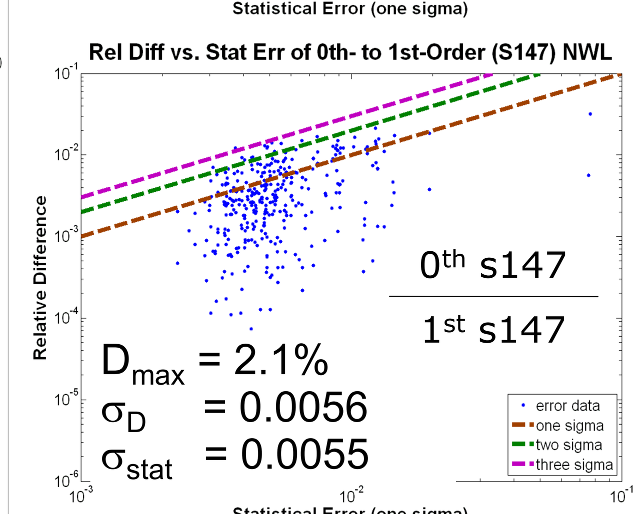
- 3 field periods
- average major radius: 7.75 m
- average minor radius: 1.7 m
- aspect ratio: 4.5
- 1/3 of geometry sliced into 352 surfaces for tallying
- Source region has distorted and degenerate hexes

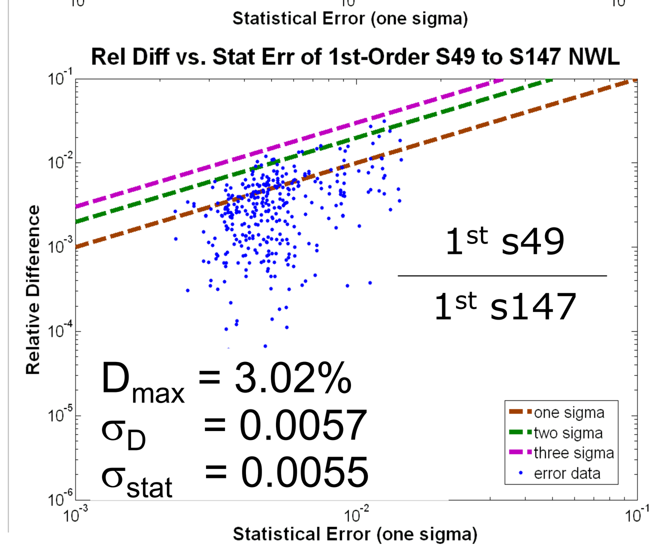
New 1st-Order vs. Previous 0th-Order

- Previous analysis done using a uniform source and a 0th-order source[3]
- New method is a 1st-order source
- Differences between the 1st-order and 0th-order:
  - Neutron source strength in each hex
    - 0th: volume averaged
    - 1st: *numerically integrated average*
  - Neutron source sampling in each hex
    - 0th: uniform in volume
    - 1st: *rejection based on source variation*

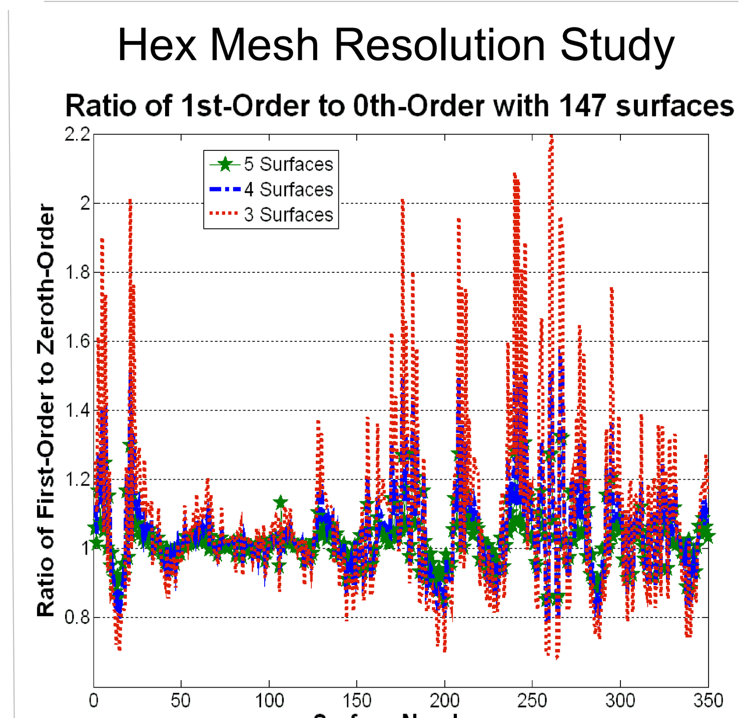
Results







- Uniform source is not accurate
- 1st-order source is correct at high resolution
- Lower resolution 1st-order source still provides good results



- 1st-order with 5, 4, & 3 flux surfaces compared to 0th-order 147 surfaces
- Increasing number of flux surfaces at low resolution improved results
- Dramatic improvement by adding only a few surfaces

Future Work

- Investigate effects of changing mesh resolution for Cylindrical mesh method.
- Continue investigating the tradeoffs between the 1st- and 0th-order Conformal Hexahedral mesh methods.
- More smoothly integrate source generation and source sampling.

Conclusions

- Methods were developed which accurately sample the neutron source for fusion machines.
- Cylindrical Mesh method was applied to ARIES-RS and found to be necessary for detailed calculations.
- Conformal Hexahedral Mesh method was shown to be correct.
- This method was successfully applied to ARIES-CS.
- Increasing the number of flux surfaces increases the source quality for few flux surfaces.
- Plasma physics simulation output is a direct input, whether the *geqdsk* format or coefficients for Jacobian transformation.

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References

[1] El-Guebaly, L.A., The ARIES Team. "Overview of ARIES-RS neutronics and radiation shielding: key issues and main conclusions." *Fusion Engineering and Design* **38**. (1997) : 139-158.

[2] Hughes, T.J.R. *The Finite Element Method: Linear Static and Dynamic Finite Element Analysis*. Dover, 2000.

[3] El-Guebaly, L., et. al. "Designing ARIES-CS Compact Radial Build and Nuclear System: Neutronics, Shielding, and Activation" submitted to *Fusion Science and Technology* (2007).