# Addition of Implicit Monte Carlo Radiation Transport and Validation of 3D Hydrodynamics in *DRACO*

*Milad Fatenejad [1], Gregory Moses [1], Timothy Collins [2], Patrick McKenty [2]*

[1] Fusion Technology Institute, University of Wisconsin – Madison; [2] Laboratory for Laser Energetics, University of Rochester

# Summary

## Abstract

Two new advancements have been made to the *DRACO* Lagrangian radiation-hydrodynamics code. First, the 3D hydrodynamics module has been validated using analytic formulas; second, we have added implicit Monte Carlo (IMC) radiation transport in 2D *r-z* geometry. New results validate simulations of the growth of 3D Richtmyer-Meshkov (RM) and Rayleigh-Taylor (RT) instabilities via comparisons with theoretical growth rates and 2D calculations. The IMC code is validated using comparisons with diffusion theory and analytic results. Finally, comparison of a 2D symmetric direct drive target implosion using IMC to an identical simulation using no radiation transport and diffusion theory will be presented.

*Work Supported By: University of Rochester, Laboratory for Laser Energetics*

We would like to thank Todd Urbatsch and Jeff Densmore at LANL for their useful discussions.

## IMC has been implemented in *DRACO* and validated against several results

- DRACO now includes an IMC model in 2D *x-y* and *r-z* geometries
- This model has been validated against flux-limited diffusion theory and the Su and Olson analytic benchmark problem
- Symmetric target implosion simulations agreed with diffusion theory
- Future work will investigate
  - The validity of IMC when zones are optically thick
  - Variance reduction techniques to allow better accuracy with fewer Monte Carlo particles

## The 3D Hydrodynamics code has been validated against theoretical results

- Two slabs at different temperatures/densities were impacted against one another
- This led to the growth of RM and RT instabilities
- The 3D growth rates agreed with 2D and analytic results in the linear regime
- The 3D version of *DRACO* is now being used to model OMEGA experiments [See TJB Collins (CO 5.0002)]
- 3D Non-linear Rayleigh-Taylor bubble competition will be investigated in the future

# Implicit Monte Carlo Radiation Transport

## Implicit Monte Carlo Radiation Transport[1] (IMC) has been added to *DRACO*

- Radiative heat transfer can play an important role in direct-drive ICF
  - Early in the implosion, the corona re-radiates energy absorbed by the target
  - For ignition targets, the hot spot reaches temperatures in excess of 10 keV
- *DRACO* models radiative heat-transfer by numerically solving the time-dependent, radiative-transfer equations for a plasma in local thermodynamic equilibrium (LTE)

$$\frac{1}{c}\frac{\partial I(r,\Omega,\nu,t)}{\partial t} + \Omega\cdot\nabla I + \sigma I = \frac{c}{4\pi}\sigma B(\nu,T)$$

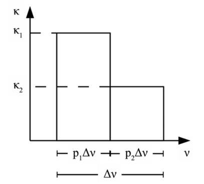
$$\frac{\partial u_e(r,t)}{\partial t} = \iint \sigma I \mathrm{d}\nu \mathrm{d}\Omega - c\int \sigma B \mathrm{d}\nu$$

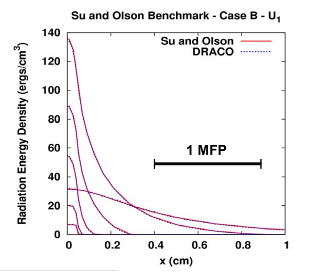
- Previously, only a multi-group, flux-limited diffusion model was available
- IMC was implemented as an alternative that can be used to validate the diffusion results

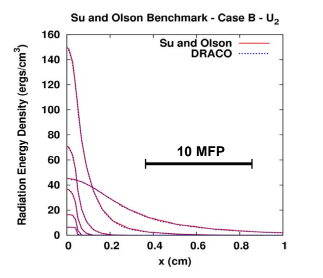
[1]Fleck and Cummings, JCP 8, 313 (1971).

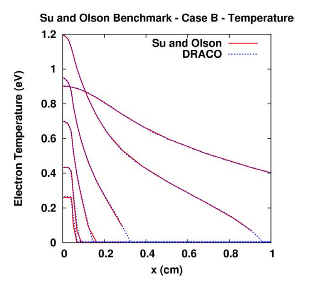
## The IMC method obtains excellent agreement with an analytic solution[1]

- Su and Olson generated a frequency dependent, transient, transport solution to the radiative transfer equations in one dimension
- In order to obtain a tractable problem, the equations must be linear, which requires that $c_v = \alpha T^3$, and use of a novel "picket-fence" opacity



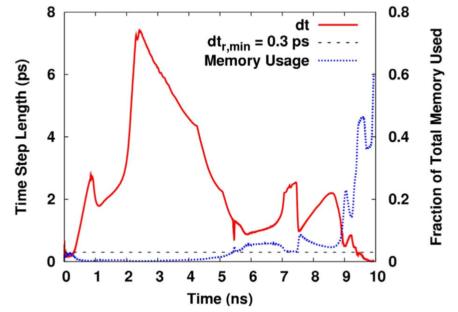




[1]Su and Olson, JQSRT 62, 279-302 (1999).

## A separate IMC time-step has been implemented to allow IMC to run when hydro time-step is small

- Near ignition, the time-step is small (~0.01 ps), and is constrained by the CFL limit
- If the time-step is too small, IMC will consume all available memory
- The IMC code can automatically run on a longer time-step when the hydro step falls below a specified value

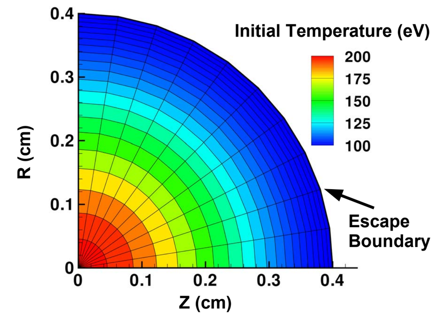


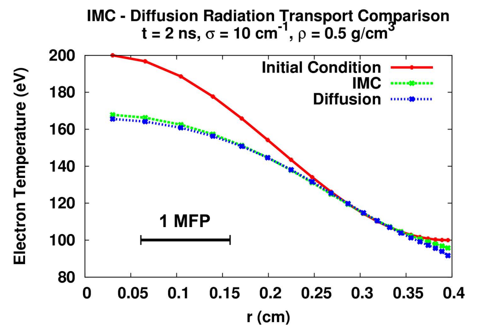

## IMC has several advantages over traditional Monte Carlo methods

- Monte Carlo methods transport photons across a spatial domain by continuous absorption and re-emission of the photons
- Older Monte Carlo methods are ill suited for solving time-dependent, radiative-transfer problems in ICF where materials can be optically thick and close to equilibrium
  - In this case, the absorption/re-emission of photons occurs on time-scales much shorter than hydrodynamic time-scales
  - Stability requirements then drive the simulation to small time-steps

## IMC has also been compared to the flux-limited diffusion model in *DRACO*

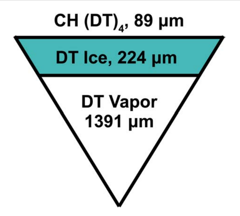
- Relaxation of a non-uniform temperature profile was simulated
- The simulation was stopped before a steady state solution was reached and the results of diffusion and IMC were compared
- Agreement was very good for problems with optically thin zones

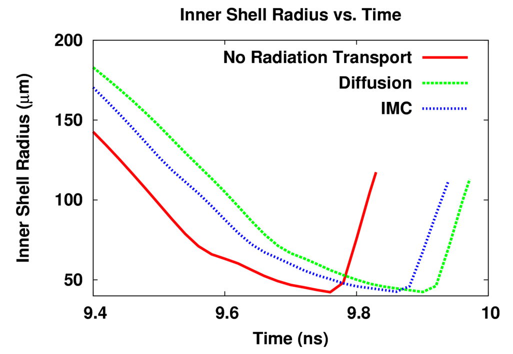


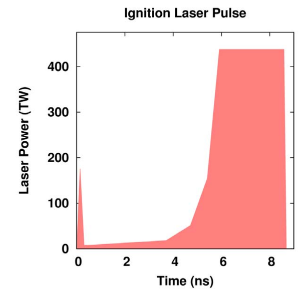




## IMC has been used to model a symmetric, direct-drive ignition target implosion

- IMC agrees well with diffusion results before ignition
- For this design, gain is comparable between simulations
- IMC advances the onset of ignition by 40 ps




## The stability constraint is removed by introducing a fictitious scattering term

- The radiation intensity equation is solved using a standard Monte Carlo method which includes scattering
- The plasma energy equation is advanced on each time-step

$$\frac{1}{c}\frac{\partial I}{\partial t} + \hat{\Omega}\cdot\nabla I + \sigma I = \frac{c b_\nu}{4\pi}\sigma_a u_r^n + \frac{b_\nu \sigma_s}{4\pi\sigma_p}\iint \mathrm{d}\nu \mathrm{d}\hat{\Omega} I$$

$$T_e^{n+1} = T_e^n + \frac{f}{c_v}\left(E/V - c\sigma_p \Delta t u_r^n\right)$$

$$\sigma_a = f\sigma,$$
$$\sigma_s = (1-f)\sigma$$

$$f = \frac{1}{1 + c\sigma_p\alpha\beta\Delta t}$$

$$u_r = aT^4$$

$$\beta = \frac{u_r}{u_e}$$

- α is the implicitness factor. This method is stable for ½ ≤ α ≤ 1
- $u_e$ is the electron energy density
- $u_r$ is the equilibrium radiation energy density
- f is the fraction of absorption, 1-f is the scattering fraction

## IMC supports Planck-weighted, multi-group opacities

- These opacities are necessary to directly compare IMC radiation transport to diffusion in target implosions
- The Planck averaged opacity, $\sigma_p$, weighted over the entire spectrum was computed using tabulated opacities

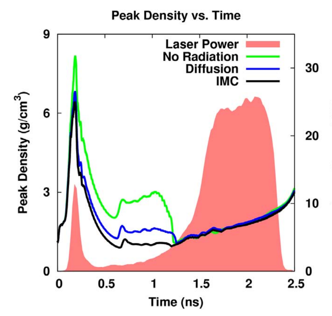
$$\sigma_p = \int_0^\infty \sigma_\nu(\nu) b(\nu) d\nu \qquad \sigma_p = \frac{15}{\pi^4}\sum_{g=1}^{N}\sigma_\nu^g\left(P(x_{g+1}) - P(x_g)\right)$$

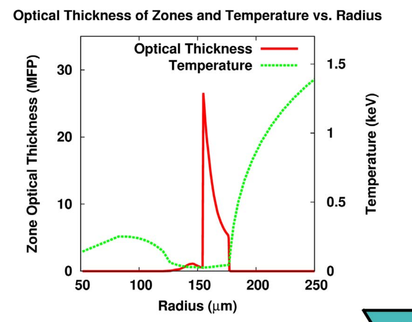
$$\sigma_\nu^g = \frac{\int_{\nu_g}^{\nu_{g+1}}\sigma_\nu(\nu)B(\nu)d\nu}{\int_{\nu_g}^{\nu_{g+1}} B(\nu)d\nu} \qquad P(x) = \int_0^x \frac{x^3}{\exp x - 1}dx\,, \quad x_g = \frac{h\nu_g}{kT}$$

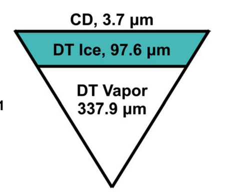
- Determination of initial photon frequency requires a cumulative distribution function (CDF)

$$\mathrm{CDF}(\nu) = \frac{\int_0^\nu \sigma_\nu(\nu)B(\nu)d\nu}{\int_0^\infty \sigma_\nu(\nu)B(\nu)d\nu} \longrightarrow \mathrm{CDF}_k = \frac{\sum_{g=1}^{k-1}\sigma_\nu^g\left(P(x_{g+1}) - P(x_g)\right)}{\sum_{g=1}^{N}\sigma_\nu^g\left(P(x_{g+1}) - P(x_g)\right)}$$

## IMC has also been used to model a symmetric, OMEGA-scale, cryogenic target implosion



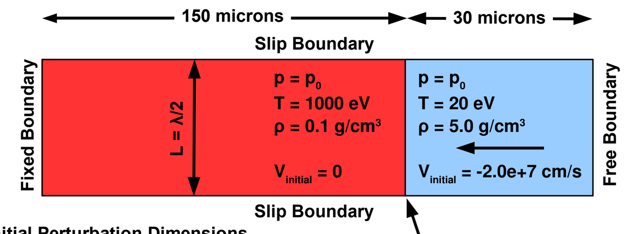

- Despite general agreement with diffusion there are known problems when IMC is used to model optically thick materials[1]

[1]Densmore and Larsen, JCP 199, 175-204 (2004).

# 3D Code Validation

## The 3D hydrodynamics model has been validated against 2D and theoretical results

- Simulations examined the growth of Richtmyer-Meshkov (RM) and Rayleigh-Taylor instabilities in the linear regime
- Comparison to theoretical and simulations results show excellent agreement



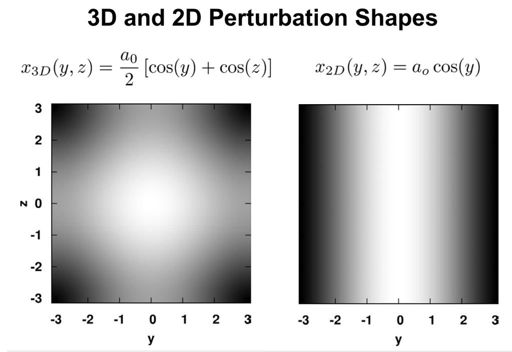

Initial Perturbation Dimensions

| lmode | k (µm⁻¹) | λ (µm) | amplitude (µm) |
|---|---|---|---|
| 5 | 0.033 | 188.5 | -1.279 |
| 10 | 0.067 | 94.25 | 0.304 |
| 20 | 0.133 | 47.12 | 4.00E-2 |
| 40 | 0.267 | 23.56 | 4.40E-4 |

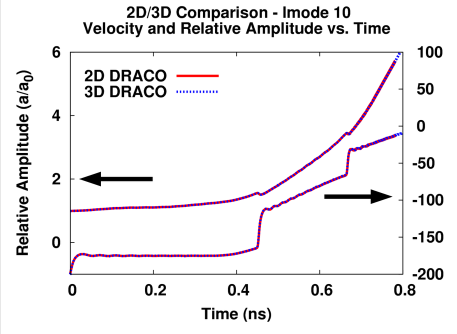
* Parameters obtained from OMEGA shot 47206

## The 3D and 2D amplitudes are equal in the linear regime when the initial amplitudes are the same

- The acceleration leads to a combination of RM and RT growth
- The sudden jumps in velocity are caused by shocks reflecting off the fixed boundary




$$x_{3D}(y,z) = \frac{a_0}{2}\left[\cos(y) + \cos(z)\right] \qquad x_{2D}(y,z) = a_o\cos(y)$$




## Comparison to theoretical RT and RM amplitudes shows excellent agreement

- As expected, the numerical amplitude matches the theoretical RM growth until acceleration begins and the RT growth dominates