AN UPPER BOUND FOR STRESS WAVES INDUCED BY VOLUMETRIC HEATING IN IFE CHAMBER WALLS

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ABSTRACT

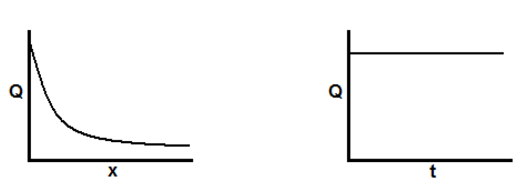
Rapid heating by x-rays and ions in Inertial Fusion Energy (IFE) chambers will produce stress waves in dry chamber walls, in some cases leading to damage that will ultimately fail the structure. These waves can affect the surface or propagate to the substrate and produce delamination. Hence, it is important that these waves be understood. Models exist for thermally induced stress waves resulting from surface heating, but models with volumetric heating have not been presented for IFE conditions. In this paper we develop models for elastic stresses caused by rapid volumetric heating in a half-space. The stress wave models are obtained analytically for heating distributions which are both uniform over a finite region and exponentially decaying over the entire depth. These two cases cover the relevant heating for a typical IFE threat. Results are given for both x-ray and ion heating using threats from a direct drive target developed for the High Average Power Laser (HAPL) target.

INTRODUCTION

Dry chamber walls in inertial fusion energy (IFE) plants will experience rapid heating as x-rays and ions are deposited in the near-surface. These have the potential for producing damaging stress waves in the solid. The presented work develops a series of results for thermoelastic waves produced by rapid, volumetric heating in a half space. The calculations are carried out without including conduction. The exclusion of conduction will increase the thermoelastic loads, so that the results represent an upper bound for the stresses. Solutions for different cases of heating and how they compare to surface heating will be shown.

MODELING

The derivation of the model is shown on a particular case. In this case, the heat decreases exponentially with depth and remains constant with time after it is turned on instantaneously (case 1).



Considering thermoelastic deformation, with x denoting perpendicular from the surface, the temperature equation becomes:

$$\frac{\partial T}{\partial t} = \frac{Q'''}{\rho c_p} = \frac{Q_0 e^{-\gamma x}}{\rho c_p}$$

The governing equation for the displacements in the solid becomes:

$$\frac{\partial^2 u(x,t)}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 u(x,t)}{\partial t^2} - \frac{(1+\nu)}{(1-\nu)}\alpha\frac{\partial T}{\partial x}$$

The wave speed is defined as: $c^2 = \frac{2(1-\nu)\mu}{(1-2\nu)\rho}$

Boundary conditions: $u_x(x,0)=0 \quad \dot{u}_x(x,0)=0$, $u_x(\infty,t)=0 \quad \sigma_x(0,t)=0$

Variables: $g = \frac{(1+\nu)}{(1-\nu)}\alpha, \quad b = \frac{Q_0}{\rho c_p}$

Laplace-transform of the governing equation:

$$\frac{\partial^2 \bar{u}(x,s)}{\partial x^2} = \frac{s^2}{c^2}\bar{u}(x,s) + \gamma\frac{gb}{s^2}e^{-\gamma x}$$

Solution:

$$\bar{u}(x,s) = C_1 e^{\frac{sx}{c}} + C_2 e^{-\frac{sx}{c}} + \gamma\frac{bgc^2}{s^4 - s^2c^2\gamma^2}e^{-\gamma x}$$

$u_x(\infty,t) = 0 \rightarrow C_1 = 0$

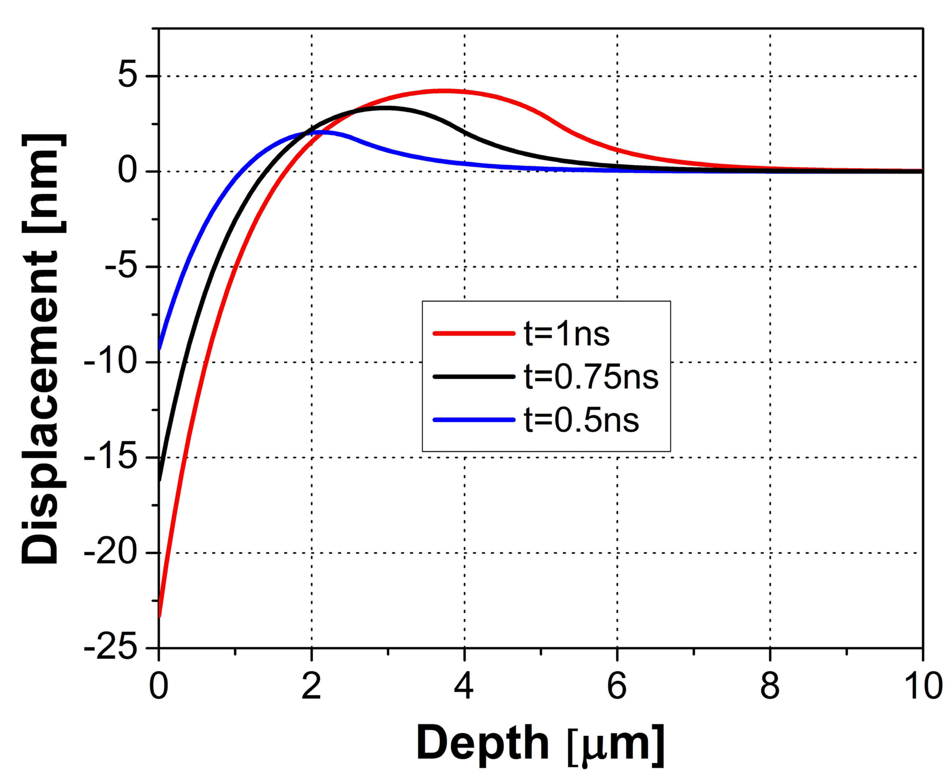
Displacement-stress relations:

$$\sigma_x = \frac{2\mu}{1-2\nu}\left[(1-\nu)\frac{\partial u(x,t)}{\partial x} - (1+\nu)\alpha T\right]$$

$$\bar{\sigma}_x = \frac{2\mu}{1-2\nu}\left[(1-\nu)\frac{\partial \bar{u}(x,s)}{\partial x} - (1+\nu)\alpha\frac{be^{-\gamma x}}{s^2}\right]$$

Maximum displacement: $u_{max} = \frac{(1-e^{-ct\gamma} - ct\gamma)}{c\gamma^2}bg$

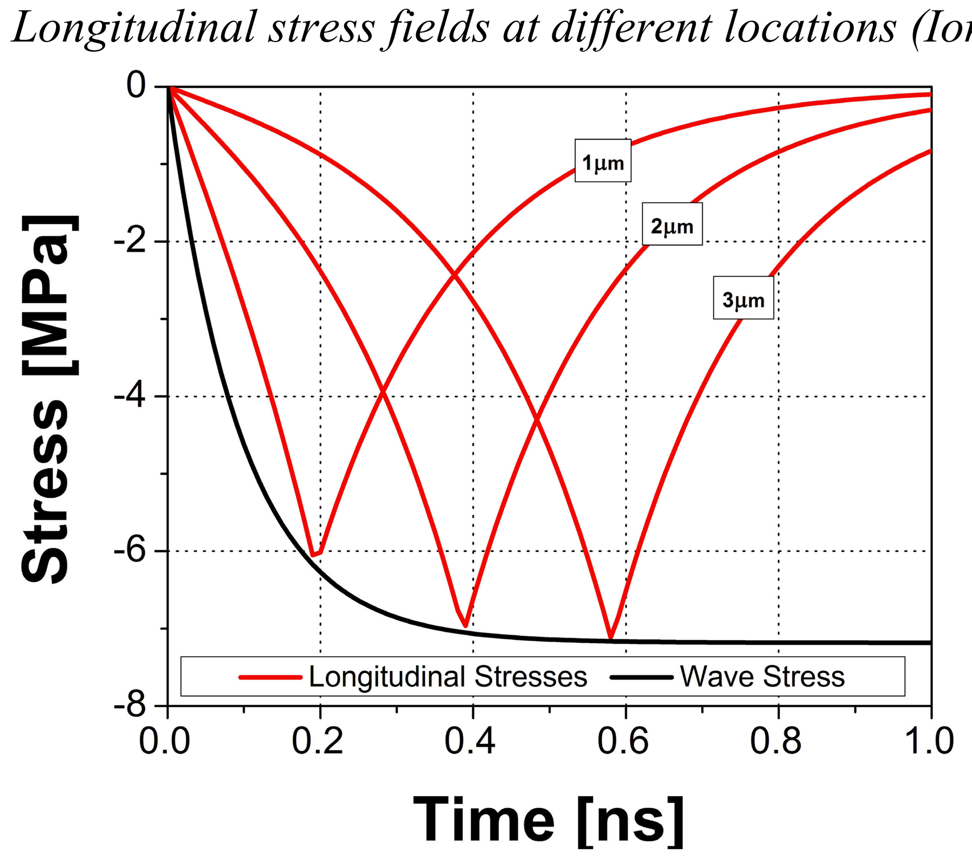
Displacement at fixed times (x-rays):



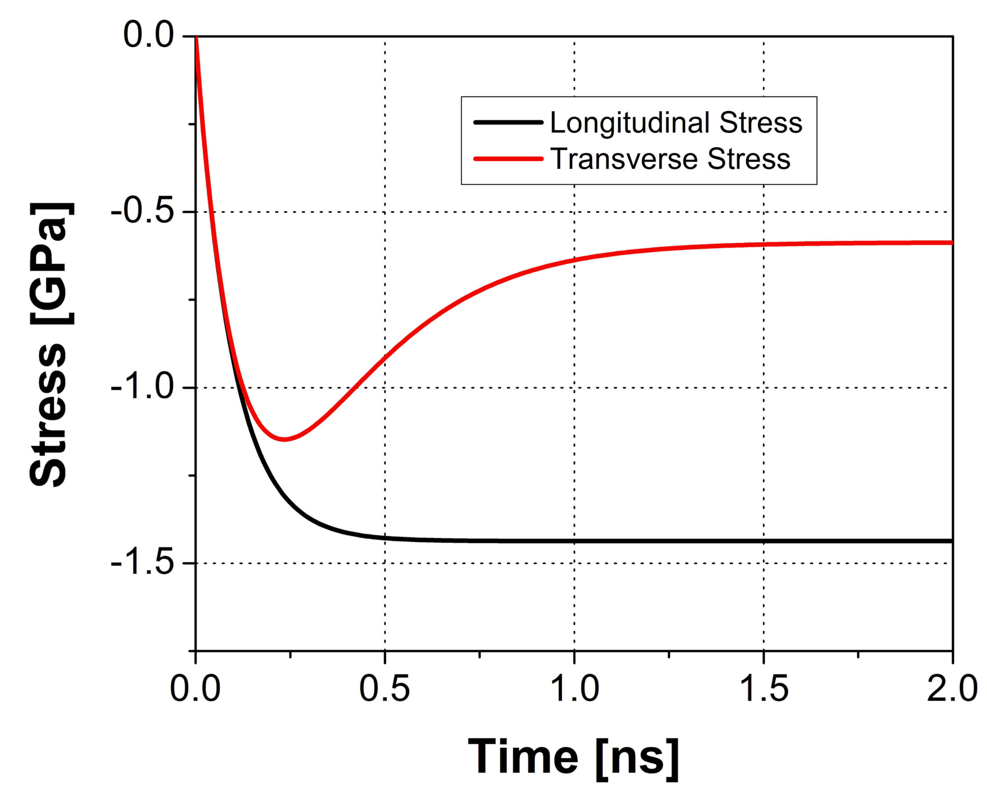
Longitudinal Stress:

$$\sigma_x = \frac{\mu\alpha b(1+\nu)[-1+e^{2ct\gamma} + (-e^{2ct\gamma} + e^{2x\gamma})}{c\gamma(-1+2\nu)} \times H(t - \frac{x}{c})]e^{(-ct-x)\gamma}$$

Longitudinal stress fields at different locations (Ions):



Stresses at the wave tip (x-rays):

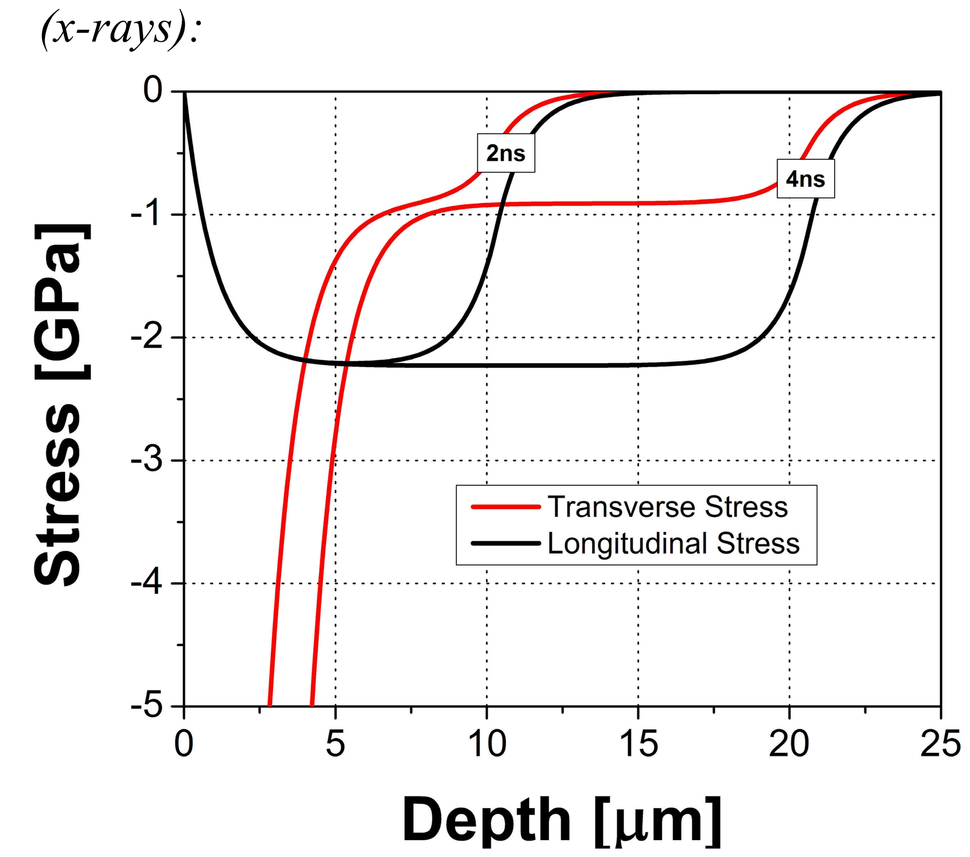


In a different case the heat is ramped over time, but still decreases exponentially with depth (case 2):

Depositing the same amount of heat as in the previous case (case 1) yields,

$$\frac{\partial T}{\partial t} = \frac{Q'''}{\rho c_p} = \frac{2Q_0 e^{-\gamma x}t}{\rho c_p \tau}$$

Longitudinal and transverse stresses at fixed times: (x-rays):



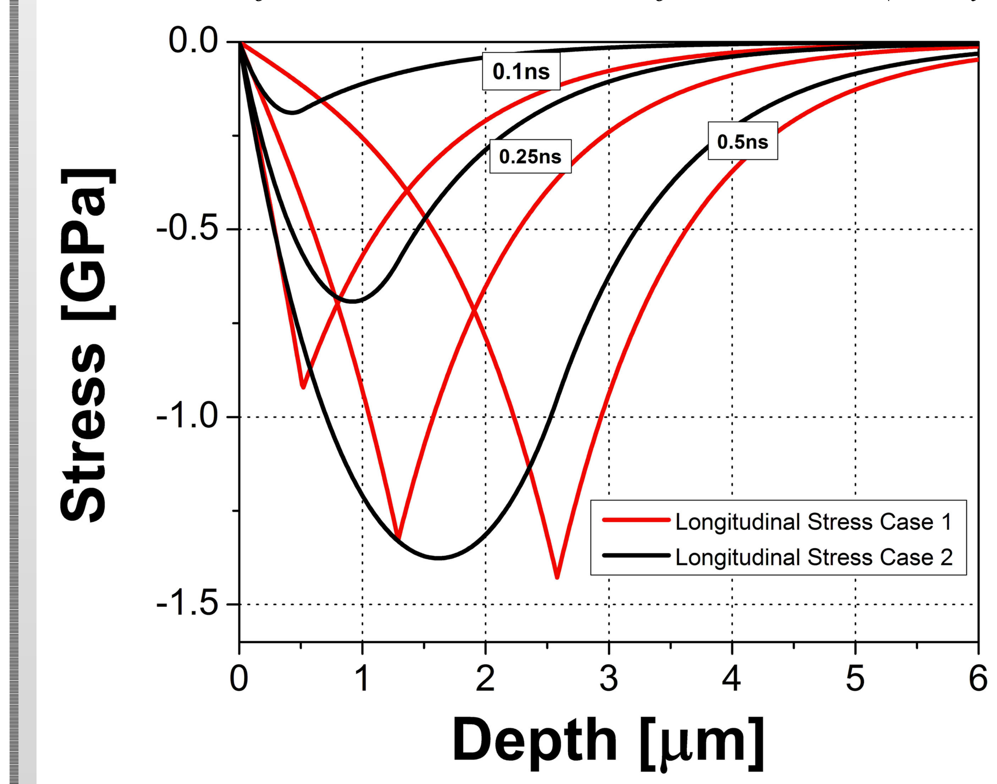
COMPARISON

The plots show that the stress caused by a step increase in the volumetric heating is slightly higher than for a ramp increase, assuming the same total heat is deposited over a pulse. In addition, the stresses increase faster for the case of a step increase.

By comparing the stresses at the end of the pulse, one can determine the exact ratio between the stresses in case 1 and case 2. Stepped heating always causes slightly higher stresses than an equivalent ramped case. In the IFE x-ray case, with a 0.5ns pulse length, the ratio is **1.07**.

$$\frac{\sigma_{step}}{\sigma_{ramped}} = \frac{c\gamma\tau(-1+e^{2c\gamma\tau})e^{-\frac{1}{2}c\gamma\tau}}{2(-1+e^{\frac{c\gamma\tau}{2}})^2(1+2e^{\frac{c\gamma\tau}{2}})}$$

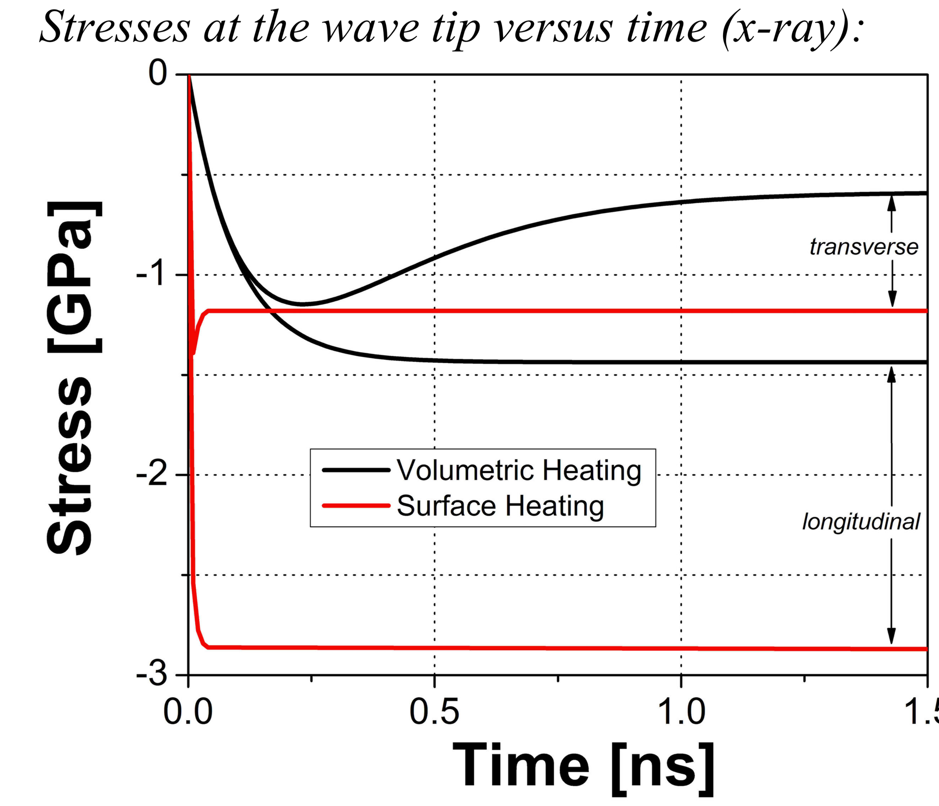
Stresses from case 1 and 2 at fixed times (x-rays):



Comparing the results for volumetric heating with previous results for surface heating (based on the schematic for case 1), assuming the same total heat is deposited, reveals a ratio of 2 for long times.

$$\lim_{t\to\infty}\frac{\sigma(surface-heating)}{\sigma(volumetric-heating)} = 2$$

Stresses at the wave tip versus time (x-ray):



CONCLUSIONS

Solutions are developed for thermoelastic stress waves due to volumetric heating. It is found that the stresses induced by volumetric heating are lower than for a case in which the same amount of heat is supplied as surface heat. It is also shown that step heating causes a larger stress than equivalent ramped heating, though the difference is relatively small.

ACKNOWLEDGEMENTS

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