Energetic Particle Loss and Field Bumpiness Due to the "D" Shaped Coils in a Tokamak Reactor

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Abstract

In conceptual designs of Tokamak reactors, the ripple of the magnetic field produced by the finite number of "D" shape toroidal field coils is found to be both radial and poloidal dependent. This ripple can be modeled by a power function of minor radius and Gaussian function of poloidal angle. The fraction of energetic alpha particles trapped in the magnetic wells between coils is estimated and its minimization is discussed.

I. Introduction

The finite number of coils used to generate the toroidal field of a Tokamak produces a bumpiness of the magnetic field strength which can cause particles to be trapped between coils and drift vertically out of the plasma to the walls. This has been considered by Anderson and Furth[1] for a field ripple which has only a radial variation. In this paper, we consider a field ripple which has both radial and poloidal variation, as is appropriate for conceptual designs[2,3] using "D" shape constant thickness magnets. UWMAK-I[2] is used as a model for this study. The loss of energetic alpha particles is estimated and its minimization is discussed.

II. Calculation

UWMAK-I has 12 constant-tension "D" shaped toroidal magnetic field coils spaced at 30° intervals. The shape of the coils is shown in Fig. 1. The cross section of the coil is rectangular. The magnetic field structure is illustrated by some of the field lines shown in Fig. 2. The plasma column is corrugated at the outer edge with a deviation $\Delta r \cong .5$ meters at the center of the gap between the coils. The magnetic field ripple $\varepsilon(r,\theta)$ produced by N coils is defined by

$$B_\phi(r,\theta,\phi) = \frac{B_o}{1 + \frac{r}{R}\cos\theta}\,[1 + \varepsilon(r,\theta)\cos(N\phi)], \qquad (1)$$

where B_ϕ is the true toroidal magnetic field strength, B_o is the mean field on the magnetic axis of major radius R. Here (r,θ,ϕ) are the usual quasi-toroidal coordinates with θ being the poloidal angle. The poloidal and radial dependences of the ripples were calculated numerically and are shown in Figs. 3 and 4 respectively. Curves I in both figures are those for 12 "D" coils. The other three curves are for the 12 extended "D" coils, 24 "D" coils and 24 extended "D" coils; these will be discussed later. A fit to the curves of Figs. 3 and 4 gives

$$\varepsilon(r,\theta) = \varepsilon_a\left(\frac{r}{a}\right)^n \exp\left(-\left(\frac{\alpha\theta}{\pi}\right)^2\right), \qquad (2)$$

where n, ε_a and α are constants determined by a particular design and a is the minor radius of the plasma. For low-β circular Tokamaks, the trajectory of a field line is r = constant, $\phi=\phi_o + q\theta$, where q is the MHD stability factor. The field strength along the field line is

$$B(r,\theta,\phi) = \frac{RB_o}{R+r\cos\theta}\left[1+\varepsilon_a\left(\frac{r}{a}\right)^n \exp\left(-\left(\frac{\alpha\theta}{\pi}\right)^2\right)\cos\left(Nq\theta+\phi_o\right)\right] \quad (3)$$

neglecting the poloidal field strength. Consider the upper half of the r-θ plane for fixed ϕ_o. Local magnetic wells are produced by the $\cos(Nq\theta+N\phi_o)$ term. These are shown in Fig. 5 for UWMAK-I conditions (N = 12, q = 1.75, A = ^{o}R/a=2.6, ε_a = .20, α = 3.2, N = 3.0). Particles can be trapped in these wells if they have sufficiently large pitch angle relative to the magnetic field. The lower boundary in θ for toroidally trapped particles can be determined from the condition

$$\frac{\partial}{\partial\theta}\left(\frac{B_\phi}{B_o}\right)\Big|_{\theta_1} = 0$$

using (3) we have

$$\frac{\sin\theta_1}{1+\frac{r}{R}\cos\theta_1} \simeq \frac{aK}{\varepsilon_a r}\left(\frac{r}{a}\right)^n \exp\left(-\left(\frac{\alpha\theta_1}{\pi}\right)\right)^2 \sin(Nq\theta_1+\phi_o) \quad , \quad (4)$$

where K is defined by[1]

$$K = \frac{\varepsilon_a N q R}{a} \quad .$$

The upper boundary θ_2 is given by $B_\phi(\theta_2) = B_\phi(\theta_1)$. The projection of the trapping regions on the r - θ plane is shown in Fig. 6 for constant ϕ_o. Unlike the case of poloidally uniform ripples the trapping regions are confined in the first and fourth quadrents of the r - θ plane instead of distributed uniformly even though K is large. These locally trapped particles will drift vertically in the gradient of the toroidal with a velocity

$$v_D = \frac{m v_\perp^2}{2e B_o R} \quad . \quad (5)$$

A 3.5 MeV alpha particle can drift to the wall in about 10^{-2}m sec which is much shorter than the time to slow down or to scatter in pitch angle and become untrapped. Consequently the fraction of alphas formed in the region of phase space corresponding to being locally trapped in the bumpy field will be lost directly from the plasma and will impinge on the first wall in localized zones.

For an isotropic source of energetic particles, the fractions of particles trapped at (r,θ,ϕ) is

$$f(r,\theta,\phi) = \sqrt{1-\frac{B(r,\theta,\phi)}{B(r,\theta_1)}}$$

where $B(r,\theta_1)$ is the field at the mirror point on the field line passing through (r,θ,ϕ). From (3)

$$B(\theta_1) \simeq \frac{R B_o}{R+r\cos\theta_1}\left[1+\varepsilon_a\left(\frac{r}{a}\right)^n \exp\left(-\left(\frac{\alpha\theta_1}{\pi}\right)^2\right) \right. .$$

Since $\cos(Nq\theta_1 + N\phi_o) \simeq 1$. Thus

$$\frac{B(r,\theta,\phi)}{B(\theta_1)} \simeq \frac{1+\varepsilon_a\left(\frac{r}{a}\right)^n \exp\left(-\left(\frac{\alpha\theta}{\pi}\right)^2\right)\cos N\phi}{1+\varepsilon_a\left(\frac{r}{a}\right)^n \exp\left(-\left(\frac{\alpha\theta}{\pi}\right)^2\right)} \quad ,$$

and $f(r,\theta,\phi) \simeq \sqrt{\varepsilon_a}\left(\frac{r}{a}\right)^{\frac{n}{2}} \exp\left(-\frac{1}{2}\left(\frac{\alpha\theta}{\pi}\right)^2\right)\sqrt{1-\cos N\phi}$.

The total fraction of particles trapped is

$$f_T = \frac{\int_o^a rdr\ S(r) \int_o^{\theta_{max}(r)} d\theta \int_o^{2\pi} d\phi\ f(r,\theta,\phi)}{2\pi^2 \int_o^a rdr\ S(r)}\ , \qquad (6)$$

where S(r) is a source function independent of θ.

Thus

$$f_T = \frac{\sqrt{\varepsilon_a} \int_o^{2\pi} \sqrt{1 - \cos N\phi}\ d\phi \int_o^a dr\ r\ S(r)\ (\frac{r}{a})^{\frac{n}{2}} \int_o^{\theta_{max}(r)} \exp(-\frac{1}{2}(\frac{\alpha\theta}{\pi})^2) d\theta}{2\pi^2 \int_o^a rdr\ A(r)}$$

For purposes of estimation, we assume

$$\frac{1}{2\pi} \int_o^{2\pi} \sqrt{1 - \cos N\phi}\ d\phi \simeq 1,$$

and $\frac{\alpha\theta max\ (r)}{\pi} >> 1, \quad S(r) = 1 - (\frac{r}{a})^2$

then $$f_T = \frac{16\ \sqrt{2\pi}}{(n+4)\ (n+8)\alpha} \sqrt{\varepsilon_a}\ .$$

For UWMAK-I conditions, $f_T = 5.6\%$; This means that 5.6% of the 3.5 MeV alpha particles will not be magnetically confined but will strike the wall in localized areas between the coils in the first quadrant in r - θ plane. This may cause additional wall damage and generation of impurities. We would like to minimize this loss by increasing the number of coils and/or extend the coils.

The curves III in Figs. 3 and 4 show that the amplitude of the ripple is reduced by a factor 6 by doubling the number of coils. It also decreases faster both radially and poloidally than those of 12 coils. Correspondingly the trapping regions are also less as shown by Fig. 7. The curves II and IV in Figs. 3 and 4 are ripples respectively for 12 and 24 extended D magnets as shown in Fig. 1. The parameters for modeling these ripples by powers and gaussian functions and their trapping fractions are listed in Table I. We would like to mention here that the numerical accuracy in the magnetic field calculation is about 0.01%.

The corrugation of the plasma column is Δr = 0.13 meters for 24 standard coils, and is negligible for 24 extended coils.

Table I indicates that the 24 extended D-coils magnet design will give us a reasonably low loss and uniform plasma column, and there appears to be enough space available for access to the blanket. The extended D-coil would also provide the convenience for blanket removal without moving the coils.

III. Acknowledgement

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References

1. O. A. Anderson and H. P. Furth, Nuclear Fusion, 12, 207 (1972).
2. Wisconsin Fusion Design Group, UWFDM-68, (1973).
3. J. File, Fifth Symposium on Engineering Problems of Fusion Research, Princeton N. J.,(1973).

Figure Captions

Fig. 1. Geometry of Toroidal Magnetic Field Coil, six arcs and one straight segment are used to approximate a half of "D" coil. The numerals indicate the centers of curvatures of the correspondent arcs. α_1 and α_2 are the integration limits of arc #2 for magnetic field calculation.

Fig. 2. Field pattern on midplane of torus, plasma is centered at 13 meters and occupies the region 8 to 10 meters. The dotted rectangulars are the cross-sections of the coils.

Fig. 3. The radial dependences of the ripples $\varepsilon(r,\theta)$. Curve I is for 12 "D" coils. Curves II, III and IV are for 12 extended "D" coils, 24 "D" and 24 extended "D" coils respectively.

Fig. 4. The poloidal dependences of the ripples $\varepsilon(r,\theta)$. Curve I is for 12 "D" coils. Curves II, III, and IV are for 12 extended "D" coils, 24 "D" and 24 extended "D" coils respectively.

Fig. 5. The variation of field strength along a field line as a function of θ. The ripples are superimposed on the true toroidal field shown by dashed curve. θ_1 and θ_2 are lower and upper mirror points of the walls.

Fig. 6. The projection of trapping regions on the r-θ plane for 12 "D" coils.

Fig. 7. The projection of trapping regions on the r-θ plame for 24 "D" coils.

Table I

Parameters of the Magnetic Field Ripples for 4 Different Magnetic Field Coils Designs, and the Total Fractions of α-Particles Trapped in Those Ripples

	ε_a	n	α	f_T (%)
12 coils	0.25	3	3	5.6
12 extended coils	0.046	3	3.2	3.4
24 coils	0.03	6	4.4	1.1
24 extended coils	0.016	12	5.0	0.3

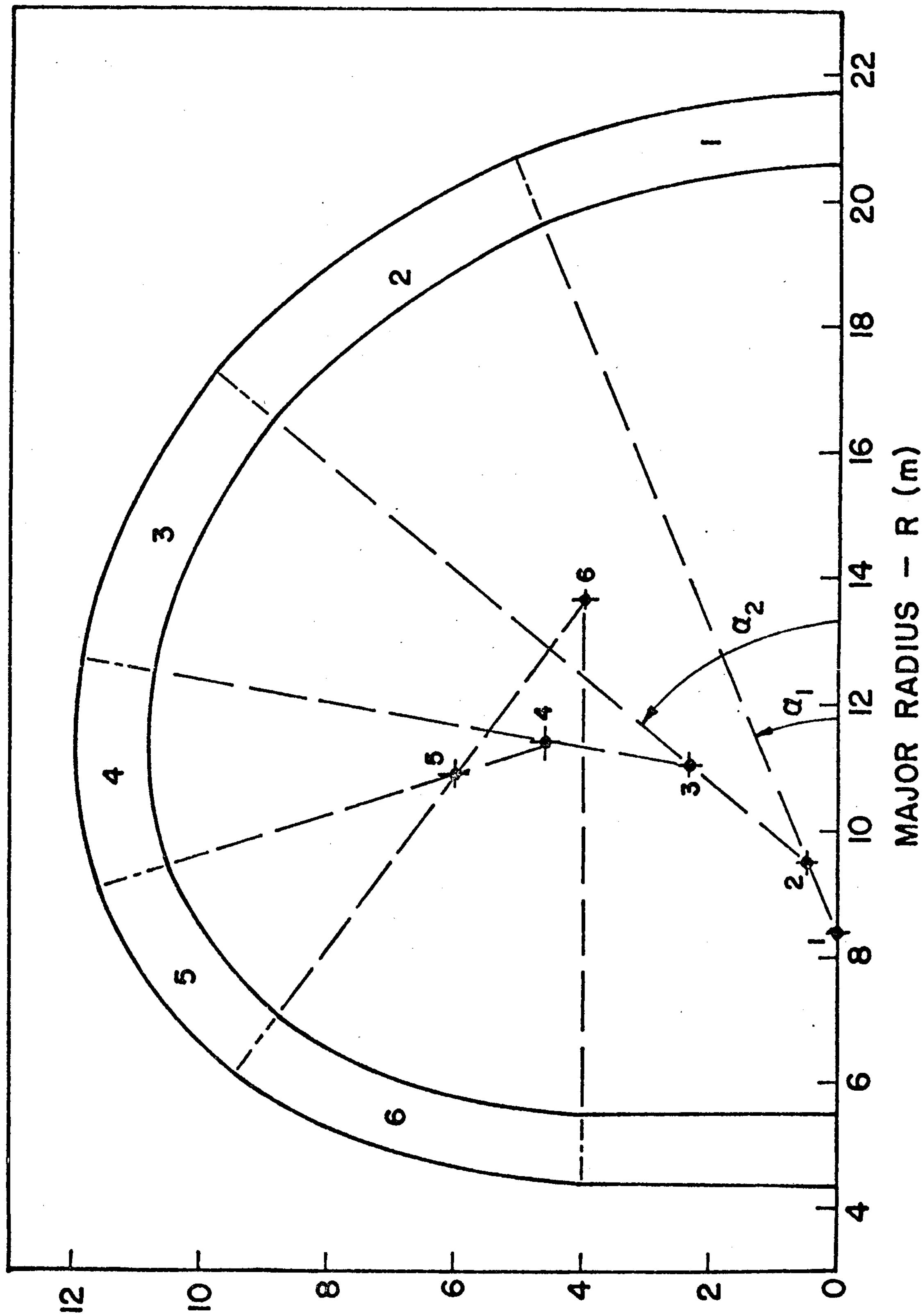
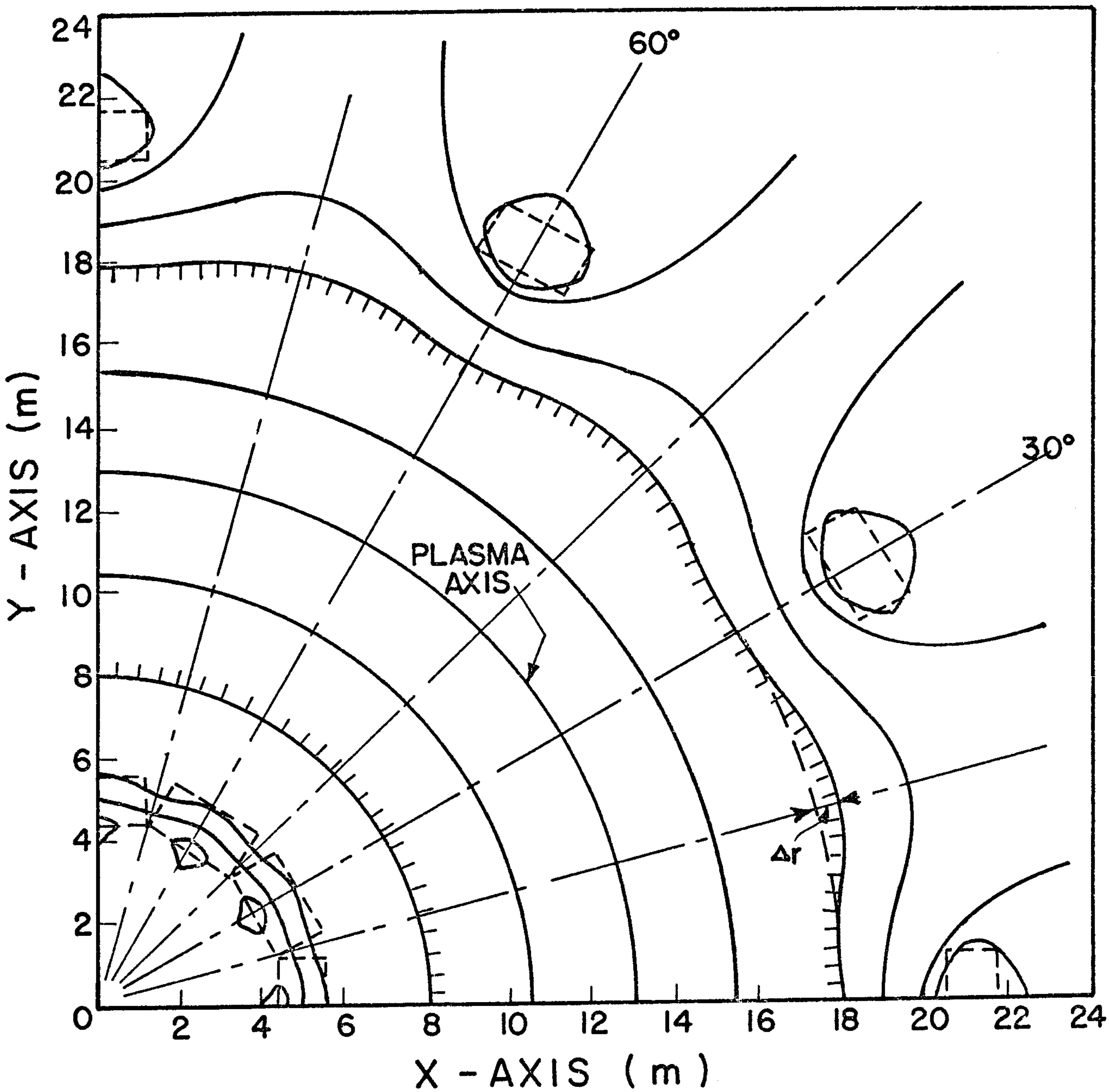

HEIGHT (m)
MAJOR RADIUS — R (m)
0
2
4
6
8
10
12
4
6
8
10
12
14
16
18
20
22
α_1
α_2
1
2
3
4
5
6

FIGURE 1



24
22
20
18
16
14
12
10
8
6
4
2
0
2
4
6
8
10
12
14
16
18
20
22
24
Y - AXIS (m)
X - AXIS (m)
60°
30°
PLASMA
AXIS
Δr

FIGURE 2

FIGURE 3

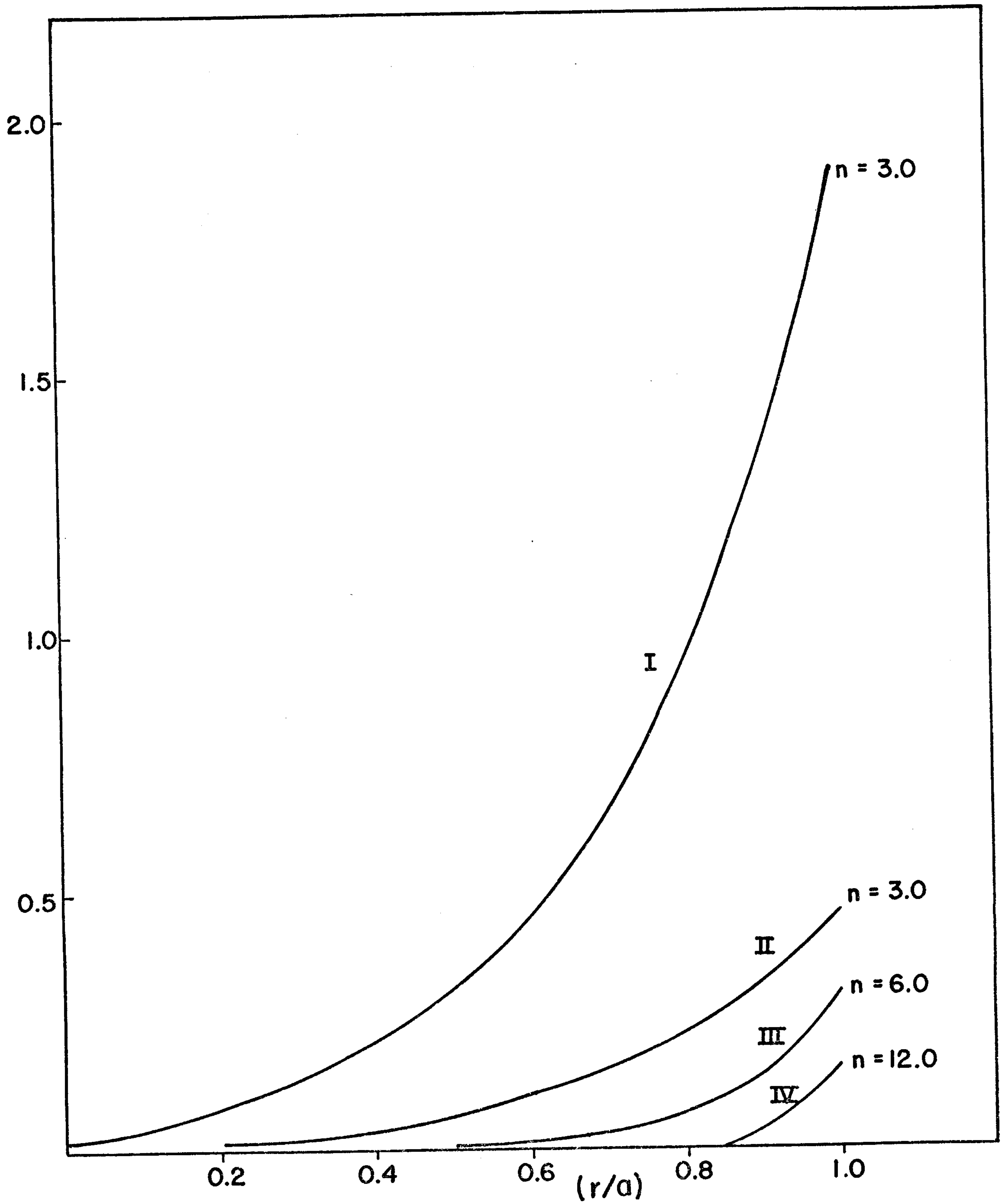
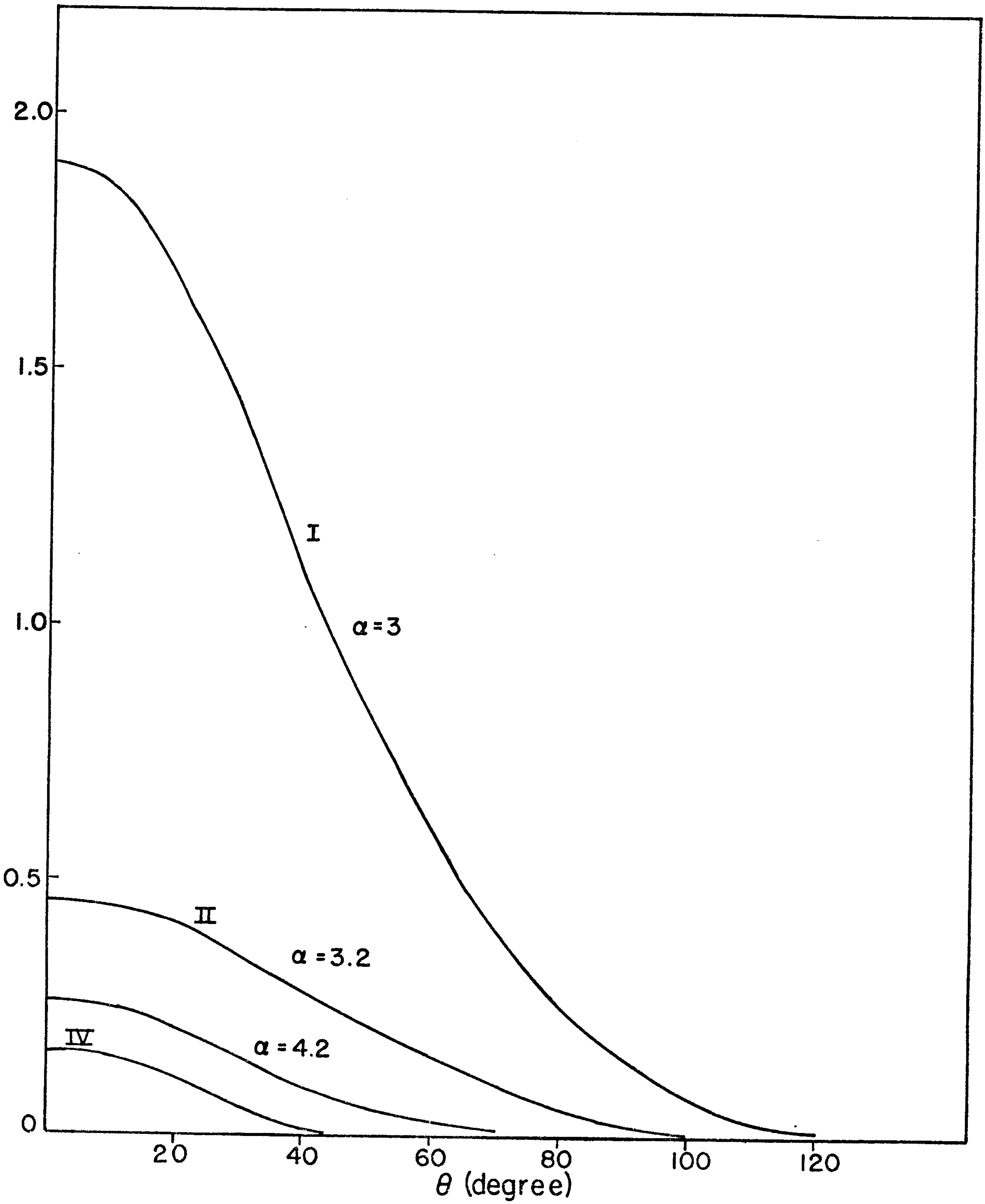


FIGURE 4



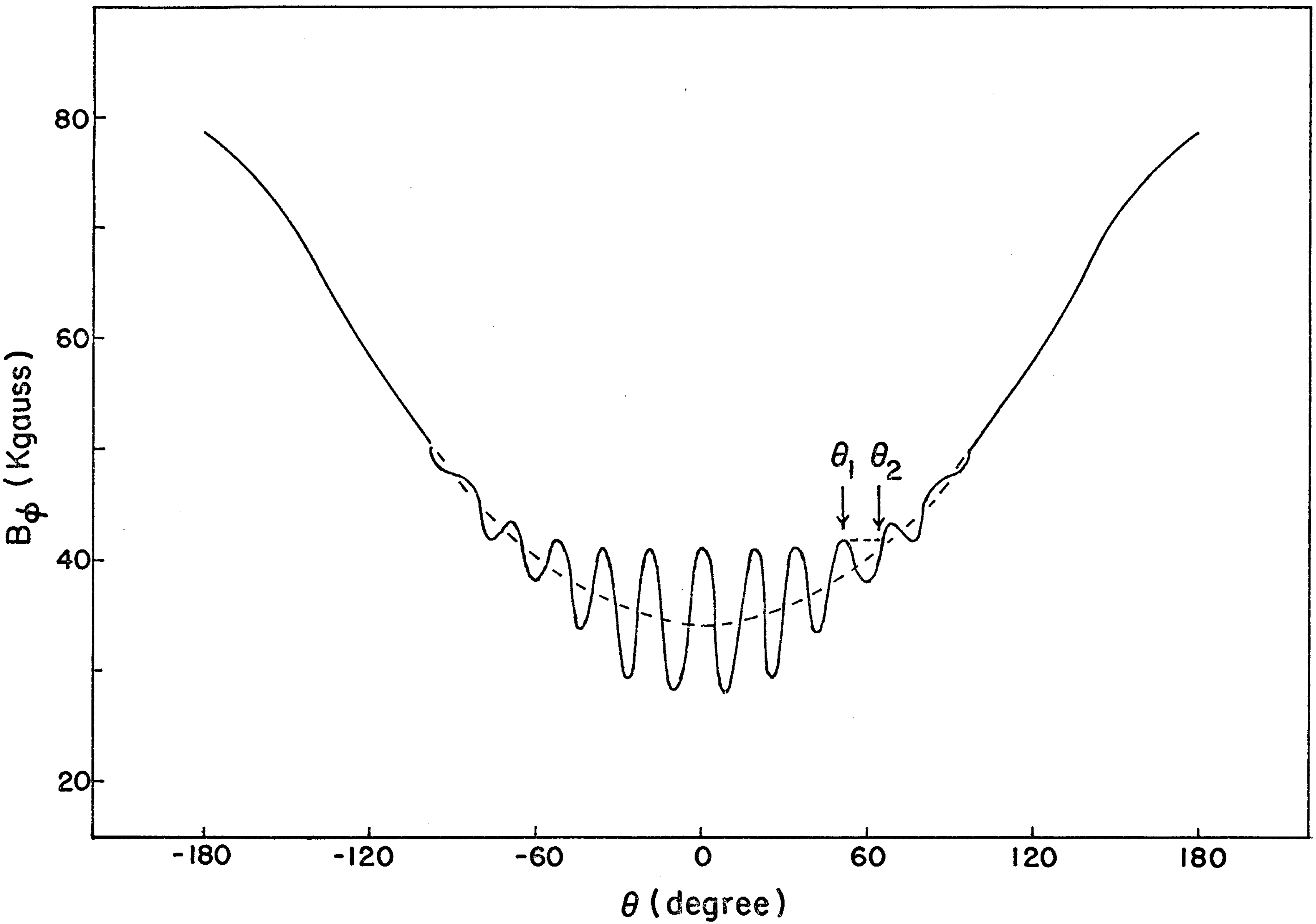

80
60
40
20
B_ϕ (Kgauss)
-180
-120
-60
0
60
120
180
θ (degree)
θ_1
θ_2

FIGURE 5

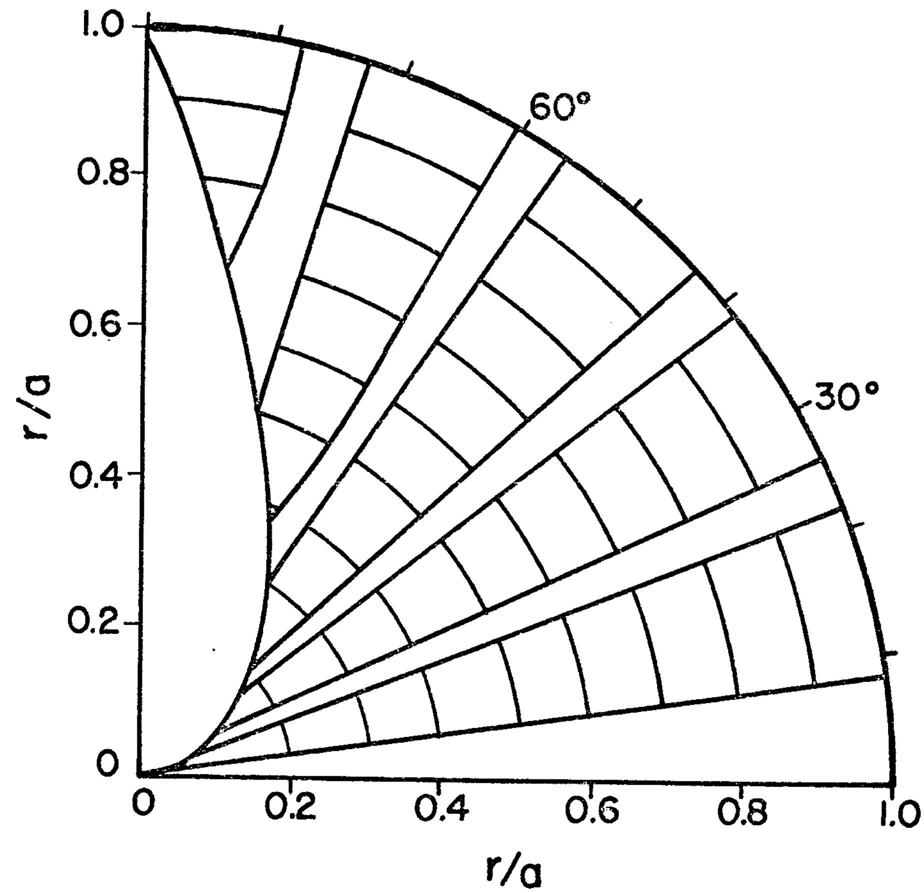


FIGURE 6

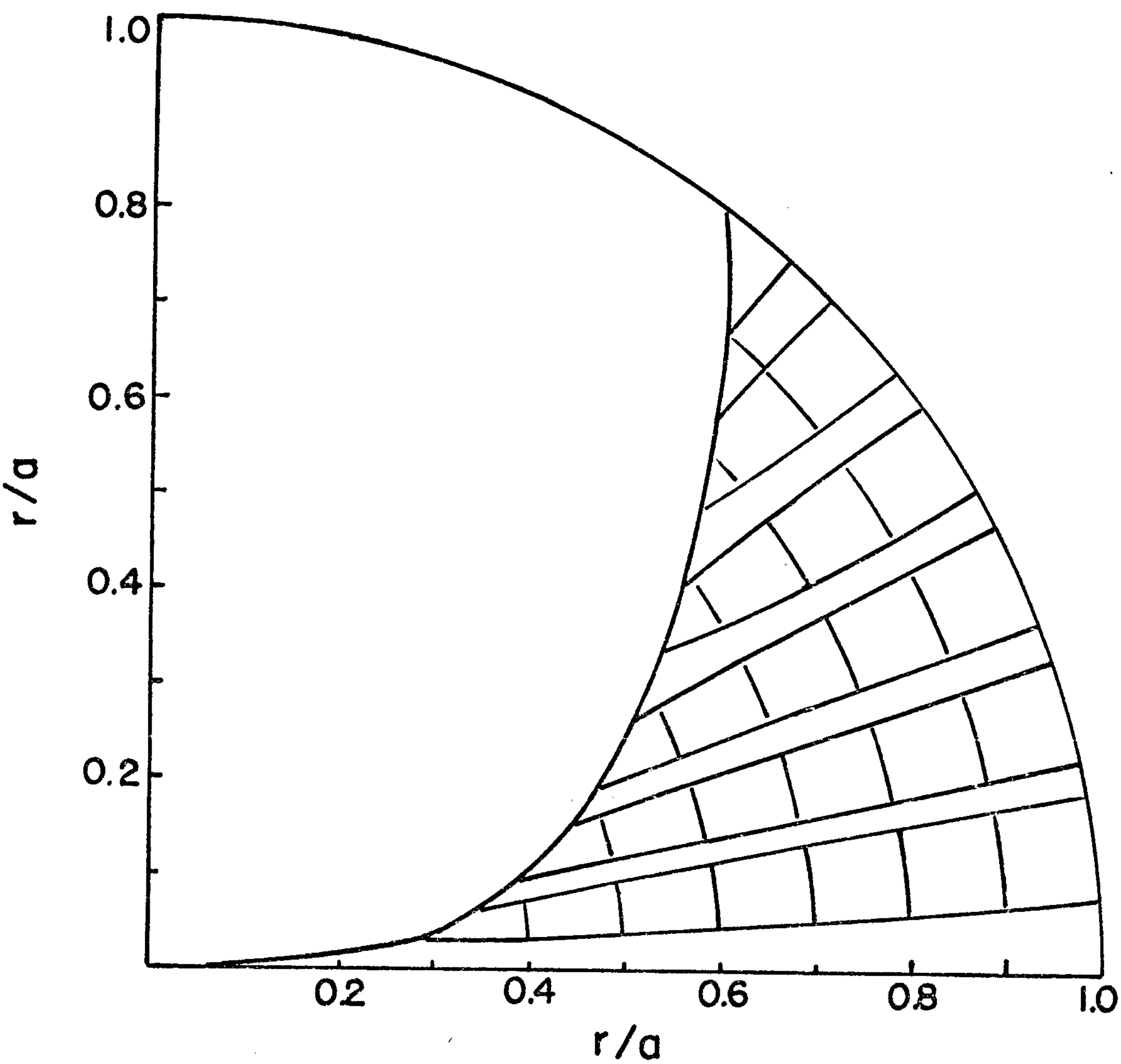


FIGURE 7