



**Application of Variational Methods to Fusion  
Reactor Blanket Studies**

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Fusion Reactor Blanket Studies

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The use of variational methods for CTR blanket and shield studies has recently been discussed<sup>1,2,3,4</sup>. In any approximate procedure, it is important to investigate the accuracy which can be expected. We report here on a set of calculations which gives such information when the tritium breeding ratio in CTR blanket systems is estimated. Specifically, the effect of changing the amount of structural material in the tritium breeding zones on the production rate of tritium is investigated. The variational results were calculated from the functional

$$I_F[\phi^*, \phi] = \frac{(S^*, \phi)(\phi^*, S)}{(\phi^*, L\phi)} \quad (1)$$

where the notation is that of reference 1 and the transport equation is represented by  $L\phi = S$ . The exact results are from direct discrete ordinates calculations using the ANISN program<sup>5</sup> and all results are for slab geometry,  $S_4 - P_3$ , with 46 neutron groups. The group structure is given elsewhere<sup>6</sup>. The blankets systems were all of the following type: 1) a 4mm first wall; 2) a 51cm homogenized breeding zone of natural lithium and structure; 3) a 15cm reflector of the basic structural material; and 4) a 5cm homogenized zone of natural lithium and structure.

Results are given in tables 1 and 2. For table 2, the trial fluxes and adjoints were calculated for the systems with 5% structure in the breeding zones. In table 1, results are also presented when the reference system has 25% structure. The synthesis<sup>7</sup> results in table 1 are obtained by using linear combinations of the 5% and 25% reference

fluxes and adjoints with coefficients obtained using a Rayleigh-Ritz procedure<sup>1</sup>. These latter results, while requiring two additional ANISN calculations, are extremely accurate over the entire range. The results using equation (1) are also quite accurate in a sizeable range about the particular reference system.

As for the results themselves, we note that tritium breeding is best in vanadium, more than adequate and about the same in stainless steel and molybdenum, and lowest for niobium systems<sup>8</sup>. In addition, the breeding ratio in the niobium system falls below 1 with ~15% structure in the breeding zones. Breeding is least affected by structure in V because of its low capture cross section and  $T_6$  actually increases.  $T_7$ , however, decreases markedly in all systems as the amount of structure is increased since the spectrum is softened by additional inelastic scattering.

## References

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Table 1

Structural Material; Stainless Steel

Volume and Weight Percent Structure in Breeding Zones	Variational		Variational		Synthesis		E X A C T		
	Reference at 5% Structure		Reference at 25% Structure		References at 5 % and 25% Structure		T <sub>6</sub>	T <sub>7</sub>	Total Breeding Ratio
	T <sub>6</sub>	T <sub>7</sub>	T <sub>6</sub>	T <sub>7</sub>	T <sub>6</sub>	T <sub>7</sub>			
0% (0.0 w/o)	0.863	0.725	0.881	0.808	0.862	0.720			
1% (13 w/o)	0.863	0.691	0.880	0.761	0.862	0.689	0.862	0.689	1.55
5% (43.8 w/o)	0.861	0.579	0.874	0.613	0.861	0.579	0.861	0.579	1.44
10% (62.2 w/o)	0.859	0.473	0.866	0.483	0.859	0.471			
15% (72.4 w/o)	0.857	0.392	0.857	0.391	0.854	0.387	0.854	0.387	1.24
20% (78.8 w/o)	0.855	0.329	0.847	0.322	0.846	0.321			
25% (83.2 w/o)	0.852	0.279	0.836	0.268	0.836	0.268	0.836	0.268	1.10
30% (86.4 w/o)	0.850	0.237	0.824	0.225	0.823	0.225			
35% (88.9 w/o)	0.846	0.202	0.811	0.190	0.807	0.189	0.807	0.189	0.99

Table 2

Structural Material; Niobium

Volume (Weight) Percent Structure in Breeding Zones	Variational Results Reference System, 5% Structure		
	T <sub>6</sub>	T <sub>7</sub>	Total Breeding Ratio
0% (0.0 w/o)	0.764	0.723	1.49
1% (14 w/o)	0.760	0.689	1.45
5% (46 w/o)	0.724	0.576	1.30
10% (64.2 w/o)	0.684	0.469	1.15
15% (74 w/o)	0.644	0.389	1.03

Structural Material; Vanadium

0% (0.0 w/o)	0.914	0.760	1.67
1% (10.4 w/o)	0.922	0.733	1.66
5% (37.7 w/o)	0.960	0.637	1.60
10% (56.1 w/o)	1.02	0.540	1.56
15% (67 w/o)	1.09	0.462	1.55

Structural Material; Molydenum

0%	0.826	0.731	1.56
1% (16.3 w/o)	0.824	0.698	1.52
5% (50.3 w/o)	0.819	0.588	1.41
10% (68.1 w/o)	0.811	0.484	1.30
15% (77.3 w/o)	0.802	0.404	1.21