Fast Ion Pressure in Fusion Plasmas

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January 1987

UWFDM-718

FUSION TECHNOLOGY INSTITUTE

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ABSTRACT

The pressure due to fast fusion-born ions is calculated using the slowing down approximation to the Fokker-Planck equation. The analysis considers multiple ion species, including impurities. The various ion species are assumed to have the same temperature, but may have arbitrary charge and mass. A simple closed form expression for the pressure is obtained. Some results for D-T and D-^{3}He plasmas are given. It is found that the fast ion pressure in a D-^{3}He plasma at a typical plasma temperature of 60 keV is about 20% of the thermal pressure; this is about the same ratio as one gets in a D-T plasma at 20 keV. Consequently, the impact of fast ion pressure on the total pressure is similar for D-T and D-^{3}He plasmas at their corresponding expected operating temperatures. At elevated temperatures, however, the fast ion pressure can become a larger fraction of the total pressure.

I. INTRODUCTION

Fusion reactions produce fast charged particles which slow down in the plasma and thereby transfer energy to the electrons and fuel ions. The subject of fast ion slowing down has been studied for some time.[1-13] The fraction of the initial energy transferred to the ions and to the electrons is of concern for plasma power balance calculations. In addition, in magnetic fusion, the pressure of the fast ions is of concern since this uses up some of the available pressure determined by the maximum beta (ratio of plasma pressure to magnetic pressure). A derivation of the pressure of fast alpha particles in a D-T plasma has been given by Rose and Clark,[2] but their result apparently contains an error in the calculation of the critical energy, E_c (defined later). An approximate formula for the fast alpha pressure in a pure D-T plasma has been given by Logan.[10] In this report, we present a derivation of an analytical formula for the fast ion pressure; this derivation is based on the slowing down approximation from the Fokker-Planck equation. In our work, we do not assume the background ions are hydrogenic, but retain explicitly the charge and mass dependence in order to apply the results to impure plasmas and advanced fuels.

II. THE SLOWING DOWN RATE

The rate of slowing down of a test particle of mass M, charge Ze, and energy E, due to Coulomb collisions with a background species of mass m_j, charge $Z_j e$, density n_j, and temperature T_j, is given by[14]

$$\frac{dE}{dt} = [-\Phi(x_j) + x_j (1 + \frac{m_j}{M} \Phi'(x_j))] \frac{4\pi n_j}{m_j V} (\frac{Z_j Z e^2}{4\pi\epsilon_0})^2 \ln \Lambda_j , \tag{1}$$

where $\Phi(x)$ is the error function,

$$\Phi'(x) = \frac{d\Phi}{dx} \,, \qquad V = \sqrt{2E/M} \,, \qquad V_j = \sqrt{2kT_j/m_j} \,, \qquad x_j = \frac{V}{V_j} \,,$$

and $\ln \Lambda_j$ is the usual Coulomb logarithm for interactions between the test particle and the background species. MKS units are used in this report. To obtain the total rate of slowing down, we sum over the various background species in Eq. (1).

For the contribution to the slowing down due to interaction with the background ions, we can use a large argument expansion for the error function. This is because the fusion born ions have a velocity V, much greater than the thermal velocity V_j, of the background ions. The velocity of the fast ions is much less than the thermal velocity of electrons, however. For the electron contribution to the slowing down we use the small argument expansion for the error function. The net slowing down rate is then given by

$$\frac{dE}{dt} = -\frac{AZ^2 \sqrt{M}}{\sqrt{E}} - \frac{BZ^2 E}{M} \,, \qquad (2)$$

where the coefficients A and B are given by

$$A = \frac{4\pi}{\sqrt{2}} \left(\frac{e^2}{4\pi\epsilon_0}\right)^2 \sum_j \left(\frac{n_j Z_j^2}{m_j} \ln \Lambda_j\right) \,, \qquad (3)$$

$$B = \frac{16\sqrt{\pi}}{3\,kT_e} \sqrt{\frac{m_e}{2\,kT_e}} \left(\frac{e^2}{4\pi\epsilon_0}\right)^2 n_e \ln \Lambda_e \,. \qquad (4)$$

The sum over j in A is over the various ionic species. The quantities with subscript e refer to the electrons. Expressions for the Coulomb logarithms, $\ln \Lambda_j$ and $\ln \Lambda_e$, are given in Appendix 1.

Equation (2) can be rewritten in the form,

$$\frac{dE}{dt} = -\frac{2E}{\tau_s} \left[1 + \left(\frac{E_c}{E}\right)^{3/2}\right] . \tag{5}$$

The critical energy E_c is given by

$$E_c = \left[\frac{3\sqrt{\pi}}{4} \frac{M^{3/2}}{n_e \sqrt{m_e}} \sum_j \left(\frac{n_j Z_j^2}{m_j} \ln \Lambda_j\right) \frac{1}{\ln \Lambda_e}\right]^{2/3} kTe . \tag{6}$$

Some authors take $\ln \Lambda_i \approx \ln \Lambda_e$ in the expression for E_c, but this is not a good approximation for fast ion slowing down in fusion plasmas since $\ln \Lambda_e \approx 17$, while $\ln \Lambda_i \approx 22$. The slowing down time, τ_s is given by

$$\tau_s = \frac{3}{4} \frac{(kTe)^{3/2}}{\sqrt{2\pi m_e} Z^2} \left(\frac{4\pi\varepsilon_0}{e^2}\right)^2 \frac{M}{n_e \ln \Lambda_e} . \tag{7}$$

When the particle energy E is above E_c the contribution of the electrons to the slowing down is larger than that of the ions. The slowing down time τ_s is actually the time scale for V to decrease due to electron drag, i.e. $\tau_s = -V/(dV/dt)_e$.

III. FRACTION OF INITIAL ENERGY GIVEN TO THE IONS AND ELECTRONS

The test particle is born at energy E_0, which is much greater than either T_i or T_e, and gives up its energy to the ions and electrons in the process of slowing down. The power transferred to the ions at time t is

$$P = AZ^2 \sqrt{\frac{M}{E(t)}} \ . \tag{8}$$

The total energy transferred to the ions is

$$W_i = \int AZ^2 \sqrt{\frac{M}{E(t)}} \ dt = -\int_{E_f}^{E_0} AZ^2 \sqrt{\frac{M}{E}} \ (\frac{dt}{dE}) \ dE \ , \tag{9}$$

where E_f is the final energy. Since $E_f \ll E_0$, its value is not important and we can take $E_f = 0$ without introducing a significant error. The fraction f_i of the initial energy given to the ions can be written as

$$f_i = \frac{W_i}{E_0} = \frac{2 \ V_c^2}{V_0^2} \int_0^{X_c} dx \ \frac{x}{1 + x^3} \ , \tag{10}$$

where $\qquad E_c = MV_c^2/2 \ , \qquad E_0 = MV_0^2/2 \ , \qquad X_c = V_0/V_c \ .$

The integral in (10) can be evaluated analytically[15] to yield

$$f_i = \frac{2}{X_c^2} \{ \frac{1}{\sqrt{3}} \tan^{-1} (\frac{2X_c - 1}{\sqrt{3}}) + \frac{1}{\sqrt{3}} \tan^{-1} (\frac{1}{\sqrt{3}}) - \frac{1}{6} \ln [\frac{X_c^2 + 2X_c + 1}{X_c^2 - X_c^2 + 1}] \} \ . \tag{11}$$

This result, or equivalent expressions, have been given earlier by Houlberg.[7]

The fraction f_e of the initial energy transferred to the electrons is simply $1 - f_i$.

IV. FAST ION PRESSURE

The fast ions, because of their finite slowing down time, develop a certain amount of pressure which has to be supported by the magnetic field.

This is in addition to the pressure of accumulated thermal "ash" in the plasma. To calculate this pressure we introduce a kinetic equation for the slowing down particles and solve it for their distribution function. Integration of the distribution function then determines the fast ion pressure.

We introduce the distribution function g, of fast ions; g is defined as the number of ions per unit energy per unit spatial volume and satisfies the steady-state kinetic equation,

$$\frac{\partial}{\partial E} \left(g \frac{dE}{dt} \right) = S(E) \; , \qquad (12)$$

where $S(E)$ is the source function in this "phase" space and dE/dt is given by Eq. (5). This equation assumes the ions have a confinement time much longer than the slowing down time. For cases in which this is not true, an additional loss term would have to be introduced in Eq. (12). The relationship of Eq. (12) to the Fokker-Planck equation is discussed in Appendix 2. If we assume the ions are born monoenergetically, then

$$S(E) = S_0 \; \delta(E - E_0) \; , \qquad (13)$$

where S_0 is the number of ions born per unit time per unit volume and δ is the Dirac delta function.

We impose the boundary condition that $g = 0$ for $E > E_0$. Equation (12) can then be integrated to yield

$$g(E) = \begin{cases} \dfrac{S_0 \tau_s}{2E(1 + (E_c/E)^{3/2})} , & E < E_0 \\[12pt] 0 , & E > E_0 \end{cases} \qquad (14)$$

Similar results have been obtained in Refs. 2, 6, and 7, if we consider the appropriate limiting cases.

The pressure p of the fast ions is

$$p = \frac{2}{3} \int_0^{E_0} dE \; g(E) \; E \qquad (15)$$

which can be rewritten as

$$p = \frac{M S_0 \tau_s V_c^2}{3} \int_0^{X_c} dx \; \frac{x^4}{1 + x^3} . \qquad (16)$$

The integral in Eq. (16) can also be evaluated analytically[15] to yield

$$p = \frac{M \tau_s V_c^2 S_0}{3} \left[\frac{X_c^2}{2} + \frac{1}{6} \ln \left(\frac{X_c^2 + 2X_c + 1}{X_c^2 - X_c + 1} \right) - \frac{1}{\sqrt{3}} \tan^{-1} \left(\frac{2X_c - 1}{\sqrt{3}} \right) - \frac{1}{\sqrt{3}} \tan \left(\frac{1}{\sqrt{3}} \right) \right] . \qquad (17)$$

For most applications to fusion born particles, the dominant term is the first term in the square brackets.

V. SOME RESULTS

To illustrate the magnitude of the fast ion pressure, we show some results calculated using the above formulae. We present the results in terms of Γ, which is the ratio of the fast ion pressure to the pressure of the background plasma. Shown in Fig. 1 is Γ versus T_e for a D-T plasma (50:50 mix-

Fig.1 Fast Ion Pressure in a D–T Fusion Reactor
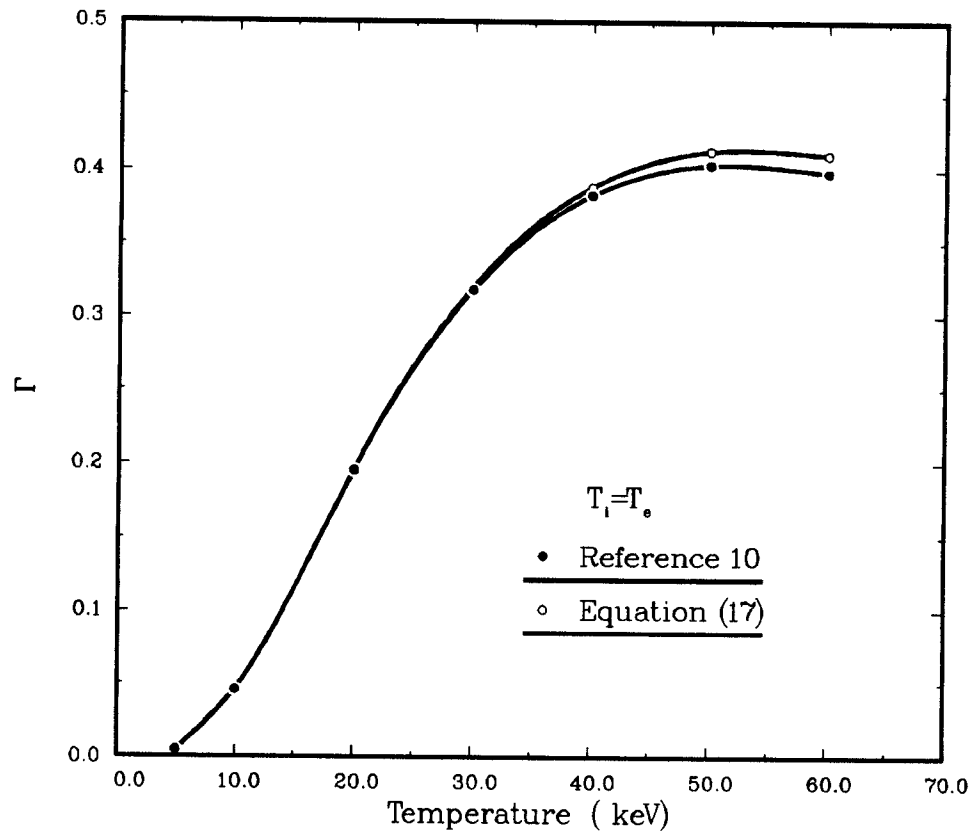
ture) along with the approximate result from Logan.[10] Logan's simple formula fits the results rather well.

The effect of impurity accumulation on the fast ion pressure in a D-T plasma is shown in Fig. 2. In this case the impurity is oxygen (Z_j = 8) and the electron temperature is 15 keV; Z_{eff} is defined as

$$Z_{eff} = \frac{\sum_j n_j Z_j^2}{\sum_j n_j Z_j} \; .$$

The normalized pressure, Γ, decreases with increasing Z_{eff} partly because the impurity ions increase the thermal ion drag on the fast ions, but also because the increased electron density, relative to the ion density, increases the ion drag. The latter effect is probably more important. In addition, the thermal pressure is increased for a given ion density and temperature. All of these effects contribute to the reduction in Γ.

The normalized pressure, Γ, for a D-^{3}He (50:50 mixture) plasma is shown in Fig. 3. For an electron temperature of about 60 keV (a typical operating temperature), Γ is about 20%, which is about what is gotten in a D-T system at about 20 keV. Consequently, the impact of the hot ion pressure on a D-^{3}He reactor is about the same as on a D-T reactor. Figure 4 shows the effect of changing the fuel mixture in a D-^{3}He plasma. The quantity Γ peaks at a fuel mixture of about 65% D and 35% ^{3}He. This is about the same fuel mixture which optimizes the ignition margin and the fusion power density.

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Fig.2 Fast Ion Pressure in a D–T Fusion Reactor

Fig.2 Fast Ion Pressure in a D–T Fusion Reactor

$T_i=T_e=15$ keV
$n_i=10^{20}m^{-3}$
Impurity=Oxygen
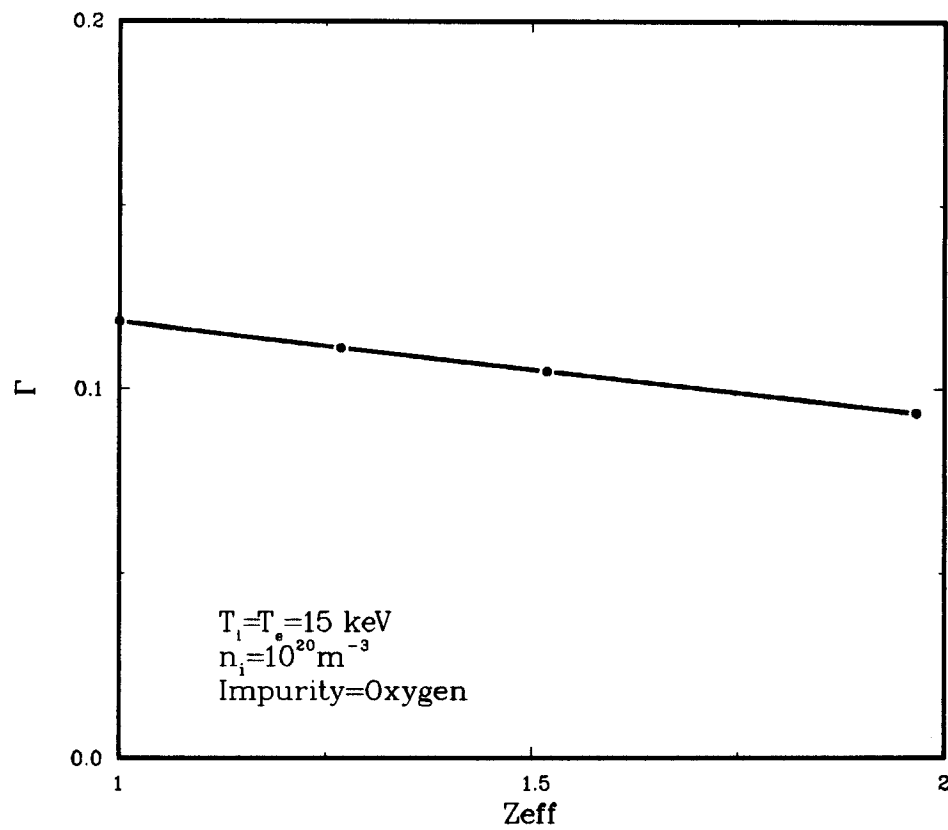
Fig.3 Fast Ion Pressure in a D-^{3}He Fusion Reactor

$n_i = 10^{20} m^{-3}$
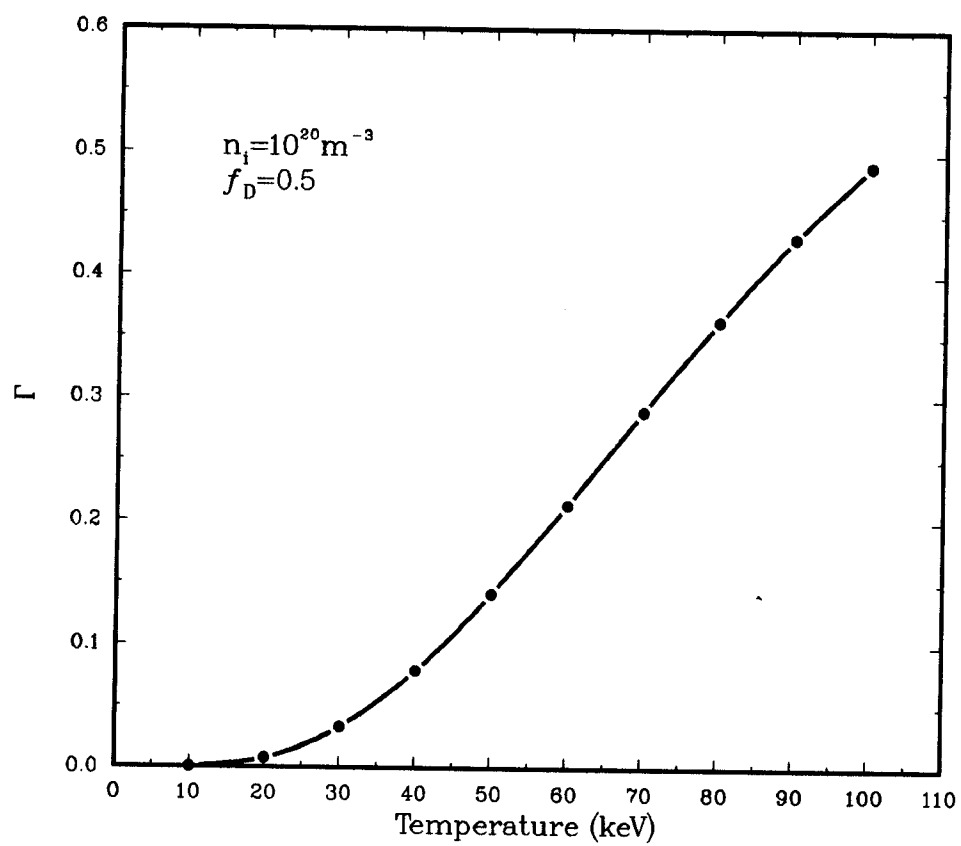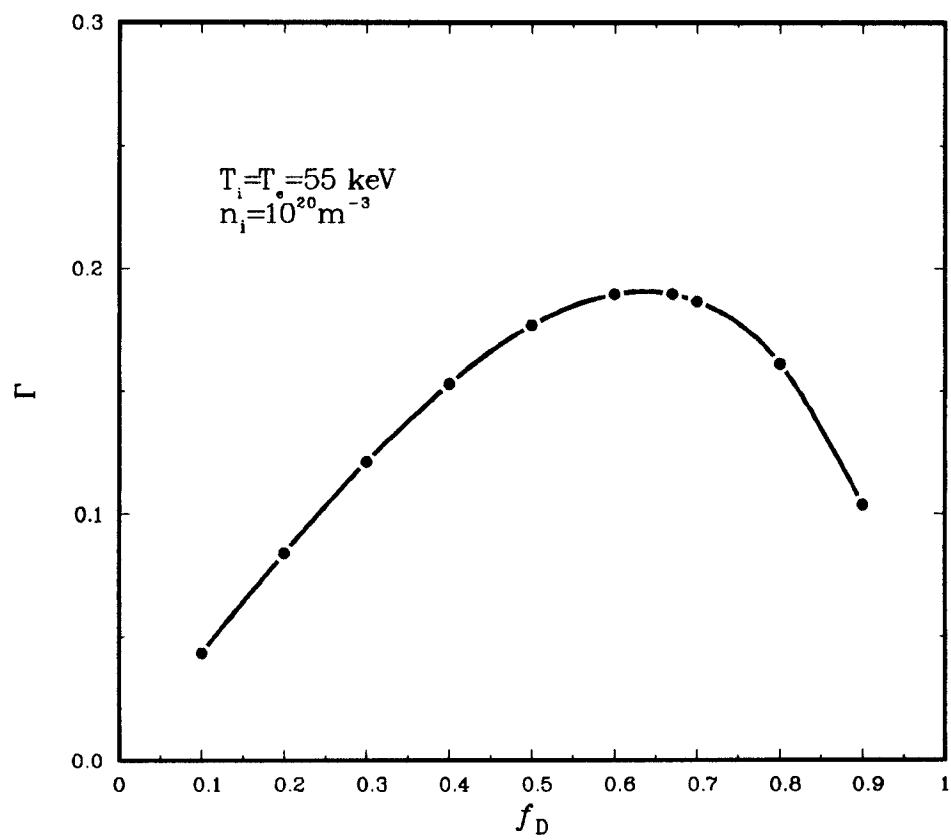$f_D = 0.5$

Fig.4 Fast Ion Pressure in a D-^{3}He Fusion Reactor

APPENDIX 1. THE COULOMB LOGARITHM

In general,

$$\Lambda = b_{max}/b_{min} \; ,$$

where the maximum impact parameter b_{max} is taken to be the electron Debye length since the background ions cannot respond quickly enough to shield a fast test ion. The minimum impact parameter b_{min} is taken to be the greater of the 90° impact parameter, b_0, or the de Broglie wavelength, λ. For fast ion-electron collisions the usual results given in the NRL Plasma Formulary[16] apply. The case of ion-ion collisions requires special attention, however. For collisions between thermal ions, the classical impact parameter, b_0, is usually greater than the de Broglie wavelength. High energy ions, however, can have a de Broglie wavelength which is shorter than b_0. This is because the de Broglie wavelength scales as $E^{-0.5}$ whereas b_0 scales as E^{-1}. For both 3.5 MeV alpha particles and 14.7 MeV protons, we have to take b_{min} equal to the de Broglie wavelength. As the particles slow down to thermal energy, this will no longer be true, but it is better to use the correct value for Λ at the high energy end in describing the slowing down process, since the initial slowing down time scale plays a role in determining the pressure of the fast ions. Houlberg[7] gives $E > 25 \, Z_j^2 Z^2 A$ keV as the rule of thumb for when the quantum mechanical b_{min} applies (A is the amu of the test ion). This covers most of the interesting energy range for our purposes. Consequently, we take

$$b_{min} = \frac{h}{m_r u} \, ,$$

where h is Planck's constant, m_r is the reduced mass, and u is the velocity of the fast test ion. Thus we can write

$$\ln \Lambda_j = \ln \left[\sqrt{\frac{2E_0 kT_e E}{n_e e^2 Mh^2}} \left(\frac{m_j M}{m_j + M} \right) \right] \, ,$$

or in more practical units,

$$\ln \Lambda_j = 17.1 + \ln \left[\left(\frac{m_j M}{m_j + M} \right) \sqrt{\frac{T_e E}{n_e M}} \right] \, ,$$

where the electron density, n_e, is measured in units of 10^{20} m^{-3}, the electron temperature and the test particle energy in units of keV, and the masses in amu.

APPENDIX 2. RELATIONSHIP TO THE FOKKER-PLANCK EQUATION

The kinetic equation used in this report, which was written down from first principles, can be obtained directly from the Fokker-Planck equation,[14]

$$\frac{\partial f}{\partial t} = -\frac{\partial}{\partial v_i}\left(f\ \frac{\langle\Delta v_i\rangle}{\Delta t}\right) + \frac{1}{2}\frac{\partial^2}{\partial v_i \partial v_j}\left(f\ \frac{\langle\Delta v_i \Delta v_j\rangle}{\Delta t}\right) + S_v\ , \qquad (2.1)$$

where S_v is the source function in velocity space. We are using Cartesian tensor notation with the summation convention.

For test ions fast relative to the background ions and for interaction of test ions with electrons, the diffusion term, $\langle\Delta v_i \Delta v_j\rangle/\Delta t$, is negligible compared with the dynamical friction term, $\langle\Delta v_i\rangle/\Delta t$. We also drop the time derivative for steady-state to get,

$$\frac{\partial}{\partial v_i}\left(f\ \frac{\langle\Delta v_i\rangle}{\Delta t}\right) = S_v\ . \qquad (2.2)$$

For an isotropic source, f is also isotropic. Since g is the number of particles per unit energy, we have

$$f\ d^3 v = 4\pi v^2 f\ dv = g\ dE = Mvg\ dv\ ,$$

or
$$f = \frac{Mg}{4\pi v}$$

is the relationship between the two distribution functions, f and g. Likewise

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$$S_v = \frac{MS}{4\pi v} \; ,$$

where S is the source per unit energy. Also

$$\frac{\langle \Delta v_i \rangle}{\Delta t} = \frac{dv_i}{dt} \; .$$

Since the slowing down process is isotropic,

$$\frac{dv_i}{dt} = \frac{v_i}{v} \frac{dv}{dt} \; , \qquad \frac{dv}{dt} = \frac{1}{Mv} \frac{dE}{dt} \; .$$

We use the above results to transform Eq. (2.2) to energy space

$$\frac{\partial}{\partial v_i} \left(\frac{Mg}{4\pi v} \frac{v_i}{v} \frac{1}{Mv} \frac{dE}{dt} \right) = \frac{MS}{4\pi v} \; ,$$

or

$$\frac{\partial}{\partial v_i} \left(g \frac{dE}{dt} \right) \frac{v_i}{v^3} = \frac{MS}{v} \; ,$$

since

$$\frac{\partial}{\partial v_i} \left(\frac{v_i}{v^3} \right) = 0 \; .$$

Thus

$$\frac{\partial}{\partial E} \left(g \frac{dE}{dt} \right) \frac{\partial E}{\partial v_i} \frac{v_i}{v^3} = \frac{MS}{v} \; .$$

But

$$\frac{\partial E}{\partial v_i} = M v_i \qquad \text{and} \qquad v_i v_i = v^2 \; ,$$

which yields the desired result

$$\frac{\partial}{\partial E} \left(g \, \frac{dE}{dt} \right) = S \ .$$

This is our kinetic equation, Eq. (12).

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