Energy Storage in Transformer, Divertor and Plasma Poloidal Fields

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Results

The following computations are performed to determine (approximately) what amount of energy is stored in the magnetic fields produced by (1) the transformer coils, (2) the divertor coils, and (3) the poloidal field of the plasma current. The energy is given for both the thermally stable and thermally unstable designs.

Plasma current stable design = 27.2×10^6 amperes

Energy stored = 8.603×10^{10} joures

= 23.90 MW-HR

Plasma current unstable design = 20.7×10^6 amperes

Energy stored = 4.983×10^{11} joules

= 13.84 MW-HR

Method of Analysis

$$\text{Energy Stored} = \frac{1}{2} \sum_{K=1}^{N} \sum_{L=1}^{N} M_{KL}\, I_K\, I_L \tag{1}$$

where M_{KL} are inductances (>0), both self and mutual, the matrix is symmetric.

$$M_{KL} = \begin{pmatrix} L_{11} & M_{12} & - - - - - & M_{1n} \\ M_{12} & L_{22} & & \vdots \\ \vdots & & L_{33} & \vdots \\ M_{1n} & & \ddots & L_{nn} \end{pmatrix}$$

The self inductances are found using [1]

$$L_{ii} = \mu_o[2R_i - a_i][(1 - \frac{k_i^2}{2}) K(k_i) - E(k_i)] + \frac{\mu_o R_i l_i}{2} \qquad (2)$$

$l_i = \frac{1}{2}$ uniform current

li = .982 for plasma (current distribution $\sim(1 - \frac{r^2}{a^2})$)

R_i = major radius to conductor

a_i = minor radius of conductor

For a rectangular coil one computes a_i from

$$a_i = \left(\frac{W_i \cdot h_i}{\pi}\right)^{1/2}$$

W_i, h_i are the width and height of conductor respectively

$\mu_o = 12.566 \times 10^{-7}$

$$k_i^2 = \frac{4R_i(R_i - a_i)}{(2R_i - a_i)^2}$$

K,E are complete elliptic integrals (computed using Hastings approximations [2])

The mutual inductances are computed using

$$M_{ij} = \mu_o\sqrt{R_i R_j} \left[\left(\frac{2}{k_{ij}} - k_{ij}\right) K(k_{ij}) - \frac{2}{k_{ij}} E(k_{ij})\right] \qquad (3)$$

$$k_{ij}^2 = \frac{4R_i R_j}{(d_{ij})^2 + (R_i + R_j)^2}$$

R_i, R_j = major radii of i^{th} and j^{th} conductors

$d_{ij} = |Z_i - Z_j|$ = distance between planes containing i^{th} and j^{th} conductors

All other terms are previously defined.

The effect of the mutual inductance on energy storage depends on the product of $I_K I_L$. If I_K and I_L are in opposite directions, then $I_K I_L < 0$ and the effect of $(M_{KL} + M_{LK}) I_K I_L$ is to decrease to stored energy. If $I_K I_a > 0$ then the stored energy is increased.

The coil positions, dimensions, currents relative to the plasma current are given in Table 1. The inductance matrix in microhenries is given in Table 2. A drawing of the configuration is shown in Figure I.

References

1. S. Romo, J. R. Whinnery, T. Van Duzer, "Fields and Waves in Communication Electronics," Chap. 5

2. C. Hastings, Jr., "Approximations for Digital Computers," p. 172, 175.

Table #1 Coil Data

Coil[1] #	Coil Location		Radius[2]	I(coil)/I(plasma)	Unstable Design
	R(m)	Z(m)	A(m)		I(coil)x10^6 AMPS
1	13.0	0.0	5.0	1.0	20.700
2	8.0	13.0	.780	1.1835	24.853
3	8.0	-13.0	.780	1.1835	24.853
4	13.0	13.5	.705	.5917	12.426
5	13.0	-13.5	.705	.5917	12.426
6	18.0	12.0	1.066	-1.1835	-24.853
7	18.0	-12.0	1.066	-1.1835	-24.853
8	22.0	8.0	.135	- .11835	- 2.485
9	22.0	- 8.0	.135	- .11835	- 2.485
10	3.889	2.996	.284	- .25189	- 5.290
11	3.889	-2.996	.284	- .25189	- 5.290
12	4.297	5.967	.159	- .21838	- 4.586
13	4.297	-5.967	.159	- .21838	- 4.586
14	4.946	8.895	.209	- .17006	- 3.571
15	4.946	-8.895	.209	- .17006	- 3.571
16	6.132	12.714	.356	- .15765	- 3.311
17	6.132	-12.714	.356	- .15765	- 3.311
18	8.036	17.335	.184	- .11968	- 2.513
19	8.036	-17.335	.184	- .11968	- 2.513

I(plasma) = 21.0 x 10^6 Amps (Thermally unstable reactor design)
= 27.2 x 10^6 Amps (Thermally stable reactor design)

1. Coil #1 = Plasma
   Coil 2-9 = Divertor
   Coil 10-19 = Transformer

2. Divertor and Transformer coils are rectangularbut an effective circular radius is computed to use in self inductance calculation (πa^2 = hxw)

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Table #2

Inductance Mateix[1] (micro-henies)

Coil #	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
1	22.49	3.223	3.223	6.632	6.632	10.66	10.66	15.11	15.11	2.038	2.038	2.038	2.038	2.039	2.039	2.039	2.039	2.039	2.039
2		25.08	.3589	11.50	.7643	7.564	1.320	5.521	2.093	.8454	.3168	1.779	.2537	4.117	.2305	12.59	.2272	8.280	.2419
3			25.08	.7643	11.50	1.320	7.564	2.093	5.521	.3168	.8454	.2537	1.779	.2305	4.117	.2272	12.59	.2419	8.280
4				51.24	1.699	23.37	3.043	15.09	4.985	1.058	.5300	1.798	.4654	3.172	.4527	6.215	.4766	9.278	.5364
5					51.24	3.043	23.37	4.985	15.09	.5300	1.058	.4654	1.798	.4527	3.172	.4766	6.215	.5364	9.278
6						68.80	5.623	34.57	9.483	1.186	.7439	1.742	.7062	2.630	.7290	4.305	.8129	6.492	.9599
7							68.80	9.483	34.57	.7439	1.186	.7062	1.742	.7290	2.630	.8129	4.305	.9599	6.492
8								149.3	16.52	1.269	.9710	1.658	.9922	2.232	1.083	3.224	1.274	4.555	1.568
9									149.3	.9710	1.269	.9922	1.658	1.083	2.232	1.274	3.224	1.568	4.555
10										13.75	9.348	3.037	.4727	1.312	.3065	.6620	.2192	.4093	.1774
11											13.75	.4727	3.037	.3065	1.312	.2192	.6620	.1774	.4093
12												19.16	.2859	3.930	.2107	1.545	.1695	.8016	.1511
13													19.16	.2107	3.930	.1695	1.545	.1511	.8016
14														21.17	.1705	4.142	.1501	1.749	.1447
15															21.17	.1501	4.142	.1447	1.749
16																23.60	.1447	5.403	.1511
17																	23.60	.1511	5.403
18																		40.91	.1702
19																			40.91

1. Matrix in symmetric (Mij = Mji)

2. The diagonal has the self inductances (L_{ii})

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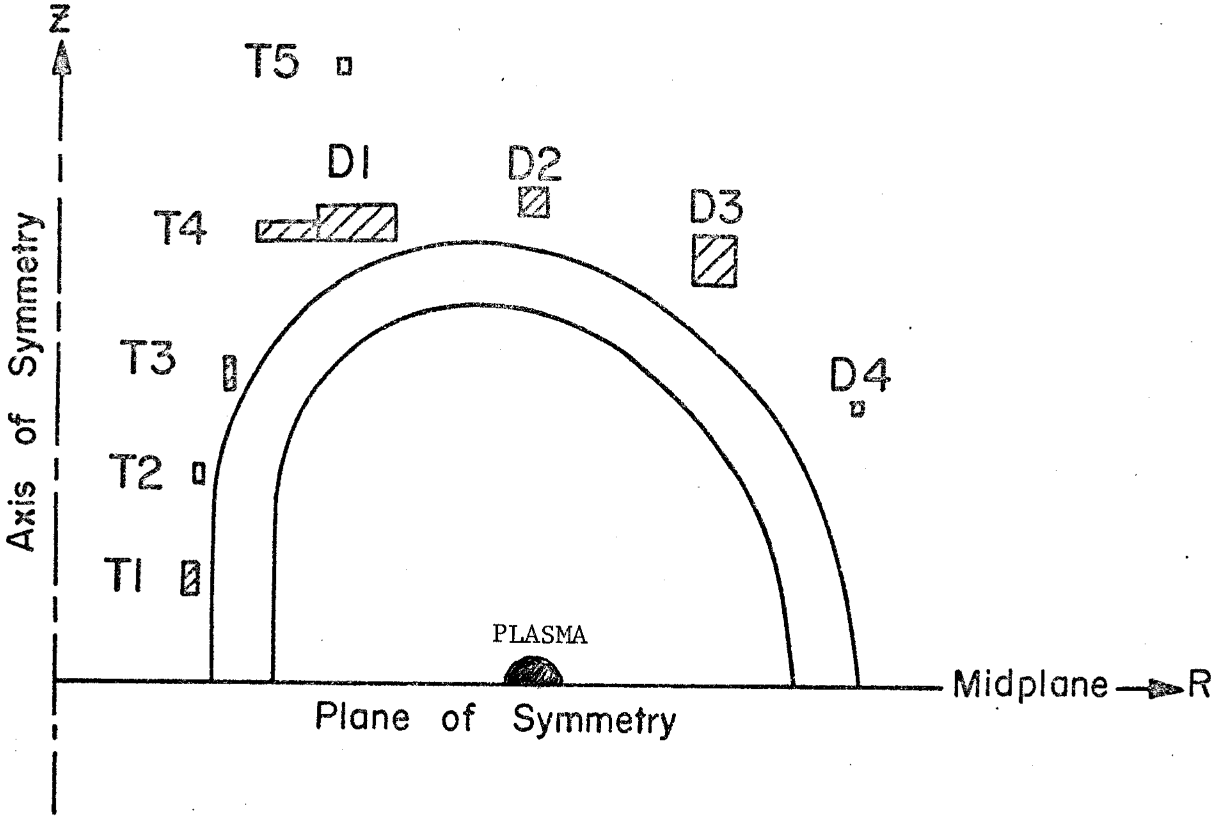
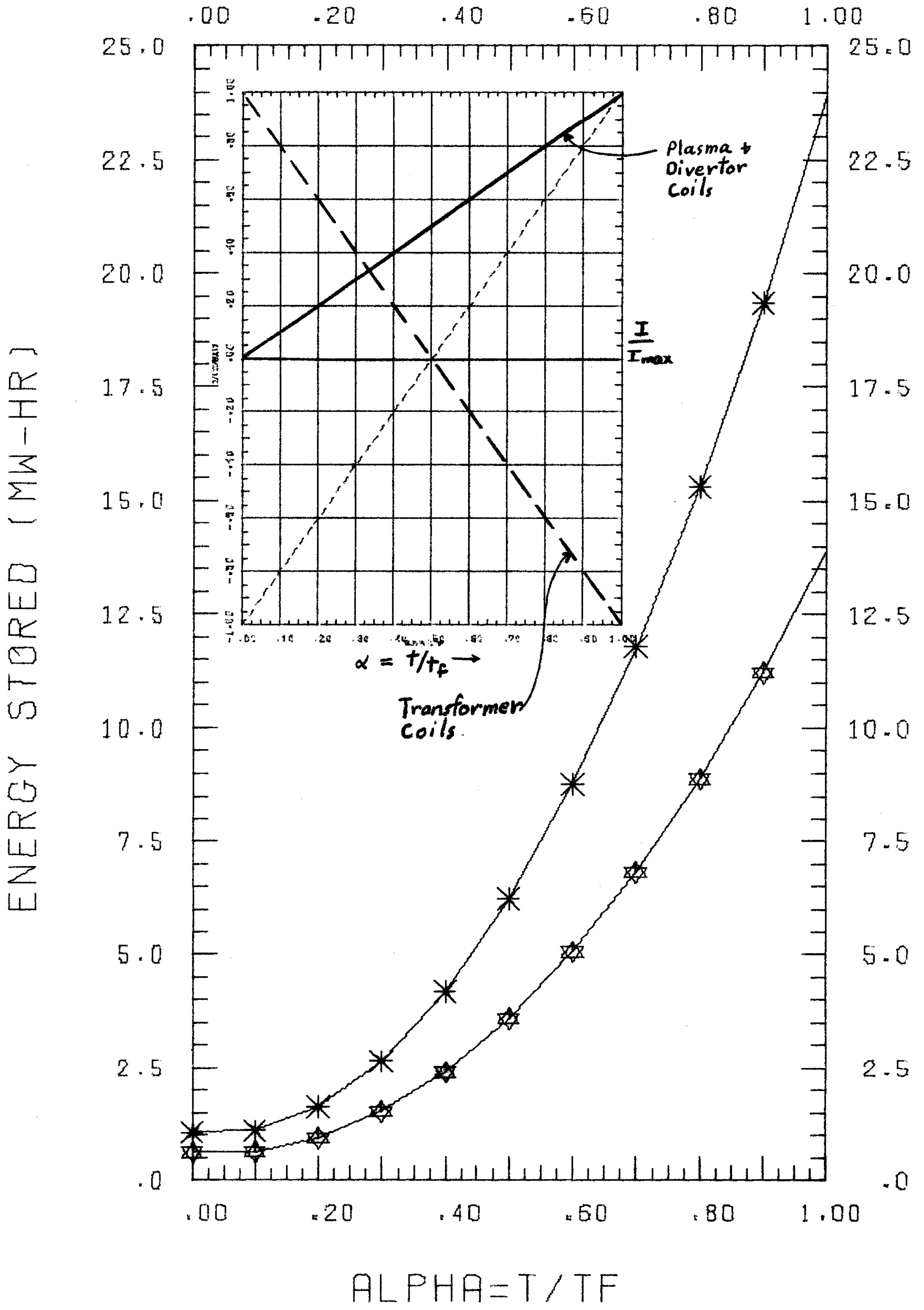


FIGURE I

Position of Divertor and Transformer Coils

ENERGY STØRED IN PØLØIDAL FIELD
STABLE DESIGN = ✳, UNSTABLE DESIGN = STAR



Program to Compute Energy

```
      DIMENSION XM(30,30),AF(50),ZF(50),AT(50),AH(50),AW(50)
      ENERGY=0.0
      XMU0=12.566E-7
      ATPST=27.2E6
      ATPUST=21.0E5
      I=1
    4 READ 1,AF(I),ZF(I),AT(I),AH(I),AW(I)
    1 FORMAT(5F10.3)
      IF(AF(I).LT.0.0) GO TO 3
      IF(AW(I).LE.0.000001) GO TO 2
      AREA=AH(I)*AW(I)
      AH(I)=SQRT(AREA/3.1415926)
      I=I+1
      AF(I)=AF(I-1)
      ZF(I)=-ZF(I-1)
      AT(I)=AT(I-1)
      AH(I)=AH(I-1)
C    AH(I) NOW CONTAINS THE EFFECTIVE RADIUS OF EQUIVALENT CIRCULAR
C     CONDUCTOR OF SAME CROSS-SECTIONAL AREA
    2 CONTINUE
      I=I+1
      GO TO 4
    3 CONTINUE
      NF=I-1
C     COMPUTE TERMS IN INDUCTANCE MATRIX
      DO 5 I=1,NF
      DO 5 J=1,NF
      IF(I.NE.J)GO TO 6
      TERM1=XMU0*(2.0*AF(I)-AH(I))
      XKS=4.0*AF(I)*(AF(I)-AH(I))/(2.0*AF(I)-AH(I))**2
      ETA=1.0-XKS
      TERM2=(1.0-XKS/2.0)*ELIPT(ETA,1)-ELIPT(ETA,2)
      ALI=0.5
      IF(I.EQ.1) ALI=0.982
      TERM3=XMU0*AF(I)*0.5*ALI
      XM(I,J)=TERM1*TERM2+TERM3
      GO TO 7
    6 TERM1=XMU0*SQRT(AF(I)*AF(J))
      DIJ=ABS(ZF(I)-ZF(J))
      DIJS=DIJ*DIJ
      XKS=4.0*AF(I)*AF(J)/(DIJS+(AF(I)+AF(J))**2)
      XK=SQRT(XKS)
      ETA=1.0-XKS
      TERM2=(2.0/XK -XK)*ELIPT(ETA,1)-2.0*ELIPT(ETA,2)/XK
      XM(I,J)=TERM1*TERM2
    7 DELE=0.5*XM(I,J)*AT(I)*AT(J)
      ENERGY=ENERGY+DELE
    5 CONTINUE
```

```
C     ENERGY FOR STABLE DESIGN
      ESTAB=ENERGY*ATPST**2
      ESTMWH=ESTAB/3.6E9
C    ENERGY IN  UNSTABLE DESIGN
      EUNSTB=ENERGY*ATPUST**2
      EUNMWH=EUNSTB/3.6E9
      PRINT 500
  500 FORMAT(1H1)
      PRINT 200,ESTAB,ESTMWH,EUNSTB,EUNMWH
  200 FORMAT(5X,'ENERGY STORED FOR STAB.DESIGN ',5X,E12.4,
     1 ' JOULES OR',5X,E12.4,' MW-HR',///,5X,'ENERGY STORED IN UNSTABLE
     2DESIGN',5X,E12.4,' JOULES OR',5X,E12.4,' MW-HR',//)
      PRINT 501
  501 FORMAT(5X,/////,5X)
      DO 10 I=1,NF
      ATS=AT(I)*ATPST
      ATUS=AT(I)*ATPUST
   10 PRINT 11,I,AF(I),ZF(I),AH(I),AT(I),ATS,ATUS,ATPUST
   11 FORMAT(5X,I3,2X,3F10.3,1P4E12.4)
C     PRINT OUT INDUCTANCE MATRIX
      PRINT 500
      DO 250 I=1,NF
      DO 250 J=1,NF
  250 XM(I,J)=XM(I,J)*1.0E6
C     INDUCTANCES ARE NOW IN MICRO HENRIES
      PRINT 253

  253 FORMAT(1H0,' ALL INDUCTANCES ARE IN UNITS OF  MICRO-HENRIES',///)
      PRINT 252,(((K,L,XM(K,L)),L=1,NF),K=1,NF)
  252 FORMAT(1X,5(2I3,E12.4))
      END
```

Function Subroutine to Compute Elliptic Integrals

```
      FUNCTION ELIPT(ETA,I)
      IMPLICIT DOUBLE PRECISION(A,B)
      DOUBLE PRECISION E,EK,EE
      DATA AK1/1.38629436112D0/AK2/0.09666344253D0/AK3/0.03590092383D0/
     1 AK4/0.03742563713D0/AK5/0.01451196212D0/BK1/0.500D0/
     2 BK2/0.12498593597D0/BK3/0.06880248576D0/BK4/0.03328355346D0/
     3 BK5/0.00441787012D0/AE1/0.44325141463D0/AE2/0.06260601220D0/
     4 AE3/0.04757383546D0/AE4/0.01736506451D0/BE1/0.24998368310D0/
     5 BE2/0.09200180037D0/BE3/0.04069697526D0/BE4/0.00526449639D0/
      AE0=1.0D0
      E=ETA
      GO TO (1,2),I
    1 IF(E.LE.1.0D-6)GO TO 3
C     ***************COMPUTES  K  ELLIPTIC INTEGRAL  ********************
      EK=AK1+(AK2+(AK3+(AK4+AK5*E)*E)*E)*E+DLOG(1.0D0/E)*(BK1+(BK2+(BK3+
     1(BK4+BK5*E)*E)*E)*E)
      ELIPT=SNGL(EK)
      RETURN
    2 IF(E.LE.1.0D-6)GO TO 4
C       **********COMPUTES  E  ELLIPTIC INTEGRAL  *********************
      BE0=0.0D0
      EE=AE0+(AE1+(AE2+(AE3+AE4*E)*E)*E)*E+DLOG(1.0D0/E)*(BE0+(BE1+(BE2+
     1(BE3+BE4*E)*E)*E)*E)
      ELIPT=SNGL(EE)
      RETURN
    3 ELIPT=SNGL(AK1)
      RETURN
    4 ELIPT=SNGL(AE0)
      RETURN
      END
```