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1. Introduction

The motion of neutral particles in a halo plasma can be calculated using various neutral particle transport codes which take account of the changes in both direction and energy that result from interactions with halo plasma ions. SPUDNUT [1] is an example of such a code and has been utilized for halo plasma calculations [2]. Here we present a very simple two energy group model which is employed to identify the role played by atomic processes and to estimate the reflection coefficient and transmissivity of the incident neutral flux in a homogeneous halo plasma. Comparisons are given between this model and calculations obtained with the SPUDNUT code.

2. The Two Energy Group Model

We divide the neutral atoms of the halo plasma into two energy groups. Group (1) contains the wall released neutrals which have velocity V_w characteristic of its release from a wall surface and have a $\cos\theta$ angular distribution at $x = 0$. The neutrals resulting from charge exchange in the halo plasma belong to group (2). They have random direction and speeds corresponding to the mean thermal speed, V_{th}, of the ions.

The halo is assumed to be a homogeneous slab with width h. We only consider a single generation process to estimate the probability that a neutral released from the wall will recycle back to the wall. The total effect of the neutral motion is considered only in the x direction. The halo is infinite in both the z and y directions. The incident flux of group (1) neutrals is $\Gamma^{(1)}(0)$ and the flux of group (1) atoms at a distance x from the edge is given by [1]

$$\Gamma^{(1)}(x) = 2\Gamma^{(1)}(0)\ E_3(\beta_w(x)) \tag{1}$$

if the wall-originated neutrals have a cos θ angular distribution. Now $E_3(\beta_w(x))$ can be fitted approximately by an exponential function (see Fig. 1), so that

$$\Gamma^{(1)}(x) \simeq \Gamma^{(1)}(0)\ \exp(-1.5\ \beta_w(x)) \tag{2}$$

where the optical depth to the distance x for group (1) neutrals is

$$\beta_w\ (x) = \int_0^x \frac{\{n_e(x')\langle\sigma v\rangle_e + n_i(x')\langle\sigma v\rangle_{cx}\}}{V_w} = \frac{n_e\langle\sigma v\rangle_e + n_i\langle\sigma v\rangle_{cx}}{V_w}\ x\ .$$

Here the assumption that the halo density and temperature are uniform has been used. We set

$$\beta_w(x) = \frac{x}{\Delta^{(1)}} \tag{3}$$

where
$$\frac{1}{\Delta^{(1)}} = \frac{1}{\lambda_i^{(1)}} + \frac{1}{\lambda_{cx}^{(1)}}$$

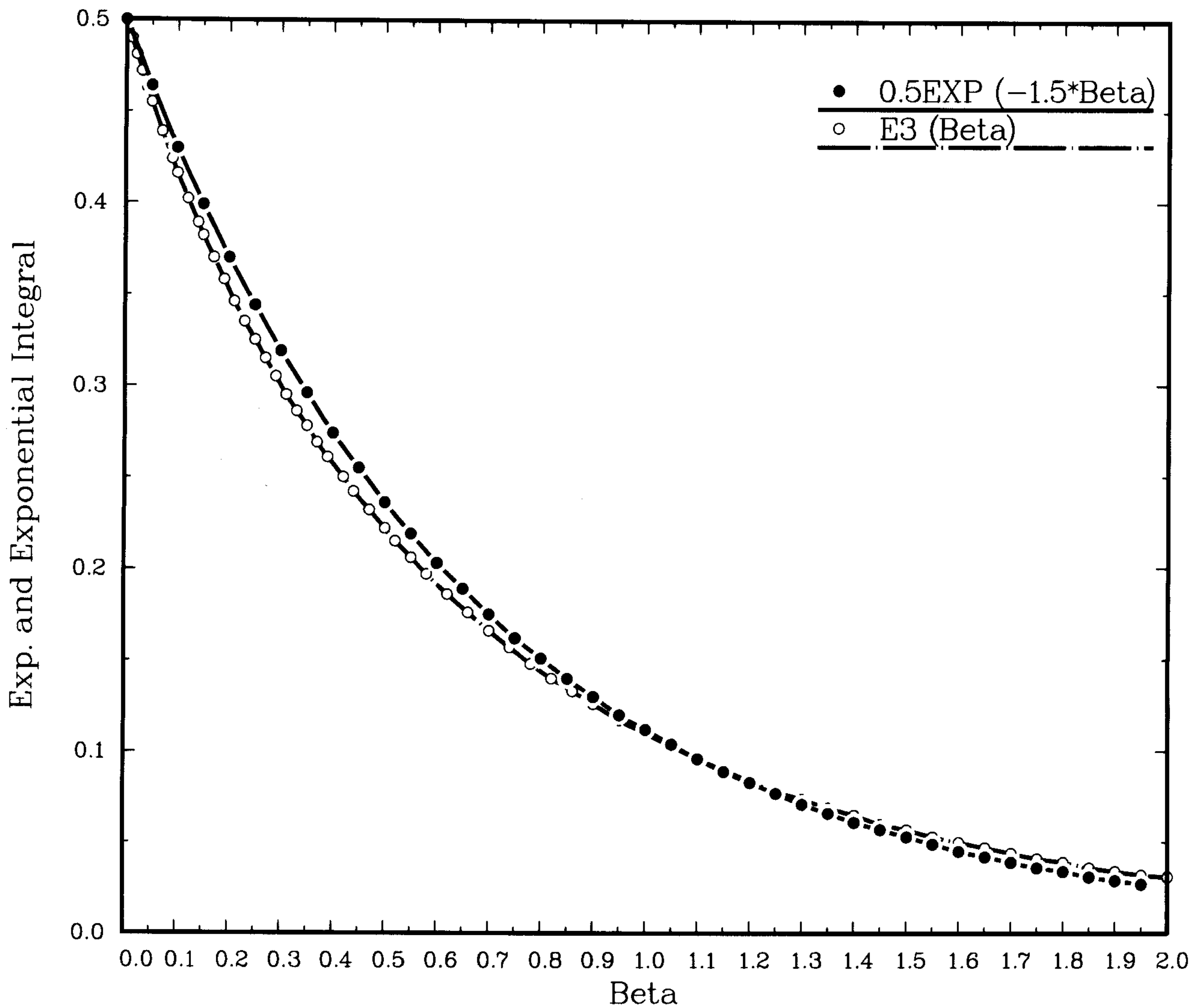
and
$$\lambda_{cx}^{(1)} = \frac{V_w}{n_i\langle\sigma v\rangle_{cx}}\ , \qquad \lambda_i^{(1)} = \frac{V_w}{n_e\langle\sigma v\rangle_e}\ .$$

From Eqs. (2) and (3), we have

$$\Gamma^{(1)}(x) \simeq \Gamma^{(1)}(0)\ \exp(-1.5\ \frac{x}{\Delta^{(1)}})\ . \tag{4}$$

The factor 1.5 in Eq. (4) can be understood by using the formula for normal

Fig. 1 Comparison of Exponential Function
and Exponential Integral E3



incidence, but with the neutral atom velocity replaced by the mean value of the x-component of the velocity. For a cos θ angular distribution, the average of the x-component of the velocity is given by

$$\langle V_{wx}\rangle = \frac{\int_0^{\pi/2} d\theta\, V_w \cos\theta \cos\theta\, 2\pi \sin\theta\, d\theta}{\int_0^{\pi/2} \cos\theta\, 2\pi \sin\theta\, d\theta} = \frac{2}{3} V_w\ .$$

With $$\Gamma^{(1)}(x) \simeq \Gamma^{(1)}(0)\exp\left(-\frac{x}{\Delta_0^{(1)}}\right)$$

and $$\Delta_o^{(1)} = \frac{\langle v_{wx}\rangle}{n_e\langle\sigma v\rangle_e + n_i\langle\sigma v\rangle_{cx}} = \frac{2}{3}\Delta^{(1)}\ ,$$

we recover Eq. (4).

In an element of extent dx at x, the rate of charge exchange is

$$\kappa^{(1)}(x) \simeq \Gamma^{(1)}(x)\frac{dx}{\lambda_{cx}^{(1)}}\ . \tag{5}$$

This produces group (2) neutrals which move with equal probability either inward or outward. Subsequent motion backward towards the wall can be treated as random walk migration, with step length equal to the charge exchange mean free path $\lambda_{cx}^{(2)}$. Group (2) neutrals have an isotropic distribution since they arise from the ion distribution. Thus

$$v_x^2 + v_y^2 + v_z^2 = v_{th}^2 \simeq 3v_x^2$$

or $$v_x \simeq \frac{v_{th}}{\sqrt{3}}\ . \tag{6}$$

We can regard the macroscopic effect of random walk migration in the x direction as a form of "diffusion." The scale length $\Delta_0^{(2)}$ can be expressed as [3]

$$\frac{1}{\Delta_0^{(2)}} = \left(\frac{1}{\lambda_{cx}^{(2)}} + \frac{1}{\lambda_i^{(2)}}\right)\left(3\,\frac{\lambda_{cx}^{(2)}}{\lambda_i^{(2)}}\right)^{1/2} C \tag{7}$$

where $C \to 1$ if $\lambda_i^{(2)} >> \lambda_{cx}^{(2)}$

$$\frac{1}{\Delta_0^{(2)}} \simeq \frac{1}{\lambda_{cx}^{(2)}}\left(3\,\frac{\lambda_{cx}^{(2)}}{\lambda_i^{(2)}}\right)^{1/2} \tag{8}$$

$$\Delta_0^{(2)} \simeq \left(\frac{1}{3}\,N^{(2)}\right)^{1/2} \lambda_{cx}^{(2)} \tag{9}$$

where $N^{(2)}$ is the average number of charge exchange collision prior to ionization and it is equal to the ratio of collision times

$$N^{(2)} \simeq \frac{\tau_i^{(2)}}{\tau_{cx}^{(2)}} \simeq \frac{\lambda_i^{(2)}}{\lambda_{cx}^{(2)}} = \frac{\langle\sigma v\rangle_{cx}^{(2)}}{\langle\sigma v\rangle_e^{(2)}} \;.$$

The atoms turning back from dx at x belong to group (2), and their flux is

$$\Gamma^{(2)}(x) \simeq \frac{1}{2}\,\kappa^{(1)}(x)\,\exp\frac{-x}{\Delta_0^{(2)}} = \frac{1}{2}\,\Gamma^{(1)}(0)\,\exp\left(-x\left(\frac{1}{\Delta_0^{(2)}} + \frac{1}{\Delta_0^{(1)}}\right)\right)\frac{dx}{\lambda_{cx}^{(1)}} \;.$$

The total flux of group (2) atoms at the wall is

$$\Gamma^{(2)} = \int_0^h \frac{1}{2}\,\frac{1}{\lambda_{cx}^{(1)}}\,\Gamma^{(1)}(0)\,e^{-x\left(\frac{1}{\Delta_0^{(1)}} + \frac{1}{\Delta_0^{(2)}}\right)}\,dx \;.$$

The reflectance, R, is

$$R = \frac{\Gamma^{(2)}}{\Gamma^{(1)}(0)} \simeq \frac{1}{2}\frac{1}{\lambda_{cx}^{(1)}}\left(\frac{\Delta_0^{(1)}\Delta_0^{(2)}}{\Delta_0^{(1)} + \Delta_0^{(2)}}\right)\left[1 - \exp\left(-\frac{h(\Delta_0^{(1)} + \Delta_0^{(2)})}{\Delta_0^{(1)}\Delta_0^{(2)}}\right)\right] . \qquad (10)$$

Similarly, the atoms moving towards the core plasma (positive x direction) also belong to group (2). Their flux at x = h is

$$\Gamma^{(2)}(h) = \int_0^h \frac{1}{2}\frac{1}{\lambda_{cx}^{(1)}}\,\Gamma^{(1)}(0)\, e^{-x/\Delta_0^{(1)}}\, e^{-(h-x)/\Delta_0^{(2)}}\, dx$$

and

$$\Gamma^{(1)}(h) \simeq \Gamma^{(1)}(0)\, e^{-h/\Delta_0^{(1)}} .$$

The transmissivity, T, is

$$T = \frac{\Gamma^{(1)}(h) + \Gamma^{(2)}(h)}{\Gamma^{(1)}(0)} \simeq e^{-h/\Delta_0^{(1)}} + \frac{1}{2}\frac{1}{\lambda_{cx}^{(1)}}\left(\frac{\Delta_0^{(2)}\Delta_0^{(1)}}{\Delta_0^{(2)} - \Delta_0^{(1)}}\right) e^{-h/\Delta_0^{(2)}}$$

$$* \left[1 - e^{-(\Delta_0^{(2)} - \Delta_0^{(1)}/\Delta_0^{(2)}\Delta_0^{(1)})h}\right] . \qquad (11)$$

3. Comparative Calculations

Now we estimate the reflectance and transmissivity for a finite width of halo hydrogen plasma, assuming that $T_i = T_e = T = 20$ eV, $n_i = n_e = n = 2 \times 10^{12}/cm^3$. The wall-originated hydrogen atoms have energy $E_w = 3$ eV. The mean free paths for group (1) and group (2) are

$$\lambda_{cx}^{(1)} \simeq \frac{V_w}{n_i \langle \sigma_{cx} \overline{v}_{wr} \rangle} , \qquad \lambda_i^{(1)} \simeq \frac{V_w}{n_i \langle \sigma_e \overline{v}_e \rangle} ,$$

$$\lambda_{cx}^{(2)} \simeq \frac{V_{th}}{n_i \langle \sigma_{cx} \overline{v}_i^{(2)} \rangle} , \qquad \lambda_i^{(2)} \simeq \frac{V_{th}}{n_i \langle \sigma_e \overline{v}_e \rangle} .$$

The relative velocity of the neutral-electron interaction can be replaced by $\overline{v}_e$, since $\overline{v}_e >> V_w$, $\overline{v}_e >> v_{th}$.

$$\Delta_0^{(1)} \simeq \frac{0.667\ V_w}{n_i(<\sigma_{cx}\overline{v}_{wr}^{(2)}> + <\sigma_e\overline{v}_e>)}$$

$$\Delta_0^{(2)} \simeq \left(\frac{1}{3(<\sigma_{cx}\overline{v}_r^{(2)}><\sigma_e\overline{v}_e^{(2)}>)}\right)^{1/2} \frac{v_{th}}{n_i}$$

where $\overline{v}_r^{(2)}$ is the average relative velocity over dual Maxwellian distributions,

$$\overline{v}_r^{(2)} = \int_0^\infty v_r^{(2)}\ f(v_r^{(2)})\ dv_r^{(2)} = \int_0^\infty \left(\frac{\mu}{2\pi kT_i}\right)^{3/2} 4\pi e^{-\mu v_r^{2(2)}/2\kappa T_i}\ v_r^{3(2)}\ dv_r^{(2)}$$

$$= 4\sqrt{\frac{kT_i}{\pi m_i}}$$

if the two masses are equal.

For our case, $V_w \simeq 2.4 \times 10^6$ cm/s, $\overline{v}_r^{(2)} \simeq 1.0 \times 10^7$ cm/s, $v_{th} = 7.6 \times 10^6$ cm/s, $v_e^{(2)} \simeq 3.2 \times 10^8$ cm/s, $\sigma_{cx} \simeq 4.5 \times 10^{-15}$ cm^2, $\sigma_e = 3 \times 10^{-17}$ cm^2 [4]. We then have

$$\Delta_0^{(1)} \simeq 18.87 \text{ cm}, \qquad \Delta_0^{(2)} \simeq 107 \text{ cm}$$

$$\frac{\Delta_0^{(1)}\Delta_0^{(2)}}{\Delta_0^{(1)} + \Delta_0^{(2)}} \simeq 16.04 \text{ cm}$$

$$\frac{\Delta_0^{(2)}\Delta_0^{(1)}}{\Delta_0^{(2)} - \Delta_0^{(1)}} \simeq 22.9 \text{ cm}$$

$$\lambda_{cx}^{(1)} \simeq 36.6 \text{ cm} .$$

We can now calculate the reflectance and transmissivity for different integrated line densities in the halo. The results are plotted in Fig. 2. A comparison with a SPUDNUT calculation for the same parameters is given in Fig. 3.

4. Discussion

From Fig. 3, we see that the two energy group model calculations give a lower reflectance and a greater transmissivity. This is because the two energy group model has not taken into account the contribution to the reflectance due to charge exchange of higher generations for group (2) while traveling from x to h. We also assumed $\lambda_i^{(2)} >> \lambda_{cx}^{(2)}$ which means that electron impact ionization loss of group (2) atoms has been neglected. Thus the transmissivity is somewhat overestimated. The errors are not great, however. This model can give a simple estimate of neutral transport in a homogeneous halo plasma.

Acknowledgement

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Fig.2 Reflectance and Transmissivity versus Integrated Line Density

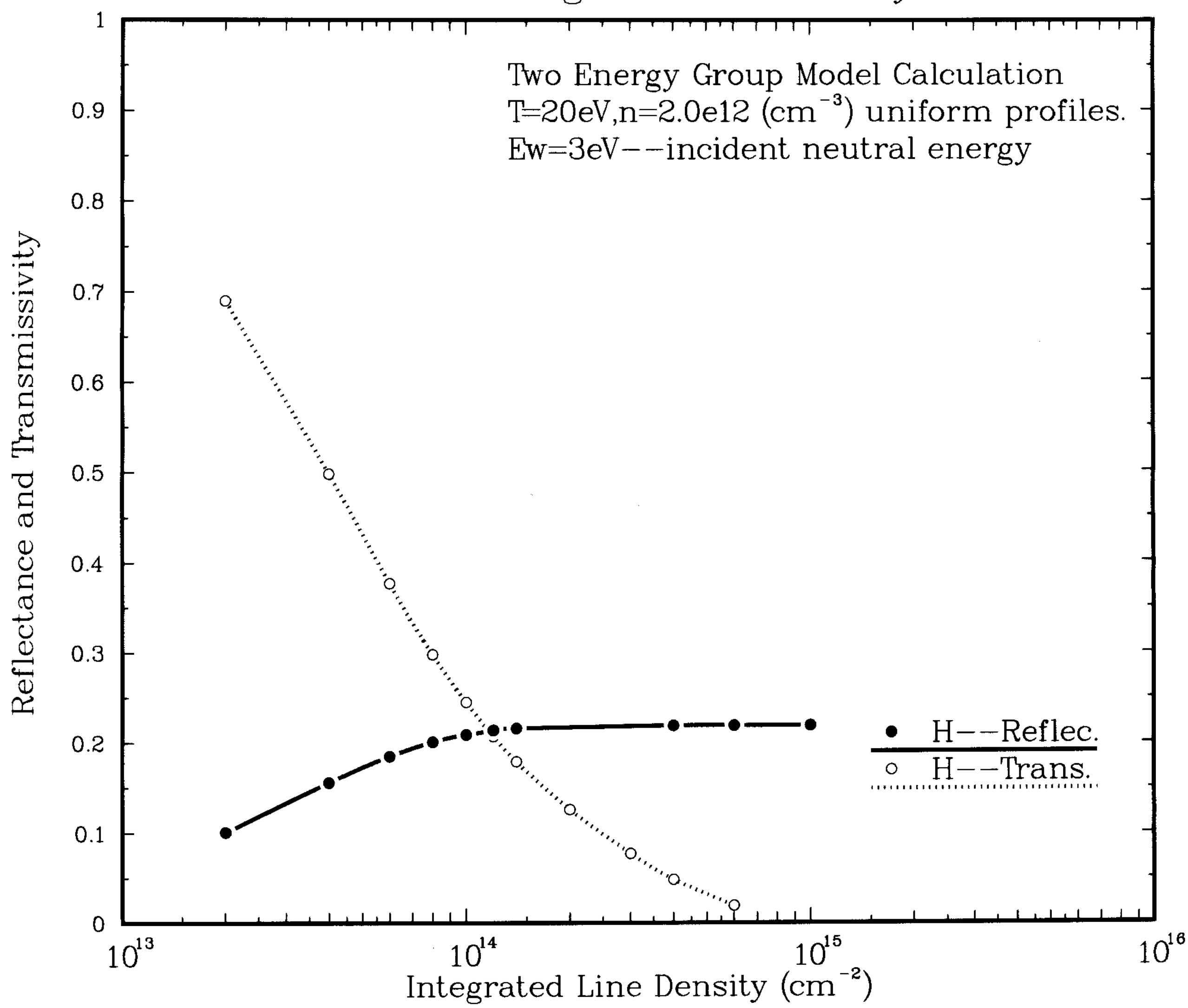
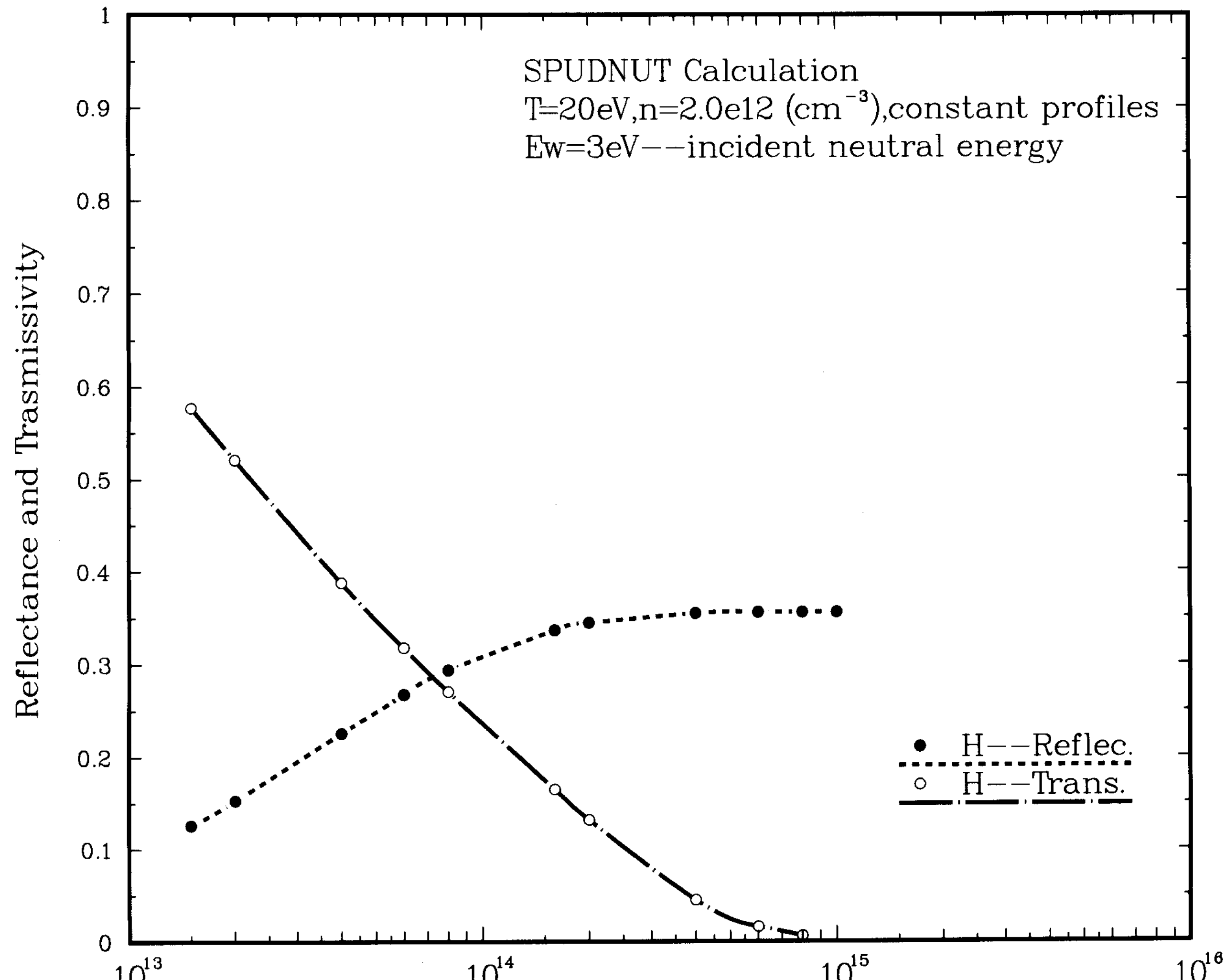


Fig.3 Reflectance and Trasmissivity versus Integrated Line Density



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