Stress Analysis of a Duplex Structure in a High Heat Flux Fusion Environment

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STRESS ANALYSIS OF A DUPLEX STRUCTURE

IN A HIGH HEAT FLUX FUSION ENVIRONMENT

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ABSTRACT

High heat flux components in fusion reactors will experience inelastic strains resulting from swelling, creep, and thermal expansion. Additionally, because of thermal and irradiation creep, the stresses will redistribute during the lifetime of the component. Current proposals for fusion first walls and divertors include structures fabricated by bonding two different metals together. The plasma side material is chosen to minimize sputtering; the coolant side material is chosen to maximize heat transfer. The structural response of such a design is not well known. Accordingly, a one dimensional inelastic stress analysis of a thin walled shell element has been performed. The stress analysis can include temperature dependent material properties, radiation induced swelling, thermal and irradiation creep, and thermal expansion. Furthermore, a simple equation has been derived for the case of a duplex plate constrained from bending. The stress distribution through the plate is followed with time. It is shown that the initial stress distribution evolves with time until some near steady state distribution is approached. The evolution is dependent on swelling and particularly on creep.

INTRODUCTION

Current proposals for high heat flux fusion components include duplex thin walled structures. The advantage of such a structure is that the plasma side material can be chosen to minimize sputtering and the coolant side material can be chosen to maximize heat transfer. The structure may be subjected to high temperatures, high temperature gradients, thermal and irradiation creep, swelling, and thermal expansion. While extensive stress analysis has been carried out for high heat flux structures[1] made of single materials such as 316 stainless steel, stress analysis for duplex structures is only just beginning. One exception has been the work of Mattas.[2] Mattas has analyzed structures with the plasma side material being beryllium or graphite and the coolant side material being 316 stainless steel or V-15Cr-5Ti.

This paper presents the development of the inelastic stress equations for a thin shell element with arbitrary material properties, and subsequently specializes it for a plate constrained from bending. As part of the development, the temperature distribution, swelling, and creep models are discussed. Finally, the results of the model are presented for a beryllium/copper plate.

INELASTIC STRESS EQUATION

Consider a thin plate constrained from bending but free to expand with no external forces. The well known stress equation for this case is

$$\sigma_x(1 - \nu^2) = \frac{E}{h} \int (e_x + \nu e_y)\, dz - E(e_x + \nu e_y) \qquad (1)$$

where σ_x is the stress in the x-direction, E is Young's Modulus, h is the plate thickness, ν is Poisson's ratio and e_x is the strain in the x-direction. One of the simplifying assumptions made in deriving Eq. (1) is that the material properties are constant through the material. What follows is a general analysis of the stress equation when material properties are not assumed constant.

Consider a shell element as shown in Fig. 1. A shell is defined as a thin curved surface whose thickness is much less than its other two dimensions. In the limit of no curvature a shell becomes a thin plate. For a given shell element assume that the membrane forces (N_i and N_j) and the bending moments (M_i and M_j) are specified. The problem then becomes how to determine the principal stresses (σ_i and σ_j) given the membrane forces and bending moments.

Hooke's Law for elastic material is given by

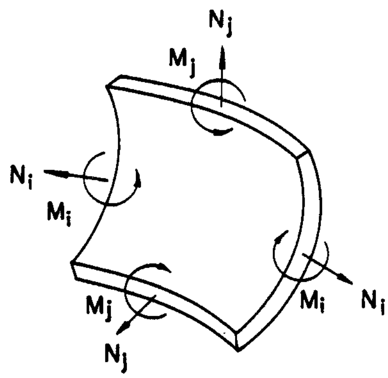

Fig. 1. A shell element with the membrane forces (N_i and N_j) and bending moments (M_i and M_j).

$$\sigma_i = \frac{E}{(1 - \nu^2)} (e_i^e + \nu e_j^e) \quad (2)$$

where e^e is the elastic strain. The subscripts i and j can be interchanged to obtain the principal stresses in the two directions. Assuming state of plane stress and no shearing strains, σ_i and σ_j are the only stresses present. Introduce next the inelastic strains by assuming that the total strain is a superposition of elastic and inelastic strains; that is,

$$e_i^t = e_i^e + e_i^i . \quad (3)$$

Substituting Eq. (3) into Eq. (2), one obtains

$$\sigma_i = \frac{E}{(1 - \nu^2)} (e_i^t - e_i^i) + \frac{E}{(1 - \nu^2)} (e_j^t - e_j^i) . \quad (4)$$

The next step is to choose an appropriate strain field which is compatible with the assumed deformations. Consider the shell element with only N_i and N_j acting on it. The strain field would not be constant through the thickness as in the plate element. Rather, the strain field would vary linearly as

$$e_i^t = K_i(1 - \frac{z}{r_j}) . \quad (5)$$

Similarly in pure bending the strain field would vary quadratically[3] as

$$e_i^t = L_i(1 - \frac{z}{r_j})z \quad (6)$$

where K_i and L_i are constants, r_j is the radius of curvature in the j-direction and z is the distance from the shell midplane. Then substituting Eqs. (5) and (6) into Eq. (4) results in

$$\sigma_i = \frac{E}{(1 - \nu^2)} [K_i(1 - \frac{z}{r_j}) + L_i(1 - \frac{z}{r_j})z - e_i^i] + \frac{E}{(1 - \nu^2)} [K_j(1 - \frac{z}{r_i}) + L_j(1 - \frac{z}{r_i})z - e_j^i] . \quad (7)$$

There are now two such equations (one for σ_i, Eq. (7), and a corresponding equation for σ_j) but with a total of four unknowns. Boundary conditions must now be imposed to assure self-consistency between applied loads and the stress distribution. These conditions are

$$N_i = \int \sigma_i(1 - \frac{z}{r_j})\, dz , \quad (8)$$

$$M_i = \int \sigma_i(1 - \frac{z}{r_j})z\, dz . \quad (9)$$

Substituting Eq. (7) into Eqs. (8) and (9) two of the four simultaneous equations in the constants K and L are

$$N_i = \int \frac{E}{(1 - \nu^2)} [K_i(1 - \frac{z}{r_j}) + L_i(1 - \frac{z}{r_j})z - e_i^i]\, dz + \int \frac{E}{(1 - \nu^2)} [K_j(1 - \frac{z}{r_i}) + L_j(1 - \frac{z}{r_i})z - e_j^i]\, dz \quad (10)$$

$$M_i = \int \frac{E}{(1 - \nu^2)} [K_i(1 - \frac{z}{r_j}) + L_i(1 - \frac{z}{r_j})z - e_i^i]z\, dz + \int \frac{E}{(1 - \nu^2)} [K_j(1 - \frac{z}{r_i}) + L_j(1 - \frac{z}{r_i})z - e_j^i]z\, dz . \quad (11)$$

In order to simplify the solving of the four equations (Eqs. (10) and (11) and their corresponding j-direction equations) we introduce the following four quantities where n = 0, 1, 2, or 3:

$$\int \frac{E}{(1 - \nu^2)} z^n\, dz = T_n \quad (12)$$

$$\int \frac{E\nu}{(1-\nu^2)} z^n \, dz = \Gamma_n \tag{13}$$

$$\int \frac{E}{(1-\nu^2)} e_i^i z^n \, dz = \phi_n \tag{14}$$

$$\int \frac{E\nu}{(1-\nu^2)} e_i^i z^n \, dz = \zeta_n \, . \tag{15}$$

Now rewriting all four simultaneous equations with Eqs. (12)-(15) substituted and the individual terms separated, one obtains

$$[T_0 - \frac{T_1}{r_j}]K_i + [\Gamma_0 - \frac{\Gamma_1}{r_i}]K_j + [T_1 - \frac{T_2}{r_j}]L_i + [\Gamma_1 - \frac{\Gamma_2}{r_i}]L_j = N_i + \phi_0^i + \zeta_0^j \tag{16}$$

$$[\Gamma_0 - \frac{\Gamma_1}{r_j}]K_i + [T_0 - \frac{T_1}{r_i}]K_j + [\Gamma_1 - \frac{\Gamma_2}{r_j}]L_i + [T_1 - \frac{T_2}{r_i}]L_j = N_j + \phi_0^j + \zeta_0^i \tag{17}$$

$$[T_1 - \frac{T_2}{r_j}]K_i + [\Gamma_1 - \frac{\Gamma_2}{r_i}]K_j + [T_2 - \frac{T_3}{r_j}]L_i + [\Gamma_2 - \frac{\Gamma_3}{r_i}]L_j = M_i + \phi_1^i + \zeta_1^j \tag{18}$$

$$[\Gamma_1 - \frac{\Gamma_2}{r_j}]K_i + [T_1 - \frac{T_2}{r_i}]K_j + [\Gamma_2 - \frac{\Gamma_3}{r_j}]L_i + [T_2 - \frac{T_3}{r_i}]L_j = M_j + \phi_1^j + \zeta_1^i \, . \tag{19}$$

Let the equations be written as

$$A_jK_i + B_iK_j + C_jL_i + D_iL_j = G_i \tag{20}$$

$$B_jK_i + A_iK_j + D_jL_i + C_iL_j = G_j \tag{21}$$

$$C_jK_i + D_iK_j + E_jL_i + F_iL_j = H_i \tag{22}$$

$$D_jK_i + C_iK_j + F_jL_i + E_iL_j = H_j \, , \tag{23}$$

where the coefficients A-H are defined by correspondence to Eqs. (16)-(19). To solve the system of four simultaneous equations by Cramer's Rule we first define a set of determinants to be

$$X_i = \det\begin{vmatrix} G_i & B_i & C_j & D_i \\ G_j & A_i & D_j & C_i \\ H_i & D_i & E_j & F_i \\ H_j & C_i & F_j & E_i \end{vmatrix} \qquad X_j = \det\begin{vmatrix} A_j & G_i & C_j & D_i \\ B_j & G_j & D_j & C_i \\ C_j & H_i & E_j & F_i \\ D_j & H_j & F_j & E_i \end{vmatrix}$$

$$Y_i = \det\begin{vmatrix} A_j & B_i & G_i & D_i \\ B_j & A_i & G_j & C_i \\ C_j & D_i & H_i & F_i \\ D_j & C_i & H_j & E_i \end{vmatrix} \qquad Y_j = \det\begin{vmatrix} A_j & B_i & C_j & G_i \\ B_j & A_i & D_j & G_j \\ C_j & D_i & E_j & H_i \\ D_j & C_i & F_j & H_j \end{vmatrix}$$

$$W = \det\begin{vmatrix} A_j & B_i & C_j & D_i \\ B_j & A_i & D_j & C_i \\ C_j & D_i & E_j & F_i \\ D_j & C_i & F_j & E_i \end{vmatrix} \, .$$

And according to Cramer's Rule

$$K_i = X_i/W \qquad L_i = Y_i/W$$

$$K_j = X_j/W \qquad L_j = Y_j/W \, .$$

The resulting equation for the principal stress becomes

$$\sigma_i = \frac{E}{(1-\nu^2)} [\frac{X_i}{W}(1 - \frac{z}{r_j}) + \frac{Y_i}{W}(1 - \frac{z}{r_j})z - e_i^i] + \frac{E\nu}{(1-\nu^2)} [\frac{X_j}{W}(1 - \frac{z}{r_i}) + \frac{Y_j}{W}(1 - \frac{z}{r_i})z - e_j^i] \, . \tag{24}$$

Equation (24) can be simplified for a plate constrained from bending with no membrane forces. In this case, the stress equation reduces to

$$\sigma_i(1 - \nu^2) = \frac{E}{W}[X_i + \nu X_j] - E[e_i^i + \nu e_j^i] \, . \tag{25}$$

Furthermore, the determinants W and X are simplified and become

$$X_i = \det\begin{vmatrix} G_i & \Gamma_0 \\ G_j & T_0 \end{vmatrix} \qquad X_j = \det\begin{vmatrix} T_0 & G_i \\ \Gamma_0 & G_j \end{vmatrix}$$

$$W = \det\begin{vmatrix} T_0 & \Gamma_0 \\ \Gamma_0 & T_0 \end{vmatrix} \, .$$

It can be shown that in the limit of homogeneous material properties for E and ν Eq. (25) reduces to Eq. (1).

INELASTIC STRAINS

We have considered inelastic strains arising from three sources: (1) thermal strains, (2) radiation induced swelling strains, and (3) thermal and radiation creep strains. Inserting the total strain

$$e_i^i = \alpha\Delta T + \frac{1}{3} S + e_i^c$$

into Eqs. (14) and (15), they become

$$\phi_0^i = \int \frac{E}{(1 - \nu^2)} (\alpha\Delta T + \frac{1}{3} S + e_i^c)\, dz$$

$$\zeta_0^j = \int \frac{E\nu}{(1 - \nu^2)} (\alpha\Delta T + \frac{1}{3} S + e_i^c)\, dz \ .$$

The quantity G_i defined previously as $G_i = N_i + \phi_0^i + \zeta_0^j$ becomes ($N_i = 0$, no membrane forces)

$$G_i = \int \frac{E}{(1 - \nu^2)} (1 + \nu)(\alpha\Delta T + \frac{1}{3} S)\, dz + \int \frac{E}{(1 - \nu^2)} (e_i^c + \nu e_j^c)\, dz \ .$$

As a consequence of $\sigma_i = \sigma_j$, $e_i^c = e_j^c$. While the assumption of a one-to-one stress ratio is not necessary to the final development of the plate stress equations, it does greatly minimize the number of terms in the final result without detracting from the physical problem. G_i now becomes

$$G_i = G_j = G = \int \frac{E}{(1 - \nu)} (\alpha\Delta T + \frac{1}{3} S + e^c) dz \qquad (26)$$

and at the same time $X_i = X_j = X$. The stress Eq. (25) is now

$$\sigma_i(1 - \nu^2) = \frac{E}{W} (1 + \nu)X - E(1 + \nu)(\alpha\Delta T + \frac{1}{3} S + e^c)$$

or $$\sigma_i = \frac{E}{(1 - \nu)} \frac{X}{W} - \frac{E}{(1 - \nu)} (\alpha\Delta T + \frac{1}{3} S + e^c) \qquad (27)$$

where

$$\frac{X}{W} = \frac{\int \frac{E}{(1 - \nu)} (\alpha\Delta T + \frac{1}{3} S + e^c)\, dz}{\int \frac{E}{(1 - \nu)}\, dz} \ . \qquad (28)$$

Equations (26) and (28) represent the simplest case to study the stress evolution in a high heat flux component with spatially varying materials properties. Its simplicity makes it suitable for extensive scoping studies of composite structures for high heat flux applications. In the following, we investigate the time dependent profile in a duplex plate constrained from bending. This plate is assumed to consist of beryllium bonded to copper.

MATERIAL PROPERTIES

Insufficient experimental evidence exists for copper to derive an empirical relationship between swelling and the irradiation parameters. In previous work[4] we have developed a rate theory model for void swelling growth. This model predicts long term swelling rates for 316 stainless steel which agree quite well with abundant experimental evidence. Using the same model but with material properties of copper, we have calculated theoretical swelling rates of copper as a function of temperature and neutron damage rate. The model predicts a maximum swelling rate of approximately 2%/dpa for copper in a rather narrow temperature range of 200°C to 450°C. Outside of this range little or no swelling occurs. To predict the swelling strains of the plasma side material beryllium, a recent model developed by Wolfer and McCarville[5] is used. Basically, the model assumes that swelling in beryllium takes place by two processes: (1) solid helium swelling where the helium precipitate grows by athermal emission of host metal atoms or loop punching, and (2) gaseous helium bubble swelling where growth occurs by the thermal absorption of vacancies. The latter process dominates at high temperatures and high helium concentrations. The helium production rate is assumed to be 14000 appm He/year; this quantity is derived from an assumed neutron wall loading of 4 MW/m^2.

For the inelastic strains associated with thermal creep, power law creep was assumed. The form of the power law creep equation is

$$\dot{e}^{th} = A\sigma^n e^{-B/T} \qquad (29)$$

where for beryllium[6] $n = 3$, $A = 3.3 \times 10^{-12}$ per second, $B = 3.8 \times 10^4$ per °K. For copper $n = 4.8$, $A = 3.37 \times 10^{-31}$, and $B = 1.41 \times 10^4$. The constants for copper were found by fitting Eq. (29) to the data by Frost et al.[7] Irradiation creep strains are less well known than thermal creep strains. Nonetheless, some estimates can be made. Ghoniem states[6] that at a temperature of half the melting point there is a stress at which irradiation creep equals thermal creep. Ghoniem assumed that for beryllium this stress is 1.4×10^7 N/m^2 (~ 2 ksi). The creep compliance is calculated to be $\psi = 4.79 \times 10^{-19}$ m^2/N/s. The irradiation creep rate for beryllium is then

$$\dot{e}^{irr} = \frac{1}{2} \psi\sigma \; . \qquad (30)$$

The irradiation creep rate for copper can be estimated by considering the stress induced preferential absorption (SIPA) of interstitials at dislocations[8] due to irradiation. Using rate theory, we have estimated a steady state creep compliance of $\psi = 5.35 \times 10^{-18}$ $m^2/N/s$. The irradiation creep rate can be found using Eq. (30).

DISCUSSION

The duplex structure we have examined consists of a layer of beryllium bonded to a layer of copper as shown in Fig. 2. In the one dimensional temperature analysis we have assumed that the bonding between the layers is ideal. The calculation of the temperature distribution assumed that the thermal conductivity k has the form $k = 1/(A + BT)$. Then using the conductivity integral approach[9] the temperature distribution can be calculated. The calculated temperature distribution is shown in Fig. 3. The backside temperature is 90°C and the surface heat flux and the neutron wall loading is 4 MW/m^2. By separating Eq. (28) into two integrals, one through the beryllium and one through the copper, we are able to find the average strain through the duplex structure. Then with Eq. (27) we can find the initial stress distribution through the structure as shown in Fig. 4. One can note the stress discontinuity at the interface between the two materials; this is due to the differences in properties between the materials.

To follow the time evolution of the stress distribution, we have followed the creep and swelling inelastic strains of each material by numerically integrating their respective strain rates with each time increment. For the copper material we have found that the stresses relax very quickly to a near steady state value of about 40 MPa. The steady state value of the stresses can be found analytically be taking the time derivative of Eq. (27) and setting it equal to zero. For the temperature calculated for the example case, there is no swelling in the copper. It should be noted that the "steady state" stresses in copper are a function of the stresses in the beryllium.

The stress evolution in the beryllium is slightly more complicated. The evolution of the stress in beryllium is very dependent on the amount of helium assumed. Early in life before a significant amount of helium induced swelling occurs, the beryllium stresses relax. From beginning-of-life until about 1 year, the stress on the plasma surface relaxes from a value of ~ 240 MPa to ~ 90 MPa. After about 1 year, the compressive stress increases because of the onset of swelling. At about 20 years the stress distribution shown in Fig. 5 is obtained.

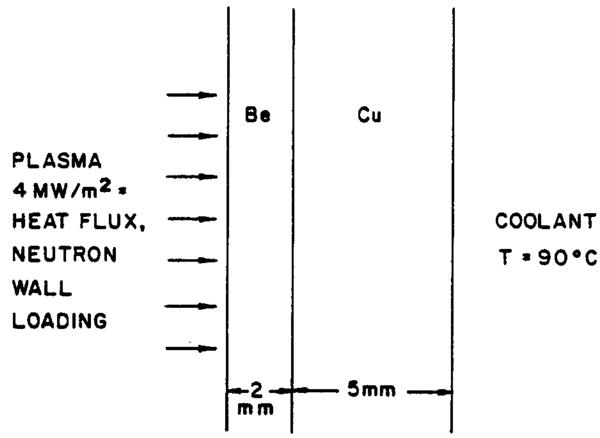

Fig. 2. Illustration of duplex wall structure.

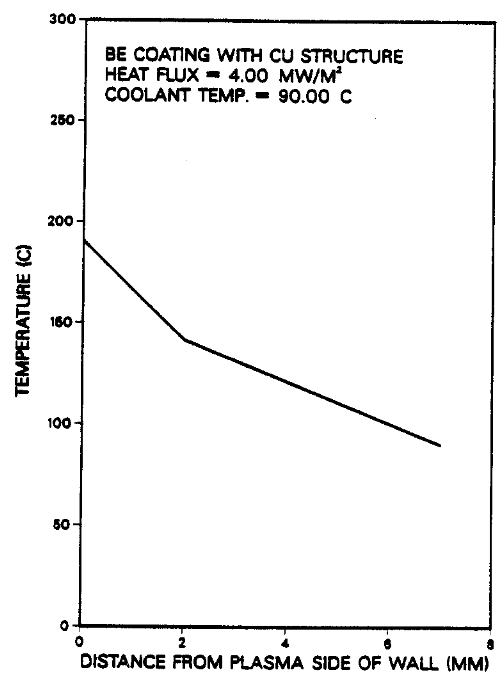

Fig. 3. Temperature distribution through the duplex structure.

CONCLUSION

The stress equation for thin walled shell elements as shown in Fig. 1 has been developed and is given in Eq. (25). A specialization of Eq. (25) has also been developed for a thin

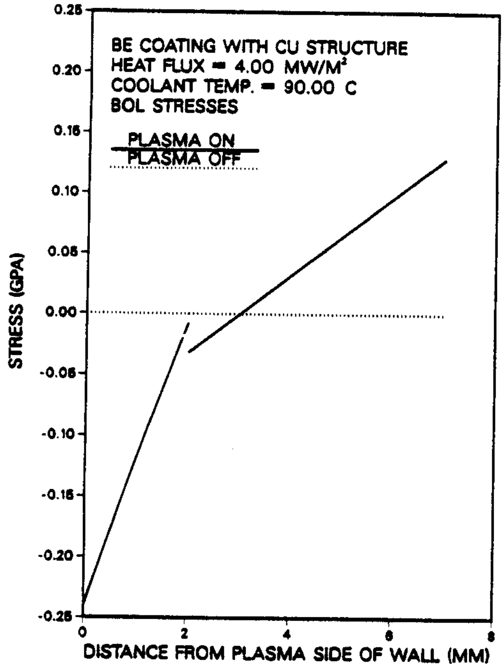

Fig. 4. Initial stress distribution.

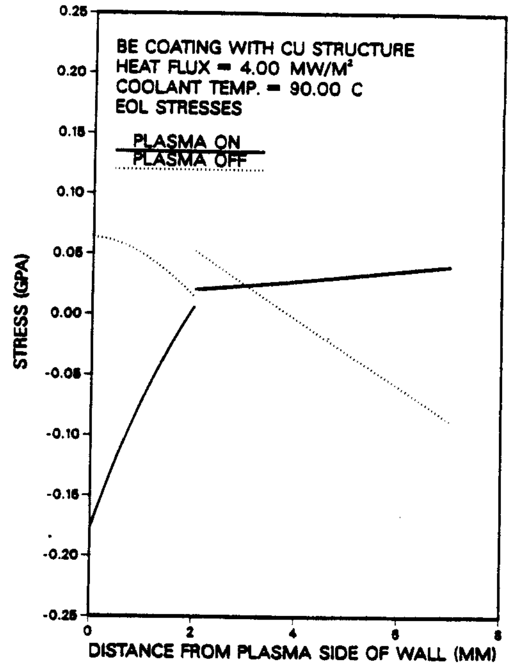

Fig. 5. Stress distribution after about 20 years.

plate constrained from bending with no membrane loads, Eq. (26). If one then considers the inelastic strains to arise from thermal, creep, and swelling strains, Eq. (27) can be derived. Equation (27) assumes isotropic material properties and strain rates. Given Eq. (27) along with appropriate strain models for beryllium and copper, the stress distribution through a duplex wall exposed to a plasma environment (Fig. 2) is calculated. We have found that the initial stress distribution (Fig. 4) quickly evolves with time due to creep relaxation and swelling. After a period of about 20 years, a significantly different stress distribution exists.

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REFERENCES

1. R.D. WATSON, R.R. PETERSON and W.G. WOLFER, "The Effect of Irradiation Creep, Swelling, Wall Erosion, and Embrittlement of the Fatigue Life of a Tokamak First Wall," Journal of Nuclear Materials, 103, 97 (1981).

2. R.F. MATTAS, "Fusion Component Lifetime Analysis," ANL/FPP/TM-160 (1982).

3. S. MIRCEA, Applications of Finite Difference Equations to Shell Elements, Pergamon Press (1967).

4. B.B. GLASGOW and W.G. WOLFER, Modeling of Void Swelling in Irradiated Steels, ASTM-STP-870, to be published.

5. W.G. WOLFER and T.J. MCCARVILLE, "An Assessment of Radiation Effects in Beryllium," Sixth Top. Mtg. on the Tech. of Fusion Energy, San Francisco, 3-7 March 1985.

6. N. GHONIEM, private communication.

7. H.J. FROST and M.F. ASHBY, Deformation-Mechanism Maps, Pergamon Press (1982).

8. W.G. WOLFER, "Correlation of Radiation Creep Theory with Experimental Evidence," Journal of Nuclear Materials, 90, 175-192 (1980).

9. D.O. OLANDER, Fundamental Aspects of Nuclear Reactor Fuel Elements, TID-26711-P1 (1976).