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INTRODUCTION

Since the reaction chamber of the Target Development Facility will be submerged in water, it is necessary to determine whether mechanical instability of the structure could develop. Two methods are used for this purpose: the ASCE/Structural Stability Research Council[1] theoretical buckling criteria and the design-based criteria of the ASME Boiler and Pressure Vessel Code.[2]

In general, a cylindrical shell's buckling resistance to external pressure depends upon its material properties as well as geometric characteristics, particularly the ratios of diameter to thickness (D/t) and length to diameter (L/D). Circumferential stiffeners can substantially reduce the effective length of the shell. Such a stiffened shell may buckle statically in one of three possible modes, as shown schematically in Fig. 1.

(i) The shell section between rings buckles into an axisymmetric "accordion pleat" shape, characterized by considerable plastic strain.

(ii) Overall collapse of the structure may occur in which the shell and reinforcing rings buckle together. Large dished-in areas extend over the length of the cylinder.

(iii) The shell section between rings may buckle into an asymmetric or lobar shape, characterized by a number of indentations or waves, circumferentially.

For the range of dimensions considered for the reaction chamber, the shell can be categorized as relatively thin and flexible with stiff reinforcing rings. Thus the determination of buckling pressure is based upon the last of the preceding three categories. In addition, it should be noted that the axial (vertical) pressure gradient is conservatively replaced by a uniform

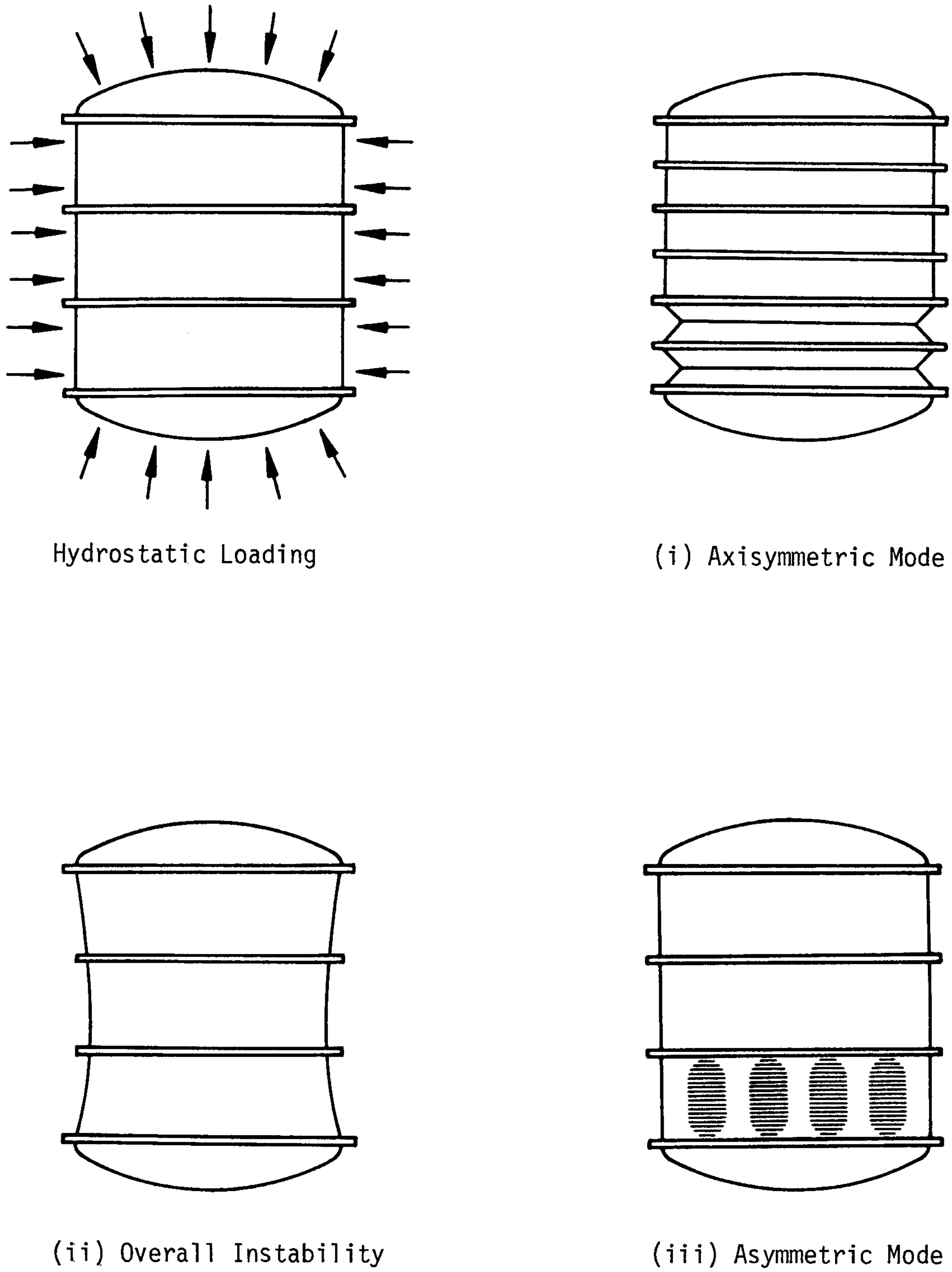

Fig. 1. Buckling Modes for a Cylindrical Shell Under Hydrostatic Pressure.

pressure over both ends and the lateral surface, with a magnitude corresponding to the mean value acting on the lowest shell section of the reaction chamber.

ASCE/SSRC CRITERION

The theoretical critical pressure for elastic buckling of stiffened shells between reinforcing rings is a function of the cylinder's length, diameter and thickness, Poisson's ratio and the elastic modulus. It does not depend upon the tangent or secant modulus or the material's yield strength. Formulas for both lateral and hydrostatic (uniform, all around) critical pressures are based upon theory initiated by von Mises.[1] For closely spaced reinforcing rings, the von Mises theory is not exact. However, it is a generally recommended procedure for determining elastic buckling pressures for ring-reinforced cylindrical shells on the basis of "predictive accuracy and conservatism." The critical buckling pressure for the uniform (hydrostatic) case which can be used for all lengths of cylinders is expressed as

$$p_c = 2E\left(\frac{t}{D}\right)\left\{\left(\frac{t}{D}\right)^2\left[\frac{(n^2+\lambda^2)^2 - 2n^2 + 1}{3(1-\nu^2)}\right] + \frac{\lambda^4}{(n^2+\lambda^2)^2}\right\}\Big/\left(n^2 + \frac{\lambda^2}{2} - 1\right) \quad (1)$$

where n is the number of circumferential waves (lobes) of the buckled shape, E and ν are the elastic modulus and Poisson's ratio, respectively, and λ is equal to $\pi D/2L$. The value of n in (1) is the integer which makes p_c a minimum. An approximation for this integer can be obtained from

$$n^4 = \frac{3\pi^2(1-\nu^2)^{1/2}}{4(L/D)^2(t/D)} \quad (2)$$

The shell geometry is usually represented by the parameter θ:

$$\theta = [12(1 - \nu^2)]^{0.25} \left(\frac{L}{D}\right)\left(\frac{D}{t}\right)^{0.50} . \tag{3}$$

For various ranges of θ, modified versions of Eq. (1) have been developed and are summarized in Table 1. These are approximations of (1), obtained by neglecting small terms, and have been suggested for design generally because they are more convenient to apply. Design curves such as Fig. 2 can be developed for critical pressure as a function of θ. In this figure, p* corresponds to the critical end pressure for a cylindrical shell loaded only in simple compression, i.e.,

$$p^* = \frac{8E \left(\frac{t}{D}\right)^2}{[3(1 - \nu^2)]^{1/2}} . \tag{4}$$

The buckling pressure is a single valued function of θ until θ approaches the magnitude of D/t, for which n = 2. This condition is beyond the range of dimensions considered for the reaction chamber. Available experimental data[3-5] exists for θ between 3 and 100 and agrees very well with predictions from these theoretical formulas. This interval does span the geometry range under consideration.

Numerical results are presented in Figs. 3 and 4 for Al 6061 and 2-1/4 Cr-1 Mo, respectively. For these curves the safety factor is unity. The higher buckling pressure for the steel is primarily attributable to the larger elastic modulus. Usually the mode shape of the buckled configuration has many circumferential lobes. For example if D/t = 200 and D/L = 3, the wave number is ten. From Fig. 3, the critical pressure for an aluminum chamber with this geometry is approximately 1000 kPa. The corresponding water depth is over 90 m, based upon a gradient of 9.80 kPa/m and an atmospheric contribution of 100

Table 1. Summary of Equations for Buckling of Cylindrical Shells by Hydrostatic Pressure ($\nu = 0.3$)

θ		L/D			
From	To	From	To	Equation	Critical Pressure (p_c)
0	0.8	0	$\frac{0.44}{\sqrt{D/t}}$	a	$\frac{3.615\ E}{(D/t)^3(L/D)^2}$
0.8	1.4	$\frac{0.44}{\sqrt{D/t}}$	$\frac{0.77}{\sqrt{D/t}}$	b	$\frac{3.615\ E}{(D/t)^2}\left[0.448\left(\frac{L}{D}\right)^2\left(\frac{D}{t}\right) + \frac{1}{(L/D)^2(D/t)}\right]$
1.4	2.0	$\frac{0.77}{\sqrt{D/t}}$	$\frac{1.1}{\sqrt{D/t}}$	c	$\frac{2.0\ E}{(D/t)(n^2 + 0.5\ \lambda^2 - 1)}\left[\frac{0.367}{(D/t)^2}\left[(n^2 + \lambda^2)^2 - 2n + 1\right] + \frac{\lambda^4}{(n^2 + \lambda^2)^2}\right]$
2.0	10.0	$\frac{1.1}{\sqrt{D/t}}$	$\frac{5.5}{\sqrt{D/t}}$	d	$\frac{2.6\ E}{(D/t)^{2.5}\left[(L/D) - 0.45(t/D)^{0.5}\right]}$
10.0	D/t	$\frac{5.5}{\sqrt{D/t}}$	$0.55\ \sqrt{D/t}$	e	$\frac{2.6\ E}{(L/D)(D/t)^{2.5}}$
D/t	4D/t	$0.55\ \sqrt{D/t}$	$2.1\ \sqrt{D/t}$	f	$\frac{2.0\ E}{(D/t)(3 + 0.5\ \lambda^2)}\left[\frac{0.367}{(D/t)^2}\left[(4 + \lambda^2)^2 - 7\right] + \frac{\lambda^4}{(4 + \lambda^2)^2}\right]$
4D/t	∞	$2.1\ \sqrt{D/t}$	∞	g	$\frac{2.2\ E}{(D/t)^3}$

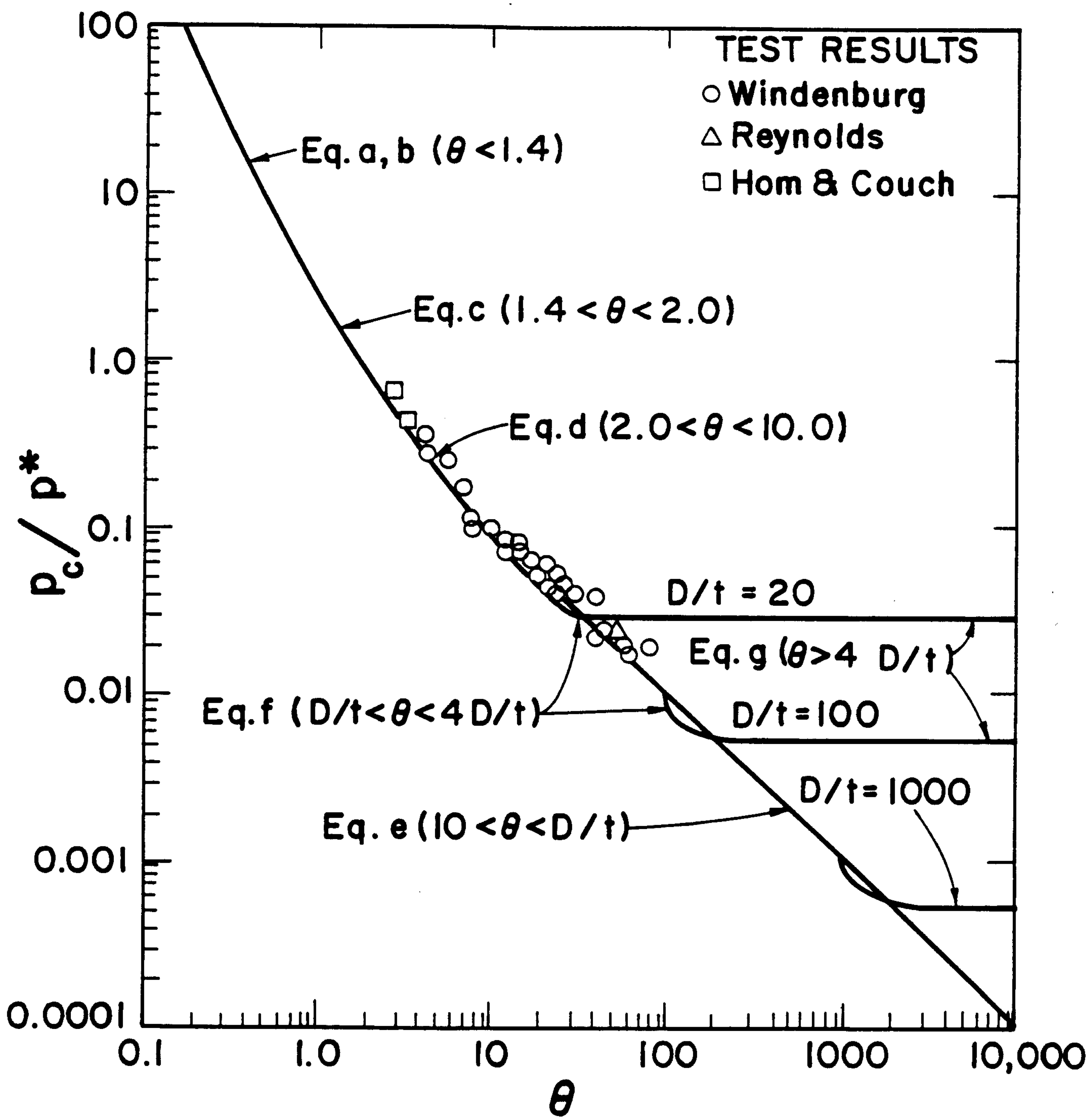

Fig. 2. Hydrostatic Buckling Pressure for Closed Cylindrical Shells (ν = 0.30).

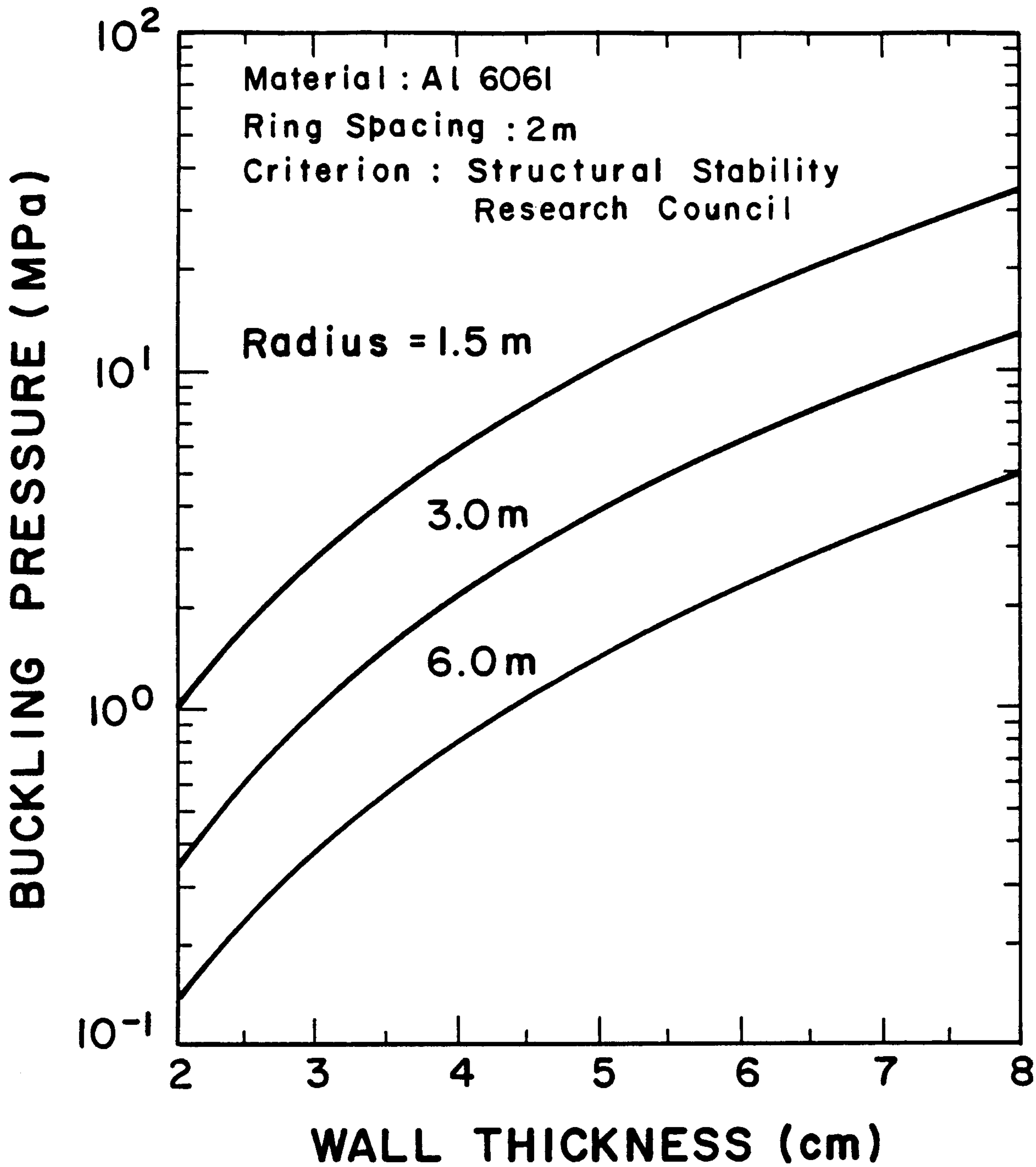

Fig. 3. Elastic Buckling of Aluminum TDF Cylindrical Shells.

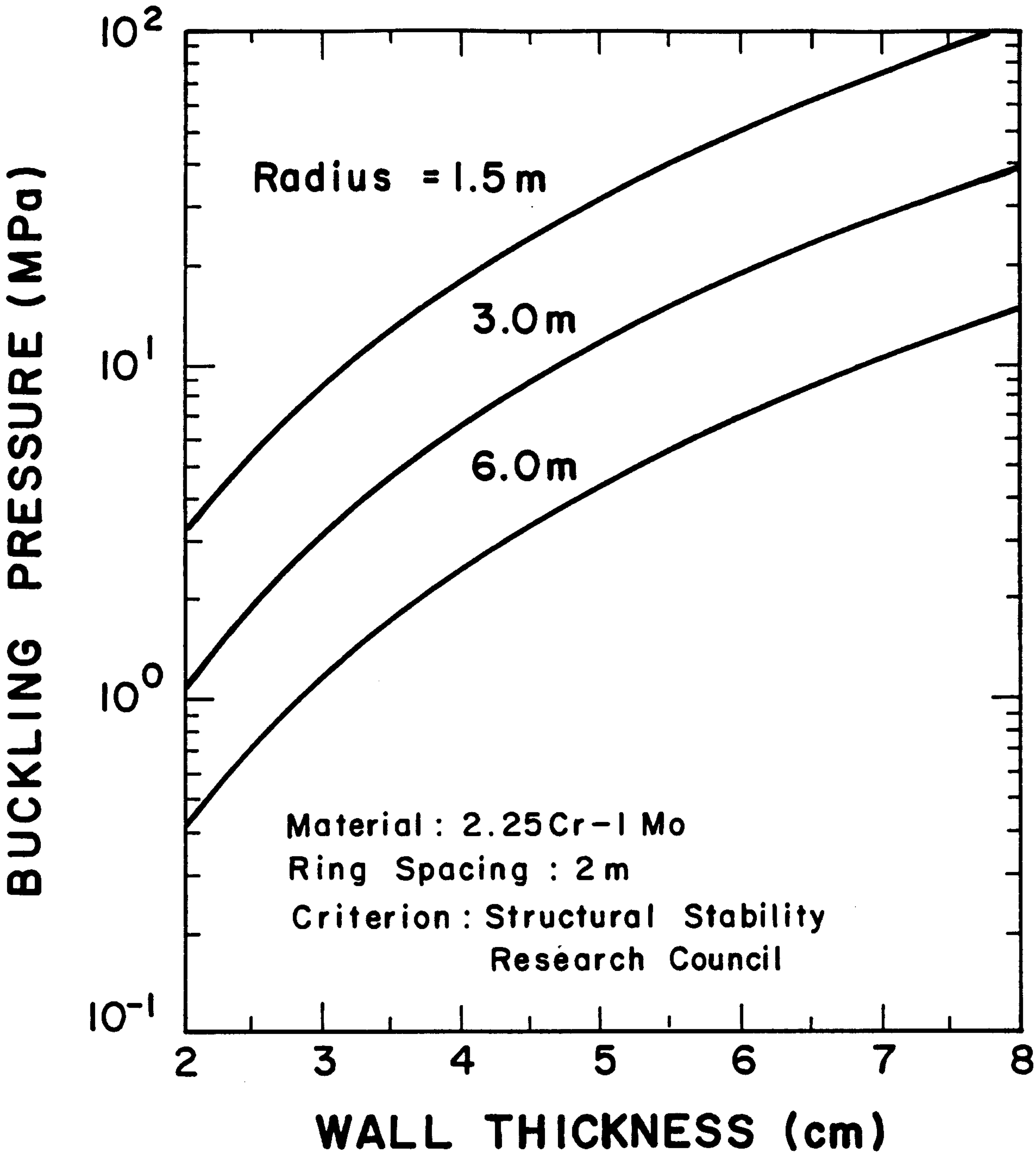

Fig. 4. Elastic Buckling of Steel TDF Cylindrical Shells.

kPa. Geometric imperfections would lower these buckling pressures. For example, if the maximum deviation from a true circular shape is 10% of the shell thickness, the reduction in the buckling pressure will be approximately 25%.

ASME DESIGN PROCEDURE

The criteria applied are for the ASME-BPV Code, Section VIII, Div. 1, Part UG-28(c). This essentially involves the use of two sets of semi-empirical graphs to facilitate the design of shells under external pressure. For discussion purposes, a typical set is shown in Fig. 5 (carbon steel, yield strength 30,000 to 38,000 psi, e.g., 2-1/4 Cr-1 Mo). The solution lines represent the reduced modulus (E_r) of the material, including temperature-dependence. The straight portions correspond to the elastic modulus while the upper curved branches show the effects of non-elastic response. Although it is not explicitly stated in the code, the following relations are implied:

$$\text{Factor A} = \sigma_i / E_r$$

$$\text{Factor B} = \sigma_i / 2$$

where σ_i is stress intensity based upon distortion energy,

$$\sigma_i^2 = \sigma_\phi^2 + \sigma_x^2 - \sigma_\phi \sigma_x + 3\tau^2 \; .$$

Here σ_ϕ and σ_x are the circumferential and axial normal stresses; τ is the shear stress (zero for axisymmetric problems). The design steps are briefly summarized.

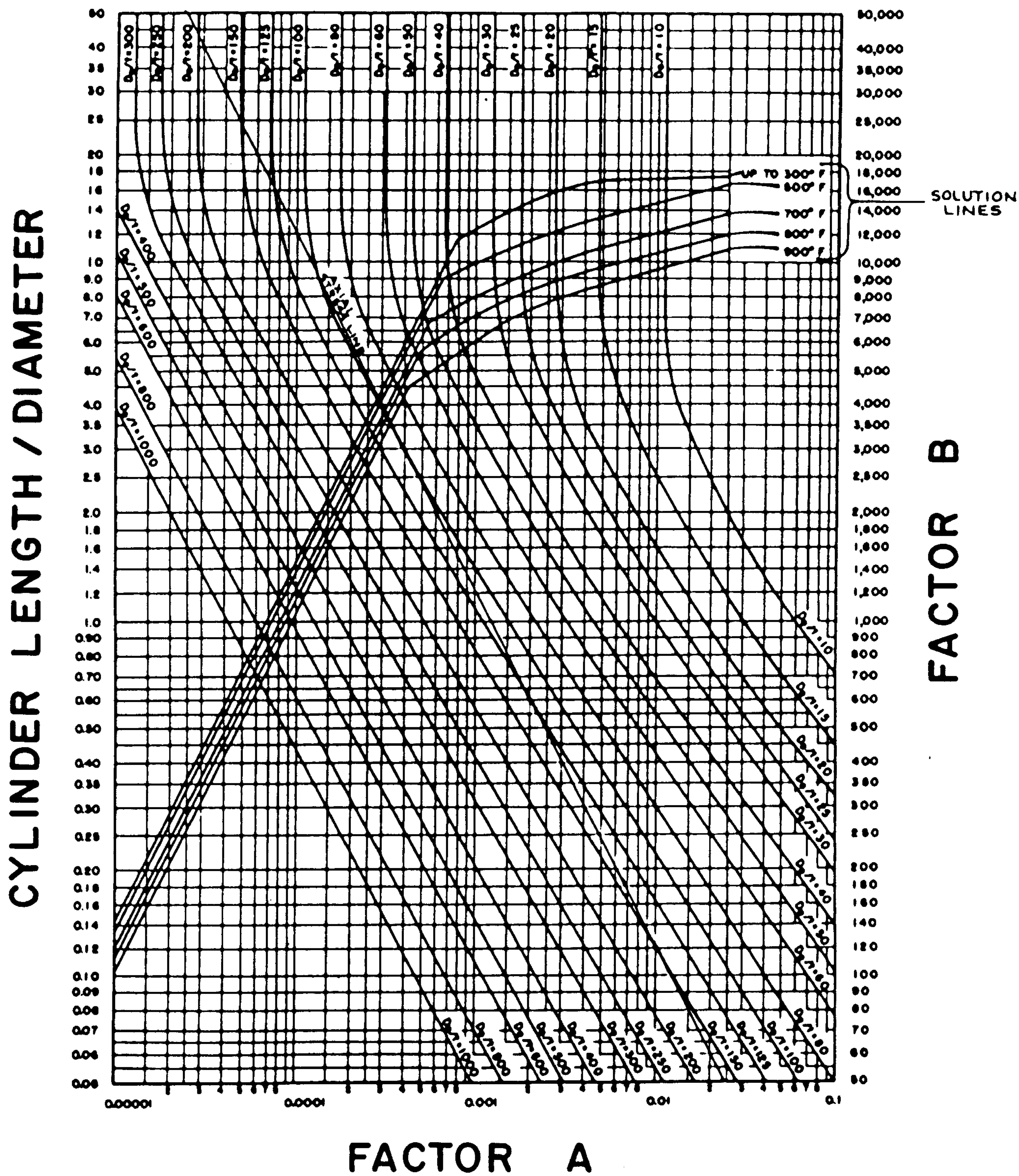

Fig. 5. ASME Design Curves for Cylindrical Shells Under Hydrostatic Pressure.

(1) Assume a thickness value (t) and determine length to diameter and diameter to thickness ratios (L/D; D/t).

(2) Determine the intersection of a constant L/D (horizontal) with constant D/t (inclined). This intersection corresponds to a particular value of Factor A.

(3) The constant A line from (2) intersects the solution lines for a particular material and temperature.

(4) From the intersection in (3), identify Factor B.

(5) Calculate the maximum allowable external pressure p_a:

$$p_a = 4B/3(D/t) \ .$$

In a specific design problem, one would compare p_a with the working pressure on the shell. If p_a is less than this value, the procedure would be repeated with increased thickness or added stiffeners to decrease the effective length.

Results based upon the ASME code are shown in Figs. 6 and 7. The general features are similar to those from the SSRC criteria, the major difference being the built-in safety factor of ASME. For comparison, an aluminum chamber with a thickness and radius of 3 and 300 cm, respectively, has an allowable external pressure of 280 kPa from Fig. 6. This would correspond to a depth of 18 m of water for an evacuated chamber. The similar result from the SSRC criteria was 90 m.

CONSIDERATIONS FOR EXTERNALLY PRESSURIZED HEADS

Among the possible geometric shapes for formed heads applicable to the TDF chamber design, the most common are spherical, ellipsoidal and torispherical. The torispherical cap has the most compact shape, is least expensive to

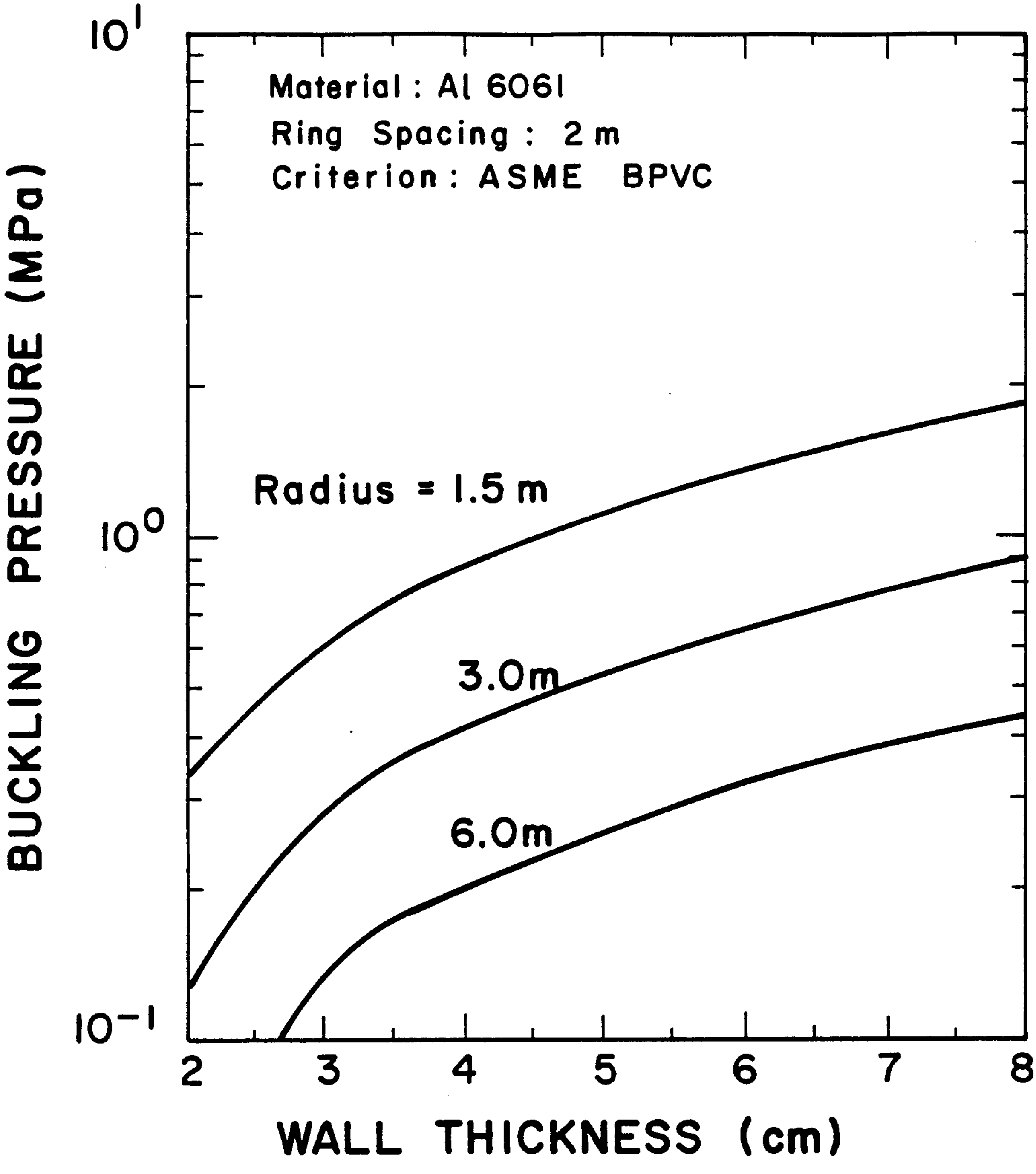

Fig. 6. Allowable External Pressure for Aluminum TDF Cylindrical Shells.

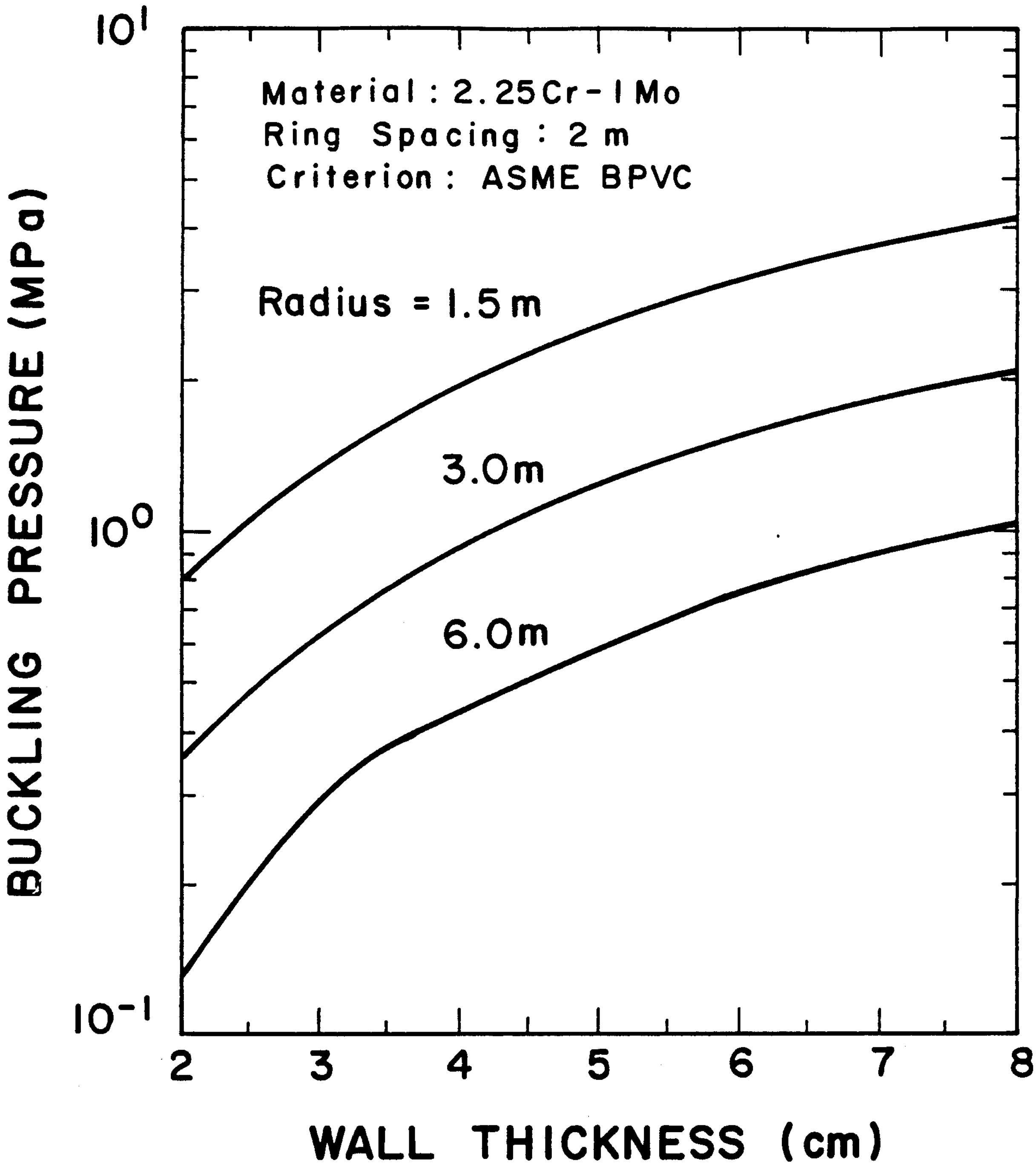

Fig. 7. Allowable External Pressure for Steel TDF Cylindrical Shells.

manufacture but also has the lowest resistance to external pressure. Using this as a worst case, critical pressure-shell thickness relations have been determined according to the ASME-BPV Code, Section VIII, Div. 1, Part UG-33(e). The code specifies that the crown radius cannot be greater than the base diameter, which corresponds to the cylindrical shell for the TDF vessel. In addition, the knuckle radius cannot be less than 6% of the cylinder diameter. (The profiles of Fig. 1 correspond to such geometric ratios.)

Results are presented in Fig. 8 for torispherical heads of 6061 aluminum (A) and 2-1/4 Cr-1 Mo steel (S) for various base radii. It should be emphasized that the critical pressure has a safety factor built into it and thus should be regarded as a design pressure. The corresponding water depth is also included, accounting for the fact that the TDF chamber would be evacuated.

CONCLUSIONS

Within the context of a scoping study, general design limits have been considered for external pressurization of the TDF reaction chamber by the water shield. For the cylindrical shell, both the ASME Boiler and Pressure Vessel Code and the Structural Stability Research Council criteria were applied. The ASME code was also used for evaluations of torispherical heads.

In general, it was found that the critical pressures for steel components were more than three times larger than aluminum. For the same thickness and material, the critical pressure for the head is lower than the corresponding cylindrical shell. However, the results indicate that for the range of dimensions being considered for the TDF, a substantial margin of safety exists for these components at practical shield water depths.

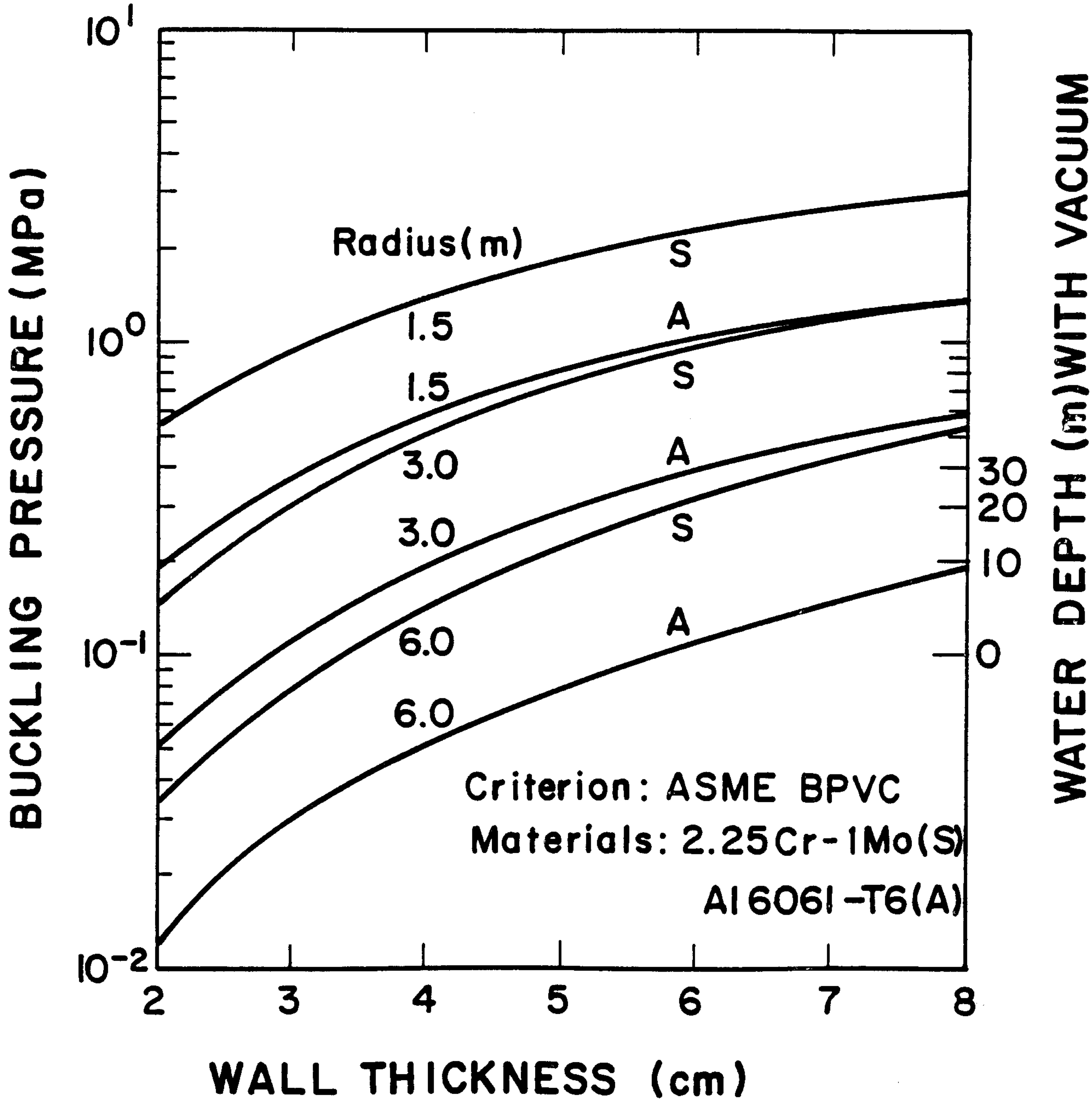

Fig. 8. Allowable External Pressure for ASME Torispherical End Caps.

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Acknowledgement

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