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## I. INTRODUCTION

The problem of calculating mass transfer rates for a pure vapor condensing at an interface with its own liquid phase has received the attention of kinetic theorists for many years. In the vicinity of the vapor-liquid interface the mean free path of the vapor molecules is comparable to the scale length of the gradients if the fluid dynamic quantities and the deviation from local equilibrium is significant. This region is called the Knudsen layer. Away from the condensing surface the opposite condition is true and the vapor can be described on the basis of fluid dynamics (see Fig. 1).

If a certain quantity of noncondensable gas exists in the system, because of collisions, it will be carried along with the vapor toward the condensing surface and accumulate there. The vapor must diffuse through the noncondensable gas in order to condense. Since the partial pressure of noncondensable gas at the liquid-vapor interface increases above that in the bulk of the mixture, it produces a driving force for gas diffusion away from the surface. This motion is counterbalanced by the motion of the vapor-gas mixture toward the surface. Because the total pressure of the system remains nearly constant the partial pressure of vapor at the interface is lower than that in the bulk mixture. This provides the driving force for vapor diffusion toward the interface.

One can categorize two types of kinetic theory treatment of the condensation problem. These are the so-called weak condensation and strong condensation approaches. Each of these have been applied to the pure vapor condensation problem and the weak condensation approach has been applied to condensation in the presence of a noncondensable gas. In this paper we will extend

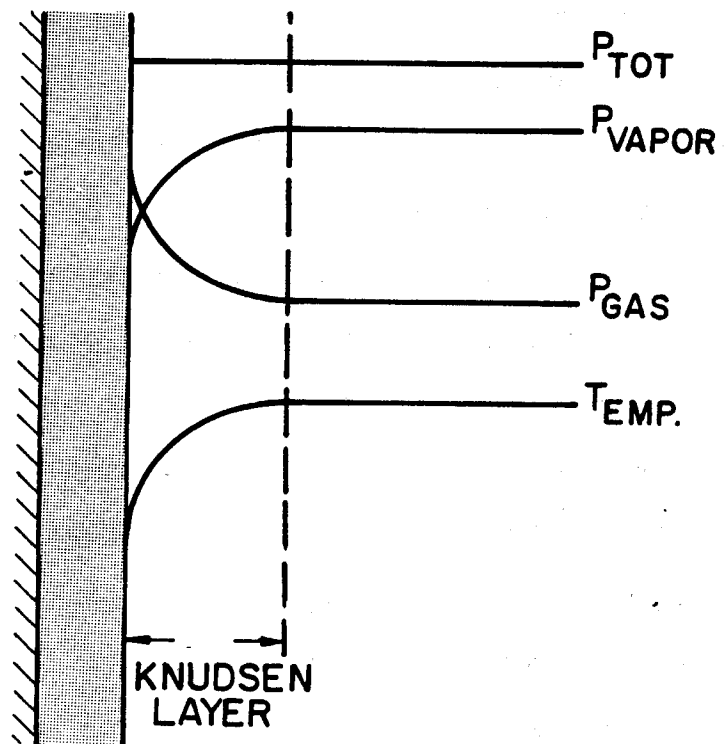


Fig. 1. The pressure and temperature distribution for vapor-gas mixture.

the strong condensation treatment to the problem of condensation in the presence of a noncondensable gas.

#### A. Weak Condensation - Pure Vapor

Figure 2 shows a one-dimensional, pure vapor condensing on its own liquid phase. For small values of the mass and energy flow induced by small temperature and density gradients (i.e.,  $\Delta T/T_S$ ,  $\Delta n/n_S \ll 1$ ), Pao<sup>(1)</sup> and Cipolla et al.<sup>(2)</sup> obtained the macroscopic temperature and pressure jumps by solving the linearized Boltzmann (BGK) equations:

$$v_x \frac{\partial \phi}{\partial x} + \alpha \phi = \alpha \pi^{-3/2} [n + T(v^2 - 1.5) + 2Uv_x] ,$$

where  $\phi$  is the perturbed distribution function defined by

$$f = f_S(1 + \phi)$$

$$f_S = n_S(2\pi RT_S)^{-3/2} \exp(-v^2) ,$$

where  $f$  is the distribution function of the gas,  $\alpha$  is a constant inversely proportional to the mean free path of the vapor, and  $v$  is the molecular velocity normalized by  $(2RT_S)^{-1/2}$ .

Gajewski et al.<sup>(3)</sup> used the discrete ordinates method to solve the full nonlinear BGK model equations and obtained the density and temperature distribution of the vapor. Ytrehus<sup>(4)</sup> used a Mott-Smith inspired moment method by assuming the distribution function in the condensing Knudsen layer

$$f(x, v) = a_S^+(x) f_S^+(v) + a_\infty^+(x) f_\infty^+(v) + a_\infty^-(x) f_\infty^-(v) + a_*^-(x) f_*^-(v)$$

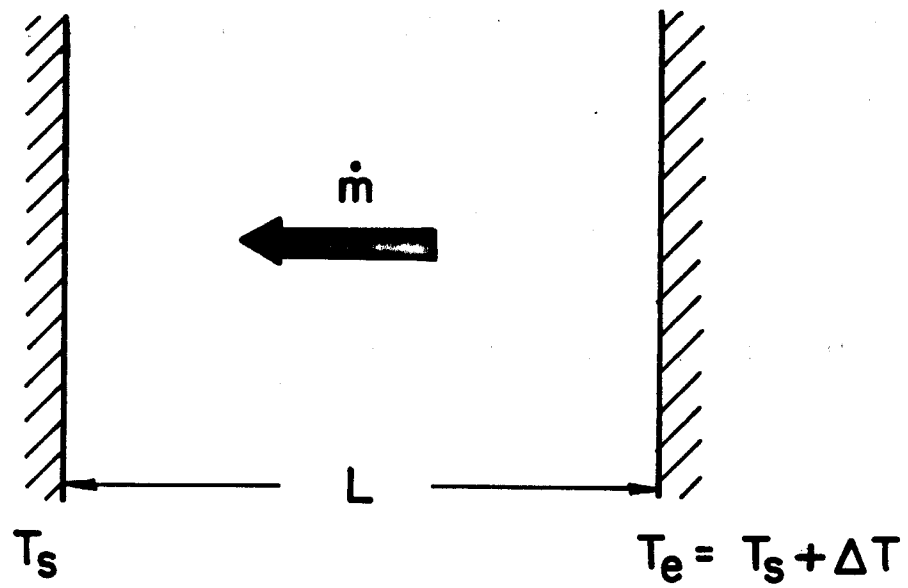


Fig. 2. Weak condensation.



where:  $f_s^+(\underline{v})$  = the half range surface Maxwellian.  
 $f_\infty^+(\underline{v}), f_\infty^-(\underline{v})$  = the half range external Maxwellian for  $v_x > 0$  and  
 $v_x < 0$ .  
 $f_*(\underline{v})$  = the self-collision distribution function.

After solving the Maxwell-Boltzmann transport equations he showed that the latent heat of the substance was found to have a critical influence on the flow behavior in the condensation problem. Onishi and Sone<sup>(5)</sup> solved the Boltzmann-Krook-Welander equation for a cylindrical condensed phase and concluded that even for very weak condensation, the linearized theory is invalid and a nonlinear analysis is required. Recently, Aoki and Cercignani<sup>(6)</sup> reviewed some of the previous papers and summarized that:

- (i) The calculated mass flow rate at the evaporating wall from the Herz-Knudsen<sup>(7,8)</sup> formula disagreed as much as 70% with the more detailed kinetic theory calculations.
  - (ii) In the two surface problem, if  $\beta = \frac{\Delta\rho/\rho_s}{\Delta T/T_s} > \beta_c$ , where  $\beta_c \approx 3.7$ , the calculated vapor temperature profile will be inverted, i.e. the temperature of the vapor at the hot wall can be below that at the cold wall.
- This phenomenon, however, has not been proven by experiments.

#### B. Weak Condensation - Vapor and Noncondensable Gas

For a vapor-gas mixture, Pao<sup>(9)</sup> used the modified linearized BKG equations to estimate the effects of the noncondensable gas on the mass flux rate and the heat flux rate. Matsushita<sup>(10)</sup> and Soga<sup>(11)</sup> solved the Gross-Krook equations for binary mixtures in the following way:

$$v_{xi} \frac{\partial f_i}{\partial x} = v_{ii}(F_i - f_i) + v_{ij}(\tilde{F}_i - f_i), \quad i = 1, 2$$

where:  $f_i$  = the distribution function for specie  $i$ ,  
 $\nu_{ii}$  = self-collision frequency,  
 $\nu_{ij}$  = cross-collision frequency,  
 $F_i, \tilde{F}_i$  = local Maxwellian distribution,

and

$$F_i = n_i (2\pi R_i T_i)^{-3/2} \exp\left\{-\frac{(v_i - U_i)^2}{2R_i T_i}\right\},$$

$$\tilde{F}_i = n_i (2\pi R_i \tilde{T}_i)^{-3/2} \exp\left\{-\frac{(v_i - \tilde{U}_i)^2}{2R_i \tilde{T}_i}\right\},$$

where  $\tilde{T}_i, \tilde{U}_i$  is the temperature, velocity after collisions.

Next introduce the reduced distribution functions  $g$  and  $h$ :

$$g(x, v_x) = \iint_{-\infty}^{\infty} f \, dv_y dv_z$$

$$h(x, v_x) = \iint_{-\infty}^{\infty} (v_y^2 + v_z^2) f \, dv_y dv_z.$$

Equations for these distribution functions are then given by:

$$v_{xi} \frac{\partial g_i}{\partial x} = A_{ii} \nu_{ii} (G_i - g_i) + A_{ij} \nu_{ij} (\tilde{G}_i - g_i) \quad (1.1a)$$

$$v_{xi} \frac{\partial h_i}{\partial x} = A_{ii} \nu_{ii} (H_i - h_i) + A_{ij} \nu_{ij} (\tilde{H}_i - h_i). \quad (1.1b)$$

Then assume:

- (1) All particles impinging onto the surface of the condensed phase are captured.

(2) The vapor emitted from the surface has a Maxwellian distribution

$$f_1(x=0) = n_{1s} (2\pi R_1 T_s)^{-3/2} \exp\left(-\frac{v_1^2}{2R_1 T_s}\right) \quad v_{x1} > 0 ,$$

$$f_1(x=L) = n_{1L} (2\pi R_1 T_e)^{-3/2} \exp\left(-\frac{v_1^2}{2R_1 T_e}\right) \quad v_{x1} < 0 .$$

(3) The surface is impermeable to the noncondensable gas, so the mean velocity of noncondensable gas will be zero or the overall gas flux will be zero.

(4) The conservation of noncondensable gas gives

$$\int_0^L n_2 dx = \bar{n}_2 L .$$

(5) The noncondensable gas reflects from the surface with a Maxwellian distribution:

$$f_2(x=0) = n_{2s} (2\pi R_2 T_s)^{-3/2} \exp\left(-\frac{v_2^2}{2R_2 T_s}\right) \quad v_{x2} > 0 ,$$

$$f_2(x=L) = n_{2L} (2\pi R_2 T_e)^{-3/2} \exp\left(-\frac{v_2^2}{2R_2 T_e}\right) \quad v_{x2} < 0 .$$

The temperature difference between these two surfaces is assumed to be small, hence the deviation from the equilibrium state is small. So they assume

$$g_i = g_{is} (1 + \phi_{gi})$$

$$h_i = h_{is} (1 + \phi_{hi})$$

where

$$g_{is} = n_{is} (2\pi R_i T_s)^{-1/2} \exp\left(-\frac{v_{ix}^2}{2R_i T_s}\right)$$

$$h_{is} = n_{is} \left(\frac{2R_i T_s}{\pi}\right) \exp\left(-\frac{v_{ix}^2}{2R_i T_s}\right)$$

and solve Eqs. (1.1a) and (1.1b) by using either finite difference methods or half range Hermite Polynomials. Their results show that (1) the presence of a small amount of noncondensable gas can cause a large buildup of the noncondensable gas near the low temperature surface; and (2) the induced resistance for mass transfer is proportional to the total amount of the noncondensable gas.

### C. Strong Condensation

The second kinetic theory approach is for intensive condensation. Makashev<sup>(12)</sup> solved the problem of strong recondensation between two infinite parallel plates over a wide range of Knudsen numbers for a one-component and two-component gas on the basis of the model Boltzmann kinetic equation. For the case of a one-dimensional, steady state and condensable gas only, Labuntsov and Kryukov<sup>(13)</sup> (hereafter denoted L&K) divided the problem into two regions (see Fig. 3):

Region I: gas dynamic region

Region II: Knudsen layer.

(1) In Region I, the flow is described by the Euler equilibrium flow with the following distribution function:

$$f_{\infty} = n_{\infty} (2\pi R T_{\infty})^{-3/2} \exp\left\{-\frac{(v - u_{\infty})^2}{2R T_{\infty}}\right\}.$$

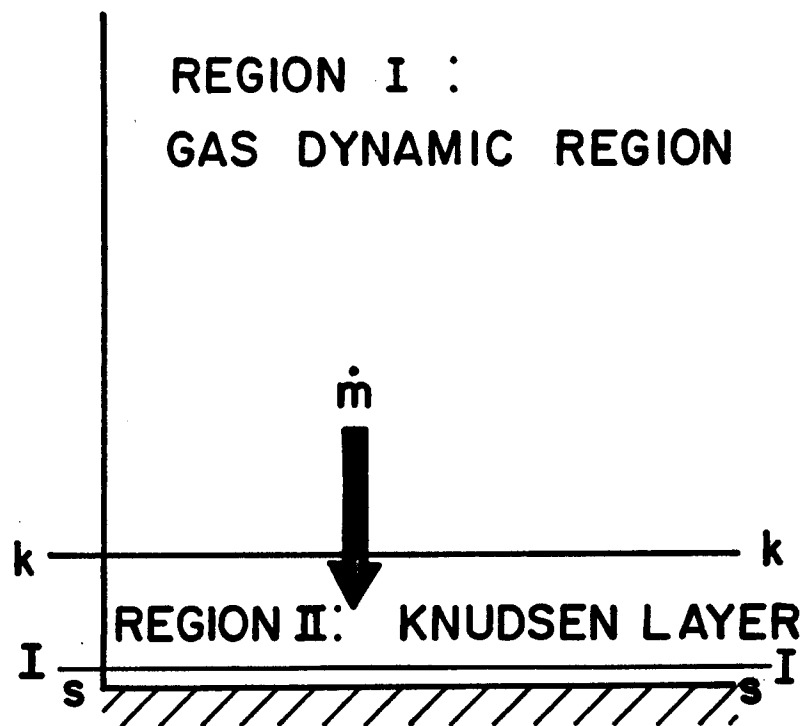


Fig. 3. Intensive condensation.

(2) In the k-k section, the vapor has a Grad's 13 moment distribution<sup>(14)</sup>

$$f_k = n_k \left( \frac{m}{2\pi k_B T_k} \right)^{3/2} \exp(-\xi^2) * \left[ 1 + \frac{1}{2} \frac{p_{11}}{p_k} (3\xi_x^2 - \xi^2) + 2 \frac{q_1 \xi_x (\frac{2}{5} \xi^2 - 1)}{p_k (2RT_k)^{1/2}} \right]$$

$$p_k = n_k k_B T_k, \quad \xi = \frac{\underline{v}}{(2RT_k)^{1/2}}$$

where:

$$\underline{v} = \underline{v} - \underline{U}_k, \quad p_{ij} = p_{ij} - \delta_{ij} p.$$

(3) The vapor emitted from the surface has a Maxwellian distribution

$$f = n_s (2\pi RT_s)^{-3/2} \exp\left(-\frac{v^2}{2RT_s}\right) \quad v_x > 0.$$

(4) Part of the flux from the k-k section will condense, so

$$f = C f_k \quad v_x < 0$$

where:

$$n(x=0) = c n_k,$$

$$T(x=0) = T_k,$$

$$U(x=0) = U_k.$$

The mass, momentum and energy fluxes are evaluated for Regions I and II and are matched at the k-k interface. For Region II we have the equations:

$$1 - C n_k \left( T_k^{1/2} \phi - \frac{U_k}{2} \psi + \frac{\phi p_{11}}{2 p_k} T_k^{1/2} - \frac{2 q_1 U_k \phi}{5 \pi p_k T_k^{1/2}} \right) = n_k U_k \quad (1.2)$$

$$\frac{1}{2} + Cn_k \left[ \left( \frac{T_k}{2} + \frac{U_k^2}{4\pi} + \frac{p_{11}}{2p_k} T_k \right) \psi - \frac{U_k T_k^{1/2}}{2\pi} \phi - \frac{4q_1 T_k^{1/2}}{5\pi p_k} \phi \right] = n_k T_k + \frac{n_k U_k^2}{2\pi} + \frac{p_{11}}{p_k} n_k T_k \quad (1.3)$$

$$\begin{aligned} \frac{1}{2} - \frac{Cn_k}{2} \left[ T_k^{3/2} \left( 1 + \frac{2p_{11}}{4p_k} \right) \phi - \left( \frac{5}{8} T_k U_k + \frac{p_{11}}{8p_k} T_k U_k + \frac{U_k^3}{16\pi} \right) \psi + \frac{T_k^{1/2} U_k^2}{8\pi} \phi \right. \\ \left. - \frac{U_k T_k^{1/2} q_1}{5\pi p_k} \phi - \frac{7T_k q_1}{5p_k} \psi \right] = \frac{5}{8} n_k T_k U_k + \frac{n_k U_k^3}{16\pi} + \frac{n_k U_k T_k p_{11}}{8p_k} + \frac{7n_k T_k q_1}{5p_k} . \end{aligned} \quad (1.4)$$

For Region I we have the equations:

$$n_k U_k = n_\infty U_\infty \quad (1.5)$$

$$n_k T_k + \frac{n_k U_k^2}{2\pi} + n_k T_k \frac{p_{11}}{p_k} = n_\infty T_\infty + \frac{n_\infty U_\infty^2}{2\pi} \quad (1.6)$$

$$\frac{5}{8} n_k U_k T_k + \frac{n_k U_k^3}{16\pi} + \frac{n_k U_k T_k p_{11}}{8p_k} + \frac{7n_k T_k q_1}{5p_k} = \frac{5}{8} n_\infty U_\infty T_\infty + \frac{n_\infty U_\infty^3}{16\pi} . \quad (1.7)$$

By assuming that at a large distance from the interface the vapor flow is of the Euler type, the authors solved the continuity equation of motion and energy of vapor and obtained the approximate equation for the fluid velocity:

$$\frac{U_k}{U_\infty} \approx 1 - \frac{(T_\infty - T_k)K/(K-1) + \frac{1}{4\pi} (U_\infty^2 - U_k^2)}{T_\infty K/(K-1) + \frac{1}{2\pi} U_\infty^2} , \quad K = \frac{C_p}{C_v} \quad (1.8)$$

where the dimensionless quantities have been introduced ( $\hat{\phantom{x}}$  stands for the real quantity):

$$T_k = \frac{\hat{T}_k}{\hat{T}_s} , \quad n_k = \frac{\hat{n}_k}{\hat{n}_s} , \quad U_k = \frac{\hat{U}_k}{(k_B \hat{T}_s / 2\pi m)^{1/2}}$$

$$p_k = \frac{\hat{p}_k}{n_s k_B T_s}, \quad p_{11} = \frac{\hat{p}_{11}}{n_s k_B T_s}, \quad q_1 = \frac{\hat{q}_1}{4 n_s k_B T_s (k_B T_s / 2\pi m)^{1/2}}$$

and

$$\phi = \exp\left(-\frac{U_k^2}{4\pi T_k}\right)$$

$$\psi = 1 + \operatorname{erf}\left[-\frac{U_k}{2(\pi T_k)^{1/2}}\right].$$

Using these equations one can specify  $n_\infty$ ,  $T_\infty$ ,  $n_s$ ,  $T_s$  and solve for  $T_k$ ,  $U_k$ ,  $C$ ,  $n_k$ ,  $p_{11}$ ,  $q_1$  and  $U_\infty$ . A comparison of the data with the experimental data of Necmi and Rose<sup>(15)</sup> showed a good agreement.

In this paper we apply a generalization of the Grad 13 Moment method suggested by L&K to the problem of strong condensation in the presence of a noncondensable gas.

## II. KINETIC TREATMENT OF INTENSIVE CONDENSATION IN THE PRESENCE OF A NONCONDENSABLE GAS

For the problem of intensive condensation in the presence of a noncondensable gas we have extended the method of Labuntsov and Kryukov by using Grad's 13 moment distribution functions for binary mixtures. This leads to 12 nonlinear equations with 12 unknowns. These are then solved using standard numerical methods. The physical situation that is modeled is shown in Fig. 3. The problem consists of a semi-infinite space of vapor-gas mixture and a condensing surface at  $x = 0$ . The imaginary planar surfaces at  $k-k$  and  $s-s$  define the Knudsen layer. To the right of the  $k-k$  surface we assume that the equations of fluid dynamics are valid.

Following the method of Labuntsov and Kryukov, the distribution functions of the gas-vapor mixture in the gas dynamic region, the  $k-k$  boundary, and the



interphase surface are:

$$f_{\infty} = n_{\infty} (2\pi RT_{\infty})^{-3/2} \exp\left\{-\frac{(v_x - U_{\infty})^2}{2RT_{\infty}}\right\},$$

$$f_k = n_k (2\pi RT_k)^{-3/2} \exp\left\{-\frac{[(v_x - U_k)^2 + v_y^2 + v_z^2]}{2RT_k}\right\} * \left\{1 + \frac{1}{2} \frac{p_{1k}}{p_k} (3\xi_x^2 - \xi^2) + \frac{2q_{1k}\xi_x(\frac{2}{5}\xi^2 - 1)}{p_k (2RT_k)^{1/2}}\right\},$$

$$f_s = n(0) (2\pi RT(0))^{-3/2} \exp\left\{-\frac{[(v_x - U(0))^2 + v_y^2 + v_z^2]}{2RT(0)}\right\} * \left\{1 + \frac{1}{2} \frac{p_{10}}{p(0)} (3\xi_x^2 - \xi^2) + \frac{2q_{10}\xi_x(\frac{2}{5}\xi^2 - 1)}{p(0)(2RT(0))^{1/2}}\right\} \quad v_x < 0,$$

$$f_s = n_s (2\pi RT_s)^{-3/2} \exp\left\{-\frac{[v_x^2 + v_y^2 + v_z^2]}{2RT_s}\right\} \quad v_x > 0.$$

Specifying the temperature and density at the surface and at infinity the 11 unknowns are:  $U_{\infty}$ ,  $n_k$ ,  $U_k$ ,  $T_k$ ,  $p_{1k}$ ,  $q_{1k}$ ,  $n(0)$ ,  $T(0)$ ,  $U(0)$ ,  $p_{10}$ ,  $q_{10}$ . Labuntsov and Kryukov further assumed that  $p_{10} = p_{1k}$ ,  $q_{10} = q_{1k}$ ,  $T(0) = T_k$  and  $U(0) = U_k$ .

In a gas mixture in addition to momentum and energy transfer we must consider the transfer of mass (diffusion) of the separate species relative to the gas as a whole, so we use the two component Grad's 13 moment distribution<sup>(16)</sup> to describe the two species of gas in the Knudsen layer.

$$f_{ix} = f_G(n_{ix}, T_x, \bar{U}_x, p_{ix}, q_{ix}, v_{dix}) = n_{ix} (2\pi R_i T_x)^{-3/2} \exp\left\{-\frac{[(v_x - \bar{U}_x)^2 + v_y^2 + v_z^2]}{2R_i T_x}\right\} * \left\{1 + \frac{1}{2} \frac{p_{ix}}{p_{ix}} (3\xi_{ix}^2 - \xi_i^2) + \frac{2q_{ix}\xi_{ix}(\frac{2}{5}\xi_i^2 - 1)}{p_{ix} (2R_i T_x)^{1/2}} + (\frac{7}{2} - \xi_i^2)\xi_{ix}(\frac{2}{R_i T_x})^{1/2} v_{dix}\right\}, \quad i=1,2$$

where:  $V_{dix}$  = diffusion velocity of specie  $i = U_{ix} - \bar{U}_x$ ,

$U_{ix}$  = fluid velocity of specie  $i$ ,

$\bar{U}_x$  = mass velocity of the mixture =  $\frac{\rho_1 U_{1x} + \rho_2 U_{2x}}{\rho_1 + \rho_2}$ ,

$\rho_i$  = mass density of specie  $i$ .

In order to get the notation straight, we use  $A_{i\pm}$  to describe a physical quantity. The first number  $i$  can be 1 or 2. The number 1 stands for the condensable gas and the number 2 for the noncondensable gas. The second  $\pm$  is the location of the gas. For a single subscript notation  $B_j$ ,  $j$  can be a number or a letter to specify the specie of the gas or the location of the gas.

For a one-dimensional steady state situation with a two component gas, we assume:

- (1) The two gas components have the same temperature.
- (2) The mean velocity of the noncondensable gas,  $U_2$ , is everywhere equal to zero.
- (3) In Region I (see Fig. 3), the mixture can be described by the Euler equilibrium flow with drifting velocity  $\bar{U}_\infty$ .

$$f_{1\infty} = n_{1\infty} (2\pi R_1 T_\infty)^{-3/2} \exp\left\{-\frac{[(v_x - \bar{U}_\infty)^2 + v_y^2 + v_z^2]}{2R_1 T_\infty}\right\}$$

$$\times \left\{1 + \left(\frac{7}{2} - \frac{\xi_1^2}{2}\right) \xi_{1x} \left(\frac{2}{R_1 T_\infty}\right)^{1/2} v_{d1\infty}\right\}$$

$$f_{2\infty} = n_{2\infty} (2\pi R_2 T_\infty)^{-3/2} \exp\left\{-\frac{[(v_x - \bar{U}_\infty)^2 + v_y^2 + v_z^2]}{2R_2 T_\infty}\right\}$$

$$\times \left\{1 - \left(\frac{7}{2} - \frac{\xi_2^2}{2}\right) \xi_{2x} \left(\frac{2}{R_2 T_\infty}\right)^{1/2} \bar{U}_\infty\right\}.$$

- (4) At the k-k boundary between the Knudsen layer and the gas dynamic region, use the Grad's 13 moment distribution:

$$f_{1k} = f_G(n_{1k}, T_k, \bar{U}_k, p_{1k}, q_{1k}, v_{d1k}) ,$$

$$f_{2k} = f_G(n_{2k}, T_k, \bar{U}_k, p_{2k}, q_{2k}, v_{d2k} = -\bar{U}_k) .$$

- (5) In the immediate vicinity of the interphase plane, denoted s-s:

- (i) The incoming particles will have the Grad's 13 moment distribution:

$$f_{1s} = f_G(n_{10}, T_0, \bar{U}_0, p_{10}, q_{10}, v_{d10}) ,$$

$$f_{2s} = f_G(n_{20}, T_0, \bar{U}_0, p_{20}, q_{20}, v_{d20} = -\bar{U}_0) \quad v_x < 0 .$$

We assume that in this thin Knudsen layer the temperature of the mixture and the fluid velocity of the condensable gas will not change significantly, so  $T_0 = T_k$  and  $U_{10} = U_{1k}$ .

- (ii) The outgoing particles will have the Maxwellian distribution:

$$f_{1s} = n_{1s} (2\pi R_1 T_s)^{-3/2} \exp\left(-\frac{v^2}{2R_1 T_s}\right) ,$$

$$f_{2s} = n_{2s} (2\pi R_2 T_s)^{-3/2} \exp\left(-\frac{v^2}{2R_2 T_s}\right) \quad v_x > 0 .$$

- (6) Assume  $p_{10} = p_{1k} = p_1$ ,  $q_{10} = q_{1k} = q_1$  and  $p_{20} = p_{2k} = p_2$ ,  $q_{20} = q_{2k} = q_2$ .

- (7) Since the surface is impermeable to the noncondensable gas, the reflected stream will have the same number density as the incoming stream.

$$\int_{-\infty}^{\infty} \int_{v_x=-\infty}^0 f_{2s} d\mathbf{v} = \int_{-\infty}^{\infty} \int_{v_x=0}^{\infty} f_{2s} d\mathbf{v} .$$

- (8) Assuming collisions can be neglected in the Knudsen layer, we are able to formulate the separate mass, momentum and energy conservation equations for condensable and noncondensable gas.
- (9) Following the work of Kolonder,<sup>(16)</sup> the collisional terms of the momentum and energy transfer equations in the gas dynamic region are treated according to perfectly elastic spheres.

$$\frac{d}{dx} (M_2) = -J_{12}^i , \quad (2.1)$$

$$\frac{d}{dx} (E_2) = -\frac{1}{2} J_{12}^{ii} , \quad (2.2)$$

where:  $M_2$  = momentum flux of specie 2,

$E_2$  = energy flux of specie 2,

$$J_{12}^i = f_1 + g_1 \bar{U} ,$$

$$\frac{1}{2} J_{12}^{ii} = f_2 + g_2 \bar{U} + h_2 \bar{U}^2 ,$$

$$f_1 = A[0.2 \tau (\rho_1 q_2 - \rho_2 q_1) + 0.04 \tau^2 (p_2 q_1 + q_1 p_2 - p_1 q_2 - q_2 p_1)] ,$$

$$g_1 = A \left\{ \frac{\rho_2}{\rho_1} [-\rho_1 \rho_2 + 0.7 \tau \rho_2 p_1 + 0.2 \tau \rho_2 p_1 - 0.1 \tau^2 (p_1 p_2 + p_1 p_2)] \right.$$

$$\left. - [\rho_1 \rho_2 - 0.7 \tau \rho_1 p_2 - 0.2 \tau \rho_1 p_2 + 0.1 \tau^2 (p_2 p_1 + p_1 p_2)] \right\},$$

$$f_2 = \left( \frac{m_1 - m_2}{m_1 + m_2} \right) A [0.2 \tau (p_1 p_2 + p_1 p_2 + p_2 p_1 + p_1 p_2) - 0.12 \tau^2 q_1 q_2],$$

$$g_2 = A \left[ \frac{\rho_2}{\rho_1} (-0.4 \tau^2 p_1 q_2 + \left( \frac{m_1 - m_2}{m_1 + m_2} \right) 0.2 \tau \rho_1 q_2 + 0.3 \left( \frac{m_1 - m_2}{m_1 + m_2} \right) \tau^2 p_1 q_2) \right.$$

$$\left. - (0.4 \tau^2 p_2 q_1 + \left( \frac{m_1 - m_2}{m_1 + m_2} \right) 0.2 \tau \rho_2 q_1 + 0.3 \left( \frac{m_1 - m_2}{m_1 + m_2} \right) \tau^2 p_2 q_1) \right],$$

$$h_2 = -A \left( \frac{m_1 - m_2}{m_1 + m_2} \right) \left( \frac{\rho_2}{\rho_1} \right) (\rho_1 \rho_2 - 0.5 \tau \rho_2 p_1 - 0.5 \tau \rho_1 p_2 - 0.75 \tau^2 p_1 p_2),$$

$$A = \frac{2\sqrt{2}\pi}{3} (m_1 + m_2)^{-1} (d_1 + d_2)^2 \sqrt{kT \left( \frac{m_1 + m_2}{m_1 m_2} \right)},$$

$$\tau = \frac{m_1 m_2}{kT(m_1 + m_2)},$$

$d_i$  = the molecular diameter,

$\rho_i = m_i n_i$ .

Integrating Eqs. (2.1) and (2.2), we have:

$$M_{2L} - M_{20} = -\int_0^L J_{12}^i dx = -\int_0^L (f_1 + g_1 \bar{U}) dx,$$

$$E_{2L} - E_{20} = -\int_0^L \frac{1}{2} J_{12}^{ii} dx = -\int_0^L (f_2 + g_2 \bar{v} + h_2 \bar{U}^2) dx.$$

Assume that only the mixture velocity  $\bar{U}$  depends upon  $x$

(i.e.,  $\bar{U}_x \approx \bar{U}_\infty (1 + a_1 e^{-Z(x)})$ ),  $Z(x) = \frac{3}{4} |j| \int_0^x \frac{dx}{\mu}$ ,  $|j|$ : mass flux and  $\mu$ : viscosity from Ref. 13) and take the average value of  $f_1$ ,  $g_1$ ,  $f_2$ ,  $g_2$  and  $h_2$ , we obtain:

$$\begin{aligned} M_{2L} - M_{20} &\approx - \int_0^L [f_1 + g_1 \bar{U}_\infty (1 + a_1 e^{-Z(x)})] dx \\ &= -(\bar{f}_1 + \bar{g}_1 \bar{U}_\infty)L + \frac{4}{3} \frac{\mu a_1}{|j|} \bar{g}_1 \bar{U}_\infty (e^{-Z(L)} - 1) \end{aligned} \quad (2.3)$$

$$\begin{aligned} E_{2L} - E_{20} &\approx - \int_0^L [f_2 + g_2 \bar{U}_\infty (1 + a_1 e^{-Z(x)}) + h_2 \bar{U}_\infty^2 (1 + 2a_1 e^{-Z(x)} \\ &\quad + a_1^2 e^{-2Z(x)})] dx = -(\bar{f}_2 + \bar{g}_2 \bar{U}_\infty + h_2 \bar{U}_\infty^2)L + \frac{4}{3} \frac{\mu a_1}{|j|} (\bar{g}_2 \bar{U}_\infty + \\ &\quad 2 \bar{h}_2 \bar{U}_\infty^2) (e^{-Z(L)} - 1) + \frac{2}{3} \frac{\mu a_1^2}{|j|} \bar{h}_2 \bar{U}_\infty^2 (e^{-2Z(L)} - 1) . \end{aligned} \quad (2.4)$$

If  $L$  is very large compared to the mean free path of the particles, then  $e^{-Z(L)}$ ,  $e^{-2Z(L)}$  will approach to zero. Equations (2.3) and (2.4) become

$$M_{2L} - M_{20} + \frac{4\mu a_1}{3|j|} \bar{g}_1 \bar{U}_\infty \approx -(\bar{f}_1 + \bar{g}_1 \bar{U}_\infty)L \quad (2.5)$$

$$E_{2L} - E_{20} + \frac{4\mu a_1}{3|j|} (\bar{g}_2 \bar{U}_\infty + \bar{h}_2 \bar{U}_\infty^2 (2 + \frac{a_1}{2})) \approx -(\bar{f}_2 + \bar{g}_2 \bar{U}_\infty + \bar{h}_2 \bar{U}_\infty^2)L . \quad (2.6)$$

Taking the ratio of Eqs. (2.5) and (2.6), we can cancel the dependence of the length  $L$ , giving

$$\frac{M_{2\infty} - M_{20} + \frac{4\mu a_1}{3|j|} \bar{g}_1 \bar{U}_\infty}{E_{2\infty} - E_{20} + \frac{4\mu a_1}{3|j|} (\bar{g}_2 \bar{U}_\infty + \bar{h}_2 \bar{U}_\infty^2 (2 + \frac{a_1}{2}))} = \frac{\bar{f}_1 + \bar{g}_1 \bar{U}_\infty}{\bar{f}_2 + \bar{g}_2 \bar{U}_\infty + \bar{h}_2 \bar{U}_\infty^2} . \quad (2.7)$$

The viscosity  $\mu$  is evaluated as:

$$\mu = \left( \frac{n_1}{v_{11} + v_{12}} + \frac{n_2}{v_{22} + v_{21}} \right) kT ,$$

$$v_{ij} = \frac{16}{3\sqrt{\pi}} \frac{(m_i + m_j)n_j}{m_i m_j} \sqrt{\frac{m_i m_j kT}{m_i + m_j}} Q_{ij} ,$$

$$Q_{ij} = \frac{\pi}{4} d_{12}^2 = \frac{\pi}{4} \left[ \frac{1}{2} (d_1 + d_2) \right]^2 .$$

Formulating the mass, momentum and energy flux conservation equations in Regions I and II and matching them in the k-k section, we get the following equations. In the Knudsen layer, because of the neglect of momentum and energy transfer between different species, the individual transport equations are:

$$1 - n_{10} (T_k^{1/2} \phi_{10} - \frac{1}{2} \bar{U}_0 \psi_{10} + \frac{1}{2} \phi_1 \frac{p_1 T_k^{1/2}}{p_{10}} - \frac{2q_1 \bar{U}_0 \phi_{10}}{5\pi p_{10} T_k^{1/2}} - \frac{v_{d10} \psi_{10}}{2} \quad (2.8)$$

$$+ \frac{v_{d10} \bar{U}_0 \phi_{10}}{4\pi T_k^{1/2}}) = n_{1k} \bar{U}_k + n_{1k} v_{d1k}$$

$$\frac{1}{2} + n_{10} \left[ \left( \frac{T_k}{2} + \frac{U_0^2}{4\pi} + \frac{p_1 T_0}{2P_{10}} \right) \psi_{10} - \frac{U_0 T_k^{1/2}}{2\pi} \phi_{10} - \frac{4q_1 \phi_{10} T_k^{1/2}}{5\pi P_{10}} + \frac{U_0 v_{d10} \psi_{10}}{2\pi} \right. \quad (2.9)$$

$$\left. - \frac{T_k^{1/2} v_{d10} \phi_{10}}{2\pi} \right] = n_{1k} T_k + \frac{n_{1k} U_k^2}{2\pi} + p_1 + \frac{n_{1k} U_k v_{d1k}}{\pi}$$

$$\frac{1}{2} - \frac{n_{10}}{2} \left[ \left( 1 + 0.5 \frac{p_1}{P_{10}} \right) T_k^{3/2} \phi_{10} - \left( \frac{5}{8} T_k U_0 + \frac{p_1 U_0 T_k}{8P_{10}} + \frac{U_0^3}{16\pi} \right) \psi_{10} + \frac{T_k^{1/2} U_0^2 \phi_{10}}{8\pi} \right. \quad (2.10)$$

$$\left. - \frac{U_0 q_1 \phi_{10} T_k^{1/2}}{5\pi P_{10}} - \frac{7q_1 \psi_{10} T_k}{5P_{10}} - \frac{3U_0^2 v_{d10} \psi_{10}}{16\pi} + \frac{U_0 v_{d10} T_k^{1/2} \phi_{10}}{2\pi} + \frac{1}{4} T_k v_{d10} \psi_{10} \right]$$

$$= \frac{5}{8} n_{1k} T_k U_k + \frac{n_{1k} U_k^3}{16\pi} + \frac{n_{1k} U_k T_k p_1}{8P_{1k}} + \frac{7n_{1k} T_k q_1}{5P_{1k}} + \frac{3n_{1k} U_k^2 v_{d1k}}{16\pi} - \frac{n_{1k} T_k v_{d1k}}{4}$$

$$n_{2s} - n_{20} \phi_{20} \left( T_k^{1/2} + \frac{1}{2} \frac{p_2 T_k^{1/2}}{P_{20}} - \frac{2q_2 U_{20} T_k^{1/2}}{5\pi P_{20} T_k^{1/2}} - \frac{U_{20}^2}{4\pi T_k^{1/2}} \right) = 0 \quad (2.11)$$

$$\frac{n_{2s}}{2} + n_{20} \left[ \left( \frac{T_k}{2} + \frac{U_{20}^2}{4\pi} + \frac{p_2 T_k}{2P_{20}} \right) \psi_{20} - \frac{4q_2 \phi_{20} T_k^{1/2} U_{20}^{1/2}}{5\pi P_{20}} - \frac{U_{20}^2 \psi_{20}}{2\pi} \right] = n_{2k} T_k - \frac{n_{2k} U_{2k}^2}{2\pi} + p_2 \quad (2.12)$$



$$\begin{aligned}
\frac{n_{2s}}{2} - \frac{n_{20}}{2} \left[ \left(1 + 0.5 \frac{p_2}{p_{20}}\right) T_k^{3/2} \phi_{20} - \left(\frac{5}{8} T_k U_{20} + \frac{p_2 U_{20} T_k}{8 p_{20}}\right) \psi_{20} - \frac{3 U_{20}^2 \phi_{20} T_k^{1/2}}{8\pi} \right. \\
\left. - \frac{U_{20} q_2 \phi_{20} T_k^{1/2} \gamma^{1/2}}{5\pi p_{20}} - \frac{7 q_2 \psi_{20} T_k}{5 p_{20}} + \frac{U_{20}^3 \psi_{20}}{8\pi} - \frac{T_k U_{20} \psi_{20}}{4} \right] = \frac{7}{8} n_{2k} T_k U_{2k} - \frac{n_{2k} U_{2k}^3}{8\pi} \\
+ \frac{n_{2k} U_{2k} T_k p_2}{8 p_{2k}} + \frac{7 n_{2k} T_k q_2 \gamma^{1/2}}{5 p_{2k}} .
\end{aligned} \quad (2.13)$$

Considering the number density conservation for the incoming and outgoing noncondensable flux to the interphase surface we can write:

$$\begin{aligned}
\frac{n_{2s}}{2} - \frac{n_{20} \psi_{20}}{2} - \frac{p_2 U_{20} \phi_{20}}{4\pi T_k^{3/2}} - \frac{4 \phi_{20} q_2 \gamma^{1/2}}{5\pi T_k^{3/2}} \left(\frac{1}{2} - \frac{U_{20}^2}{4\pi T_k}\right) + \frac{n_{20} U_{20} \phi_{20}}{4\pi T_k^{1/2}} \left(\frac{U_{20}^2}{2\pi T_k} - 3\right) = 0 .
\end{aligned} \quad (2.14)$$

In the gas dynamic region, we obtain the mixture transport equations as the following:

$$n_{1k} \bar{U}_k + n_{1k} v_{d1k} = n_{1\infty} U_{1\infty} \quad (2.15)$$

$$\begin{aligned}
n_{1k} T_k + \frac{n_{1k} \bar{U}_k^2}{2\pi} + \frac{n_{1k} T_k p_1}{p_{1k}} + \frac{n_{1k} \bar{U}_k v_{d1k}}{\pi} + n_{2k} T_k - \frac{n_{2k} U_{2k}^2}{2\pi} + p_2 \\
= n_{1\infty} T_\infty + \frac{n_{1\infty} \bar{U}_\infty^2}{2\pi} + \frac{n_{1\infty} \bar{U}_\infty v_{d1\infty}}{\pi} + n_{2\infty} T_\infty - \frac{n_{2\infty} U_{2\infty}^2}{2\pi}
\end{aligned} \quad (2.16)$$

$$\begin{aligned}
& \frac{5}{8} n_{1k} T_k \bar{U}_k + \frac{n_{1k} \bar{U}_k^3}{16\pi} + \frac{n_{1k} \bar{U}_k T_k p_1}{8p_{1k}} + \frac{7n_{1k} T_k q_1}{5p_{1k}} + \frac{3n_{1k} \bar{U}_k^2 v_{d1k}}{16\pi} - \frac{n_{1k} T_k v_{d1k}}{4} \\
& + \frac{1}{\gamma^{1/2}} \left( \frac{7}{8} n_{2k} T_k U_{2k} - \frac{n_{2k} U_{2k}^3}{8\pi} + \frac{n_{2k} U_{2k} T_k p_2}{8p_{2k}} + \frac{7n_{2k} T_k q_2 \gamma^{1/2}}{5p_{2k}} \right) \\
& = \frac{5}{8} n_{1\infty} T_\infty \bar{U}_\infty + \frac{n_{1\infty} \bar{U}_\infty^3}{16\pi} + \frac{3n_{1\infty} \bar{U}_\infty^2 v_{d1\infty}}{16\pi} - \frac{n_{1\infty} T_\infty v_{d1\infty}}{4} + \frac{1}{\gamma^{1/2}} \left( \frac{7}{8} n_{2\infty} T_\infty U_{2\infty} - \frac{n_{2\infty} U_{2\infty}^3}{8\pi} \right),
\end{aligned} \tag{2.17}$$

and the approximate mixture velocity ratio:

$$\frac{\bar{U}_k}{\bar{U}_\infty} = 1 - \frac{(T_\infty - T_k)K/(K-1) + \frac{1}{4\pi} (\bar{U}_\infty^2 - \bar{U}_k^2)}{T_\infty K/(K-1) + \frac{1}{2\pi} \bar{U}_\infty^2} = 1 + a_1. \tag{2.18}$$

Equation (2.7) gives

$$\begin{aligned}
& \frac{1}{\gamma} \left( n_{2\infty} T_\infty - \frac{n_{2\infty} U_{2\infty}^2}{2\pi} - n_{2k} T_k + \frac{n_{2k} U_{2k}^2}{2\pi} - p_2 \right) - y_1 = \left\{ \frac{1}{\gamma^{3/2}} \left( \frac{7}{8} n_{2\infty} T_\infty U_{2\infty} \right. \right. \\
& \left. \left. - \frac{n_{2\infty} U_{2\infty}^3}{8\pi} - \frac{7}{8} n_{2k} T_k U_{2k} + \frac{n_{2k} U_{2k}^3}{8\pi} - \frac{n_{2k} U_{2k} T_k p_2}{8p_{2k}} - \frac{7n_{2k} T_k q_2 \gamma^{1/2}}{5p_{2k}} \right) - y_2 \right\} y_3
\end{aligned} \tag{2.19}$$

where:  $K = \frac{c_p}{c_v}$ ,  $\phi_{10} = \exp\left(-\frac{\bar{U}_0^2}{4\pi T_k}\right)$ ,  $\psi_{10} = 1 + \operatorname{erf}\left[-\frac{\bar{U}_0}{2(\pi T_k)^{1/2}}\right]$

$$\hat{v}_{d10} = \hat{U}_{10} - \frac{\hat{m}_1 \hat{n}_{10} \hat{U}_{10}}{\hat{m}_1 \hat{n}_{10} + \hat{m}_2 \hat{n}_{20}} = \frac{\hat{m}_2 \hat{n}_{20}}{\hat{m}_1 \hat{n}_{10}} \hat{U}_0.$$

$$U_{20} = \gamma^{1/2} \bar{U}_0,$$

$$U_{2k} = \gamma^{1/2} \bar{U}_k,$$

$$U_{2\infty} = \gamma^{1/2} \bar{U}_\infty,$$

$$\phi_{20} = \exp\left(-\frac{U_{20}^2}{4\pi T_k}\right),$$

$$\psi_{20} = 1 + \operatorname{erf}\left[-\frac{U_{20}}{2(\pi T_k)^{1/2}}\right],$$

$$y_1 = G_1 W,$$

$$y_2 = 0.125 G_2 W + 0.125 H_2 W \bar{U}_\infty (2 + 0.5 a_1),$$

$$y_3 = \frac{8(F_1 + G_1 \bar{U}_\infty)}{(F_2 + G_2 \bar{U}_\infty + H_2 \bar{U}_\infty^2)},$$

$$G_1 = \frac{n_{2\infty}}{n_{1\infty}} \gamma \left( -\frac{n_{1\infty} n_{2\infty}}{\gamma} + \frac{0.7 n_{1\infty} n_{2\infty}}{1 + \gamma} + \frac{0.2 n_{2\infty} p_1}{(1 + \gamma) T_\infty} - \frac{0.1 n_{1\infty} p_2}{(1 + \gamma)^2 T_\infty} \right. \\ \left. - \frac{0.1 n_{1\infty} n_{2\infty}}{(1 + \gamma)^2} \right) - \left( \frac{n_{1\infty} n_{2\infty}}{\gamma} - \frac{0.7 n_{1\infty} n_{2\infty}}{\gamma(1 + \gamma)} - \frac{0.2 n_{1\infty} p_2}{\gamma(1 + \gamma) T_\infty} \right. \\ \left. + \frac{0.1 n_{2\infty} p_1}{(1 + \gamma)^2 T_\infty} + \frac{0.1 n_{1\infty} n_{2\infty}}{(1 + \gamma)^2} \right),$$

$$F_1 = \frac{0.8}{T_\infty \gamma (1 + \gamma)} (n_{1\infty} q_2 - n_{2\infty} q_1 \gamma) + \frac{0.16}{T_\infty^2 (1 + \gamma)^2} (p_2 q_1 + \\ + q_1 n_{2\infty} T_\infty - p_1 q_2 - q_2 n_{1\infty} T_\infty),$$

$$G_2 = \gamma n_{2\infty} q_2 \left( -\frac{1.6}{T_\infty (1 + \gamma)^2} + \frac{1 - \gamma}{(1 + \gamma)^2} \frac{0.8}{T_\infty \gamma} + \frac{1.2(1 - \gamma)}{T_\infty (1 + \gamma)^3} \right) \\ - n_{2\infty} q_1 \left( \frac{1.6}{T_\infty (1 + \gamma)^2} + \frac{1 - \gamma}{(1 + \gamma)^2} \frac{0.8}{T_\infty} + \frac{1.2(1 - \gamma)}{T_\infty (1 + \gamma)^3} \right),$$

$$F_2 = \frac{2\pi(1 - \gamma)}{\gamma(1 + \gamma)} \left[ \frac{0.2}{T_\infty (1 + \gamma)} (p_1 p_2 + p_1 n_{2\infty} T_\infty + p_2 n_{1\infty} T_\infty + n_{1\infty} n_{2\infty} T_\infty^2) \right. \\ \left. - \frac{0.96}{\pi} \frac{q_1 q_2 \gamma}{T_\infty^2 (1 + \gamma)^2} \right],$$

$$H_2 = -\frac{1 - \gamma}{1 + \gamma} n_{2\infty}^2 \left( 0.5 - \frac{0.75 \gamma}{(1 + \gamma)^2} \right),$$

$$W = \frac{4a_1 T_\infty \gamma}{3(1 + \gamma)(n_{1\infty} + \gamma n_{2\infty})} \left[ \frac{1}{0.25\sqrt{2} \frac{n_{2\infty}}{n_{1\infty}} + \frac{2}{(1 + \frac{1}{\gamma})^{1/2} (1 + \frac{d_2}{d_1})^2}} + \frac{1}{0.25\sqrt{2} \frac{n_{1\infty}}{n_{2\infty}} + \frac{2}{(1 + \gamma)^{1/2} (1 + \frac{d_1}{d_2})^2}} \right].$$

The following nondimensional quantities have been introduced:

$$\begin{aligned} n_{ix} &= \frac{\hat{n}_{ix}}{n_{1s}}, & p_i &= \frac{\hat{p}_i}{n_{1s} k_B \hat{T}_s}, & p_{ix} &= \frac{\hat{p}_{ix}}{n_{1s} k_B \hat{T}_s} = \frac{\hat{n}_{ix} \hat{T}_x}{n_{1s} \hat{T}_s}, \\ q_i &= \frac{\hat{q}_i}{4n_{1s} k_B \hat{T}_s (k_B \hat{T}_s / 2\pi m_1)^{1/2}}, & i &= 1, 2; & n_{2s} &= \frac{\hat{n}_{2s}}{n_{1s}}, & \gamma &= \frac{\hat{m}_2}{\hat{m}_1}, \\ v_{d10} &= \frac{\hat{v}_{d10}}{(k_B \hat{T}_s / 2\pi m_1)^{1/2}}, & T_x &= \frac{\hat{T}_x}{\hat{T}_s}, & U_x &= \frac{\hat{U}_x}{(k_B \hat{T}_s / 2\pi m_1)^{1/2}}. \end{aligned}$$

Specifying  $n_{1\infty}$ ,  $n_{2\infty}$ ,  $T_\infty$  and the surface conditions we can solve for  $n_{10}$ ,  $n_{20}$ ,  $n_{1k}$ ,  $n_{2k}$ ,  $\bar{U}_k$ ,  $T_k$ ,  $p_1$ ,  $q_1$ ,  $p_2$ ,  $q_2$ ,  $U_{1\infty}$  and  $n_{2s}$  from the 12 equations.

### III. METHOD OF SOLUTION AND RESULTS

The 12 nonlinear equations are solved by using a modification of the Levenberg-Marquardt algorithm in which the derivatives of the functions are calculated by a forward-difference approximation.<sup>(17)</sup>

In order to test the model, a calculation was done for the parameters shown in Table 1 and the result compared with the data of Labuntsov and Kryukov. In the presence of a negligible amount of noncondensable gas, the condensation rate should approach that predicted by L&K. Table 2 lists some of the nondimensional data from the calculations. Notice that the conden-

Table 1. Results of Calculations of Gas Dynamic Parameters  
for Vapor Condensation

$n_{\infty}$	1.7111	5.1365	3.2279	6.1327	1.3495
$T_{\infty}$	1.9162	2.5319	1.8546	1.6303	1.8535
$-U_{\infty}$	2.4128	5.2513	3.3942	3.9509	1.8826
$n_k$	1.9834	5.4867	3.7125	6.2354	1.6317
$T_k$	1.6496	2.3667	1.7323	1.6033	1.5296
$-U_k$	2.0815	4.9162	3.1732	3.8858	1.5570
$p_1/p_k$	0.0686	0.1123	0.0607	0.0252	0.0549
$q_1/p_k$	-0.1744	-0.2045	-0.1109	-0.0380	-0.15575
$a_1$	-0.1373	-0.0638	-0.0651	-0.0165	-0.1729

Table 2. Parameters for H<sub>2</sub>O Condensation With and Without Noncondensable Gas

	<u>H<sub>2</sub>O</u>	<u>H<sub>2</sub>O-H<sub>2</sub></u>	<u>H<sub>2</sub>O-He</u>	<u>H<sub>2</sub>O-Ne</u>	<u>H<sub>2</sub>O-Air</u>
$n_{2\infty}/n_{1\infty}$	0	0.05	0.05	0.05	0.05
$n_{1\infty}$	1.7	1.7	1.7	1.7	1.7
$T_{\infty}$	1.9	1.9	1.9	1.9	1.9
$P_{\infty}$	3.320	3.392	3.392	3.392	3.392
$U_{1\infty}$	-2.377	-2.370	-2.301	-1.804	-1.581
$n_{1k}$	1.970	1.972	1.963	1.719	1.522
$n_{2k}$	0	0.1921	0.2494	0.4865	0.5636
$T_k$	1.636	1.627	1.616	1.509	1.456
$P_k$	3.223	3.521	3.575	3.328	3.0366
$\bar{U}_k$	-2.051	-2.022	-1.939	-1.357	-1.127
$n_{10}$	1.964	1.966	1.956	1.719	1.536
$n_{20}$	0	0.1837	0.2416	0.4823	0.5593
$P_0$	3.213	3.498	3.551	3.322	3.051
$P_1$	0.2162	0.2179	0.1696	0.3268	0.5842
$q_1$	-0.5193	-0.5524	-0.5992	-1.004	-1.124
$P_2$	0	-0.1184	-0.1307	-0.08119	-0.07219
$q_2$	0	0.1165	0.2174	0.3993	0.3791
$n_{25}$	0	0.1723	0.2388	0.4812	0.5568

sation rate is equal to  $n_{1\infty}U_{1\infty}$  and we get a good correspondence between  $H_2O-H_2$  and  $H_2O$  only systems for  $n_{2\infty}/n_{1\infty} = 0.05$ . Table 3 is a listing of some real physical quantities corresponding to Table 2.

Figures 4, 5 and 6 show the influence of noncondensable gas concentration on Li,  $H_2O$  and Pb vapor condensation rate for  $n_{1\infty} = 1.7$ ,  $T_{\infty} = 1.9$ . Figure 7 is a plot for Pb-Ar, Pb-Kr and Pb-Xe gases condensing with different bulk temperatures for  $n_{1\infty} = 1.7$  and  $n_{2\infty}/n_{1\infty} = 0.1$ . From these results we can make the following observations:

- (1) As we increase the number density of noncondensable gas, the condensation rate decreases.
- (2) For the same  $n_{2\infty}/n_{1\infty}$  ratio, the noncondensable gas with the larger atomic mass will cause a larger decrease in the condensation rate. This is because the accumulation of noncondensable gas near the surface ( $n_{20}$ ) increases.
- (3) The heavier the noncondensable gas, the lower the temperature  $T_0 (= T_k)$ , i.e. the condensation rate decreases as the superheat of the mixture near the condensing surface decreases.

These basic calculations indicate the strong influence of the atomic mass of the noncondensable gas on the condensation rate. This method will be applied in the future to other condensable-noncondensable gas mixtures.

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Table 3. Listing of Some Real Physical Quantities  
for Different Vapor-Gas Systems

	<u>H<sub>2</sub>O-H<sub>2</sub></u>	<u>H<sub>2</sub>O-He</u>	<u>H<sub>2</sub>O-Ne</u>	<u>H<sub>2</sub>O-Air</u>
$n_{1\infty}/n_{1s}$	1.7	1.7	1.7	1.7
$n_{2\infty}/n_{1\infty}$	0.05	0.05	0.05	0.05
$P_{\infty}$ (atm)	0.408	0.408	0.408	0.408
$T_s$ (°C)	50	50	50	50
$T_k$ (°C)	252.52	248.97	214.41	197.29
$T_{\infty}$ (°C)	340.70	340.70	340.70	340.70
$-\bar{U}_{1\infty}$ (cm/s)	$3.6519 \times 10^4$	$3.5456 \times 10^4$	$2.7798 \times 10^4$	$2.4361 \times 10^4$



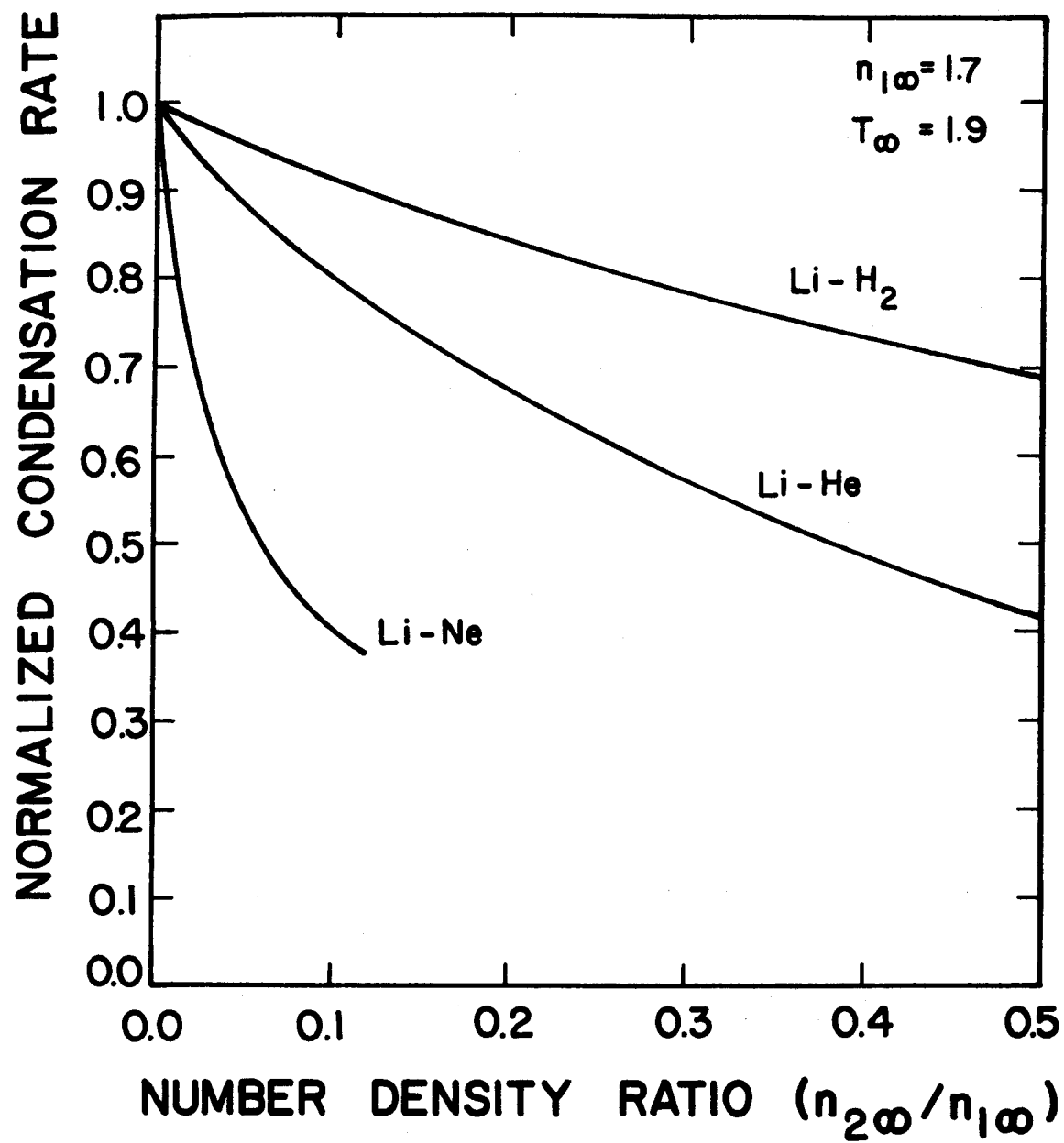


Fig. 4. Effect of noncondensable gas on Li condensation rate.

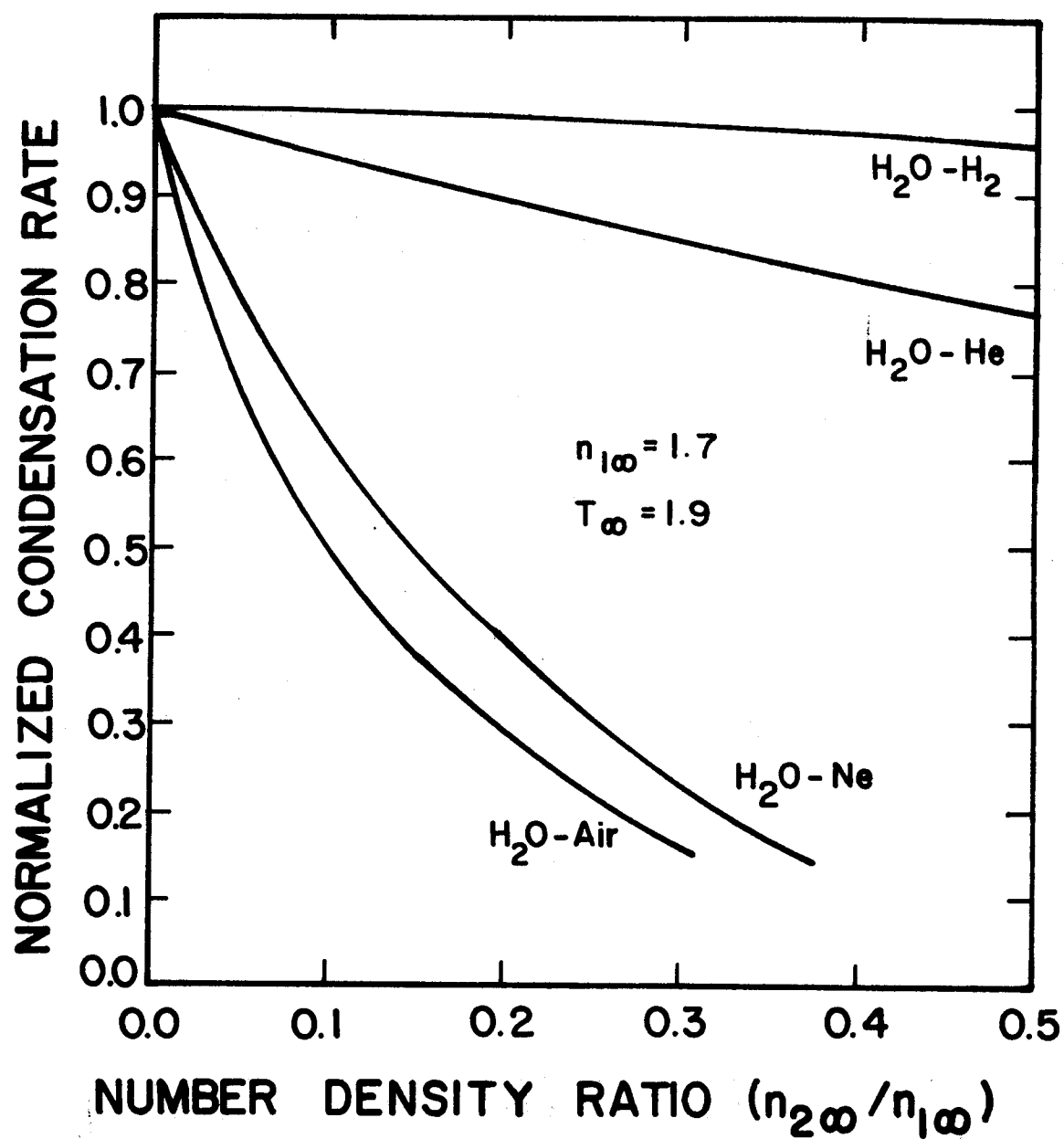


Fig. 5. Effect of noncondensable gas on  $H_2O$  condensation rate.

# NORMALIZED CONDENSATION RATE VS. NO. DENSITY RATIO

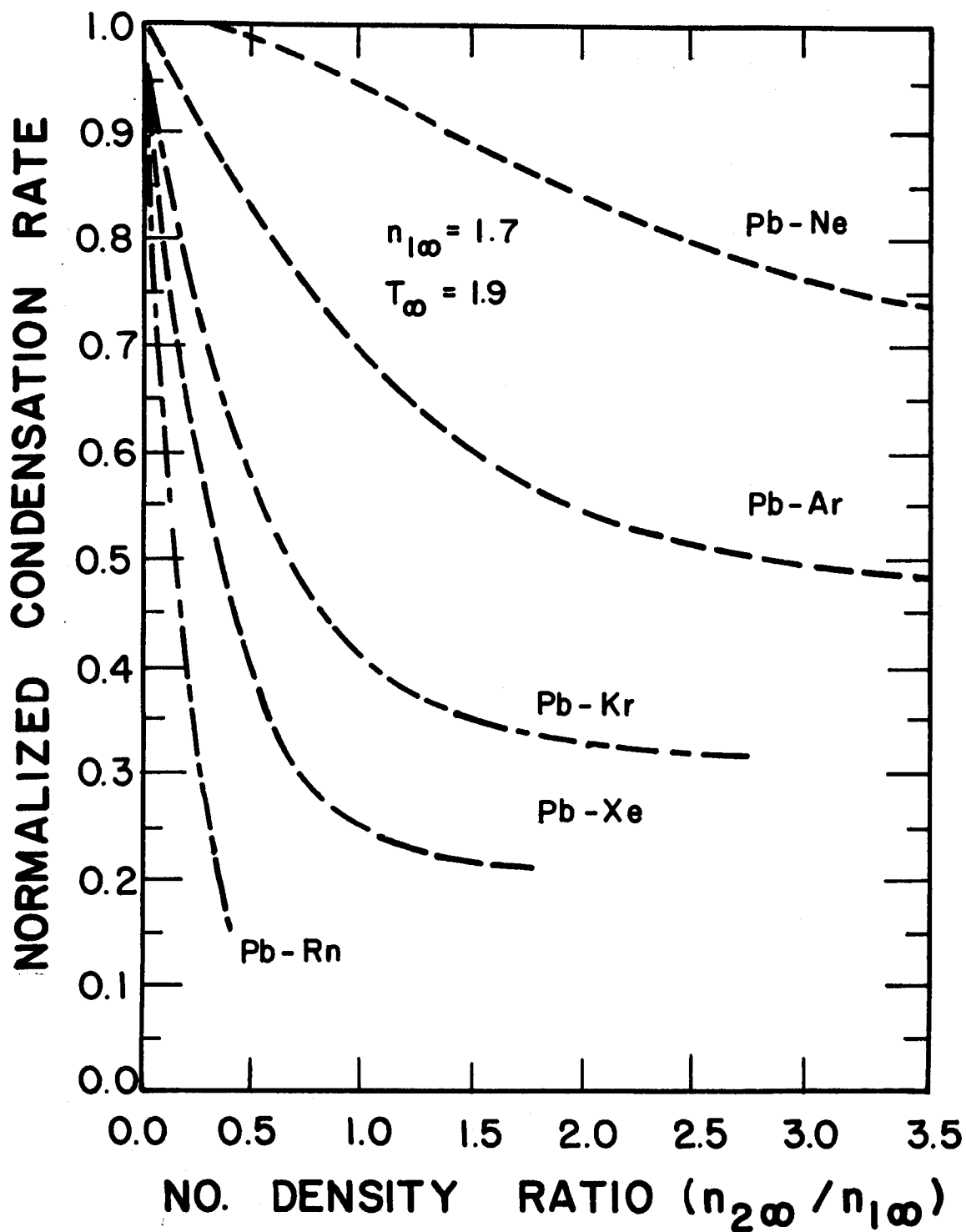


Fig. 6. Effect of noncondensable gas on Pb condensation rate.

# CONDENSATION RATE VS. BULK TEMPERATURE (Pb-Ar , Pb-Kr, Pb-Xe)

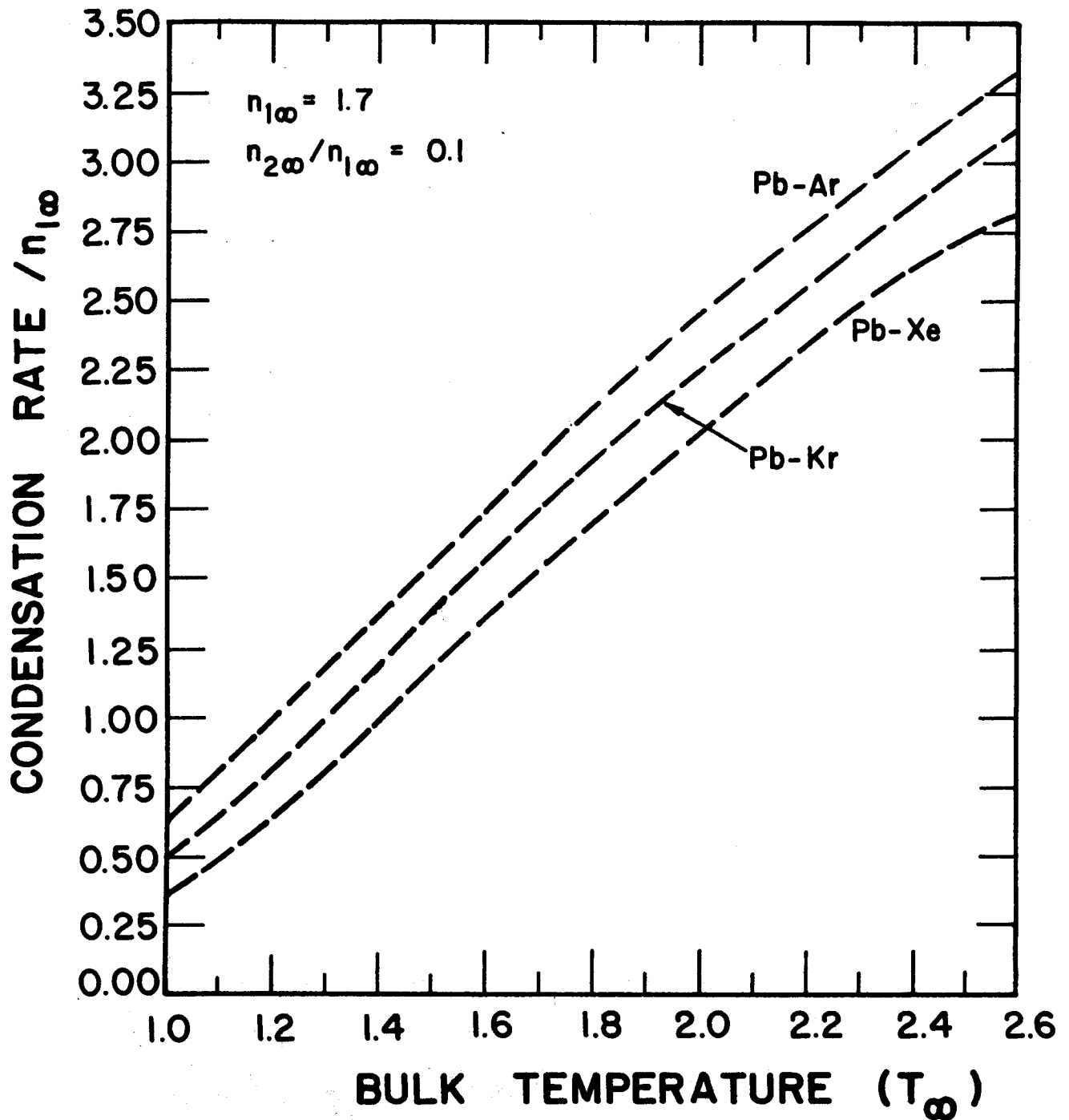


Fig. 7. Pb vapor condensation with different bulk temperatures.

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