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BUILDUP FACTORS FOR MAGNET SHIELDING  
IN TANDEM MIRROR FUSION REACTORS

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ABSTRACT

An approximate analytical approach is presented to estimate the shield required for protecting the superconducting magnets against streaming radiation in the penetrations of a tandem mirror reactor. The first step in the approach involves using a one-dimensional model to represent the actual three-dimensional problem. The problem is simplified further by constructing a set of buildup factors to determine the radiation effects in the magnets in analytical form. The method was applied to the MARS axicell design and was found to give damage rates a factor of  $\sim 1.3$  higher than those obtained from the detailed three-dimensional calculation.

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INTRODUCTION

In tandem mirror fusion reactors a number of penetrations are required for plasma fueling, heating and pumping.<sup>1,2</sup> Some of these penetrations, such as the neutral beam injector ducts, are large in size and must fit between the superconducting magnets used for plasma confinement. Radiation streaming into these ducts can lead to adverse radiation effects in the magnets. It is, therefore, essential to provide sufficient shielding to reduce radiation effects in the magnets to acceptable limits. This imposes a constraint on the spacing between magnets. The shielding requirements depend on the fusion power density, injection angle, duct size, magnet size and first wall radius. Hence iterations are required between plasma physics and neutronics calculations to produce a workable design. Detailed three-dimensional neutronics calculation is needed for each set of plasma parameters. This is very expensive and time consuming. In this work, we present an approach to determination of the shielding requirements using a simple analytical formulation. These formulas can be integrated with the plasma physics models to perform a complete parametric study.

APPROXIMATE TREATMENT OF RADIATION STREAMING

A schematic showing a duct with injection angle  $\theta_1$  that fits between two superconducting magnets in a tandem mirror fusion reactor is given in Fig. 1. The points on the duct wall closest to the left and right superconducting magnets are denoted I and II, respectively. The peak radiation effects in these magnets will result from the neutrons impinging on the duct wall around these

points. The first step in the present approach is to calculate the neutron wall loadings at these points. This gives a measure of the energy current of the fusion neutrons coming directly from the plasma.

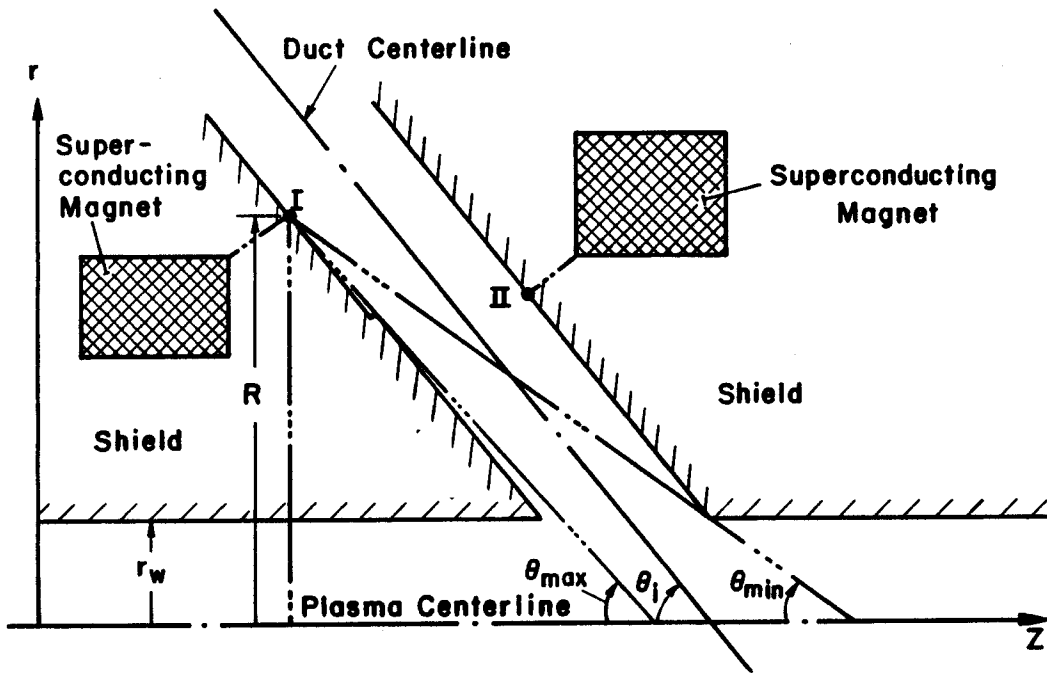


Fig. 1. A Schematic of a Neutral Beam Injector Duct Fitting Between Two Superconducting Magnets in a Tandem Mirror Reactor.

Assuming a line source of fusion neutrons along the plasma centerline, the neutron wall loading at either point I or II on the duct wall is given by

$$\Gamma = \left( r_w \Gamma_w / 2R \right) \left| \cos (\theta_i - \theta_{\min}) - \cos (\theta_i - \theta_{\max}) \right| , \quad (1)$$

where  $\Gamma_w$  is the wall loading at a first wall radius  $r_w$  in the plasma chamber,  $R$  is the radius of the point under consideration, and  $\theta_i$  is the angle between the duct wall and the plasma centerline. The angles  $\theta_{\min}$  and  $\theta_{\max}$  correspond to the boundaries of the plasma zone that will contribute direct neutrons to the point of interest on the duct wall as shown in Fig. 1. This formula can be used for any duct shape which will impact only the values of  $\theta_{\min}$  and  $\theta_{\max}$ .

The calculated wall loading gives only the contribution from the primary source neutrons produced in the plasma. It does not include the secondary component resulting from neutrons having collisions in the plasma chamber and succeeding in streaming into the duct. Although as many secondary neutrons as primary neutrons can stream into the duct, these secondary neutrons have a much softer spectrum and their contribution to the wall loading is small. In addition, the softer spectrum leads to smaller shield penetration.

The problem can then be modeled as a one-dimensional problem with a neutron source at the duct center that gives the same wall loading (at the point on the duct wall which has the largest contribution to the radiation effects in the magnet) as that resulting from the actual neutron source. Notice that

modeling the duct in a one-dimensional calculation will also account for the contribution of secondary neutrons and gamma photons resulting from primary streaming neutrons impinging on all of the duct wall. Using this approach will result in conservatively large shield thickness requirements because in the one-dimensional model most source neutrons are incident perpendicularly to the duct wall while in the actual case they are incident at an angle. This is expected to overestimate the required shield thickness by a distance on the order of a transport mean free path which is the distance beyond which a neutron "forgets" its original direction. However, the amount of overestimate will be decreased due to neglecting the contribution from secondary neutrons streaming from the plasma chamber in the wall loading calculation. Quantitative estimates of the impact of the different assumptions used in this approach will be presented shortly.

### BUILDUP FACTORS FOR MAGNET SHIELDING

The need for performing one-dimensional calculations can be eliminated by constructing a set of buildup factors for different shielding materials. These buildup factors can be used to determine the radiation effects in the magnet in analytical form. The shielding requirements for a superconducting magnet are determined by the radiation effects on different components of the magnet. The atomic displacement (dpa) rate in the stabilizer should not exceed a certain value in order to limit the resistivity increase as required for cryostability. The radiation dose absorbed in the insulator is also limited to preserve its mechanical and electrical integrity. The peak magnet heat load resulting from neutron and gamma heating should also be limited to ensure that the nuclear heat load on the magnet does not cause excessively high cryogenic refrigeration and plant cost. The relative impact of these different effects on the shield design depends on the specific limits which depend in turn on the operational and magnet design conditions. It is clear that while some of these radiation effects are related to the neutron flux alone, others are determined by both neutron and gamma fluxes.

Buildup factors have been calculated for the different magnet response functions which impact the shield design. The shield composition used in this work is that used in the Mirror Advanced Reactor Study (MARS) design.<sup>1</sup> It consists of 80 v/o W (0.9 d.f.), 10 v/o Fe-1422, and 10 v/o H<sub>2</sub>O. The buildup factor for the *i*th response function from a point source is defined as

$$B_{pi}(\Sigma_t(E_0), r) = R_{bi}(r)/R_{ui}(r) \quad , \quad (2)$$

where  $R_{ui}(r)$  is the uncollided *i*th response function at distance *r* from the source given by

$$R_{ui}(r) = \Sigma_i(E_0) e^{-\Sigma_t(E_0)r} / 4\pi r^2 \quad . \quad (3)$$

$\Sigma_t(E_0)$  is the total macroscopic cross section of the shield at the source neutron energy  $E_0$ .  $R_{bi}(r)$  is the *i*th buildup response function for a shield thickness *r*.  $\Sigma_i(E_0)$  is the macroscopic cross section used to calculate the *i*th response function in the particular magnet component at the source energy. For example, if the response function of interest is the nuclear heating in the magnet,  $\Sigma_i(E_0)$  is the macroscopic kerma factor for the magnet at  $E_0$ .

Several one-dimensional P<sub>3</sub>-S<sub>8</sub> calculations have been performed in spher-

ical geometry using the discrete ordinates code ONEDANT<sup>3</sup> with a coupled 46 neutron - 21 gamma group cross section library based on the VITAMIN-C data library<sup>4</sup> and the MACKLIB-IV-82 response library.<sup>5</sup> A point source emitting isotropic 14.1 MeV neutrons was used followed by a shield of varying thickness and a superconducting magnet with a design similar to that used for the MARS high field magnets. The variation with shield thickness of the different magnet response functions was obtained. Equation (2) was then used to calculate the point buildup factors for the copper stabilizer dpa rate, the magnet heat load, and the radiation dose rate in the organic insulator. The results are shown in Fig. 2.

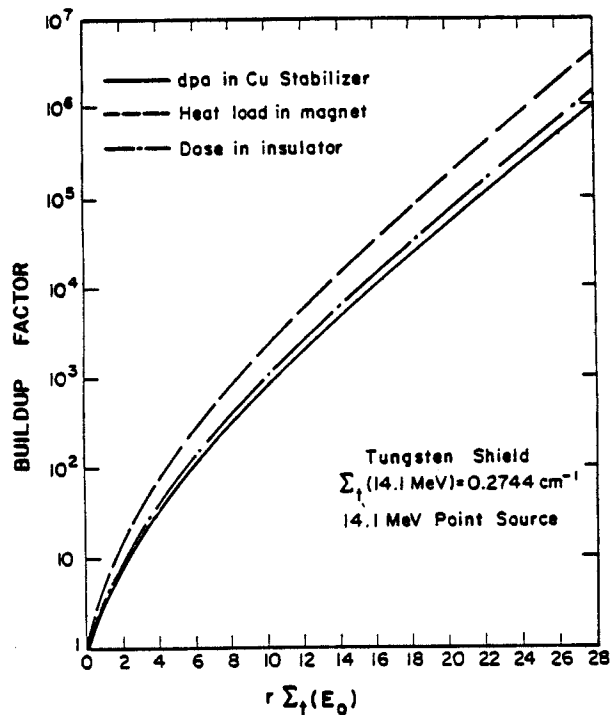


Fig. 2. Buildup Factors in a Tungsten Shield for Isotropic Point 14.1 MeV Neutron Source.

As the shield thickness increases from zero to a few mean free paths, the energy spectra of neutrons and gamma photons change considerably. Different buildup factors are obtained for the different response functions depending on the relative gamma contribution and the energy dependence of the cross sections for the different response functions. The results show that after a few mean free paths, the neutron and gamma spectra assume fixed shapes. This stems from the fact that the mean free path for the 14.1 MeV source neutrons is larger than that for lower energy neutrons and gammas. This results in the same buildup factor variation with shield thickness regardless of the response function. However, this will not be true if a light shield material is used with its large gamma mean free path. In this case different asymptotic buildup variations will be obtained for response functions which include the gamma contribution. Nevertheless, the same asymptotic buildup variation is still obtained for neutron response functions.

For computational purposes it is convenient to express the buildup factor as the sum of exponentials in the standard Taylor form used with gamma buildup factors.<sup>6</sup> The point buildup factor for the *i*th response function is, therefore, given by

$$B_{pi}(\Sigma_t(E_0) r) = \sum_{j=1}^J \alpha_{ij} e^{-\lambda_{ij} \Sigma_t(E_0) r} \quad (4)$$

In general, the parameters  $\alpha_{ij}$  and  $\lambda_{ij}$  depend on the source energy, shielding material, and response function. For the sake of generality, the lambdas in

Eq. (4) are assumed to vary for the different response functions. However, for heavy shielding materials, where similar asymptotic variations of the buildup factor are obtained for the different response functions, a single set of lambdas can be used with the subsequent simplification of the buildup factor tables.

The FITPULS<sup>7</sup> code was used to fit the buildup factors given in Fig. 2 to the Taylor form represented by Eq. (4) using a nonlinear least-squares method. The buildup factor parameters for the tungsten shield and 14.1 MeV source energy are given in Table 1 for the different response functions. Use of three exponentials was found to give an adequate fit for the tungsten shield buildup factors with less than 10% deviation. Similar buildup factor tables can be generated for different shielding materials.

Table 1. Buildup Factor Parameters for Tungsten Shield

	$\lambda_1 = -0.3563$	$\lambda_2 = -0.2691$	$\lambda_3 = -0.1094$
Response Function	$\alpha_1$	$\alpha_2$	$\alpha_3$
dpa in Cu	54.54	-81.62	28.50
Heat Load in Magnet	217.70	-369.10	158.80
Dose in Insulator	76.03	-117.31	43.17

The point buildup factor can be used to calculate the radiation effects in the magnet resulting from neutron sources of different shapes. The buildup ith response function at a point  $\underline{r}$  resulting from a unit isotropic source at point  $\underline{r}'$  is given by

$$R_{bi}(|\underline{r}-\underline{r}'|) = R_{ui}(|\underline{r}-\underline{r}'|) B_{pi}(\Sigma_t(E_0)|\underline{r}-\underline{r}'|) , \quad (5)$$

where  $B_{pi}(\Sigma_t(E_0)|\underline{r}-\underline{r}'|)$  is given by Eq. (4) and  $R_{ui}(|\underline{r}-\underline{r}'|)$  is the uncollided response function given by Eq. (3). For a source  $S(\underline{r}')$  distributed over a volume  $V$ , the buildup ith response function is, therefore, given by

$$R_{bi}(\underline{r}) = \int_V S(\underline{r}') R_{bi}(|\underline{r}-\underline{r}'|) d\underline{r}' . \quad (6)$$

For a line source emitting 1 n/cm-s and surrounded by a shield of thickness  $a$ , the integration in Eq. (6) yields

$$R_{bi}(a) = \Sigma_1(E_0)/2\pi a \sum_{j=1}^J \alpha_{ij} K_{i1} \{ (1 + \alpha_{ij}) \Sigma_t(E_0) a \} . \quad (7)$$

In general, the macroscopic cross section  $\Sigma_1(E_0)$  depends on the magnet design, particularly for response functions which have contributions from more than one species. However, the buildup factors and, consequently, their parameters are nearly independent of the magnet design as they depend on the energy variation of the macroscopic cross section for the particular response function and not on its magnitude. Such energy variation is nearly the same for the different materials used in current magnet designs.



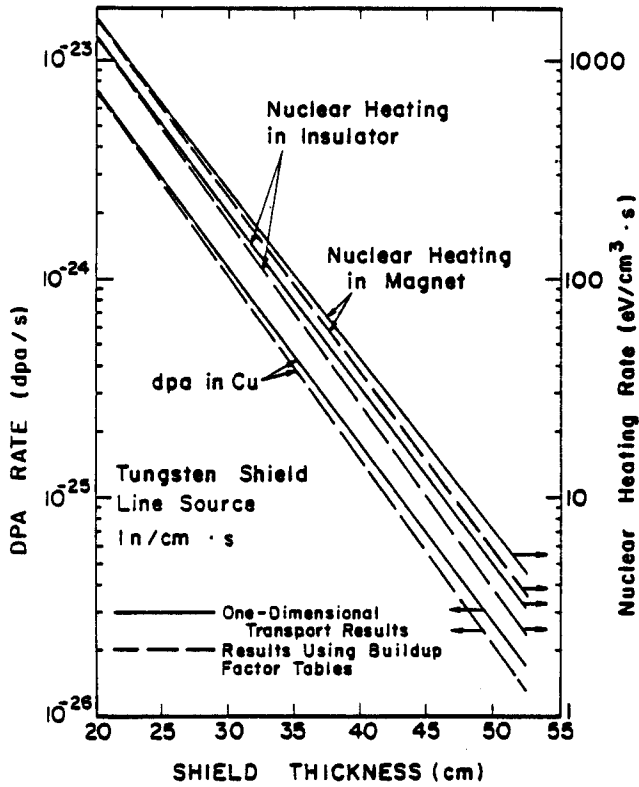


Fig. 3. Comparison of the Response Functions Obtained Using the Buildup Factor Table and Those from the One-Dimensional Transport Calculation for a Line Source.

Equation (7) was used to calculate the different response functions for a line source with different shield thickness. The macroscopic cross sections for Cu dpa, magnet heating, and insulator heating at 14.1 MeV used in the calculation are  $3.74 \times 10^{-21}$  dpa  $\text{cm}^2$ ,  $2.74 \times 10^5$  eV  $\text{cm}^{-1}$ , and  $4.92 \times 10^5$  eV  $\text{cm}^{-1}$ , respectively. These values were calculated for the MARS high field magnet design. The results are compared in Fig. 3 to the exact results obtained by performing one-dimensional cylindrical geometry ONEDANT calculations. The results agree to within 25%. The deviation can result in a difference of only a quarter of a mean free path in the required shield thickness. Using the simple analytical equations based on the buildup factors for shielding analysis is, therefore, justified. For a planar source emitting  $1 \text{ n/cm}^2 \cdot \text{s}$ , which is incident isotropically on the surface of a shield of thickness  $a$ , Eq. (6) yields

$$R_{bi}(a) = \sum_i(E_o) \sum_{j=1}^J \alpha_{ij} E_i \{ (1 + \lambda_{ij}) \Sigma_t(E_o) a \} . \quad (8)$$

The good agreement between the results of this analytic expression based on the buildup factor and the plane geometry one-dimensional calculation is shown in Fig. 4.

#### APPLICATION TO MARS AXICELL MAGNET SHIELDING

In the MARS design version employing a high field axicell to mirror confine plasma particles, neutral beams are introduced between the split axicell superconducting and normal insert coils. This creates a shielding problem as the neutrons stream up the ducts and produce excessive radiation damage in the front corner of the superconducting coil. The ducts are oriented at  $90^\circ$  to the axis and the duct opening of 32.5 cm by 48 cm looks directly at the peak 14.1 MeV neutron source creating a maximum neutron wall loading of  $4.08 \text{ MW/m}^2$  at the first wall of the plasma chamber of radius 30 cm. At the point on the duct wall nearest the front corner of the superconducting coil, a space of only 33.75 cm is available for shielding in addition to a 5.8 cm equivalent thickness of steel provided by the magnet case and cryostat.

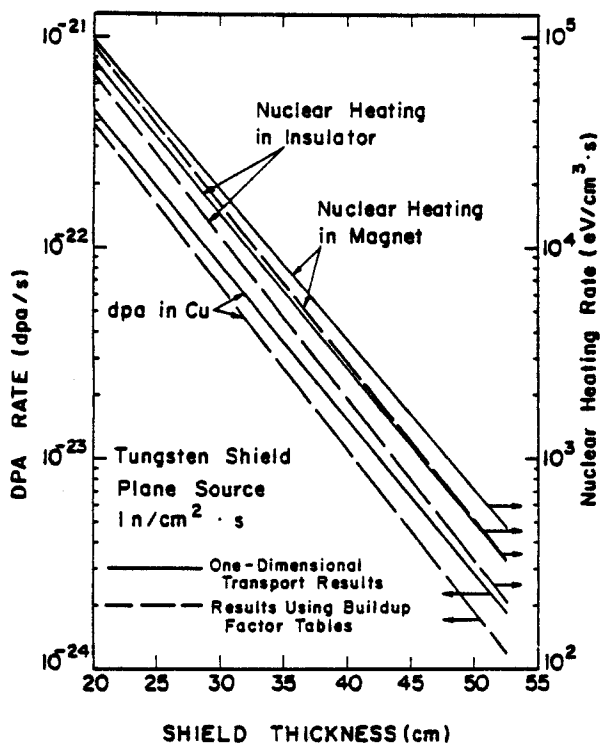


Fig. 4. Comparison of the Response Functions Obtained Using the Buildup Factor Table and Those from the One-Dimensional Transport Calculation for a Plane Source.

results of 20000 histories indicated that a peak dpa rate of  $3.6 \times 10^{-4}$  dpa/per full power year (FPY) in the copper stabilizer will result from radiation streaming into the duct. This is about an order of magnitude larger than the design limit. The statistical uncertainty in this result is  $\sim 50\%$  and the run took 80 minutes on the Cray computer.

The approach presented here was applied to this problem. As a first step the neutron wall loading at the point on the duct wall closest to the magnet corner was calculated. This point is at a radius of 130 cm. For this duct geometry,  $\theta_{\max} = \theta_i = 90^\circ$ , and  $\theta_{\min} = 72^\circ$ . Substituting in Eq. (1), yields a wall loading of  $0.023 \text{ MW/m}^2$ .

The problem was then modeled as a one-dimensional plane geometry problem with the source normalized to give the above calculated wall loading. The neutron source incident on the wall was assumed to have an anisotropic distribution with all neutrons incident uniformly over the angle range  $0 \leq \cos \theta \leq \cos \theta_{\min}$ , where  $\theta$  is the angle between the source direction and the normal to the surface. This accounts for the angular distribution of streaming direct source neutrons as they impinge on the duct wall. The steel in the magnet case and cryostat was replaced by an equivalent thickness of tungsten shield. Previous one-dimensional calculations indicated that the 1/10 folding distances for steel and tungsten are 15 and 12 cm, respectively. Hence, the steel is replaced by an equivalent 4.65 cm W shield. Therefore, a total tungsten shield thickness of 38.4 cm was used in the one-dimensional calculations.

In order to assess the shielding problem posed by the duct, a three-dimensional radiation transport model was implemented using the Monte Carlo code MCNP<sup>8</sup> and a cross section data library based on ENDF/B-V. A trapping surface was located at the duct entrance where all crossing particles were counted according to energy and angle bins. This was used to represent a surface source in the modeling of the duct itself. The geometrical model used is shown in Fig. 5 where only the corner of the coil was modeled as other radiation damage effects in the magnet are less severe. In order to reduce the statistical uncertainties in the quantities of interest, the angle of source neutrons was biased toward the corner of the magnet and splitting surfaces were used to increase the number of particles as they move in the direction of interest. Considering the design limits on the different radiation effects as determined by the magnet design and the reactor's operational conditions, it was found that the main limiting factor which will drive the shield design is the dpa rate in the copper stabilizer. The calculational

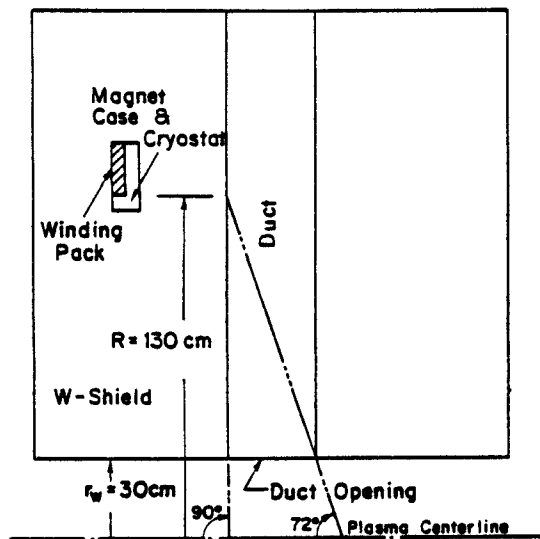


Fig. 5. Geometrical Model for the Three-Dimensional Neutronics Calculation of the MARS Axicell.

of the two assumptions made in the analysis is to overestimate the damage rate by a factor of 1.8.

The problem is simplified further by using Eq. (4) for the buildup factor with Table 1 instead of performing the one-dimensional calculation. The resulting dpa rate is  $4.72 \times 10^{-4}$  dpa/FPY. This implies that using the buildup factor tables tends to underestimate the damage rate by 26%. The damage rate obtained using the complete analytical approach presented here is a factor of 1.3 larger than that obtained from the much more expensive three-dimensional result which also involves large statistical uncertainty. Therefore, the present approach will result in a conservative estimate of the required shield thickness which is only about a quarter mean free path larger than that obtained from the detailed three-dimensional analysis.

The damage rate obtained in the magnet exceeds the design limit of  $3.8 \times 10^{-5}$  dpa/FPY. In order to determine the required spacing between the duct wall and the magnet for the radiation effects not to exceed the design criteria, it will be essential to perform several expensive three-dimensional calculations. However, using the analytical approach presented here, one can determine the required shield thickness with minimal extra computational cost. The required shield thickness was found to be 52.7 cm. This implies that the magnet must be moved away from the duct wall by at least an extra 14.3 cm. The spacing between the two axicell superconducting magnets must, therefore, be increased by 28.6 cm to accommodate the shield required for adequate protection of the magnets against streaming neutrons. These results represented a major contributing factor to the abandonment of the use of neutral beam injection and split axicell coils in later versions of MARS.

The resulting dpa rate in the Cu stabilizer is  $2.15 \times 10^{-4}$  dpa/FPY. This is less than a factor of two lower than the result of the three-dimensional calculation. This underestimation is attributed to neglecting the contribution to the wall loading from the secondary neutrons streaming from the plasma chamber into the duct.

Another one-dimensional calculation was performed with an isotropic source at the center of the duct which produces the same wall loading at the duct wall. The resulting dpa rate is  $6.42 \times 10^{-4}$  dpa/FPY. This indicates that neglecting the source anisotropy tends to overestimate the radiation effects by a factor of three. However, comparing this result with the three-dimensional results, it is clear that the combined effect

## SUMMARY

A one-dimensional model of the shielding problem for superconducting magnets near a penetration in a fusion reactor is presented. For the resulting problem an adaptation of the buildup factor method of radiation shielding calculations has been applied to the neutron and gamma shielding problem.

Point source results from a spherical multigroup discrete ordinates model of a tungsten shield have been fitted to the Taylor form of the buildup factor for nuclear heating, displacements per atom (dpa) in copper, and dose in a polyimide insulator responses. These buildup factors have been applied to uncollided flux models for line and plane sources as well as a realistic geometry tandem mirror penetration.

The results are conservative, yielding responses on the order of a factor of 1.3 higher than those obtained from a detailed three-dimensional model. This amounts to only a few centimeters additional shield and the quick inexpensive results which allow easy interpolation during design iterations have been found a valuable tool in shield design for fusion reactors.

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