Tokamak Scaling in Terms of Critical Fields and Plasma Stability

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Golovin, Dnestrovsky and Kostomarov[1] have reviewed tokamak
scaling from the point of view of reactor power, wall loading and
magnetic field limitations. It is also instructive to consider
scaling in terms of the plasma stability factor (q), blanket and
shield thickness (T_B) and β_p where $\beta_p = 2\mu nk(T_e+T_i)/B_p^2$. In the
present analysis we seek to determine the plasma radius (a),
aspect ratio (A = R/a), and maximum field at the toroidal field
superconductor (B_m) for a reactor operating with uniform plasma
density (n = 1.10^{14}/cm^3) and temperature ($T_e = T_i$ = 15 keV). To
carry out this simple analysis we use the definition q = aB_t/RB_p
where R is the major radius and the subscript refers to the
toroidal field (t) or poloidal field (p) at R.

Using y = a/r_w, where r_w is the radius to the vacuum wall,
we write

$$B_m = \frac{RB_t}{R - T_B - r_w} \qquad . \tag{1}$$

Coupling this with the definition of q and applying two different
stability limit conditions on β_p such that[2] $\beta_p \leq A$ or $\beta_p \leq A^{1/2}$,
we find

$$B_m = \frac{q(4\mu n T_i A)^{1/2}}{1 - T_B/Aa - 1/Ay} \quad \text{or} \quad \frac{q(4\mu n T_i A^{3/2})^{1/2}}{1 - T_B/Aa - 1/Ay} \quad . \tag{2), (3}$$

Figure 1 is a plot of B_m at $q = 2.0$, $y = 0.8$ for different T_B and A which shows that no solutions exist for Eq. (3) and NbTi superconductors having critical fields around 100 kG. Solutions do exist for Nb_3Sn, which essentially has unlimited critical fields but is very expensive and stress loading may limit its use to fields near that of NbTi. .

Assuming cost and stress considerations limit B_m to 100 kG, we seek solutions to Eq. (2) and (3) for various values of the blanket thickness and stability factor. These are plotted in Fig. 2 to give the plasma radius as a function of the aspect ratio for the conditions shown. Solutions do not exist for small values of A since the major radius must be larger than the blanket thickness plus vacuum wall radius and allow room for superconductor windings, structure and insulation as well as an iron excitation core. The curves indicate that, for a reasonable size reactor, we need $\beta_p \sim A$ and $q \sim 1.5$ or less; otherwise the reactor size and power output becomes excessive. For comparison purposes, the star in Fig. 2 represents a reactor power level of 1500 MW(th) and the power increases as $n^2 a^3 A$. On the other hand β_p is proportional to n and the magnetic field needed for confinement is proportional to $n^{1/2}$.

The above analysis neglects the α pressure. It should be noted that, at the temperature used here, $p_\alpha \sim 120 f_b p$ where f_b is the fractional burnup, and p_α and p are the α and plasma pressures, respectively. At 1% burnup, $p_\alpha \approx p$, so the β_p is doubled. Since β_p is limited by equilibrium and stability considerations, the α pressure can seriously reduce the maximum ion pressure that can be confined. This also points to the need to make the upper limit on β_p as large as possible.

References

1. I.N. Golovin, Yu.N. Dnestrovsky and D.P. Kostomarov, "Tokamak as a Possible Fusion Reactor - Comparison With Other C.T.R. Devices", Proceedings B.N.E.S. Nuclear Fusion Reactor Conference, p.194, Culham 1969.

2. V.S. Mukhovatov, V.D. Shafranov, "Plasma Equilibrium in a Tokamak", Nuclear Fusion 11, 605 (1971).

Figure showing FIELD AT SUPERCONDUCTOR (kG) versus ASPECT RATIO.

$T_e = T_i = 15 \, keV$

$q = 2.0$

$y = 0.8$

$\beta_p = A^{1/2}$ ------

$\beta_p = A$ ———

PLASMA RADIUS (m)   BLANKET THICKNESS (m)

1.5 — 2.5	
2.0	2.5
{1.5 / 2.5}	{1.5 / 2.5}
2.0	1.5
2.5	1.5
1.5	2.5
2.0	2.5
{1.5 / 2.5}	{1.5 / 2.5}
2.0	1.5
2.5	1.5

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