Perturbation Theory for Neutron and Photon Transport Calculations in Controlled Fusion Blankets and Shields

R.W. Conn

June 1973

UWFDM-48

FUSION TECHNOLOGY INSTITUTE

UNIVERSITY OF WISCONSIN

MADISON WISCONSIN

Perturbation Theory for Neutron and Photon Transport Calculations in Controlled Fusion Blankets and Shields

R.W. Conn

Fusion Technology Institute
University of Wisconsin
1500 Engineering Drive
Madison, WI 53706

http://fti.neep.wisc.edu

June 1973

UWFDM-48

PERTURBATION THEORY FOR NEUTRON AND PHOTON TRANSPORT CALCULATIONS IN CONTROLLED FUSION BLANKETS AND SHIELDS*

by

Robert Conn
Nuclear Engineering Department
The University of Wisconsin
Madison, Wisconsin 53706

June 1973

FDM 48

*Paper given at Special Session on The Use of Perturbation Theory and Variational Methods, Annual Meeting of American Nuclear Society, Chicago, Illinois, June, 1973. Trans. Am. Nucl. Soc. 16 (1973) 343.

There has been a great deal of excitment in the fusion community in the past four years, due mainly to the recent successes of the Tokamak machines. This has led to an increased interest in technology studies on Controlled Thermonuclear Reactors (CTR) and raised the problems of neutron-photon transport and nuclear data to levels of greater importance and interest. This paper will present an overview of the CTR technology problem and indicate the role and importance of transport calculations in this area. This will then lead naturally to a discussion of the use of variational methods for CTR blanket and shield studies.

The recent surge of research on Tokamak devices followed the report by the Russians in 1969 of their successes with the T-3 Tokamak.[1] The important result that followed from this and other experiments is that the plasma parameters appear to follow favorable, and theoretically understood, scaling laws. In particular, the plasma density, temperature and confinement time are predicted to get better in larger machines leading to the hope that scaling in size will get us to the ignition experiments.

Based on these very real advances, people started thinking seriously four or so years ago about the technology problems a controlled thermonuclear reactor would pose assuming the plasma problems can be overcome. Work began in several groups around the country and we at Wisconsin have had a fairly large group active in this area. It was our feeling, and that of others as well, that to assess the technological problems posed by a power-producing CTR, it would be best to choose a specific plasma containment concept and develop a design for a power reactor based on that approach. Since the Tokamaks have had such success recently, it was logical to examine this concept first and try to assess

just what technological problems would be presented by such a reactor.

Figure 1 indicates the basic ideas involved in the Tokamak confinement concept. Essentially, the plasma acts as a single turn secondary of a transformer. A current pulse through the primary windings induces a voltage toroidally around the secondary. This serves to break down the gas in the chamber and then to drive a current in the plasma thus produced. The plasma loop is confined by a strong toroidal field produced by the solenoidal windings. In addition, a poloidal magnetic field is generated by the current carried in the plasma. Actually, with just these two fields, the plasma would expand outward and it is necessary to have either a conducting shell, in which image currents keep the plasma in place, or extra coils, as indicated in the figure, to produce a third, vertical field.

While all present experiments use either hydrogen or deuterium, the first fusion reactor will almost certainly be a mixture of deuterium and tritium since such a D-T plasma should ignite at ion temperatures between 4.5 KeV and 6 KeV and operate at between 10 and 30 KeV. These are much lower temperatures than are possible with any other fuel mixture, such as D-D or D-^{3}He. Figure 2 recalls the basic D-T reaction and also indicates how one can breed the tritium required as fuel. The reaction products of the D-T reaction are a 3.5 MeV alpha particle and a 14.1 MeV neutron. The α-particle, being charged, is trapped in the plasma by the magnetic field and serves to heat the plasma by slowing down via collision with the electrons and ions of the plasma. The neutrons however escape freely and must be slowed down to extract their energy and must be captured predominantly in ^{6}Li or ^{7}Li to produce tritium. This function is performed by a region surrounding the plasma

zone commonly called a blanket region. Behind this blanket is a shield designed to protect the large, superconducting magnets that produce the main toroidal field.

Figure 3 shows a cross-sectional view of a CTR Tokamak reactor taken from our recent conceptual design study at the University of Wisconsin.[2] The central zone is the reacting plasma and is followed, moving outward, by a blanket, a shield, and the main, superconducting, toroidal field coils. The slots through the blanket and shield are there to allow particles which diffuse out of the plasma to be diverted away from the first wall. This is achieved by the superconducting divertor field coils which create a magnetic field configuration such that the desired particle diversion is achieved. The particles, and their energy, are removed by collector plates in the zone above and below the blanket and shield.

With this overview, it is not difficult to see the importance of neutron and photon transport for CTR reactors. A schematic of a CTR blanket and shield is shown in figure 4 which sketches in somewhat more detail the general structure that has evolved for such systems. In what will be discussed, we have in mind using liquid lithium as the heat transfer fluid, although lithium bearing salts, in particular, Li-Be-F (Flibe), and helium cooling are also candidates. The main point to keep in mind is the overall structure of the problem. The plasma is a source of 14 MeV neutrons and the blanket and shield together are media ranging in thickness between 1.5 and 2.5 meters. Thus, the problem is quite similar, in structure, to shielding problems although reactions both near to, and far from, the source are important in CTR calculations.

To summarize then, figure 5 indicates that the role of the blanket region in a CTR reactor is to amplify and extract the energy of the incident neutrons and to breed tritium. Energy amplification comes through capture reactions, such as $^{6}Li(n,\alpha)T$ reaction in which 4.786 MeV is released per event. On the other hand, the role of a shield in CTR systems is to reduce the energy deposition, and thereby the refrigeration load, in the main magnets. It is also desirable to reduce radiation damage to the magnet stabilizer, usually copper, and to the superinsulation, namely, mylar. Actually, our recent studies have indicated that reduction of the energy deposition is the most important factor. We find that between 1/2 and 1 kilowatt is required to remove each watt of power deposited in the magnets.

To assess and compare different blanket and/or shield designs, it is necessary to have some quantitative indication of the performance of these zones. In shielding calculations, the quantitative indication is generally provided by some dose rate in a region far from the source. In CTR problems, these performance indices can be somewhat more varied and a list of them is given in figure 6. The first two indices are related to blanket performance, as indicated previously. The third and fourth indices, relating to charged particle production and transmutation rates, are very important for materials radiation damage, heating and reactor safety and hazards analyses. The fifth through seventh indices relate to the performance of the shield design, vis-a-vis protecting the superconducting magnets. The final index, cost, brings in the idea of optimum design, that is, one must optimimize the first seven indices consistent with either minimizing the total cost or, at least, not exceeding some upper dollat limit.

The decisions that one must make in CTR blanket and shield design are shown in figure 7. In the selection of structural materials, one must consider such diverse criteria as the maximum operating temperature, neutronics properties, after heat problems, and the questions of material availability and ease of fabrication. The selection of a coolant is governed by criteria such as heat transfer capability, pumping cost and materials compatability. One must also select the chemical form of the breeding material, namely, natural lithium, lithium enriched in ^{6}Li, or lithium bearing salts. This choice is required whether or not the material used for breeding is also used as the coolant. Finally, one must decide on the size and composition of the many zones that comprise a blanket and shield design.

What one wants to know in the end is just how sensitive the various performance indices are to the design decisions outlined above. A separate question, though also very important, is how sensitive the performance indices are to uncertainties in the nuclear data employed.

It is at this point that the potential usefulness of variational methods in CTR blanket and shield studies becomes apparent. The performance indices of interest, except cost, are all reaction rates, or linear functionals in the flux. Hence, once a variational expression for a particular performance index is obtained, one can use it to examine the sensitivity of that performance index to either design decisions or uncertainties in nuclear data. Let me proceed, therefore, to outline the variational theory required for CTR problems and then indicate, by some examples, how the theory can be applied.[3]

In general, we are interested in a functional, $G[\phi]$, as indicated in figure 8. The appropriate equations of motion are the Boltzmann transport equation and the corresponding adjoint equation. $G'[\phi]$ denotes the functional derivative of G with respect to ϕ. A variational expression for $G[\phi]$ is the general Roussopoulos functional shown on the next slide. This functional is stationary about $G[\phi]$ when ϕ and ϕ^* satisfy the Euler equations indicated. That is, the expression for the functional, $I_B[\phi^*,\phi\ ]$, when the exact solution of

$$L\phi_{ex} = S$$

and

$$L^*\phi^*_{ex} = G'[\phi_{ex}]$$

are inserted into I_B, $G[\phi_{ex}]$.
When trial functions ϕ_t and ϕ^*_t are used, they can be expressed in terms of the differences, $\delta\phi$ and $\delta\phi^*$, from their exact ϕ_{ex} and ϕ^*_{ex}. In this case, $I_B[\phi^*_t,\phi_t]$, reduces to the expression given in the figure, with the error term a bilinear form in $\delta\phi^*$ and $\delta\phi$, i.e., second order.

It is often convenient to express the functional, I_B, in terms of trial, or reference fluxes and adjoints and also in terms of reference operators, L_o and L^*_o, which produced the trial functions. The procedure for doing this is outlined in figure 9. We express L and L^* in terms of reference operators L_o and L^*_o and perturbing operators ΔL and ΔL^*. The reference trial functions, ϕ_{ref} and ϕ^*_{ref}, are solutions of the equations as shown in the slide. Using this breakup, the expression for I_B is as shown. In practice, this is the equation one works with.

In CTR blanket and shield work, the functionals most frequently encountered are linear functionals (reaction rates.) The appropriate variational expression in this case are shown in the next figure. Another variational form that is widely used is the Schwinger variational functional, which can be readily obtained by the procedure outlined in the figure. The important difference between I_F and I_B is that I_F is independent of the normalization of ϕ and ϕ^*. Pomraning has derived a generalization of the Schwinger form for general functionals, $G[\phi]$.[4]

It can sometimes be helpful to use a linear combination of trial fluxes and adjoints, as for example, is commonly done in synthesis methods. This procedure is outlined in figure 11. Rendering I_B stationary with respect to the expansion coefficients c_i and D_i leads to the compact expression given at the bottom. $\underline{\underline{M}}$ is a matrix with elements M_{ij} as shown in the slide.

Now with this outline of the general formulation, let us turn to specific application of these ideas to CTR problems. The next figure, 12, lists two functionals, for a reaction rate and a ratio of reaction rates, and the corresponding functional derivatives for each case. The functional derivative is the source to the adjoint problem and therefore is important to know. For the reaction rate, this is straightforward. For a ratio of reaction rates, which is not a linear functional, $G'[\phi]$ depends on ϕ. However, since we ultimately evaluate a variational expression, it is possible to use the trial flux in the expression for $G'[\phi]$, evaluate a reference adjoint using this functional derivative as the adjoint source, and then use the resulting adjoint together with the trial flux

in the variational expression. The value of the functional will of course still have second order errors.

Figure 13 indicates two additional functionals of interest, the ratio of the total flux at two different points in the blanket or shield, and the neutron and gamma heating rate in a particular zone, . This last functional is also a linear functional of the flux, as is a reation rate, but has Kerma factors rather than cross sections multiplying the flux. The expressions given for H(X) is a sum over isotopes i and energy groups g of the neutron plus gamma kerma factors, $K_{n,i}^{g}$ and $K_{\gamma,i}^{g}$, respectively, times the group flux, $\phi^{g}(x)$. The adjoint source therefore is directly proportional to these kerma factors.

The calculation of neutron and gamma heating rates is a central problem in the CTR blanket and shield area, as it is for fast reactor blanket problems. However, unlike fission reactors, where the fission products provide most of the energy and fission neutron heating is clearly small by comparison, the energy available from fusion is primarily that in the fusion neutrons, and the secondary gammas from neutron capture. The 3.5 MeV of energy in the alpha particle (out of a total of 20-22 MeV per fusion) is mostly deposited in the plasma and subsequently radiated to the first wall. Thus, it is most important to know neutron and gamma Kerma factors.

Since the term "Kerma factor" or "fluence-to-Kerma factor" may not be totally familar, it is defined in figure 14 KERMA is an acronym for kinetic energy released in materials. The general expression for the neutron plus gamma heating rate is given on the slide. It involves a sum over materials j of density factors N_j times what is defined to be

the microscopic Kerma factor, namely

$$K_j(E) = \sum_i \sigma_{ij}(E)\, E_{ij}(E) .$$

Here, $\sigma_{ij}(E)$ is the reaction cross section of type i and $E_{ij}(E)$ is the energy released in material j per reaction i induced by a neutron or gamma of energy E. Actually, it is convenient to define separately neutron and gamma Kerma factors as given in figure 15. The energy released per reaction is obtained by solving the Kinematic equations for each particular reaction. The gamma Kerma factor, as indicated on the slide, has three explicit pieces corresponding, respectively, to photoelectric interactions, pair production, and Compton absorption.

Abdou and Maynard[5] at Wisconsin have carried out a very careful study of this problem and the next two figures indicate some of their results. Figure 16 is a graph of microscopic neutron Kerma factors for Fe, Cr, and Ni, the prime constituents of stainless steel, as a function of neutron energy. These would then be combined to form Kerma factors for stainless and used to calculate neutron heating rates in the structural material of the blanket and shield.

Figure 17, shows neutron Kerma factors for ^6Li and ^7Li, the main materials for tritium breeding, as a function of neutron energy. Both these slides also demonstrate explicitly what the form of the adjoint source would be if the functional of interest were, say, the neutron heating rates in natural lithium. We would simply construct the macroscopic kerma factor for natural lithium from the microscopic data shown and this would be, explicitly, the adjoint source. It is of course distributed in space according to where the

lithium occurs in the blanket.

Let me turn now to some specific results of an illustrative nature to indicate both the viability of the variational approach for CTR problems and to indicate just how the approach can be used.[3] Figure 18 indicates that we require as trial functions only one reference flux but perhaps several reference adjoints. That is, we require a reference adjoint corresponding to each functional of interest. Thus, if we are interested in the tritium breeding ratio and the total heat deposition in a CTR blanket, we would need a reference flux plus two reference adjoints, one corresponding to the total tritium production rate and the other corresponding to the total energy released. Importantly, once these trial functions are in hand, we can examine the effects of many design decisions or nuclear data uncertainties without performing additional neutron-photon transport calculations. This is a most important point.

To indicate explicitly what expressions must be evaluated, the discrete ordinates form of the Roussopoulos functional is given in figure 19. The notation is standard and the mathematical form of I_B written in terms of ΔL, ΔS, ϕ_{ref} and ϕ^*_{ref} has been used. Clearly, if transfer matrices or the source are unaffected by the perturbation of interest, then those parts of the functional are automatically zero.

The nuclear data required for neutron-photon transport studies, in general, and variational studies in particular, are given in figure 20. These areas are common to the needs of shielding analysts generally. The second and third topics refer to particular types required for either tritium production, heating, or radiation damage

considerations.

Variational calculations were performed on what has been called the "standard blanket", shown in figure 21, which has been used previously for comparison of calculations performed at different labs or with different nuclear data. The quantities we estimated where the tritium production in Li^7 and the helium production in the first and second walls. These two reaction rates were chosen because they are important to the tritium breeding question and to radiation damage problem, respectively, and because they are sensitive only to the high energy part of the flux. Therefore, the trial flux and trial adjoints were obtained from S_4-P_1 transport calculations using the ANISN program with energy groups covering the range from 8 to 15 MeV. These calculations should be taken as illustrative and more general ones are being completed at the present time.

The next figure, 22, indicates results using the variational method to illustrate the assessment of changes in the nuclear data. Here, the absorption cross section in the 1st and 2nd wall was increased by 1 barn. This increases the optical thickness of these zones by about 25%. The exact result is that of a direct ANISN calculation on the altered system. In both cases, the variational results do quite well. Similar changes were made in the transfer matrices with similar results.

In figure 23, we show the results when niobium is replaced by vanadium as the structural material in the first and second walls. Again the variational scheme does well. The large 67% difference between the reference and exact results is due to the difference in the (n,α) cross sections between Nb and V. Just using $\sum_{n,\alpha}$ for V in the

expression ($\sum_{n,\alpha}$,ϕ) reduces this difference to 4.4% and the bilinear term in the variational principle picks up the rest.

Let me close by indicating some of the fruitful areas to pursue in this area of variational methods for CTR blanket and shield studies. In figure 24, I have indicated that survey calculations and sensitivity studies, where one anticipates examining the effects of many changes in either design decisions or nuclear data, are areas where variational methods can be used to distinct advantage. The use of these methods in optimization studies can also bear fruit, as shown in some recent work by Greenspan at Princeton.[6]

Finally, with the cost of multi-dimensional calculations being what they are, the possible use of variational methods on those problems is a most interesting area. Just to indicate what the actual CTR cross section might be, the last figure shows, in only some detail, a cross-sectional view of the U.W. Tokamak fusion reactor. There is even more detail in the blanket and shield than is shown. So far, of course, only 1-D cylindrical calculations have been performed for this, and other similar systems.

REFERENCES

1. L. A. Artsimovich et al., in Plasma Physics and Controlled Nuclear Fusion Research (Proc. 2nd Int. Conf., Novosibirsk, 1968) IAEA, Vienna 1 (1969) 17.

2. University of Wisconsin Fusion Feasibility Group, M. Abdou et al., "Preliminary Conceptual Design of a Tokamak Reactor," Texas Symposium on the Technology of Controlled Thermonuclear Reactor Experiments and the Engineering Aspects of Fusion, Nov. 1972 (to be published by USAEC).

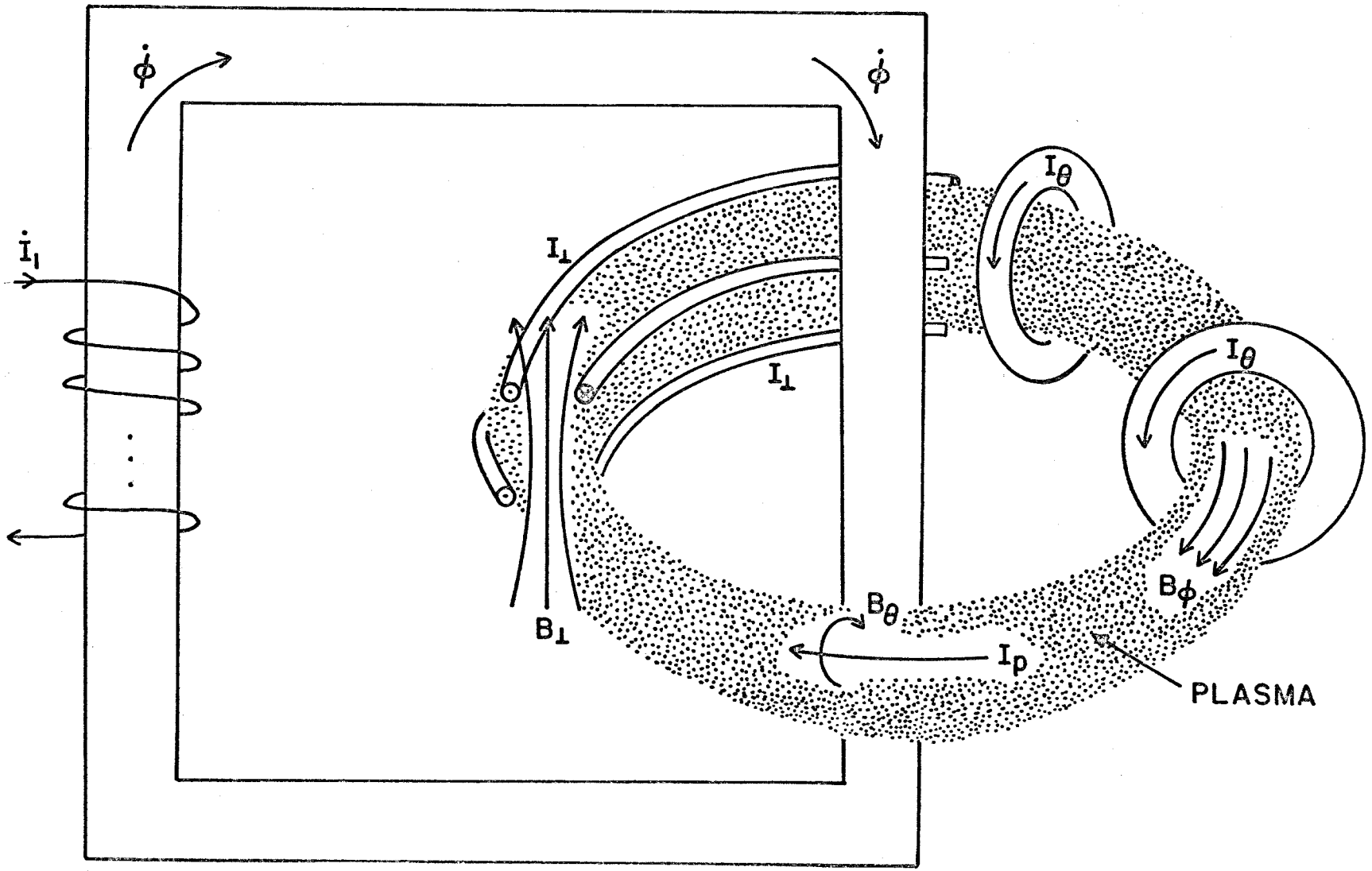
3. R. W. Conn and W. M. Stacey, Jr., "Variational Methods for CTR Blanket Studies, ibid.; Also, R. W. Conn and W. M. Stacey, Jr., Nucl. Fusion 13 (1973) 185.

4. G. C. Pomraning, S.I.A.M. 13 (1965) 511.

5. M. A. Abdou and C. W. Maynard, Tran. Am. Nucl. Soc. 16 (1973) 129.

6. E. Greenspan, "A Method for the Optimization of Fusion Reactor Neutronic Characteristics," MATT-981, Princeton Plasma Physics Lab, April, 1973.

TOKAMAK PLASMA CONFINEMENT



Conn Figure 1

D-T Fuel Cycle:

$$D + T \rightarrow {}^{4}He\,(3.5\,MeV) + n\,(14.1\,MeV)$$

$$^{7}_{3}Li + n \rightarrow T + {}^{4}He + n - 2.466\,MeV$$

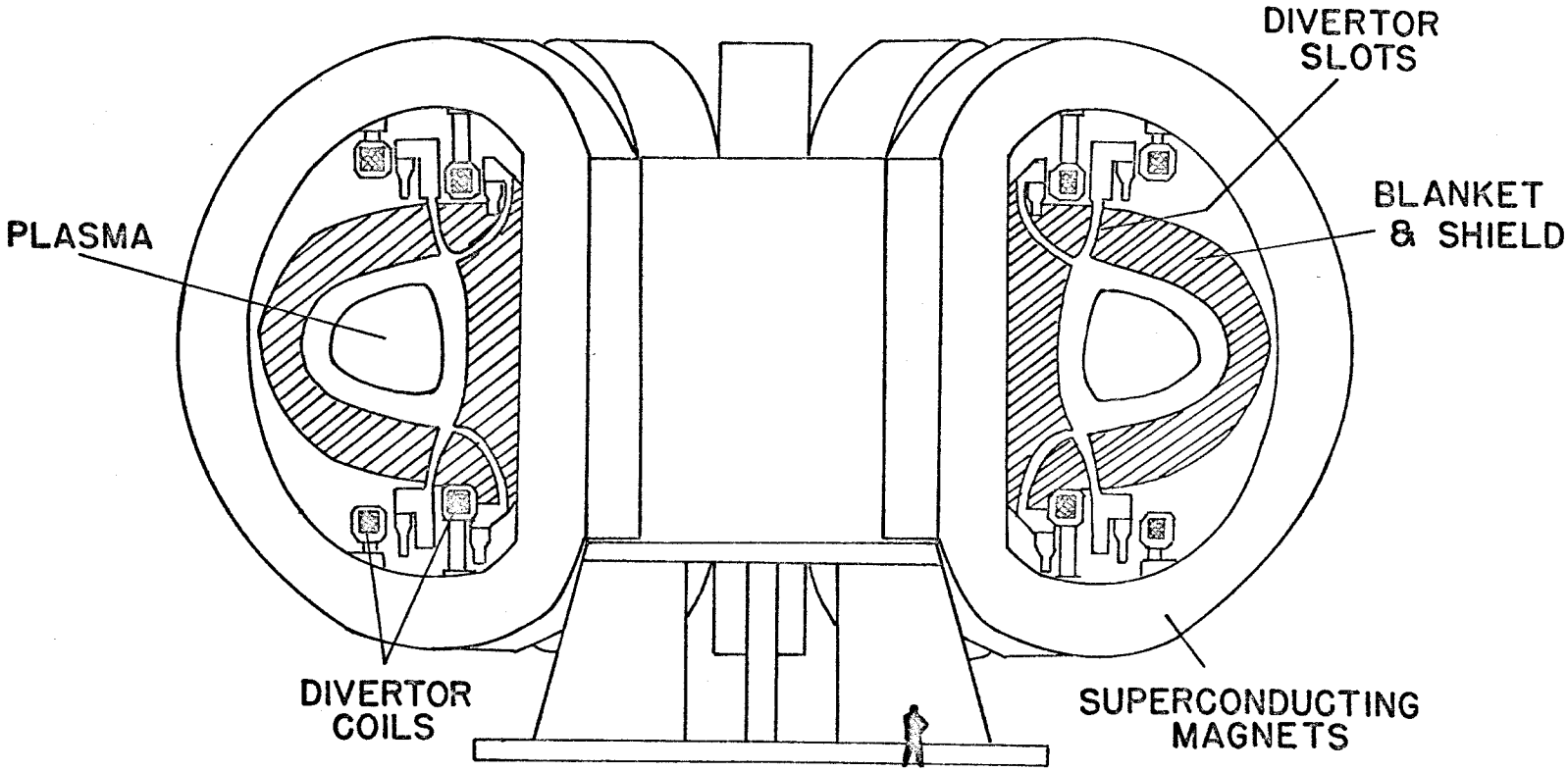
$$^{6}_{3}Li + n \rightarrow T + {}^{4}He + 4.786\,MeV$$

Resource Materials: Deuterium and Lithium

Total Energy Per Fusion Reaction: 20-22 MeV/fusion

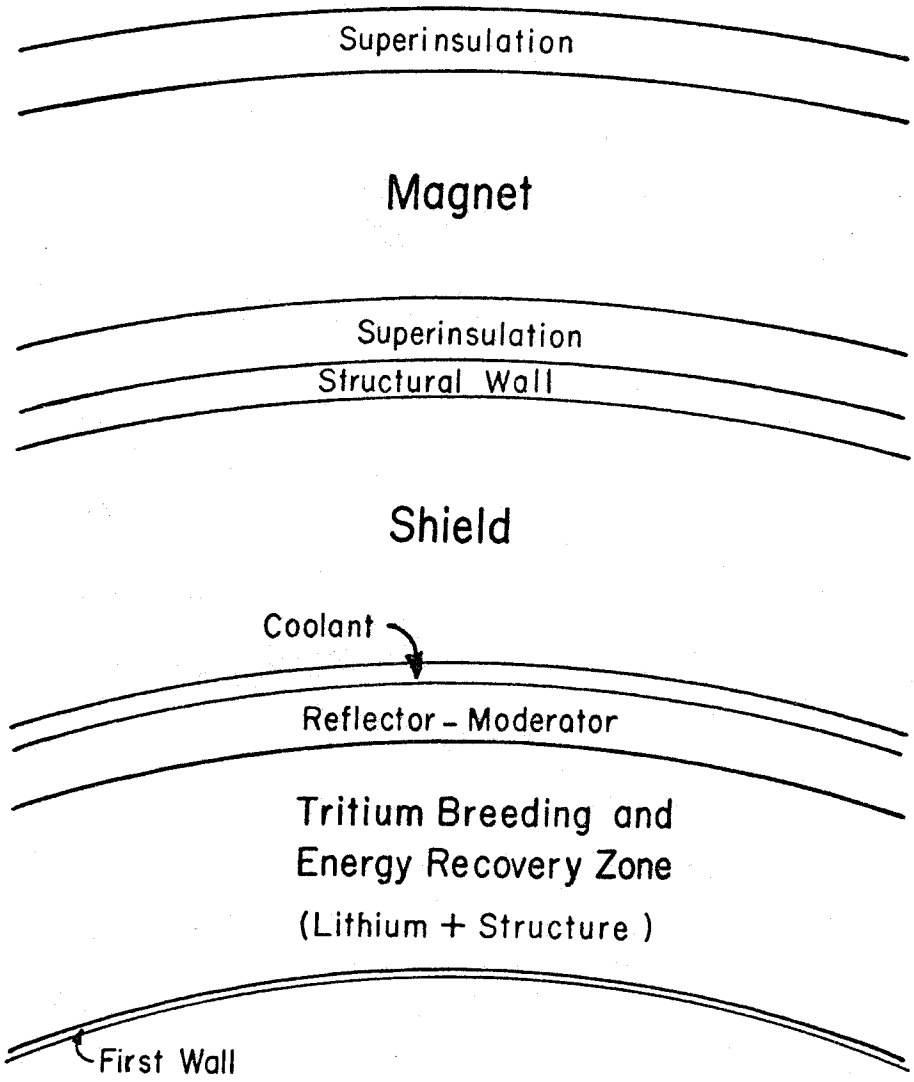
Conn Figure 2

U.W. TOROIDAL FUSION REACTOR



WORK SPONSORED BY MEMBERS OF WISCONSIN
ELECTRIC UTILITIES RESEARCH FOUNDATION
AND ATOMIC ENERGY COMMISSION

Conn Figure 3



SCHEMATIC OF BLANKET AND SHIELD REGIONS

Conn Figure 4

Role of Blanket Region in CTR Reactor:

1. Amplify and Extract Energy of Fusion Neutrons
2. Breed Tritium

Role of Shield in CTR Reactor:

1. Reduce Energy Deposition (Refrigeration Load) in Magnets
2. Reduce Radiation Damage to Stabilizer and Superinsulation

Conn Figure 5

Neutron-Photon Performance Indices of CTR Blankets and Shields:

1. Total Energy Deposition in Blanket/14.1 MeV
2. Tritium Breeding Ratio
3. Charge Particle Production Rates in Structure (H and He; for Radiation Damage)
4. Neutron-Induced Activity and Radioactive After Heat
5. Neutron and Gamma Heat Deposition in Magnets
6. Radiation Damage to Magnet Stabilizer Material
7. Radiation Damage to Superinsulation
8. Cost

Conn Figure 6

Decisions in CTR Blanket and Shield Design

1. Selection of Structural Material
2. Selection of Coolant
3. Selection of Chemical Form of the Breeding Material
4. Selection of Size and Composition of Various Zones

Sensitivity Studies

1. Sensitivity to Design Decisions
2. Sensitivity to Uncertainties in Nuclear Data

Conn Figure 7

VARIATIONAL THEORY FOR CTR PROBLEMS

FUNCTIONAL OF INTEREST:

$$G[\phi]$$

EQUATION OF MOTION: TRANSPORT EQUATION

$$L\phi = S$$

ADJOINT EQUATION:

$$L^{*}\phi^{*} = G'[\phi]$$

Conn Figure 8

ROUSSOPOULOS VARIATIONAL FUNCTIONAL

$$I_B[\phi^*,\phi] = G[\phi] + (\phi^*,(S-L\phi))$$

$$L\phi = S$$

$$L^*\phi^* = G'[\phi]$$

FUNCTIONAL WITH APPROXIMATE ϕ, ϕ^*:

$$\phi_T = \phi_{EX} + \delta\phi$$

$$\phi_T^* = \phi_{EX}^* + \delta\phi^*$$

$$I_B[\phi_T^*,\phi_T] = G[\phi_{EX}] - (\delta\phi^*, L\delta\phi)$$

Conn Figure 9

ESTIMATE OF LINEAR FUNCTIONALS: REACTION RATES

$$G[\phi] = (W,\phi)$$

$$I_B[\phi^*,\phi] = (W,\phi) + (\phi^*, S-L\phi)$$

FRACTIONAL OR SCHWINGER FORM:

$$\phi = A_1\phi$$

$$\phi^* = A_2\phi$$

USE $$\frac{\partial I_B}{\partial A_I} = 0; \; I = 1,2$$

$$I_F[\phi^*,\phi] = \frac{(S,\phi^*)\ (W,\phi)}{(\phi^*, L\phi)}$$

Conn Figure 10

COMBINATION OF SEVERAL TRIAL FUNCTIONS

$$\phi_T = \sum_{I=0}^{N-1} C_I \phi_T^I = \underline{C}^T \cdot \underline{\phi}$$

$$\phi_T^* = \sum_{I=0}^{N-1} D_I \phi_T^{*,I} = \underline{D}^T \cdot \underline{\phi}^*$$

USING $\frac{\partial I_B}{\partial C_I} = 0, \quad \frac{\partial I_B}{\partial D_I} = 0$ LEADS TO

$$I[\underline{\phi}^*, \underline{\phi}] = (\underline{\phi}^*, S)^T \cdot \underline{\underline{M}}^{-1} \cdot (W, \underline{\phi})$$

$$M_{IJ} = (\phi_T^{*,I}, L\phi_T^J); \quad I,J = 0,1,\ldots,N-1$$

Conn Figure 11

FLUENCE-TO-KERMA FACTORS

KERMA: KINETIC ENERGY RELEASED IN MATERIALS

$H(\vec{R})$ = HEATING RATE = NEUTRON HEATING RATE + GAMMA HEATING RATE

$$H(\vec{R}) = \int \phi(E,\vec{R}) \sum_J N_J(\vec{R}) \sum_I \sigma_{IJ}(E) E_{IJ}(E) dE$$

E_{IJ} = ENERGY RELEASED PER REACTION I IN ELEMENT J

MICROSCOPIC KERMA FACTOR = $K_J(E) = \sum_I \sigma_{IJ}(E) E_{IJ}(E)$

Conn Figure 12

CALCULATION OF KERMA FACTORS

NEUTRON KERMA FACTOR = K_N

$$K_N(E) = \sum_I \sigma_I(E) E_I(E)$$

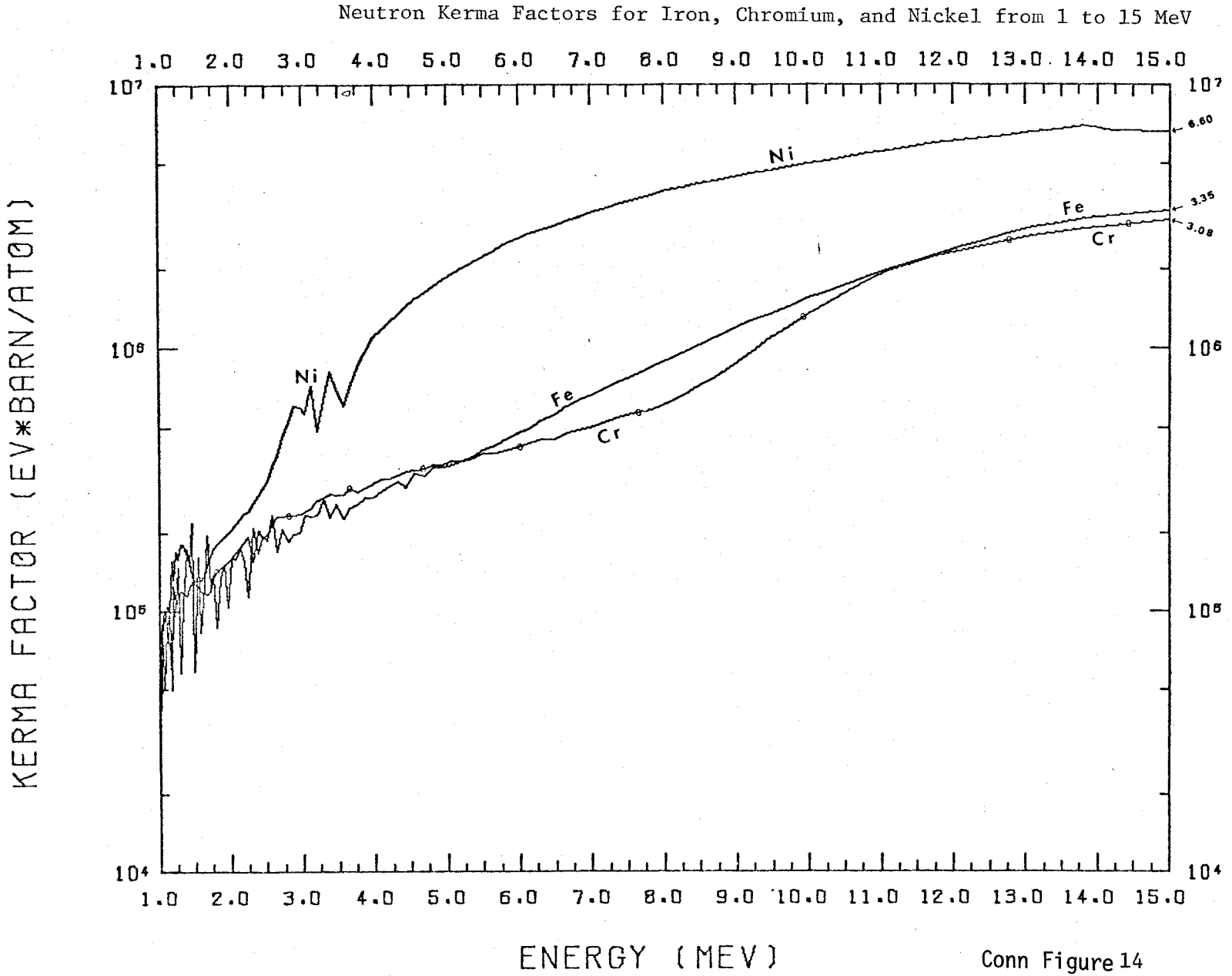
I DENOTES NEUTRON REACTION TYPE; E.G. ELASTIC, INELASTIC, (N,2N), ETC.

$E_I(E)$ OBTAINED FROM SOLUTION OF KINEMATICS EQUATION

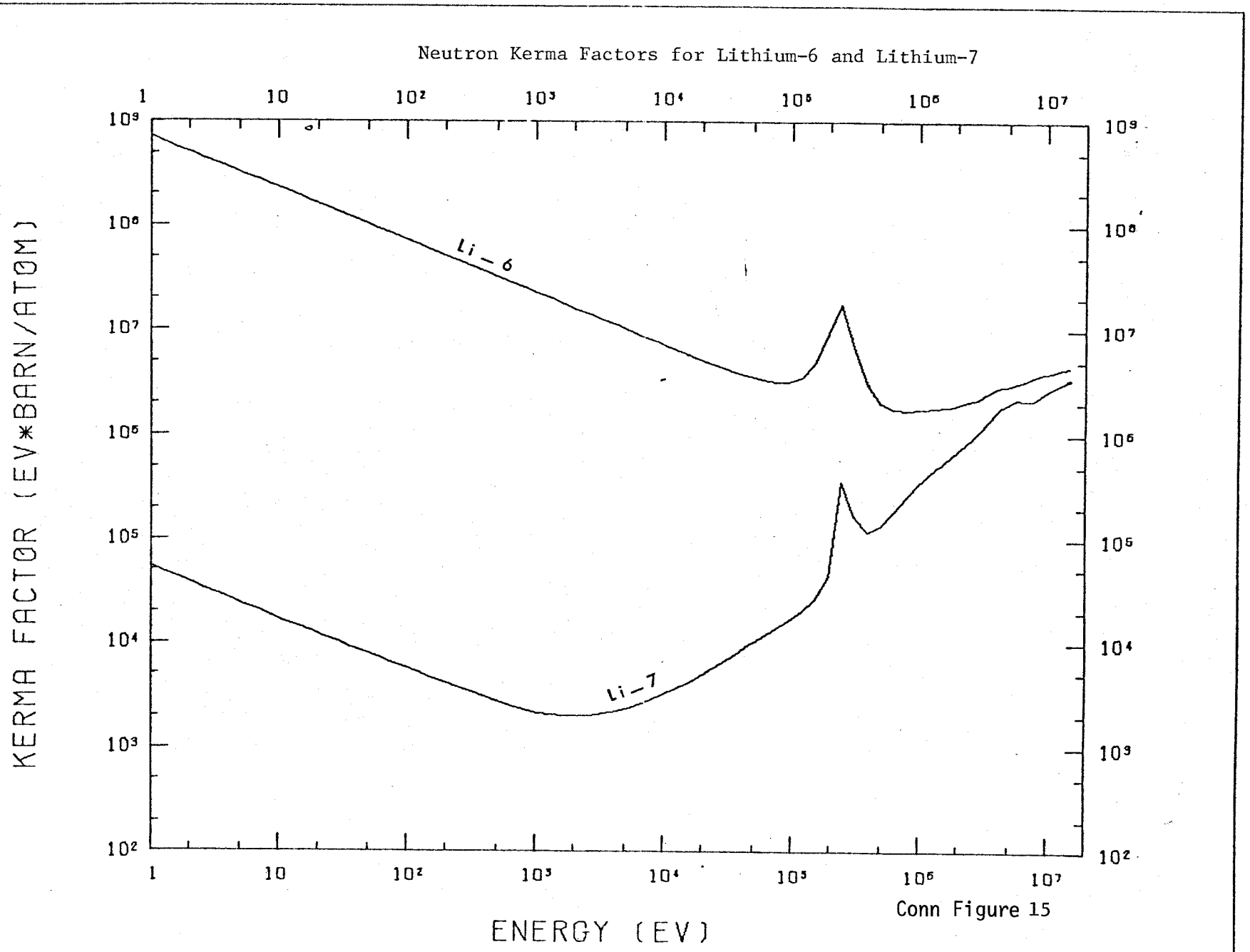
GAMMA KERMA FACTOR = K_γ

$$K_\gamma(E) = \sigma_{PE}(E)E + \sigma_{PP}(E)(E - 1.02) + \sigma_{CA}E$$

Conn Figure 13



Conn Figure 14



Conn Figure 15

(A) REACTION RATE OF I-TH TYPE

$$G[\phi] = (\Sigma_I, \phi)$$

$$G'[\phi] = \Sigma_I(x)$$

(B) RATIO OF REACTION RATES

$$G[\phi] = \frac{(\Sigma_I, \phi)}{(\Sigma_J, \phi)}$$

$$G'[\phi] = \frac{\Sigma_I - G[\phi]\Sigma_J}{(\Sigma_J, \phi)}$$

Conn Figure 16

(c) RATIO OF FLUXES AT TWO POINTS IN BLANKET OR SHIELD

$$G[\phi] = \frac{(\delta(x-x_0),\ \phi)}{(\delta(x-x_1),\ \phi)}$$

$$G'[\phi] = \frac{\delta(x-x_0) - G[\phi]\ \delta(x-x_1)}{(\delta(x-x_1),\ \phi)}$$

(D) NEUTRON AND GAMMA HEATING RATES

$$H(x) = \sum_I N_I \sum_{G=1}^{G} \{K_{N,I}^{G}\ \varepsilon_N + K_{\gamma,I}^{G}\ \varepsilon_\gamma\}\ \phi^G(x)$$

$$G'[\phi] = \varepsilon_R \sum_I N_I \sum_{G=1}^{G} \{ K_{N,I}^{G}\ \varepsilon_N + K_{\gamma,I}^{G}\ \varepsilon_\gamma\}$$

Conn Figure 17

TRIAL FUNCTIONS REQUIRED FOR VARIATIONAL STUDIES

1. ONE REFERENCE FLUX

2. ONE REFERENCE ADJOINT FOR EACH PHYSICAL QUANTITY TO BE ESTIMATED

Conn Figure 18

DISCRETE ORDINATES FORM OF ROUSSOPOULOS PRINCIPLE

$$I_B[\phi^*_{ref},\phi_{ref}] = G[\phi_{ref}] + \int dx \sum_{g=1}^{G}\sum_{j=1}^{N} W_j \phi^{*,g}_{ref,j}(x)\Delta S^g_j(x)$$

$$- \int dx \sum_{g=1}^{G}\sum_{j=1}^{N} W_j \phi^{*,g}_{ref,j}(x)\left[\Delta\Sigma^g_T(x)\phi^g_{ref,j}(x)\right.$$

$$\left. - \sum_{g'\le g}^{G}\sum_{j'=1}^{N}\sum_{\ell=0}^{\max[\bar{L}',L]}\left(\frac{2\ell+1}{2}\right)\cdot\Delta\Sigma^{g'\to g}_{s,\ell}(x)\phi^{g'}_{ref,j'}(x)P^{\ell}_{jj'}\right]$$

where

$$\Delta S^g_j(x) = \bar{S}^g_j(x) - S_j(x)$$

$$\Delta\Sigma^g_T(x) = \bar{\Sigma}^g_T(x) - \Sigma^g_T(x)$$

$$\Delta\Sigma^{g'\to g}_{s,\ell} = \bar{\Sigma}^{g'\to g}_{s,\ell}(x) - \Sigma^{g'\to g}_{s,\ell}(x)$$

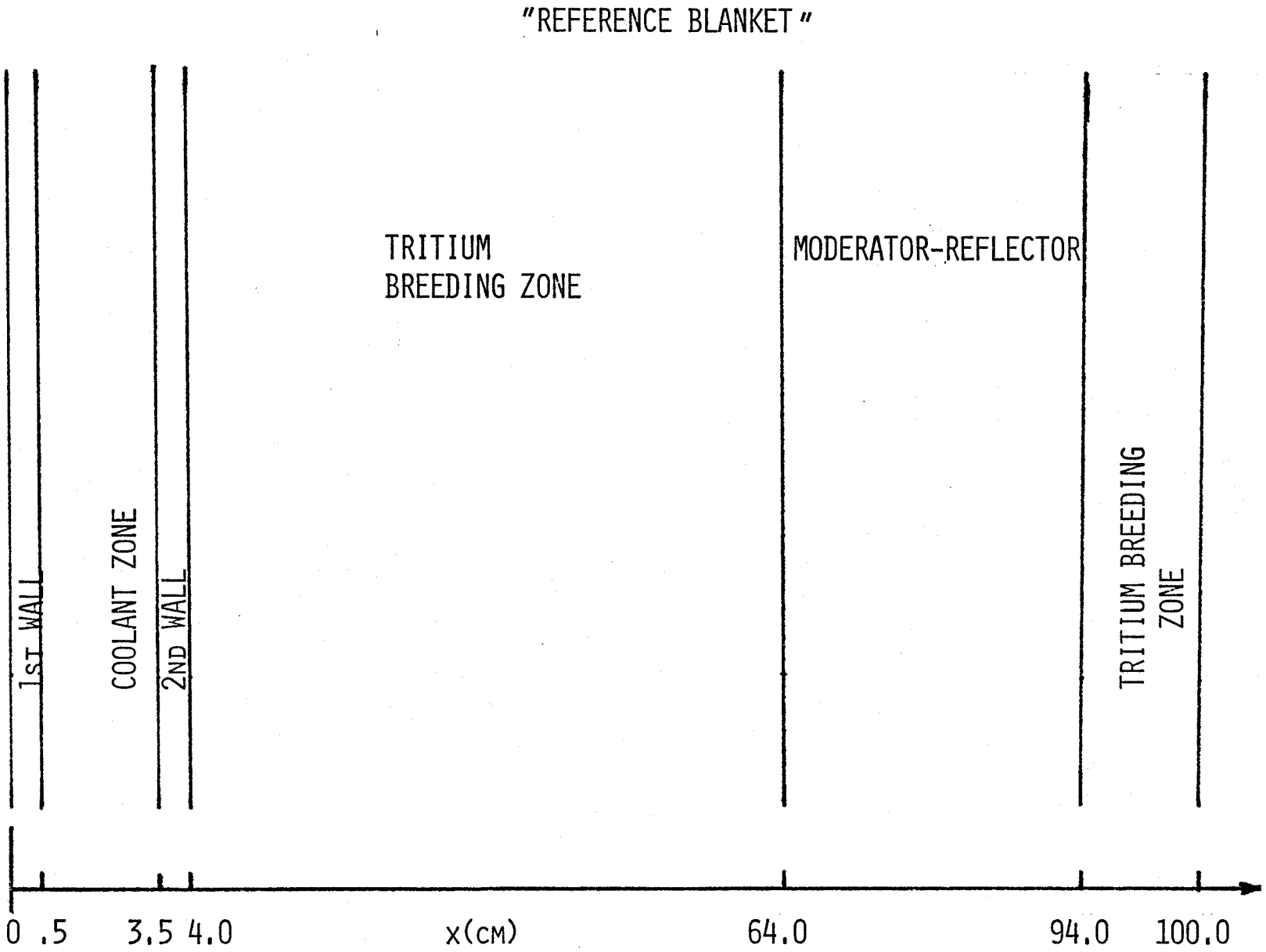
$$P^{\ell}_{jj'} = W_{j'} P_\ell(\mu_j) P_\ell(\mu_{j'}) \cdot$$

Conn Figure 19

NUCLEAR DATA NEEDED

1. Multi-Group Cross Sections
   - A. Neutron Multi-Group Cross Sections
   - B. Gamma Multi-Group Cross Sections
   - C. Gamma Production Cross Sections
2. Group Cross Sections For Reactions of Interest
   eg. (n,T), (n,α), (n,p), etc.
3. Displacement Cross Sections
4. Kerma Factors
   - A. Gamma Kerma Factors
   - B. Neutron Kerma Factors

Conn Figure 20



Conn Figure 21

TABLE 1

ALTERATION: ADD 1 BARN TO σ_a^g AND σ_T^g FOR Nb- USE AS MATERIAL FOR 1ST AND 2ND WALLS.

CALCULATION	SYSTEM	TOTAL TRITIUM PROD. RATE FROM Li^7	% DIFFERENCE FROM EXACT RESULT
ANISN	REFERENCE	.4136	13.3
VARIATIONAL	ALTERED	.3648	___
ANISN	ALTERED (EXACT)	.3649	___

		HELIUM PRODUCTION RATE IN 1ST AND 2ND WALLS $\times 10^{-4}$	% DIFFERENCE FROM EXACT RESULT
ANISN	REFERENCE	10.685	9
VARIATIONAL	ALTERED	9.718	.93
ANISN	ALTERED (EXACT)	9.809	___

Conn Figure 22

TABLE 3

ALTERATION: REPLACE Nb BY V AS THE 1ST AND 2ND WALL STRUCTURAL MATERIAL.

CALCULATION	SYSTEM	TOTAL TRITIUM PROD. RATE FROM Li^7	% DIFFERENCE FROM EXACT RESULT
ANISN	REFERENCE	.4136	7.3
0^{TH} ORDER PERT. TH.	ALTERED	.4136	7.3
VARIATIONAL	ALTERED	.4402	1.2
ANISN	ALTERED (EXACT)	.4459	___

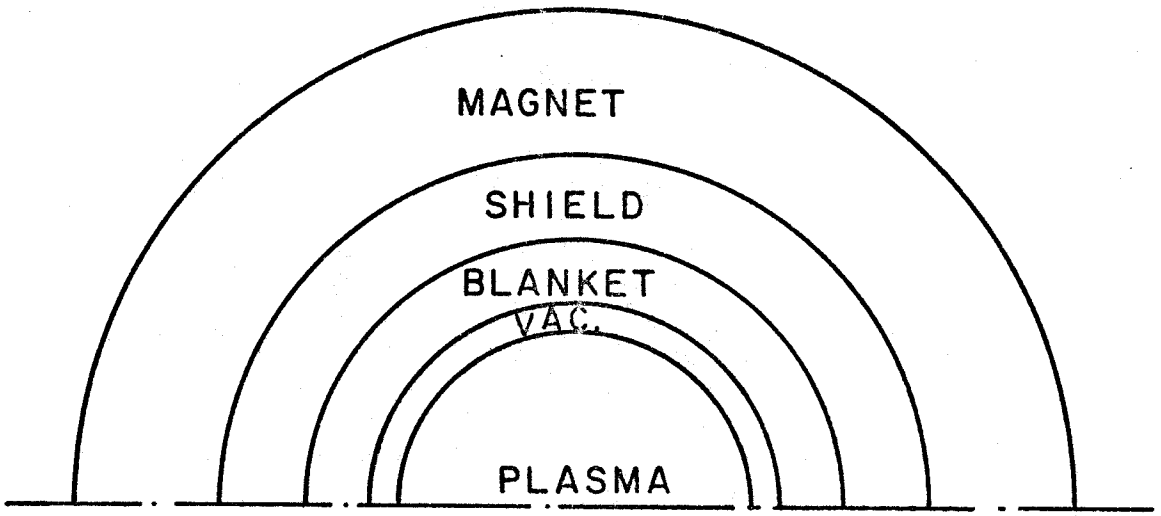
		HELIUM PRODUCTION RATE IN 1ST AND 2ND WALLS: $\times 10^{-4}$	% DIFFERENCE FROM EXACT RESULT
ANISN	REFERENCE	10.685	67
0^{TH} ORDER PERT. TH.	ALTERED	31.17	4.4
VARIATIONAL	ALTERED	32.20	1.2
ANISN	ALTERED (EXACT)	32.61	___

Conn Figure 23

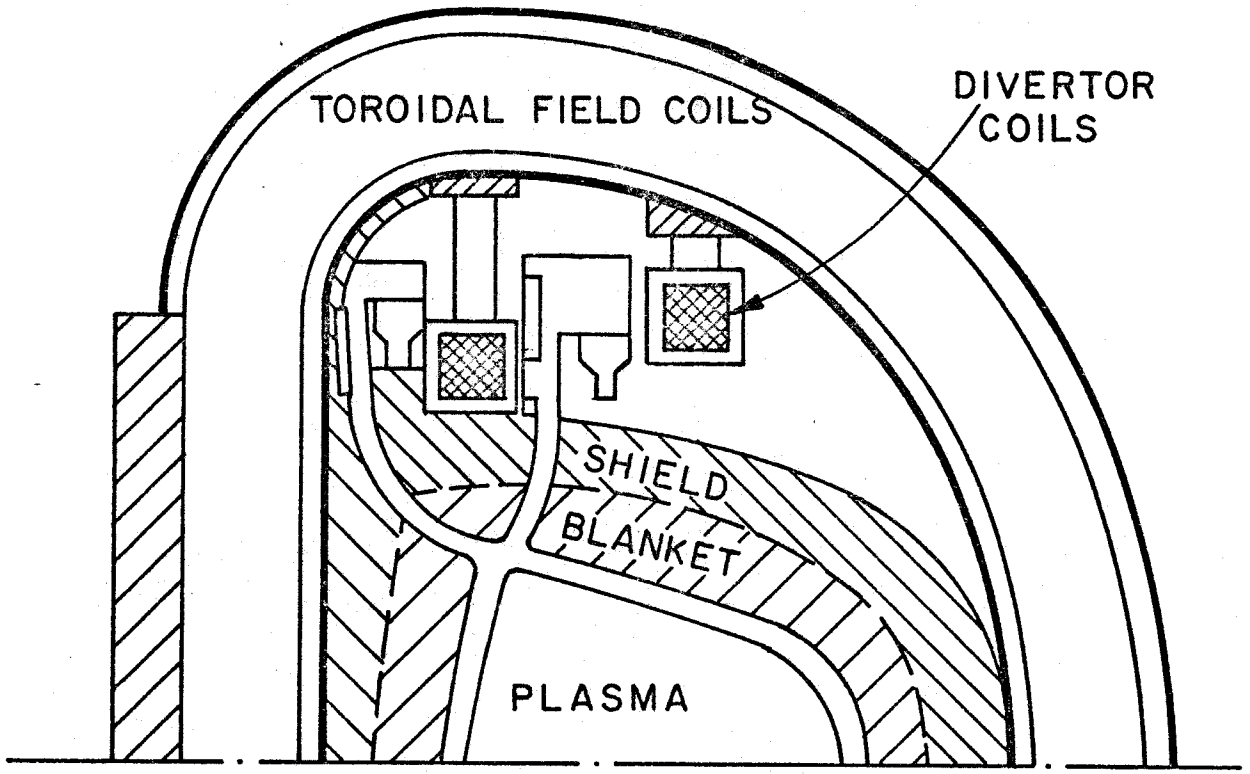
FRUITFUL AREAS TO PURSUE: VARIATIONAL METHODS FOR CTR BLANKET AND SHIELD STUDIES

1. SURVEY CALCULATIONS
2. SENSITIVITY STUDIES
3. OPTIMIZATION STUDIES
4. APPLICATION TO MULTIDIMENSIONAL PROBLEMS

Conn Figure 24



SIMPLIFIED FUSION REACTOR CROSS SECTION



UNIV. OF WISC. TOROIDAL FUSION REACTOR CROSS SECTION

Conn Figure 25