Electron Bounce Resonance Heating in a Mirror Confined Plasma

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For a mirror confined plasma electrons are usually confined in the electrostatic potential well with a characteristic bounce frequency ω_b. If there is a wave with frequency $\omega = \omega_b$ in the system, there can be a resonant interaction between the wave and electrons. The basic resonant mechanism is collisionless (bounce) Landau damping. The power transfer from the wave to electrons is calculated from a quasilinear theory. Since the electron bounce frequency ω_b is usually of the order of the ion gyrofrequency Ω_i in a mirror machine, significant electron bounce resonance heating can occur from waves in the plasma in the ion cyclotron frequency range. Applications of the theory to current mirror experiments and reactor designs are discussed.

1. INTRODUCTION

In the recent tandem mirror TMX experiment [1], the plug electron temperature was found to be higher than the central cell temperature by about a factor of two. The classical Coulomb collision model [2] seems not to be able to explain this temperature difference. The current explanation [1] is that a thermal barrier is created between the central cell and plug by a microinstability in the ion cyclotron frequency range [either drift cyclotron loss cone (DCLC) mode or Alfvén ion cyclotron (AIC) mode] and this reduces the AIG thermal contact between the central cell and plug electrons. However, a density dip was not found to support this argument.

Here, we propose another possible mechanism for the disparity in central cell and plug electron temperatures. Since the electron bounce frequency (ω_b) is close to the ion cyclotron frequency (Ω_i) in TMX, electrons can be heated by ion cyclotron frequency range instabilities of either the DCLC or AIC type [3]. We will show later that this bounce resonance heating power can be comparable to that of the electron Coulomb drag [2] and thus could explain the temperature difference in TMX.

Electron bounce resonance heating may also be an important heating scheme for the tandem mirror reactor. In order to improve the reactor Q value, which is defined as the ratio of the fusion power output to the total power input, and reduce the plug to central cell density ratio, plug electron heating is indispensable for a tandem mirror reactor. Usually this is proposed to be accomplished by applying an external wave at the electron cyclotron frequency Ω_e, or its harmonics. This is of course known as electron cyclotron heating (ECH). But ECH technology is still in the developing stage. In a reactor, however, the electron bounce frequency ω_b is usually in the range of the ion cyclotron frequency Ω_i (e.g., in the WITAMIR-I [4] design $\omega_b \cong 2\Omega_i$). Thus, electron bounce resonance heating can conceivably be accomplished with present day ion cyclotron heating (ICH) technology.

Electron bounce resonance in a mirror confined plasma has been discussed by Sprott [5]. Our calculation is different in that the inhomogeneous equilibrium electrostatic potential and transit time magnetic damping are taken into account in our model. In Section 2, we discuss the particle orbits in a parabolic potential well and quasilinear theory in the presence of an external wave. The application of the electron bounce resonance heating to the TMX experiment and tandem mirror reactor designs is discussed in Section 3. Conclusions are given in Section 4.

2. ELECTRON BOUNCE RESONANCE HEATING

2.1. Particle Orbit

Since only parallel (to the magnetic field line) motion is involved in the electron bounce resonance heating, we need to study the parallel dynamics. The equation of motion along the magnetic field line can be written as

$$\frac{dv_\parallel}{dt} = \frac{d^2s}{dt^2} = -\frac{\mu}{m}\frac{\partial B}{\partial s} - \frac{q}{m}\frac{\partial \phi}{\partial s} + \frac{qE_0}{m}\sin(ks - \omega t) \quad , \tag{1}$$

where s is the distance along the field line, $\mu = mv_\perp^2/2B$ is the magnetic moment, the $v_\parallel$ ($v_\perp$) is the electron speed parallel (perpendicular) to the magnetic field line, m and q are the electron mass and charge respectively, ϕ is the equilibrium electrostatic potential, and B is the magnetic field strength. An external wave with parallel electric field strength E_0, parallel wave vector k, and frequency ω is also included in Eq. (1). To proceed, we assume a parabolic profile for both the magnetic field B and potential ϕ:

$$B = B_o(1 + s^2/L^2) \quad , \tag{2}$$

and

$$\phi = \phi_o \, (1 - s^2/L^2) \quad , \tag{3}$$

where L is the typical parallel scale length in the mirror. The difference between magnetic and potential scale lengths is neglected, but can be included very easily. Assuming $|qkE_o/m\omega_b^2| << 1$ (only slight heating within one bounce period), we obtain the zeroth order equation

$$\frac{d^2s}{dt^2} = -(\frac{v_{\perp o}^2}{L^2} + \frac{2e\phi_o}{mL^2}) \; s \quad , \tag{4}$$

where $v_{\perp o}$ is the perpendicular speed at the midplane s = 0. From Eq. (2), we have the bounce frequency ω_b

$$\omega_b^2 = \frac{v_{\perp o}^2}{L^2} + \frac{2e\phi_o}{mL^2} \quad , \tag{5}$$

and the constant of motion

$$\varepsilon = \frac{1}{2}\,\omega_b^2 s_o^2 = \frac{1}{2}\,v_\parallel^2 + \frac{1}{2}\,\omega_b^2 s^2 \quad . \tag{6}$$

The constant of motion ε is simply the parallel energy and is closely related to the second adiabatic invariant J. Destruction of the constancy of J is essential to achieving bounce resonance heating. From Eq. (1), we obtain the time rate of change of the parallel energy

$$\dot{\varepsilon} \equiv \frac{d\varepsilon}{dt} = v_\parallel \, \frac{qE}{m} \quad . \tag{7}$$

The solution to Eq. (4) is simply

$$s = s_0 \sin \psi \quad , \tag{8}$$

where $\psi = \omega_b t + \psi_0$ is the bounce phase and $\dot{\psi} = \omega_b$.

Employing the Bogoliubov asymptotic expansion method [6], we assume the first order solution to Eq. (1) has the form

$$s = s_0(t) \sin[\omega_b t + \psi_0(t)] + s_1 \quad , \tag{9}$$

where s_0 and ψ_0 are assumed to have a first order time dependence, and s_1 is the first order, nonresonant coherent solution. At resonance, namely $\omega = \ell\omega_b$ where ℓ is an integer, we have

$$\dot{\psi}_0 = \frac{qE_0}{s_0 \omega_b m} J_\ell' \, (ks_0) \cos \ell\psi \quad , \tag{10}$$

$$\dot{s}_0 = \frac{qE_0}{\omega_b m} \frac{\ell}{ks_0} J_\ell \, (ks_0) \sin \ell\psi \quad , \tag{11}$$

where J_ℓ is the Bessel function of order ℓ, and $J_\ell'(x) = dJ_\ell/dx$. Note that Eq. (11) can also be obtained from Eq. (7). The nonresonant solution s_1 will not be presented here.

Equations similar to Eqs. (10) and (11) have been obtained by many authors [7-9] in studying ion cyclotron resonance motion in the presence of the external waves. Hsu [9] has shown that the nonlinear particle motion governed by Eqs. (10) and (11) is stochastic if $|qkE_0/m\omega_b^2| \gtrsim 1$ at $\ell = 1$. For the cases we are interested in, namely the TMX experiment and reactor design,

$|qkE_0/m\omega_b^2|$ is usually much less than unity, and the collisionless particle orbit is deterministic coherent (i.e., not stochastic) if we neglect nonideal effects such as collisions and finite frequency spread of the wave or bounce frequency. Collisions could make the particle motion stochastic if $(\Delta v_\parallel/v)_c \gtrsim (\Delta v_\parallel/v)_w$, where $(\Delta v_\parallel/v)_c$ and $(\Delta v_\parallel/v)_w$ are the change of pitch angle in one bounce period due to pitch angle scattering and wave heating, respectively. If we employ an electrostatic approximation for E_0, i.e. $E_0 \sim k\tilde{\phi}$, where $\tilde{\phi}$ is the fluctuation potential, we can obtain from Eq. (9) that $(\Delta v_\parallel/v)_w \sim \tilde{\phi}/\phi_0$. We can estimate from small-angle diffusive Coulomb scattering that $(\Delta v_\parallel/v)_c \simeq (\nu_e/\omega_b)^{1/2}$, where ν_e is the electron collision frequency. The criterion $(\Delta v_\parallel/v)_c \gtrsim (\Delta v_\parallel/v)_w$ thus becomes $\tilde{\phi}/\phi_0 \lesssim (\nu_e/\omega_b)^{1/2}$. We will show in Section 3 that this criterion is roughly satisfied in both the TMX experiment and reactor designs. If the collision frequency is not large enough to make the particle orbits stochastic, a finite frequency spread of the wave frequency is probably needed. However, exactly how much frequency spread is needed to make the particle motion stochastic is beyond the scope of this paper.

2.2 Quasilinear Theory

To calculate the power transfer from the wave to the electrons, we solve the Vlasov equation as is usually done in quasilinear theory [10]. The Vlasov equation in terms of the parallel energy ε and bounce phase ψ can be written as

$$\frac{\partial f}{\partial t} + \frac{\partial}{\partial t}(\dot{\varepsilon} f) + \frac{\partial}{\partial \psi}(\dot{\psi} f) = 0 \quad , \tag{12}$$

where $f = f(\varepsilon,\psi)$ is the particle distribution function. In the absence of the external wave, $\dot{\varepsilon} = 0$. Then, we have the zeroth order steady state equation

$$\frac{\partial}{\partial\psi} (\dot{\psi} f_0) = 0 \quad ,$$

and obtain $f_0 = f_0(\varepsilon)$. The first order equation is

$$\frac{\partial f_1}{\partial t} + \frac{\partial}{\partial\varepsilon} (\dot{\varepsilon} f_0) + \frac{\partial}{\partial\psi} (\dot{\psi} f_1) = - \frac{\partial}{\partial\varepsilon} (\dot{\varepsilon} f_1) \quad . \tag{13}$$

The nonlinear term on the right hand side of Eq. (13) is the usual quasilinear term. Averaging Eq. (13) over the bounce phase, we obtain the time evaluation of the first order distribution f_1

$$\frac{\partial \langle f_1 \rangle}{\partial t} = - \langle \frac{\partial}{\partial\varepsilon} \dot{\varepsilon} f_1 \rangle \quad , \tag{14}$$

where $\langle A \rangle \equiv \int_0^{2\pi} d\psi A/2\pi$.

To calculate the quasilinear diffusion coefficient, we need to know the f_1 obtained from Eq. (13) by neglecting the nonlinear term on its right hand side and integrating along the unperturbed particle orbit. Thus,

$$f_1 = - \int_{-\infty}^{t} dt' \, \dot{\varepsilon} \, \frac{\partial f_0}{\partial\varepsilon} \quad , \tag{15}$$

and

$$\frac{\partial \langle f_1 \rangle}{\partial t} = \frac{\partial}{\partial\varepsilon} \langle \dot{\varepsilon} \int_{-\infty}^{t} dt' \, \dot{\varepsilon} \rangle \frac{\partial f_0}{\partial\varepsilon} = \frac{\partial}{\partial\varepsilon} D \, \frac{\partial f_0}{\partial\varepsilon} \, . \tag{16}$$

As usual, to calculate D we need only to know the resonant part of f_1; the nonresonant part of f_1 will describe the coherent (nonresonant) oscillation of the particles. From Eq. (15), after integrating along the unperturbed particle orbit, we obtain the resonant part of f_1

$$\mathrm{Re}\ f_1 = -\pi \frac{q\tilde{E}_{k\omega}}{m} \frac{\omega_b}{k} \sum_{n,\ell} \ell J_n(ks_o) J_\ell(ks_o)\ e^{i(\ell-n)\psi}\ \delta(\ell\omega_b - \omega) \frac{\partial f_o}{\partial \varepsilon} \quad , \tag{17}$$

and

$$D = \pi \left(\frac{\omega_b}{k}\right)^2 \frac{q^2 |\tilde{E}_{k\omega}|^2}{m^2} \sum_\ell \ell^2 J_\ell^2\ (ks_o)\ \delta(\ell\omega_b - \omega) \quad , \tag{18}$$

where $\tilde{E}_{k\omega}$ is the amplitude of $E = \tilde{E}_{k\omega}\ \exp[i(ks - \omega t)]$. To obtain Eq. (18), we have assumed there is only one mode. If there are many modes Eq. (18) should include a summation over all the modes.

The power transfer from the wave to the electrons can be calculated from Eqs. (16) and (18)

$$P = \pi \int_0^\infty dv_\perp^2 \int_0^\infty d\varepsilon\ m\varepsilon \frac{\partial \langle f_1 \rangle}{\partial t} \quad . \tag{19}$$

For $\ell = 1$ (i.e., $\omega = \omega_b$), $ks_o \ll 1$ and assuming f_o to be a Maxwellian, we have

$$P = \frac{\pi}{2} \langle n \rangle \frac{q^2 |\tilde{E}_{k\omega}|^2}{m\omega} \left(\frac{2|q|\phi_o}{T_e}\right) \frac{\omega}{\omega_{bo}} \exp\left[- \frac{2|q|f_o}{T_e} \left(\frac{\omega}{\omega_{bo}} - 1\right)\right] \quad , \tag{20}$$

where $\langle n \rangle$ is the plasma density averaged along the field line, $\omega_{bo} = (2|q|\phi_o/mL^2)^{1/2}$ is the relevant electron bounce frequency and T_e is the electron temperature. To obtain Eq. (20) we have assumed that $|q|\phi_o \gg \frac{1}{2} mv_{\perp o}^2$, which is usually satisfied for a mirror confined plasma since electrons are an electrostatically confined species. Note that the resonance condition $\omega = \omega_b$ can be satisfied only if $\omega \geq \omega_{bo}$; thus $P = 0$ for $\omega < \omega_{bo}$ and $P \neq 0$ for $\omega \geq \omega_{bo}$. Since in general $|q|\phi_o \sim (3 - 5)\ T_e$, the resonance zone is very narrow, $(\frac{\omega}{\omega_b} - 1) \lesssim 0.2$, in frequency space. Higher harmonic ($\ell > 1$) bounce resonances can also be calculated from Eq. (19), but will not be discussed here.

2.3. Transit Time Magnetic Damping

In Sections 2.1. and 2.2., we have assumed that there is a parallel electric field. Similar calculations can be carried out for a parallel magnetic fluctuation. In that case, $m^{-1}\ \mu\partial\tilde{B}/\partial s$ will play the role of qE_o/m, where $\tilde{B}$ is the parallel magnetic fluctuation. This is the usual transit time magnetic damping [11]. For a transverse magnetic field $\vec{E} = E_y\hat{e}_y \exp[i(ks + k_x x - \omega t)]$ with $\hat{e}_y$ the unit vector in the y direction, k_x and x the wave vector and distance in the x direction, the power transfer from wave to the electrons for $\ell = 1$ is

$$P = \frac{\pi \langle n \rangle q^2}{8m\omega} \left(\frac{kv_T}{\omega}\right)^2 (k_x\rho_e)^2\ |E_y|^2 \left(\frac{2e\phi_o}{T_e}\right) \frac{\omega}{\omega_b}\ a^2 e^{-a} \quad , \tag{21}$$

where $v_T = \sqrt{2T_e/m}$, $\rho_e = v_T/\Omega_e$, and $a = 2e\ \phi_o(\omega/\omega b_o - 1)/T_e$. From Eq. (21) we see that at $\omega = \omega_{bo}$, $P = 0$. This is because at $\omega = \omega_{bo}$, the resonant particles have $v_{\perp o} = 0$ and thus, $\mu\partial\tilde{B}/\partial s = 0$.

3. APPLICATIONS

The theory developed in Section 2 can be used to discuss the temperature difference between the plug and central cell electrons in TMX. We assume TMX plug plasma parameters to be as follows: $n = 1 \times 10^{13}\ cm^{-3}$, $T_e = 100$ eV, $|q|\phi_o = 500$ eV, B = 1 T, and L = 37.5 cm. Then, we have $\omega_{bo} = 3.5 \times 10^7\ sec^{-1}$, which is close to the observed unstable mode frequency ω. Note that the parallel density scale length L_n corresponding to the potential scale length L = 37.5 cm is $L_n \cong L/\sqrt{5} = 16.8$ cm, which is about the L_n measured in the TMX, since from Eq. (6), we have $n(s) = n(s=0)\ \exp(\frac{1}{2}\ \omega_b^2 s^2) \cong n(s=0)\ \exp(-|q|\phi_o s^2/L^2 T_e)$ and $|q\phi_o| \simeq 5\ T_e$. Since $\omega_{bo} \simeq \omega$, it is possible that the unstable mode could heat plug electrons through the bounce resonance heating mechanism. In order to have an appreciable effect on the electron temperature due to bounce resonance heating, the wave heating power has to be comparable to that due to electron Coulomb drag. We will show that this is roughly the case. Since there is an uncertainty about which mode -- DCLC or AIC -- is observed in the experiment, we will estimate the power transfer from the wave to electrons for both modes. For simplicity, we define a normalized power transfer $\overline{P}$ [2]

$$\overline{P} = P/nT_e\nu_d \quad . \tag{22}$$

Since $T_i \simeq 100\ T_e$, P will be comparable to the electron Coulomb drag power if $\overline{P} \simeq 100$.

If the mode is a DCLC mode, a small $k_\parallel \equiv k$ can produce a parallel electric field $\tilde{E}_\parallel \simeq k_\parallel\tilde{\phi}$, where $\tilde{\phi}$ is the fluctuation potential. From Eqs. (20) and (22), we have

$$\overline{P} \simeq (k_\parallel L)\ \frac{\omega}{\nu_d}\ \left(\frac{|q|\tilde{\phi}}{T_e}\right)^2 \quad . \tag{23}$$

If averaged over the region where the electrons bounce we have $k_\parallel \simeq 10^{-3}\ \text{cm}^{-1}$ and $|q|\tilde{\phi} \simeq T_e$, then since $\omega/\nu_d \simeq 10^5$, we have $\overline{P} \simeq 10^2$, which indicates the electron bounce heating power can be comparable to the electron Coulomb drag power. Also since $(\nu_e/\omega_b)^{1/2} \sim 10^{-1}$ and $\tilde{\phi}/\phi_o \sim 1/5$ for $|q|\tilde{\phi} \sim T_e$, the stochastic criterion $(\Delta v_{\parallel}/v)_c \gtrsim (\Delta v_{\parallel}/v)_w$ is roughly satisfied. If the mode is an AIC mode, then the parallel electric field is zero but there is a parallel magnetic fluctuation. From Eqs. (21) and (22), we then have

$$\overline{P} = \frac{\pi}{8} \left(\frac{kv_T}{\omega}\right)^2 (k_x L)^2 \left(\frac{\rho_e}{\rho_i}\right)^2 (k_y \rho_i)^2 \left(\frac{e\tilde{\phi}}{T_e}\right)^2 \frac{\omega}{\nu_d} a^2 e^{-a} \quad , \tag{24}$$

where k_y is the azimuthal wave vector, and ρ_i is the ion gyroradius. To obtain Eq. (24), we have assumed $E_y \simeq k_y \tilde{\phi}$. With $|q|\tilde{\phi}/T_e \simeq 1$, $k_x L \cong 10$, $k_y \rho_i \simeq 1$, $a^2 e^{-a} \simeq 1$, and $k = \omega_{pi}/c$, where ω_{pi} is the ion plasma frequency, we again have $\overline{P} \simeq 100$. Thus, we see that either the AIC mode or DCLC mode could heat electrons substantially through the bounce resonance heating mechanism. However, the estimate for $\overline{P}$ depends sensitively on the various wave vectors and other parameters. A reduction of the wave vector by an order of magnitude will reduce $\overline{P}$ to the order of unity and thus to a level where it could not explain the experiment results. Further comparison with the experiment cannot be made until more information is available from the experiment.

The most important application of the electron bounce resonance heating may be in the reactor design area. With the WITAMIR-I parameters, $\omega/\nu_d \simeq 5 \times 10^9$, and if we choose $k_{\parallel} L \simeq \pi/2$ then we have $\overline{P} \cong 10^9 \, (|q|\tilde{\phi}/T_e)^2$. A $\overline{P} \cong 10^3$ will be sufficient to deliver a total electron heating power on the order of the 100 MW, which is required for the reactor. Thus, $|q|\tilde{\phi}/T_e$ only has to be about 10^{-3}. For $T_e \simeq 100$ keV this implies a potential fluctuation $\tilde{\phi} \simeq 100$ V. If $L = 1$ m then $E_{\parallel} \simeq 1$ V/cm. Thus we see that a small parallel electric field is sufficient to heat electrons in the reactor. Since $\omega_{bo} \simeq (2 - 3)\, \Omega_i$ in the reactor, a fast magnetosonic wave could be used to provide that parallel electric field and thereby heat electrons, and the perpendicular electric field could at the same time heat ions. With WITAMIR-I parameters, $(\nu_e/\omega_{bo})^{1/2} \simeq 10^{-3}$ and $\tilde{\phi}/\phi_0 \simeq 10^{-3}$; thus, the stochastic criterion $(\Delta v_{\parallel}/v)_c \gtrsim (\Delta v_{\parallel}/v)_w$ is marginally satisfied. The most appealing potential advantage of the electron bounce resonance heating is that it only requires present day ICF heating technology.

4. CONCLUSIONS

We have calculated the power transfer from waves to electrons through the bounce resonance mechanism. The theory is advanced as a possible explanation for temperature differences between plug and central cell electrons in TMX. We have shown that either a DCLC mode or the AIC mode could heat electrons substantially and might thereby cause the electron temperature difference between central cell and plug observed in the experiment. The most important application of electron bounce resonance heating will probably be in the reactor design area. In this regard we have shown that a small parallel electric field, which may be provided by the fast magnetosonic wave, can deliver the required electron heating power in a reactor.

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REFERENCES

[1] TMX Group, Summary of Results from the Tandem Mirror Experiment (TMX), Lawrence Livermore Lab. Rep. UCRL-53120 (Feb. 1981).

[2] COHEN, R. H., BERNSTEIN, I. B., DORNING, J. J., ROWLANDS, G., Nucl. Fusion 20 (1980) 1421.

[3] BALDWIN, D. E., Rev. Mod. Phys. 49 (1977) 317.

[4] BADGER, B., et al., WITAMIR-I: A University of Wisconsin Tandem Mirror Reactor Design, University of Wisconsin Fusion Engineering Program Report UWFDM-400 (Sept. 1980).

[5] SPROTT, J. C., Phys. Fluids 15 (1972) 2247.

[6] MINORSKY, N., Nonlinear Oscillations, Van Nostrand, New York (1962).

[7] SIGMAR, D. J., CALLEN, J. D., Phys. Fluids 14 (1972) 1423.

[8] SHAING, K. C., Phys. Fluids 24 (1981) 1299.

[9] HSU, J. Y., Phys. Fluids 25 (1982) 159.

[10] KAUFMAN, A. N., Phys. Fluids 15 (1972) 1063.

[11] STIX, T. H., The Theory of Plasma Waves, McGraw-Hill, New York (1962).