



# Neoclassical Ripple Transport in Tokamaks

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## Abstract

The usual ripple transport calculations lead to a  $\nu^{-1}$  scaling of the transport coefficients with collision frequency  $\nu$ . This paper extends and clarifies this scaling by taking into account the fact that the dominant contributions to transport come from particles in the high energy tail [ $E \sim (4 - 6)T$ ], which restricts the  $\nu^{-1}$  scaling range to  $\nu(T) > (E/T)^{5/2} \delta\omega_d(T) \sim 100 \delta\omega_d(T)$ . In addition, transport coefficients are derived in the low collisionality regime where the radial step size is determined by the distance a particle drifts before it "collisionlessly detraps" from a ripple well. The maximum transport rate is found to be about two orders of magnitude smaller than usually assumed, primarily because of the restriction of the  $\nu^{-1}$  scaling regime to the much higher collision frequencies.

## I. Introduction

The discrete nature of toroidal magnetic field coils spoils the symmetry of a tokamak and produces small modulations of the toroidal magnetic field that are called ripples. Particles trapped in local ripple magnetic wells drift off the original flux surface and enhance the particle and heat fluxes. Local ripple wells exist in the region where  $\alpha^* = \alpha |\sin \theta| < 1$ , with  $\alpha \equiv \epsilon/Nq\delta$  and  $\theta$  the poloidal angle. In the collisional regime (i.e.  $\nu_{\text{eff}} \equiv \nu/\delta > \omega_d = \alpha v_d/r$ , where  $\nu$  is the collisional frequency,  $v_d$  the  $\vec{v} \times \vec{B}$  drift speed, and  $r$  the minor radius) particles are detrapped from the ripple wells through small-angle collisions. The transport coefficients in this regime scale like  $\nu^{-1}$  and have been studied by many authors.<sup>1-3</sup> In the collisionless regime ( $\nu_{\text{eff}} < \omega_d$ ) particles can be detrapped from the ripple wells collisionlessly, due to the fact that as particles drift along a  $|B|$  surface they see a diminishing ripple well depth. The transport coefficients in this regime are proportional to  $\nu$ . In the case of  $\nu_{\text{eff}} < \omega_d$ , particles can also drift out of the system before they are detrapped. The smooth transition of the transport coefficients from the collisional to collisionless regime is accomplished by averaging over a Maxwellian distribution function. Particles with energy  $E$  will be in collisional (collisionless) regime if  $\nu_{\text{eff}}(E) > (<) \omega_d(E)$ . Since the dominant contribution to collisional transport comes from particles on the high energy tail of the distribution function ( $E/T \sim 4 - 6$ , with  $T$  the plasma temperature), the maxima of the transport coefficients occur at  $\nu_{\text{eff}}[E \sim (4 - 6)T] = \omega_d[E \sim (4 - 6)T]$ , which is almost two orders of magnitude larger than previous predictions;<sup>2</sup> the maxima of the transport coefficients are thus reduced by a similar factor. Specific formulae for transport in the

collisionless regime, and the appropriate averaging of this region with the collisional regime are developed below.

This paper is organized as follows. In Sec. II we discuss collisionless transport. The smooth transition of the transport coefficients from collisional to collisionless regime is discussed in Sec. III. Concluding remarks are given in Sec. IV.

## II. Collisionless Transport

Before we begin calculating transport coefficients, we first discuss the ripple well model we will use in the later calculations. For a tokamak with  $N$  toroidal field coils, the magnetic field can be written approximately as\*

$$B = B_0(1 - \epsilon \cos \theta - \delta \cos N\phi) \quad , \quad (1)$$

where  $B_0$  is the magnetic field on the axis,  $r$ ,  $\theta$ ,  $\phi$  are the usual toroidal coordinates,  $\epsilon \equiv r/R$ ,  $R$  is the major radius, and  $\delta$  is the ripple depth. The minimum (maximum) magnetic field along the field line can be found from the equation  $\partial B/\partial \theta = 0$ , which leads to

$$\epsilon \sin \theta + Nq\delta \sin (N\phi_0 + Nq\theta) = 0 \quad , \quad (2)$$

where  $q$  is the safety factor, and  $\phi_0 \equiv \phi - q\theta$  is the angular variable label for a particular field line. Equation (2) can be satisfied only if  $\alpha^* = \epsilon(\sin \theta)/Nq\delta < 1$ , which is the criterion for the existence of a local magnetic well due to the ripple. One set of consecutive maxima and minima obtained from Eq. (2) is

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\*The magnetic field in some stellarator configurations can also be written in this form; thus this work is relevant to transport in some stellarator configurations as well.

$$N\phi_0 + Nq\theta_m = - \sin^{-1} \alpha^* , \quad (3)$$

and

$$N\phi_0 + Nq\theta_{\pm} = \pm\pi + \sin^{-1} \alpha^* , \quad (4)$$

where  $\theta_{\pm}$  are two consecutive maxima and  $\theta_m$  is the minimum inbetween. To obtain Eqs. (3) and (4), we have assumed  $\alpha^*$  does not vary to lowest order in  $1/Nq$  across one ripple well and is evaluated at  $\theta = \theta_m$ . The effective ripple well depth  $\delta_{\text{eff}} = [B(\theta_-) - B(\theta_m)]/B_0$  is thus

$$\delta_{\text{eff}} = \delta [\sqrt{1 - \alpha^{*2}} - \alpha^* (\pi/2 - \sin^{-1} \alpha^*)] , \quad (5)$$

and the corresponding sinusoidal ripple well with well depth  $\delta_{\text{eff}}$  and length  $2(\theta_m - \theta_-)$  is

$$B = \bar{B}_0 \left[ 1 - \delta_{\text{eff}} \cos \left( \pi \frac{N\phi_0 + Nq\theta + \sin^{-1} \alpha^*}{\pi - 2 \sin^{-1} \alpha^*} \right) \right] , \quad (6)$$

where  $\bar{B}_0 = B_0 [1 - \epsilon \cos \theta_m - \delta \alpha^* (\pi/2 - \sin^{-1} \alpha^*)]$ , is shown in Fig. 1.

In the flux coordinates where

$$\vec{B} = \vec{\nabla}_{\phi_0} \times \vec{\nabla}_{\psi} , \quad (7)$$

with  $\psi$  is the poloidal flux function, the steady-state bounce averaged drift kinetic equation can be written as

$$\frac{c}{e} \left[ \left( \frac{\partial J}{\partial \psi} / \frac{\partial J}{\partial E} \right) \frac{\partial f}{\partial \phi_0} - \left( \frac{\partial J}{\partial \phi_0} / \frac{\partial J}{\partial E} \right) \frac{\partial f}{\partial \psi} \right] = \frac{v}{B_0} \frac{\partial}{\partial \mu} \left( J_{\mu} \frac{\partial f}{\partial \mu} \right) / \frac{\partial J}{\partial E} , \quad (8)$$

where  $e$  is the electric charge of the particles,  $c$  is the speed of light, and



J is the second adiabatic invariant defined as

$$J = \oint v_{\parallel} \frac{B d\theta}{\vec{B} \cdot \vec{\nabla}\theta} , \quad (9)$$

for a particle with parallel (to the magnetic field line) speed  $v_{\parallel}$ . With the ripple well model given in Eq. (6), we have

$$J = \frac{16}{\pi} \frac{B}{\vec{B} \cdot \vec{\nabla}\theta} \sqrt{\frac{\mu B_0 \delta_{\text{eff}}}{m} \frac{\pi - 2 \sin^{-1} \alpha^*}{Nq}} [E(k) - (1 - k^2) K(k)] , \quad (10)$$

where  $m$  is the mass of the particle,  $K$  and  $E$  are complete elliptic integrals of first and second kind respectively, and  $k^2$  is the pitch angle variable defined as

$$k^2 = \frac{E - \mu B_0 [1 - \epsilon \cos \theta_m - \delta \sqrt{1 - \alpha^{*2}}]}{\mu B_0 \delta_{\text{eff}}} . \quad (11)$$

For particles trapped in the bottom of the well,  $k^2 = 0$ , while for barely trapped particles,  $k^2 = 1$ . Assuming  $1 \ll \alpha \ll Nq$ , we can rewrite Eq. (8) explicitly utilizing Eqs. (10) and (11):

$$\dot{\psi}_0 \sin \theta_m \frac{\partial f}{\partial \psi} - \dot{\theta}_0 \cos \theta_m \frac{\partial f}{\partial \theta_m} = \frac{v}{B_0} \frac{\partial}{\partial \mu} (J \mu \frac{\partial f}{\partial \mu}) / (\frac{\partial J}{\partial E}) , \quad (12)$$

where

$$\dot{\psi}_0 = - \frac{c}{e} \frac{\mu B_0 \epsilon}{q} , \quad (13)$$

and

$$\dot{\theta}_0 = \frac{c}{e} \frac{\mu B_0 \varepsilon}{q R r B_p} , \quad (14)$$

with  $B_p$  being the poloidal magnetic field and  $\mu$  the magnetic moment. From now on, we will drop the subscript  $m$  in  $\theta_m$  for simplicity in notation.

Assuming  $\dot{\theta}_0 \sim \nu \gg \dot{\psi}_0 \sin \theta / r R B_p$ , to the lowest order in  $\dot{\theta}_0$  (or  $\nu$ ), we have

$$-\dot{\theta}_0 \cos \theta \frac{\partial f_0}{\partial \theta} = \frac{\nu}{B_0} \left[ \frac{\partial}{\partial \mu} \left( J \mu \frac{\partial f_0}{\partial \mu} \right) \right] \left( \frac{\partial J}{\partial E} \right)^{-1} . \quad (15)$$

A solution to Eq. (15) can be chosen as a Maxwellian distribution

$f_0 = f_M(\psi, E)$  with  $\partial f_0 / \partial \theta = 0$ . The linearized drift kinetic equation is then

$$\dot{\psi}_0 \sin \theta \frac{\partial f_M}{\partial \psi} - \dot{\theta}_0 \cos \theta \frac{\partial f_1}{\partial \theta} = \frac{\nu}{B_0} \left[ \frac{\partial}{\partial \mu} \left( J \mu \frac{\partial f_1}{\partial \mu} \right) \right] \left( \frac{\partial J}{\partial E} \right)^{-1} . \quad (16)$$

Next, we make an auxiliary ordering. Assuming  $\dot{\theta}_0 \alpha \gg \nu_{\text{eff}}$ , we obtain

$$\dot{\psi}_0 \sin \theta \frac{\partial f_M}{\partial \psi} = \dot{\theta}_0 \cos \theta \frac{\partial f_1}{\partial \theta} . \quad (17)$$

The solution to Eq. (17) can be written as

$$f_1 = - \frac{\dot{\psi}_0}{\dot{\theta}_0} \cos \theta \frac{\partial f_M}{\partial \psi} + g , \quad (18)$$

with  $\partial g / \partial \theta = 0$ . To obtain Eq. (18), we assumed  $\cos \theta \approx 1$ . The function  $g$  can be determined from the constraint that the first order perturbed distribution function  $f_1$  not contribute to the equilibrium density. Thus, we obtain

$$f_1 = \frac{\dot{\psi}_0}{2\dot{\theta}_0} (\theta^2 - \theta_T^2/3) \frac{\partial f_M}{\partial \psi} \quad (19)$$

where  $\theta_T$  is defined through  $k^2(\theta=\theta_T) = 1$ . Physically,  $\theta_T$  is the poloidal angle at which particles with a particular pitch angle  $k_0^2 = k^2(\theta = 0)$  at  $\theta = 0$  will be detrapped at  $\theta_T$ .

The relationship between  $\theta_T$  and  $k_0^2$  can be obtained from Eq. (11). We define

$$k_0^2 = \frac{E - \mu B_0(1 - \varepsilon - \delta)}{2\mu B_0 \delta} \quad (20)$$

Since along the particle's drift trajectory there is no variation of the quantity  $B_0(1 - \varepsilon \cos \theta - \delta \sqrt{1 - \alpha^{*2}})$ , we have

$$k^2 = k_0^2 \frac{\delta}{\delta_{\text{eff}}} \approx k_0^2 (\sqrt{1 - \alpha^{*2}})^{-1} \quad (21)$$

To obtain Eq. (21), we have used the approximation  $\delta_{\text{eff}} \approx \delta \sqrt{1 - \alpha^{*2}}$  for simplicity. From Eq. (21), we obtain

$$|\theta_T| = \frac{1}{\alpha} \sqrt{1 - k_0^4} \quad (22)$$

Those particles trapped at the bottom of the well at  $\theta = 0$  will be detrapped at  $\theta_T = 1/\alpha$ , while those particles that are barely trapped will be detrapped immediately, i.e.  $\theta_T = 0$ .

To the next order of the drift kinetic equation, we have

$$- \dot{\theta}_0 \cos \theta \frac{\partial f_2}{\partial \theta} = \frac{v}{B_0} \left( \frac{\partial J}{\partial E} \right)^{-1} \frac{\partial}{\partial \mu} \left( J \mu \frac{\partial f_1}{\partial \mu} \right) \quad (23)$$

Noting that

$$\mu \frac{\partial}{\partial \mu} \cong -\frac{1}{2\delta} \frac{\partial}{\partial k_0^2} , \quad (24)$$

we can simplify Eq. (23) and obtain

$$\frac{\partial f_2}{\partial \theta} = -\frac{\nu}{\dot{\theta}_0} \frac{1}{2\delta} \left[ \frac{\partial f_1}{\partial k_0^2} + k_0^2 \frac{\partial^2 f_1}{\partial (k_0^2)^2} \right] . \quad (25)$$

Using Eq. (19), we have

$$\frac{\partial f_2}{\partial \theta} = \frac{\nu}{6\delta} \frac{\dot{\psi}_0}{\dot{\theta}_0^2} \frac{\partial f_M}{\partial \psi} \left[ -\frac{k_0^2 (2 - k_0^4)}{\alpha(1 - k_0^4)^{3/2}} \theta_T + \frac{k_0^6}{\alpha^2 (1 - k_0^4)} \right] . \quad (26)$$

The particle and heat fluxes can be written as

$$\Gamma = + \int_{-\theta_T}^{\theta_T} \frac{d\theta}{\pi} \int d^3v \frac{\dot{\psi}_0}{|\nabla\psi|} \theta^2 \frac{\partial f_2}{\partial \theta} , \quad (27)$$

and

$$Q = \int_{-\theta_T}^{\theta_T} \frac{d\theta}{\pi} \int d^3v \frac{1}{2} m v^2 \frac{\dot{\psi}_0}{|\nabla\psi|} \theta^2 \frac{\partial f_2}{\partial \theta} . \quad (28)$$

To obtain Eqs. (27) and (28), we used the fact that  $f_2 \cong \theta \partial f_2 / \partial \theta$ . Using the approximation

$$\int d^3v \cong \frac{4\pi\sqrt{\delta}}{m^{3/2}} \int_0^\infty \sqrt{E} dE \int_0^\infty dk_0^2 ,$$

we obtain

$$\Gamma = -\frac{8}{45} \frac{1}{\alpha^5} \frac{1}{\sqrt{\delta}} \frac{1}{m^{3/2}} \int_0^{\infty} dE \sqrt{E} v \left( \frac{\dot{\psi}_0}{\dot{\theta}_0} \right)^2 \frac{1}{|\nabla\psi|} \frac{\partial f_M}{\partial \psi} , \quad (29)$$

and

$$Q = -\frac{8}{45} \frac{1}{\alpha^5} \frac{1}{\sqrt{\delta}} \frac{1}{m^{3/2}} \int_0^{\infty} dE E^{3/2} v \left( \frac{\dot{\psi}_0}{\dot{\theta}_0} \right)^2 \frac{1}{|\nabla\psi|} \frac{\partial f_M}{\partial \psi} . \quad (30)$$

Transforming the  $\psi$  coordinate to the usual  $r$  coordinate and noting that

$$v = v_t \frac{3}{4} \sqrt{\pi} A(x) x^{-3/2} ,$$

where

$$v_t = \frac{4}{3} \frac{\sqrt{2\pi} n e^4 \ln \Lambda}{\sqrt{m} T^{3/2}} ,$$

$$A(x) = \left( n + n' - \frac{n}{2x} \right) ,$$

$$n = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t} \sqrt{t} dt ,$$

$$n' = \frac{dn}{dx} ,$$

and  $x = E/T$ , we obtain

$$\Gamma = -0.015 \frac{r^2}{\sqrt{\delta} \alpha^5} v_t \int_0^{\infty} dx \frac{e^{-x}}{x} A(x) n \left[ \frac{1}{n} \frac{dn}{dr} + \frac{e}{T} \frac{d\phi}{dr} + \left( x - \frac{3}{2} \right) \frac{1}{T} \frac{dT}{dr} \right] , \quad (31)$$

and

$$Q = -0.015 \frac{r^2}{\sqrt{\delta} \alpha^5} v_t \int_0^{\infty} dx e^{-x} A(x) n T \left[ \frac{1}{n} \frac{dn}{dr} + \frac{e}{T} \frac{d\phi}{dr} + \left( x - \frac{3}{2} \right) \frac{1}{T} \frac{dT}{dr} \right] . \quad (32)$$

Equations (31) and (32) are valid for both electrons and ions. For electrons, both  $\nu_{ee}$  and  $\nu_{ei}$  have to be included, but  $\nu_t = \nu_{ii}$  for ions.

The electric field that appears in Eqs. (31) and (32) should be determined from the quasineutrality requirement by setting the electron particle flux (both neoclassical and anomalous) to that of the ions. Since the anomalous electron transport is not fully understood, we will leave  $d\phi/dr$  as an unknown quantity. An electric field will also affect the particle drift orbit and hence transport. This latter effect is not included in this paper.

Before closing this section, we give a heuristic derivation of the transport coefficients in Eqs. (31) and (32). For  $\alpha \gg 1$ , the radial step size before detrapping occurs is  $\Delta r \sim r/\alpha^2$ . The time scale for collisional detrapping is  $\Delta t \sim \delta/\nu$ , and the fraction of particles participating in the transport processes is  $f_p \sim \sqrt{\delta}$ . Thus, we can estimate a random walk diffusion coefficient ( $D \sim f_p (\Delta r)^2 / \Delta t$ ) to be

$$D \sim \frac{\nu}{\sqrt{\delta}} \frac{r^2}{\alpha^4} . \quad (33)$$

The extra  $1/\alpha$  factor in Eq. (31) is due to the fact that ripple trapped particles exist only where  $|\theta| < 1/\alpha$ .

### III. Overall Ripple Transport Coefficients

For a given Maxwellian distribution, particles with different energies participate in different transport processes. Ripple trapped particles can exist only if

$$\frac{E}{T} > \left[ \left( \frac{\nu_t}{\delta} \right) / \left( \frac{N \nu_t \sqrt{\delta}}{R} \right) \right]^{1/2} = (\nu_{\text{eff}} / \omega_{b\delta})^{1/2} \equiv a , \quad (34)$$

where  $\nu_t = \sqrt{2T/m}$  is the thermal speed of particles. Particles with energy  $E$

such that

$$a < \frac{E}{T} < b = \left[ \left( \frac{v_t}{\delta} \right) / \left( \frac{v_t^2}{2\Omega R} \frac{\alpha}{r} \right) \right]^{2/5} = (v_{\text{eff}}/\omega_d)^{2/5}, \quad (35)$$

will participate in collisional ripple-trapping transport,<sup>2,3</sup> where  $\Omega = eB/mc$ . In the collisional regime, boundary layer effects<sup>4</sup> should be also taken into account. The boundary layer will cause a smooth transition to the collisional ripple plateau regime.<sup>5</sup> Particles with energy  $E/T > b$  will participate in the collisionless transport. In cases where particles become detrapped before drifting out of the system as discussed in Sec. II, they will contribute to both the particle and heat fluxes as given in Eqs. (31) and (32). In the case where they can drift out of the system before being detrapped, they will not contribute to the diffusive particle and heat fluxes in the energy range given in Eq. (35); they give a small, direct loss of particles and energy.

Thus, the total particle and heat fluxes for a Maxwellian distribution can be written as

$$\begin{aligned} \Gamma = & -0.34 G(\alpha) \frac{\delta^{3/2}}{v_t} \left( \frac{cT}{eBR} \right)^2 n \int_a^b dx \frac{x^4 e^{-x}}{A(x)} \left[ \frac{1}{n} \frac{dn}{dr} + \frac{e}{T} \frac{d\phi}{dr} + \left( x - \frac{3}{2} \right) \frac{1}{T} \frac{dT}{dr} \right] \\ & - 1.72 \bar{G}(\alpha) \frac{\delta^{3/2}}{v_t} \left( \frac{cT}{eBR} \right)^2 an \int_a^b dx \frac{x^3 e^{-x}}{[A(x)]^{1/2}} \left[ \frac{1}{n} \frac{dn}{dr} + \frac{e}{T} \frac{d\phi}{dr} + \left( x - \frac{3}{2} \right) \frac{1}{T} \frac{dT}{dr} \right] \\ & - 0.015 \frac{r^2}{\sqrt{\delta} \alpha^5} v_t n \int_b^\infty dx \frac{e^{-x}}{x} A(x) \left[ \frac{1}{n} \frac{dn}{dr} + \frac{e}{T} \frac{d\phi}{dr} + \left( x - \frac{3}{2} \right) \frac{1}{T} \frac{dT}{dr} \right], \quad (36) \end{aligned}$$

and

$$\begin{aligned}
Q = & -0.34 G(\alpha) \frac{\delta^{3/2}}{v_t} \left(\frac{cT}{eBR}\right)^2 nT \int_a^b dx \frac{x^5 e^{-x}}{A(x)} \left[ \frac{1}{n} \frac{dn}{dr} + \frac{e}{T} \frac{d\phi}{dr} + \left(x - \frac{3}{2}\right) \frac{1}{T} \frac{dT}{dr} \right] \\
& - 1.72 \bar{G}(\alpha) \frac{\delta^{3/2}}{v_t} \left(\frac{cT}{eBR}\right)^2 anT \int_a^b dx \frac{x^4 e^{-x}}{[A(x)]^{1/2}} \left[ \frac{1}{n} \frac{dn}{dr} + \frac{e}{T} \frac{d\phi}{dr} + \left(x - \frac{3}{2}\right) \frac{1}{T} \frac{dT}{dr} \right] \\
& - 0.015 \frac{r^2}{\sqrt{\delta} \alpha^5} v_t nT \ell \int_b^\infty dx e^{-x} A(x) \left[ \frac{1}{n} \frac{dn}{dr} + \frac{e}{T} \frac{d\phi}{dr} + \left(x - \frac{3}{2}\right) \frac{1}{T} \frac{dT}{dr} \right], \quad (37)
\end{aligned}$$

where  $G(\alpha)$  and  $\bar{G}(\alpha)$  are geometric factors associated with ripple transport,<sup>3</sup> and boundary layer effects,<sup>4</sup> respectively, and  $\ell = 0$  for particles that can drift out of the system, or  $\ell = 1$  otherwise. The ion transport coefficients for the heat flux  $Q$ , which can be written as

$$Q = -x_n T \frac{dn}{dr} - x_\phi n e \frac{d\phi}{dr} - x_T n \frac{dT}{dr}, \quad (38)$$

are shown in Fig. 2 for a particular set of the plasma parameters, which are similar to those used by Tani et al. in their Monte Carlo simulations of ripple transport.<sup>6</sup> Notice that  $x_\phi = x_n$ .

From Fig. 2, we can see that the maxima of the transport coefficients occur at the collision frequency where  $1 \sim [v_{eff}/\omega_d]_{E \sim (4-6)T}$ , which is about two orders of magnitude larger in collisionality than previous predictions,<sup>2</sup> which implied  $1 \sim (v_{eff}/\omega_d)_{E=T}$ . The reason for this difference is that since the dominant contribution to the collisional transport comes from high energy particles [ $E \approx (4 - 6)T$ ], when these particles start to participate in the collisionless transport the  $1/v$  growth stops and this limits the maxima of the transport coefficients.

#### IV. Concluding Remarks

Ripple transport in the collisionless regime, namely  $v_{eff} < \omega_d$  has been calculated from the bounce averaged drift kinetic equation. We find that the



transport coefficients are proportional to the collision frequency in this regime. The smooth transition of the transport coefficients from the collisional to collisionless regime is obtained by splitting up a Maxwellian distribution into different collisionality regimes according to their energy. The result is that the collisionality regime and magnitude of the maxima of the transport coefficients are determined by high energy particles ( $E/T \sim 4 - 6$ ). This is because they give the dominant contributions to the collisional transport, and when they start to participate in collisionless transport ( $v_{\text{eff}}[E \sim (4 - 6)T] < \omega_d[E \sim (4 - 6)T]$ ), the  $1/v$  scaling of the transport coefficients stop and the maxima of the transport coefficients are thus limited to values about two orders of magnitude smaller than usual estimates imply.<sup>3</sup> These results indicate that the ripple-trapping transport losses in large, high-temperature plasmas confined in tokamaks or stellarators are much less severe than previously thought.

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## Figure Captions

Figure 1. Schematic of model magnetic field for ripple well.

Figure 2. Variation of ripple-trapping transport coefficients with collision frequency as calculated from Eqs. (36)-(38) for the parameters shown. The dashed lines indicate the variation that would occur if the contributions due to the collisionless regime particles ( $E/T > b$ ) were neglected. The usually assumed ripple-trapping transport coefficients are shown for reference with a dash-dot line. Note that the maxima in the latter curves are about two orders of magnitude larger than the results obtained here, and occur at a much lower collisionality.

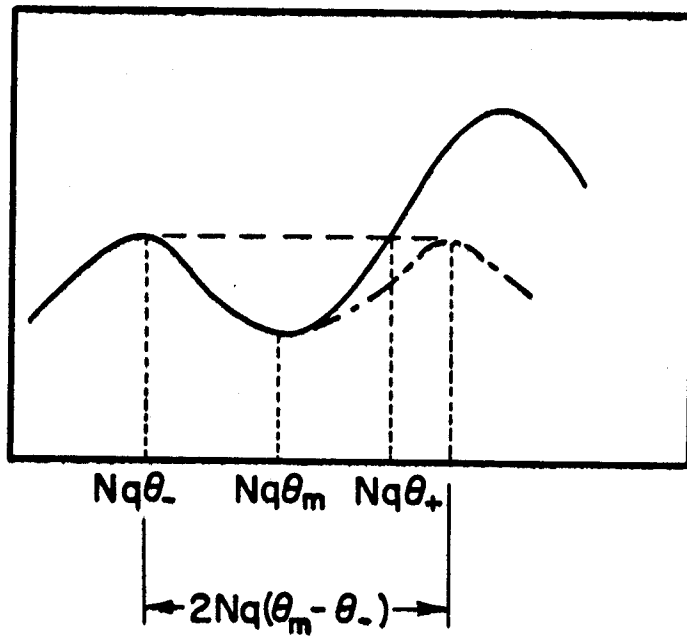


Figure 1

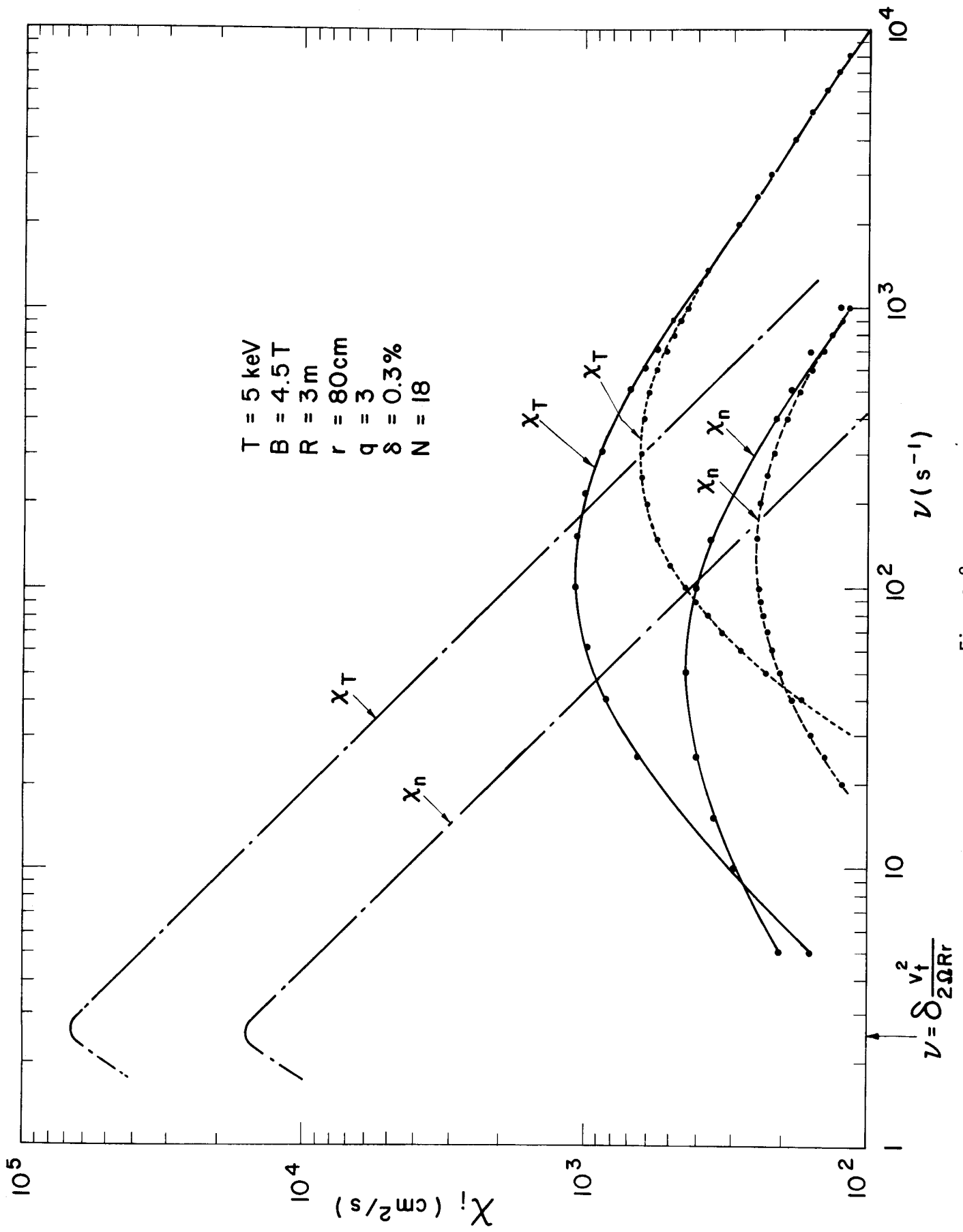


Figure 2