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Abstract

Plasma flowing through a decreasing magnetic field, as in a bundle divertor, experiences a drop in potential. The decrease is calculated using a 1-D model in which the ion distribution is calculated self-consistently. This is compared with the results of an earlier model developed for bundle divertors; the two solutions are in good agreement for usual values of the mirror ratio. This calculation, however, gives a continuous, well-behaved solution as the mirror ratio approaches unity, whereas the previous model gave a discontinuous result.

I. Introduction

Bundle divertors for tokamaks employ magnetic coils to divert a bundle of magnetic flux near the plasma surface to a "burial chamber" wherein the ions flowing along the flux bundle are neutralized and pumped away. Such divertors should hopefully allow disposal of impurities sputtered off of the first wall surface and provide a method for removal of the helium "ash". The dimensions of a typical divertor are a fraction of an ion mean free path, and so ions flowing into a divertor may be treated as collisionless, while the typical length of travel in the main plasma is such that ions there are in the collisional regime. Particles flowing into a bundle divertor experience an increase in the magnetic field intensity (due to the divertor coil), followed by a decrease in the field intensity. This results in a lower potential in the divertor chamber relative to the "scrape-off zone" of the main tokamak plasma. This potential drop impedes the backflow of ions from the divertor back into the main tokamak plasma.[1,2]

A model deriving this potential drop has previously been advanced by Emmert.[2] This earlier model considers the plasma in the scrape-off zone as Maxwellian in the forward direction up to the point at which it encounters the bundle divertor coils. In this paper this assumption is not made, and the results are similar but slightly different. The method used herein, however, does not allow modeling an initial magnetic field intensity increase as does the previous model of Emmert. Hopefully, in the future this difficulty can be corrected.

II. Derivation of the Plasma Potential

The technique used to derive the plasma potential parallels that used by Emmert, Wieland, Mense and Davidson to derive plasma sheath and presheath potentials.[3] For the sake of simplicity, a symmetric 1-D system is assumed, with plasma flow to the divertor chamber in both the +x and -x directions, as shown in Fig. 1. In practice, the actual chamber would be a torus, and the diverted flux would travel into a single burial chamber.

Outflowing ions are modeled as produced by a source function, S, which has a nonzero value only in the region $-L < x < L$. Within this region there is presumed to be a constant x-directed magnetic field, B_0, which slowly and monotonically decreases beginning for some $x < -L$ and $x > +L$. The energy of a gyrating ion will be

$$E = \frac{1}{2} M v_x^2 + q\phi(x) + \mu B(x) \tag{1}$$

where M is the ion mass, q is the ion charge, $\phi(x)$ is the electrostatic potential, μ is the magnetic moment of the ion gyration, and B(x) is the magnetic field intensity.

Consider a phase space (x,E,μ), all mutually orthogonal. The kinetic equation in this phase space is

$$\frac{\partial}{\partial x} \left(g \frac{B_0}{B(x)} v_x\right) = S \tag{2}$$

where g is the ion distribution function, $B_0/B(x)$ models the expansion of the tube of flux, and

$$v_x \, (x,E,\mu) = \sqrt{\frac{2}{M} (E - q\phi(x) - \mu B(x))} \quad . \tag{3}$$

It is necessary to add a variable, $\sigma = +1, -1$, indicating whether the ion

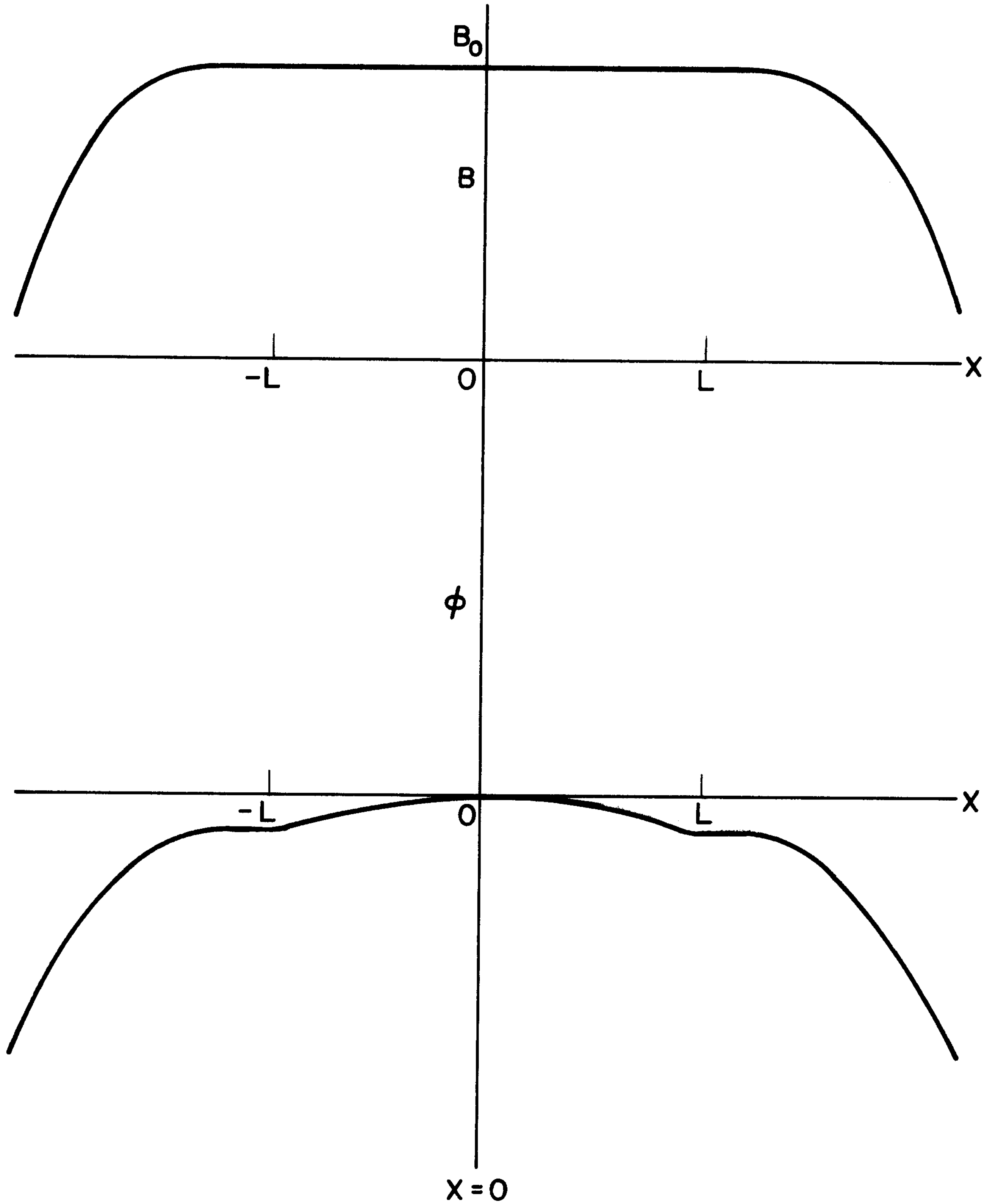

FIGURE 1 THE GEOMETRY AND COORDINATE SYSTEM.

is traveling in the +x or -x direction, respectively. As a boundary condition it will be assumed that there is no backflow of ions into the source region from outside, that is

$$g(L,E,\mu,-1) = g(-L,E,\mu,+1) = 0 \quad . \tag{4}$$

For the other boundary condition it is assumed that there is no net flow at the center of plasma, i.e.,

$$g(0,E,\mu,+1) = g(0,E,\mu,-1) \quad . \tag{5}$$

This is evident from symmetry.

It is assumed that ions are equally likely to be born traveling in one direction as in the other:

$$S(x,E,\mu,+1) = S(x,E,\mu,-1) \equiv S(x,E,\mu) \tag{6}$$

and the potential is defined as zero at the center of the plasma:

$$\phi(x=0) = 0 \quad . \tag{7}$$

For $E \geqslant \mu B_0$, $-L \leqslant x \leqslant L$

$$g(x,E,\mu,+1) = \frac{1}{v_x} \int_{-L}^{x} S(x',E,\mu)\, dx' \tag{8}$$

and

$$g(x,E,\mu,-1) = \frac{1}{v_x} \int_{x}^{L} S(x',E,\mu)\, dx' \quad . \tag{9}$$

For $E \leqslant \mu B_0$, $0 \leqslant x \leqslant L$

$$g(x,E,\mu,+1) - g(x_0,E,\mu,+1) = \frac{1}{v_x} \int_{x_0}^{x} S(x',E,\mu)\, dx' \tag{10}$$

and

$$g(x,E,\mu,-1) = \frac{1}{v_x} \int_{x}^{L} S(x',E,\mu)\, dx' \tag{11}$$

where
$$q\phi(x_0) + \mu B_0 = E \quad . \tag{12}$$

For $x \geqslant L$

$$g(x,E,\mu,+1) = \frac{B(x)\ v_x(L)}{B_0\ v_x(x)}\, g(L,E,\mu,+1) \tag{13}$$

and
$$g(x,E,\mu,-1) = 0 \quad . \tag{14}$$

The ion density is

$$n_i(x) = \sum_\sigma \int d\mu \int dE\ g(x,E,\mu,\sigma) \quad . \tag{15}$$

So for $x \geqslant L$

$$n_i(x) = \frac{2B(x)}{B_0} \int_0^\infty d\mu \int_{E_{min}}^\infty dE \int_0^L dx' \ \frac{S(x',E,\mu)}{v_x(x)} \tag{16}$$

where
$$E_{min} = q\phi(L) + \mu B_0 \quad . \tag{17}$$

Reversing the order of integration gives

$$n_i(x) = \frac{2B(x)}{B_0} \int_0^L dx' \int_0^\infty d\mu \int_{E_0}^\infty dE\ \frac{S(x',E,\mu)}{v_x(x)} \tag{18}$$

where
$$E_0 = q\phi(x') + \mu B_0 \quad . \tag{19}$$

The source, S, is chosen so that it would give a Maxwellian distribution in the absence of a potential, i.e.,

$$S(x,E,\mu) \propto \exp\left(\frac{q\phi}{kT_i}\right) \exp\left(- \frac{E}{kT_i}\right) \quad . \tag{20}$$

Normalization requires

$$\sum_{\sigma} \int_0^\infty d\mu \int_{q\phi(x)+\mu B_0}^\infty dE\ S(x,E,\mu) = S_0 h(x) \tag{21}$$

where $S_0 h(x)$ is the number of ions created per unit volume per unit time at x, S_0 being a constant and h(x) a function allowing for a nonuniform source with

$$\frac{1}{L} \int_0^L h(x)\ dx = 1\ . \tag{22}$$

Using Eqs. 21 and 22 one obtains

$$S(x,E,\mu) = \frac{S_0 B_0 h(x)}{2(kT_i)^2} \exp\left(\frac{q\phi(x)}{kT_i}\right) \exp\left(-\frac{E}{kT_i}\right)\ . \tag{23}$$

Substituting this expression for S into Eq. 18 gives

$$\begin{aligned} n_i(x) = \int_0^L dx' \int_0^\infty d\mu \int_{q\phi(x')+B_0\mu}^\infty dE\ \frac{S_0 h(x') B(x)}{(kT_i)^2\ v_x(x,E,\mu)} \\ \exp\left(\frac{q\phi(x')}{kT_i}\right) \exp\left(-\frac{E}{kT_i}\right)\ . \end{aligned} \tag{24}$$

Performing the inner two integrations gives

$$\begin{aligned} n_i(x) = \int_0^L dx' \frac{S_0 h(x')}{2} \sqrt{\frac{2\pi M}{kT_i}}\ [E(Z\tau(\psi - \psi')) \\ - \frac{1}{\sqrt{\gamma}} E(Z\tau\gamma(\psi - \psi'))] \end{aligned} \tag{25}$$

where

$$\left.\begin{aligned}
\psi &= -\frac{e\phi(x)}{kT_e} \\
\psi' &= -\frac{e\phi(x')}{kT_e} \\
e &= \text{elementary charge} \\
Z &= q/e = \text{atomic number of the ions} \\
\tau &= T_e/T_i = \text{ratio of electron to ion temperature} \\
\gamma &= \frac{B_0}{B_0 - B(x)} \\
E(x) &= \exp(x)\ \text{erfc}(\sqrt{x}) \quad .
\end{aligned}\right\} \tag{26}$$

The electron density is given by the Boltzmann relation,

$$n_e(x) = n_0 \exp\left(\frac{e\phi(x)}{kT_e}\right) \tag{27}$$

where n_0 is the electron density at $x = 0$.

S_0 and n_0 can be related by the requirement that the total charge flux out of the source region must be zero at steady-state. This can be accomplished by considering the potential that an electrostatically floating wall placed at $x = L$ would reach. That is,

$$S_0 ZL = n_0 \sqrt{\frac{kT_e}{2\pi m}}\ \exp(-\psi_w) \tag{28}$$

where $\psi_w = -\frac{e\phi_w}{kT_e}$ for ϕ_w = wall potential,

and m is the electron mass. The value of ψ_w for this case has been determined by Emmert et al. and found to be[3]

$$\psi_w = \ln \left[\sqrt{\frac{M}{m}\frac{Z}{4\pi}} \; \frac{1}{(Z + T_i/T_e)} \; \frac{\pi}{2\exp(-\psi_1)D(\sqrt{\psi_1})} \right] \tag{29}$$

where D(x) is the Dawson function[4]

$$D(x) = \int_0^x e^{t^2} \, dt \tag{30}$$

and

$$\psi_1 = -\frac{e\phi_1}{kT_e} ,$$

ϕ_1 being the potential at the edge of the source, just outside the plasma sheath, defined by the equation

$$1 = \frac{2}{\sqrt{\pi Z\tau}} \exp[-(1 + Z\tau)\psi_1] \, D(\sqrt{\psi_1}) + \mathrm{erf}(\sqrt{Z\tau\psi_1}) \quad . \tag{31}$$

Substituting Eq. 28 into Eq. 25 to eliminate S_0 gives the result,

$$n_i(x) = \int_0^L dx' \, \frac{n_0 h(x')}{2ZL} \sqrt{\frac{M}{m}\tau} \; \exp(-\psi_w) \, [E(Z\tau(\psi - \psi')) - \frac{1}{\sqrt{\gamma}} E(Z\tau\gamma(\psi - \psi'))] \quad . \tag{32}$$

Letting $Zn_e = n_i$, using Eq. 27, gives

$$\exp(-\psi) = \frac{1}{2}\sqrt{\frac{M}{m}\tau} \; \exp(-\psi_w) \int_0^1 dr' \, [E(Z\tau(\psi - \psi')) - \frac{1}{\sqrt{\gamma}} E(Z\tau\gamma(\psi - \psi'))] \tag{33}$$

where

$$dr' = \frac{1}{L} h(x') \, dx' \quad . \tag{34}$$

For the case in which the magnetic field remains constant ($\gamma \to + \infty$) the second term,

$$\frac{1}{\sqrt{\gamma}} \; E(Z\tau(\psi - \psi')) \ ,$$

vanishes[5], and the equation reduces to that previously derived by Emmert et al. for the plasma equation in the electric presheath.[3]

Equation 33 was solved numerically, the normalized potential, ψ', being determined using the relation derived by Emmert et al.,[3]

$$\sqrt{\frac{M}{m} Z\pi} \; \left(\frac{\tau r'}{1 + Z\tau}\right) \exp(-\psi_w) = 4 \exp(-\psi') \, D(\sqrt{\psi'}) \ . \tag{35}$$

III. Discussion and Conclusions

The results of this computation are shown in Figs. 2 and 3. Also shown are the results of computations using the bundle divertor model of Emmert.[2] The two models are in reasonable agreement for usual values of $B(x)/B_0$, e.g. 0.5 - 0.2, but the calculation presented goes smoothly into the presheath drop[3] at $x = L$ as $B(x)/B_0 \to 1$, while the previous model gives a discontinuous result. One may conclude from this calculation that the discontinuity at $B(x)/B_0 \to 1$ in the earlier model is an artifact; it probably results from the discontinuity in the distribution function used in that model. The earlier model, nevertheless, is a good approximation for the usual mirror ratios encountered in bundle divertors.

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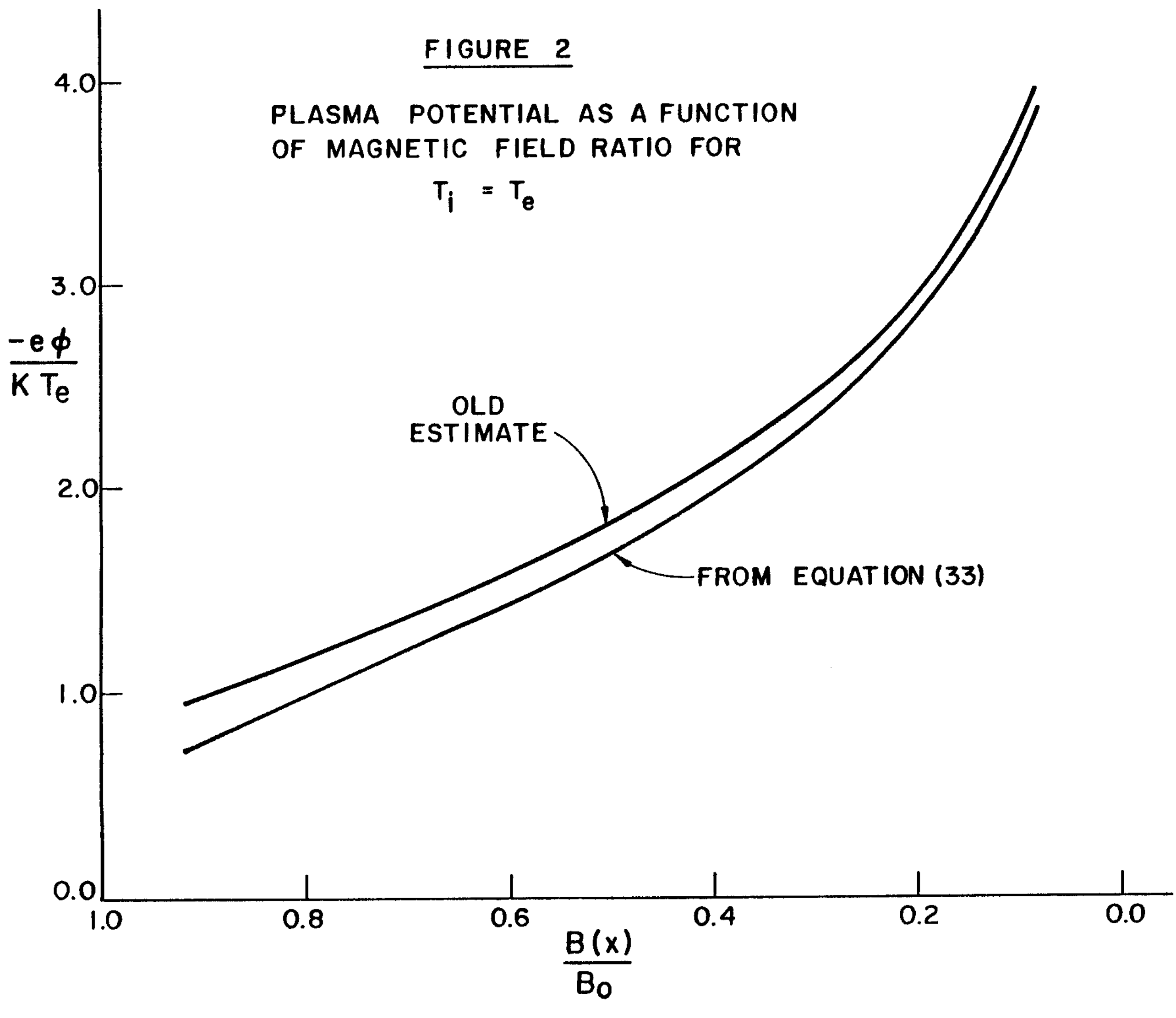
FIGURE 2
PLASMA POTENTIAL AS A FUNCTION OF MAGNETIC FIELD RATIO FOR
$T_i = T_e$
4.0
3.0
2.0
1.0
0.0
$\frac{-e\phi}{K T_e}$
OLD ESTIMATE
FROM EQUATION (33)
1.0
0.8
0.6
0.4
0.2
0.0
$\frac{B(x)}{B_0}$

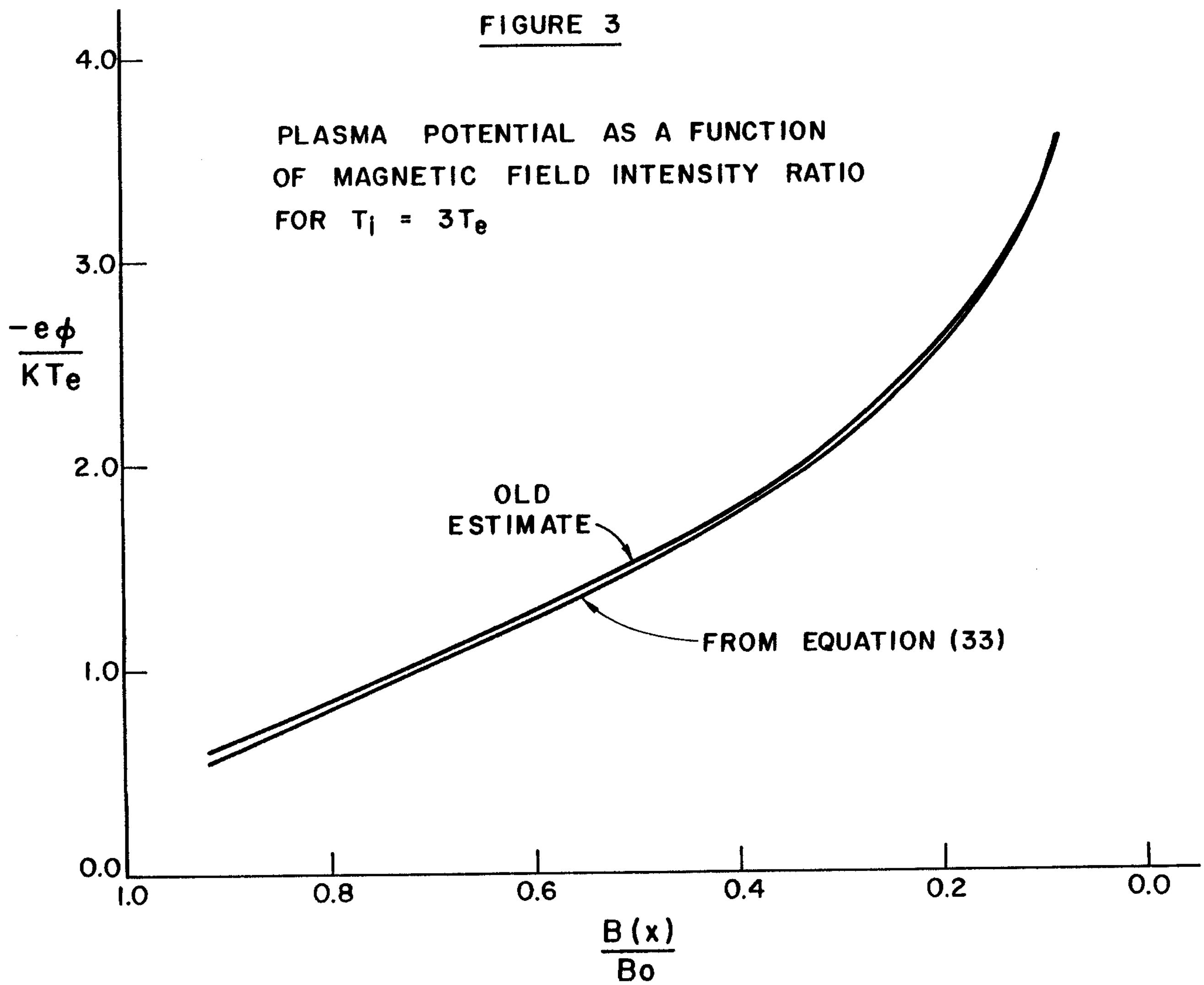
FIGURE 3
PLASMA POTENTIAL AS A FUNCTION
OF MAGNETIC FIELD INTENSITY RATIO
FOR T_i = $3T_e$
4.0
3.0
2.0
1.0
0.0
$\frac{-e\phi}{KT_e}$
OLD ESTIMATE
FROM EQUATION (33)
1.0
0.8
0.6
0.4
0.2
0.0
$\frac{B(x)}{Bo}$

References

1. J. D. Callen, G. A. Emmert, A. W. Bailey, M. E. Benchikh-Lehocine, J. N. Davidson, R. Hulse, A. Boozer, L. Foote, R. Hawryluk, D. Heifetz, W. Hsu, D. Mikkelsen, J. Ogden, K. Owens, M. Petravic, D. Post, P. Rutherford, J. Schmidt, F. Seidl, C. Singer, S. Suckewer, J. Weisheit, and M. Yamada, 8th International Conference on Plasma Physics and Controlled Nuclear Fusion Research, Brussels (1980).

2. G. A. Emmert, "Modeling of Bundle Divertors," UWFDM-343 (1980).

3. G. A. Emmert, R. M. Wieland, A. T. Mense, and J. N. Davidson, Phys. Fluids 23, 803 (1980).

4. W. Gautschi, in Handbook of Mathematical Functions, edited by M. Abramowitz and I. Stegun, National Bureau of Standards, Applied Mathematics Series No. 55, (U.S. Government Printing Office, Washington, D.C., 1964), p. 319.

5. Ibid., p. 298.