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May 1980

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FUSION TECHNOLOGY INSTITUTE

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1. Introduction

The central cell field of the tandem mirror reactor (TMR) will be achieved with discrete solenoids spaced at regular intervals to provide space for feeding and diagnostic devices. The solenoids are superconducting and operate in a steady dc mode. To provide long term operation, the solenoid will be designed for cryostatically stable operation.

2. Stresses

The central cell solenoids for the TMR may be wound in pancakes or in disks similar to the NUWMAK conductor arrangement(1). If the windings are mechanically independent, the hoop forces in the solenoids are given by the local Lorentz forces. The tangential stress in an unsupported conductor due to the magnetic force is given by

$$\sigma(r) = j(r) \cdot B(r) \cdot r \; . \qquad (2.1)$$

The total force on the current carrying winding is distributed among the components of the conductor and the structural material. The stress is given by the rule of mixtures

$$\sigma = \sum_n \sigma_n \alpha_n \qquad (2.2)$$

where the strain $\varepsilon_n = \sigma_n/E_n$ is constant for all n components. α_n is the relative volume fraction and E is Young's modulus. Eq. (2.2) is applicable in the elastic region of the materials, i.e. where Hooke's law is valid.

The NUWMAK conductor consists of five components:

1. NbTi superconductor
2. High purity Al-stabilizing material
3. High strength Al to encapsulate the first two components.

These three components form the conductor itself.
The fourth component is epoxy to electrically insulate the conductor and to couple the conductor mechanically with the fifth component, the structural material of the disk. In the case of the NUWMAK conductor, the structural material is high strength aluminum, Al-2219 T 87, the same as the third component of the conductor.

The maximum magnetic load for our solenoids under consideration is given in a simplified manner by

$$F_{mag} = I \cdot B_{max} \cdot a_1 \qquad (2.3)$$

where I is the current, a_1 is the inner radius of the solenoid winding and B_{max} is the field at a_1. Other equations for the magnetic load may be found in Ref. (2).

The load is balanced by tangential hoop forces. In the case of a supported conductor this load is balanced by the conductor itself and the structural material. Therefore, we have a balance equation

$$F_{mag} = F_c + F_s = \sigma_c A_c + \sigma_s A_s \qquad (2.4)$$

where the indices c and s denote conductor and structure, respectively. Dividing by A_c gives

$$\sigma_{mag} = \frac{F_{mag}}{A_c} = \sigma_c + \sigma_s \frac{A_s}{A_c} . \qquad (2.5)$$

Together with Eq. (2.3) and the conductor current density

$$j_{cond} = I/A_c \qquad (2.6)$$

we get

$$j_{cond} \cdot B_{max} \cdot a_1 = \sigma_c + \sigma_s \frac{A_s}{A_c} . \qquad (2.7)$$

In the case under consideration we have

$$\sigma_c = \varepsilon \, (E_1 \, \alpha_1 + E_3 \, \alpha_3) + \sigma_2 \, \alpha_2 \qquad (2.8)$$

and

$$\sigma_s = \varepsilon \, (E_4 \, \gamma_4 + E_5 \, \gamma_5) \qquad (2.9)$$

where the α_i's are the relative fractions of the i-th component in the conductor area only and the γ's are the relative fractions in the structure area only. Eq. (2.8) takes into account that the high purity aluminum (component 2) in the conductor is in the plastic region. For the α's and the γ's we have

$$1 = \alpha_1 + \alpha_2 + \alpha_3 \tag{2.10}$$

and

$$1 = \gamma_4 + \gamma_5 \ . \tag{2.11}$$

Eq. (2.7) together with (2.8,9) results in:

$$j_{cond} \cdot B_{max} \cdot a_1 \leqslant \varepsilon \ (E_1 \ \alpha_1 + E_3 \ \alpha_3) + \sigma_2 \ \alpha_2 + \varepsilon \ (E_4 \ \gamma_4 + E_s \ \gamma_s) \cdot \frac{A_s}{A_c} \ . \tag{2.12}$$

This relation means that the magnetic load has to be supported by the conductor and the structure. Therefore, it is favorable to consider Eq. (2.12) as an inequality.

According to Ref. (2) some other expressions can be used for the circumferential stresses σ_c, the left side of Eq. (2.12). One relation is

$$\sigma_c = \frac{j_{cond}}{4} \ (B_1 + B_2) \ (a_2 + a_1) \ . \tag{2.13}$$

This is an average over the coil thickness. a_1 is the minor and a_2 is the major radius of the solenoid. B_1 and B_2 are the magnetic fields at a_1 and a_2. Two other relations for the stresses were considered; both are radius-dependent. The first one is

$$\sigma_c = \frac{a_i^2 \cdot j_{cond}}{4a\ell n\alpha} \cdot (B_1+B_2)\ (\alpha^2-1) \qquad (2.14)$$

where $a_1 \leqslant a \leqslant a_2$ and $\alpha = a_2/a_1$. The second one is

$$\sigma_c = a_1 \cdot j_{cond} \cdot \frac{B_1(\alpha^2+\alpha-2)+B_2(2\alpha^2-\alpha-1)}{6(\frac{a}{a_1})\ \ell n\alpha} . \qquad (2.15)$$

A comparison of the different formulas for the stresses is given in section 7 for the central cell magnets.

3. Critical Current Density

The critical current density j_c in the superconductive material itself has to be taken at the maximum magnetic field B_{max}, working at the conductor, and at the operating temperature T_{op}. The critical current density in the composite conductor j_{cond} is given by

$$J_{cond} = \alpha_1 \cdot f \cdot J_c\ (B_{max}, T_{op}) \qquad (3.1)$$

where α_1 is the fraction of superconducting material in the conductor and f means the safety factor (f < 1).

4. Stabilization

To protect the solenoids during a quench, the criterion of cryostatic stability must be fulfilled. It is given by[3]:

$$I^2 \cdot \rho_n = \delta \cdot h \cdot \Delta T \cdot A_n \cdot S_n \qquad (4.1)$$

where I is the current, δ is the stability parameter, h the heat transfer coefficient, ΔT the temperature difference, A_n the area of the stabilizing material and S_n is the cooled surface area of the conductor. For $\delta \leqslant 1$ we have full stabilization.

Eq. (4.1) can be written in the form

$$j^2{stab} \cdot \rho_{stab} \leqslant \frac{pP}{\alpha_{stab} \cdot A_c} \cdot q_r \qquad (4.2)$$

which is more appropriate for our consideration. J_{stab} is the current density working in the stabilizing material. ρ_{stab} is the resistivity therein. α_{stab} is the relative fraction of the stabilizing metal in the conductor area A_c, pP is the cooled portion of the perimeter P and q_r is the recovery heat flux. For pool boiling (bath cooling) a value of 0.3 W/cm^2 at 4.2 K and 0.5 W/cm^2 at 1.8 K is taken.

Considering the case of a 3-component conductor we have

$$j_{stab} = j_{cond} \left(1 + \frac{\alpha_1}{\alpha_2+\alpha_3}\right) = \frac{j_{cond}}{\alpha_2+\alpha_3} \qquad (4.3)$$

taking into account Eq. (2.10). The resistivity ρ of the conductor is given by the relation

$$\frac{1}{\rho} = \frac{\alpha_1}{\rho_1} + \frac{\alpha_2}{\rho_2} + \frac{\alpha_3}{\rho_3} \qquad (4.4)$$

where ρ_1 is the resistivity of the superconductor after the transition to normal conduction. In general $\rho_1 >> \rho_{21}$ ρ_3 and we get

$$\frac{1}{\rho_{stab}} = \frac{\rho_2\ \rho_3(\alpha_2+\alpha_3)}{\rho_2\ \alpha_3 + \rho_3\ \alpha_2} \quad \text{with} \quad \alpha_{stab} = \alpha_2 + \alpha_3 . \qquad (4.5)$$

With (4.3) and (4.5) we get from (4.2):

$$j^2_{cond} \cdot \frac{\rho_2}{\alpha_2 + \frac{\rho_2}{\rho_3}\alpha_3} \leq \frac{pP}{A_c} \cdot q_r \ . \qquad (4.6)$$

This is the cooling condition for full stabilization.

5. Magnet Protection

The protection integral is given by

$$\int_0^\infty j^2_{cond}(t)dt = \int_{T_b}^{T_{max}} \frac{c(T)\cdot\delta}{\rho(T)} dT \qquad (5.1)$$

where c(T) δ is the heat capacity of the normal conducting material(2). If we have n components in a composite conductor, then the protection integral is given by

$$\int_0^\infty j^2_{cond}(t)dt = \int_{T_b}^{T_{max}} \left[\sum_{k=1}^{n} \frac{\alpha_k}{\rho_k(T)}\right] \cdot \left[\sum_{k=1}^{n} \alpha_k\, c_k(T)\, \delta_k\right] dT\ . \qquad (5.2)$$

We consider first the time integral on the left side of Eq. (5.1) or (5.2) for two different cases:

- Discharge in a constant resistor R_D
- Discharge with a constant voltage.

In the first case we suppose an exponential decay of the initial current density j of the conductor

$$j(t) = j_o\, e^{-t/\tau_R} \qquad (5.3)$$

where the time constant τ_R is given by

$$\tau_R = \frac{2 \cdot E_s}{I_o \cdot U_o} \; . \tag{5.4}$$

E_s is the stored energy, I_0 is the initial current and U_0 is the voltage. Then the time integral is

$$\int_0^\infty j^2(t)dt = \frac{j_o^2 \tau_R}{2} = \frac{j_o^2 \cdot E_s}{I_o \cdot U_o} \overset{!}{=} I_{R_D} \; . \tag{5.5}$$

In the second case we consider the linear decay of the initial current density of the conductor given by

$$j(t) = j_o \; (1 - \frac{t}{\tau_R}) \text{ for } 0 \leqslant t \leqslant \tau_R \tag{5.6}$$

where τ_R is given by (5.4). In this case the time integral is

$$\int_0^\infty j^2(t)dt = \frac{1}{3} \, j_o^2 \; \tau_R = \frac{2}{3} \, I_{R_D} \overset{!}{=} I_u \; . \tag{5.7}$$

The time integral I_u for constant voltage discharge is only two-thirds of the time integral I_{R_D} for constant resistor discharge.

6. Design Procedure

We have the equations

$$1 = \alpha_1 + \alpha_2 + \alpha_3 \qquad (2.1)$$

$$1 = \gamma_4 + \gamma_5 \qquad (2.11)$$

$$\frac{A_s}{A_c} \cdot \varepsilon\,(E_4\,\gamma_4 + E_5\,\gamma_5) \geqslant j_{cond} \cdot B_{max} \cdot a_1 - [\varepsilon(E_1\alpha_1 + E_3\alpha_3) + \sigma_2\alpha_2] \qquad (2.12)$$

$$j_{cond} = \alpha_1 \cdot f \cdot j_c\,(B_{max}, T_{op}) \qquad (3.1)$$

$$\frac{pP}{A_c} \cdot q_r \geqslant j^2_{cond}\,\frac{\rho_2}{\alpha_2 + \frac{\rho_2}{\rho_3}\,\alpha_3}\,. \qquad (4.6)$$

In addition we have

$$j_{cond} = j_{ov}\,(1 + \frac{A_s}{A_c}) \qquad (6.1)$$

where j_{ov} is the overall density in the magnet. These equations connect geometrical data ($A_c, A_s, \alpha_1, \alpha_2, \alpha_3, \gamma_4, \gamma_5, a_1$) with mechanical material data ($E_1, \sigma_2, E_3, E_4, E_5$), with electrical data ($\rho_2, \rho_3, j_c, j_{cond}, j_{ov}, B_{max}$) and cooling data ($T_{op}, q_r, pP$). The working strain ε and the safety margin f have to be chosen.

A working strain of $\varepsilon = 0.003$ is chosen to keep the resistivity of the high purity Al below 10^{-8} ohm-cm(4). The magnet system with given inner radius a_1 should be worked at T_{op} = 4.2 K which gives the recovery flux q_r. If we choose the materials, then the data $E_1, \sigma_2, E_3, E_4, E_5, \rho_2$ and ρ_3 are fixed. The maximum magnetic field determines the critical current density of the superconductor together with the preselected T_{op}. So the nine variables $\alpha_1, \alpha_2, \alpha_3, \gamma_4, \gamma_5, A_s/A_c, j_{cond}, pP/A_c, j_{ov}$ remain, connected by six equations. In principle six variables can be calculated in dependence of the three remaining.

In practice conductor current densities of some thousand A/cm^2 are convenient. We choose a conductor current density of 4000 A/cm^2 and so α_1 is some percent of the conductor area for B_{max} from 6 T to 8 T (see Eq. 3.1). Now five equations remain to determine $\alpha_2, \alpha_3, \gamma_4, \gamma_5, A_s/A_c, pP/A_c$ and j_{ov}. We fulfill Eq. (2.10) by choosing a reasonable α_2, which means a reasonable stabilizer to superconductor ratio (α_2/α_1). Then we have the equations (2.11) and (6.1) and the inequalities (2.12) and (4.6) for $\gamma_4, \gamma_5, A_s/A_c, pP/A_c$ and j_{ov}, where Eq. (4.6) yields a lower limit of the cooling parameter pP/A_c. If we fulfill Eq. (2.11) by choosing a reasonable value of γ_4 or γ_5, we can calculate a lower limit of A_s/A_c from Eq. (2.12) and the corresponding overall current density j_{ov} from Eq. (6.1). The magnetic fields, the magnetic forces and the self and mutual inductance are calculated by means of the EFFI code[5]. In addition we have to calculate the stored self energy

$$E_s = 0.5\ L\ I^2 \tag{6.2}$$

and the total energy in the central cell given by the relation

$$E_{tot} = \frac{1}{2} \sum_{q=1}^{N} \sum_{p=1}^{N} L_{pq}\ I_q\ I_p\ . \tag{6.3}$$

7. Results

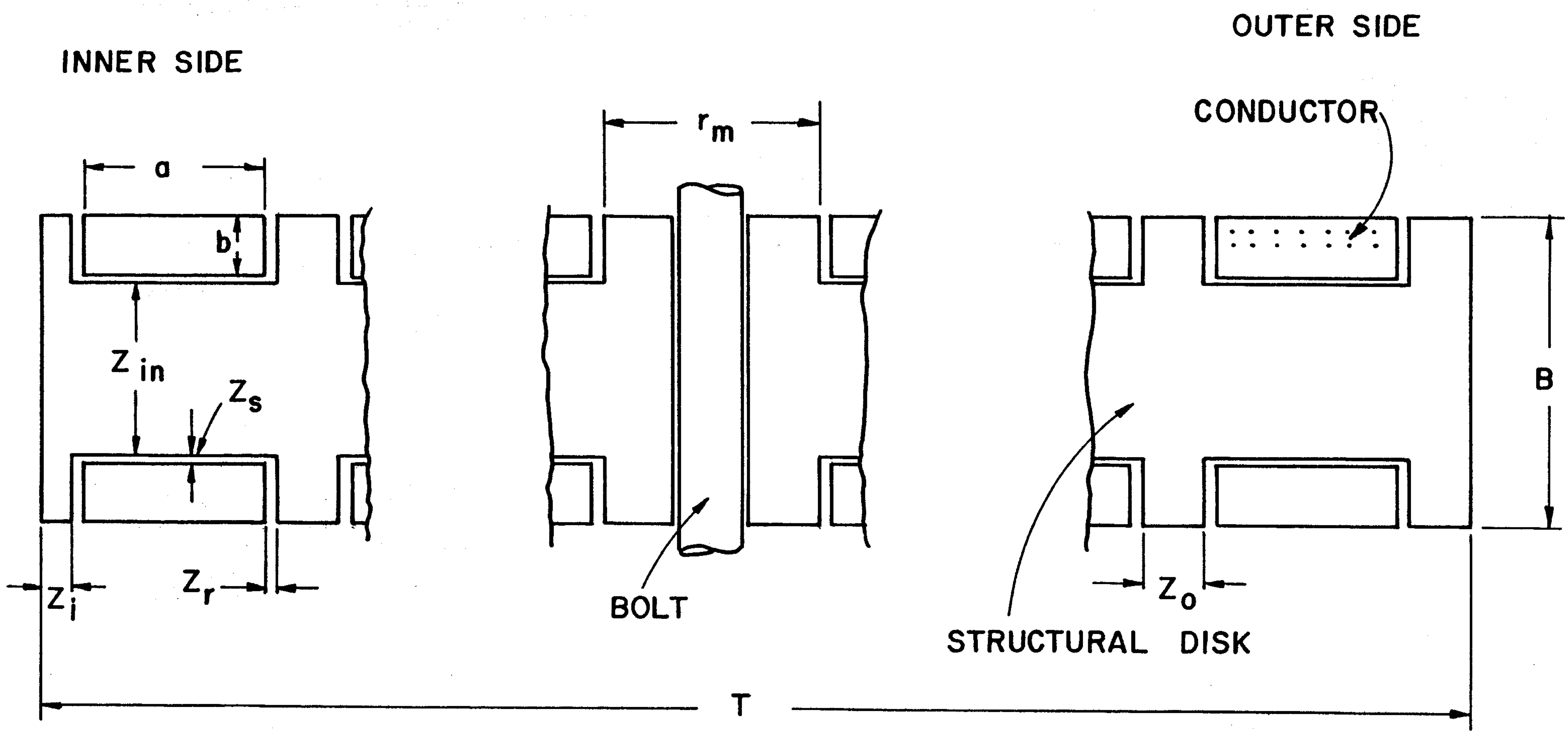
The result of such a calculation for a disk-conductor assembly (see Fig. 1) is given in Table 1. The resistivity of the high purity aluminum is

$$\rho = 10^{-10}\ \Omega m$$

and of the high strength aluminum

$$\rho = 1.5 \times 10^{-8}\ \Omega m\ .$$

We have used these values in the temperature and magnetic field range considered[6]. The Young's moduli are:



DISK CONFIGURATION

Fig. 1

Table 1

Central Cell Magnets

		Unit	
Number of solenoids	N_s		34
Coil separation (winding/ winding)	Δc	m	3.23
Coil separation (midplane/ midplane)	Δz	m	4.63
Axial coil length	W	m	1.4
Radial coil thickness	T	m	1.0
Inner coil radius	a_1	m	3.3
Outer coil radius	a_2	m	4.3
Mean coil radius	$\overline{A}$	m	3.8
Volume of one coil	V_c	m^3	33.427
Average density of a coil	ρ_c	T/m^3	2.64
Mass of one coil	M_c	T	88.087
Volume of N coils	V_{tot}	m^3	1136.502
Mass of N coils	M_{tot}	T	2994.968
Operating temperature	T_{op}	K	4.2
Conductor current density	j_{cond}	A/cm^2	4000
Overall current density	j_{ov}	A/cm^2	950
Magnetic field on axis	B_o	T	3.6
Maximum field at conductor	B_{max}	T	6.14
Field ripple on axis	$\Delta B/B_o$	%	< 3.5
Field ripple at plasma radius	$\Delta B/B_o$	%	< 4.1
Operating current	I	A	12091
Number of turns	N		1100
Ampere-Turns	NI	A-turns	$13.3x10^6$
CONDUCTOR			
Dimensions (radial x axial)		cm x cm	3.0 x 1.0
Superconductor			NbTi
Superconductor, fraction in conductor		%	3

Table 1 (cont.)

Stabilizer			High purity Al
Stabilizer, fraction in conductor		%	75
Skin			High strength Al-2219 T 87
Skin, fraction in conductor		%	22
Percentage of wetted perimeter		%	24
Conductor assembly			Disk structure
Structural material			High strength Al-2219 T 87
Working strain in structure	ε		3.0×10^{-3}
Working stress in structure	σ	MPa	246
Yield stress at 4.2 K		MPa	505
Number of conductors per disk			2x22
Number of disks per magnet			25
INDUCTANCES			
Inductance	$L/10^{-6}$ H/N^2	L/H	% coupling
Self	9.807	11.86647	
Mutual 1. Neigb.	1.397	1.6904	14.245
2.	0.3487	0.4219	3.555
3.	0.1265	0.1531	1.290
4.	0.05794	0.0701	0.591
5.	0.0309	0.0374	0.315
6.	0.01829	0.0221	0.187
7.	0.01168	0.0141	0.119
8.	0.007899	0.0096	0.081
Stored self energy in one central cell magnet			0.8674 GJ

Table 1 (cont.)

Total stored energy in the central cell		40.9625 GJ
DISK		
Cross section (radial x axial)	cm x cm	100.x5.52
Z_o	cm	1.2
Z_i	cm	0.5
Z_s	cm	0.1
Z_r	cm	0.2
Z_m	cm	3.32
R_m	cm	2.3
Area of disk unit	552 cm^2	100%
Superconductor	3.96 cm^2	0.72%
Stabilizer	99.0 cm^2	17.94%
Skin	29.04 cm^2	5.26%
High strength structural Al	380.46 cm^2	68.95%
Structural epoxy	20.03 cm^2	3.63%
Space between disks	19.27 cm^2	3.5%

For NbTi: 100×10^9 N/m^2

For Al: 82×10^9 N/m^2

And for epoxy: 10×10^9 N/m^2.

We have compared the stress relations (2.12,13,14,15) and have found that the relation used in (2.12), $j_{cond}\, B_{max}\, a_1$ for σ_c, gives the highest values for the stresses. The relation (2.13) gives 25% lower stresses and the radius-dependent relations (2.14) and (2.15) give respectively 15% and 13% lower stresses at the inner side of the solenoids, and 35% and 33%, respectively lower stresses at the outer side compared with the values used in our design (see Fig. 2). The protection integral is calculated for two cases.

In the case of our central cell solenoids we have a conductor current density of 4000 A/cm^2, a stored self energy of 0.8674 GJ, a self inductance of 11.8665H and a current of 12091A. Then we get

$$\frac{j_o^2 E_s}{I_o} = 1.148 \times 10^{20} \frac{A^2 s}{m^4} \cdot V$$

discharge with a voltage of $U_o = 2$ kV gives

$$I_{R_D} = 0.574 \times 10^{17} \frac{A^2 s}{m^4}$$

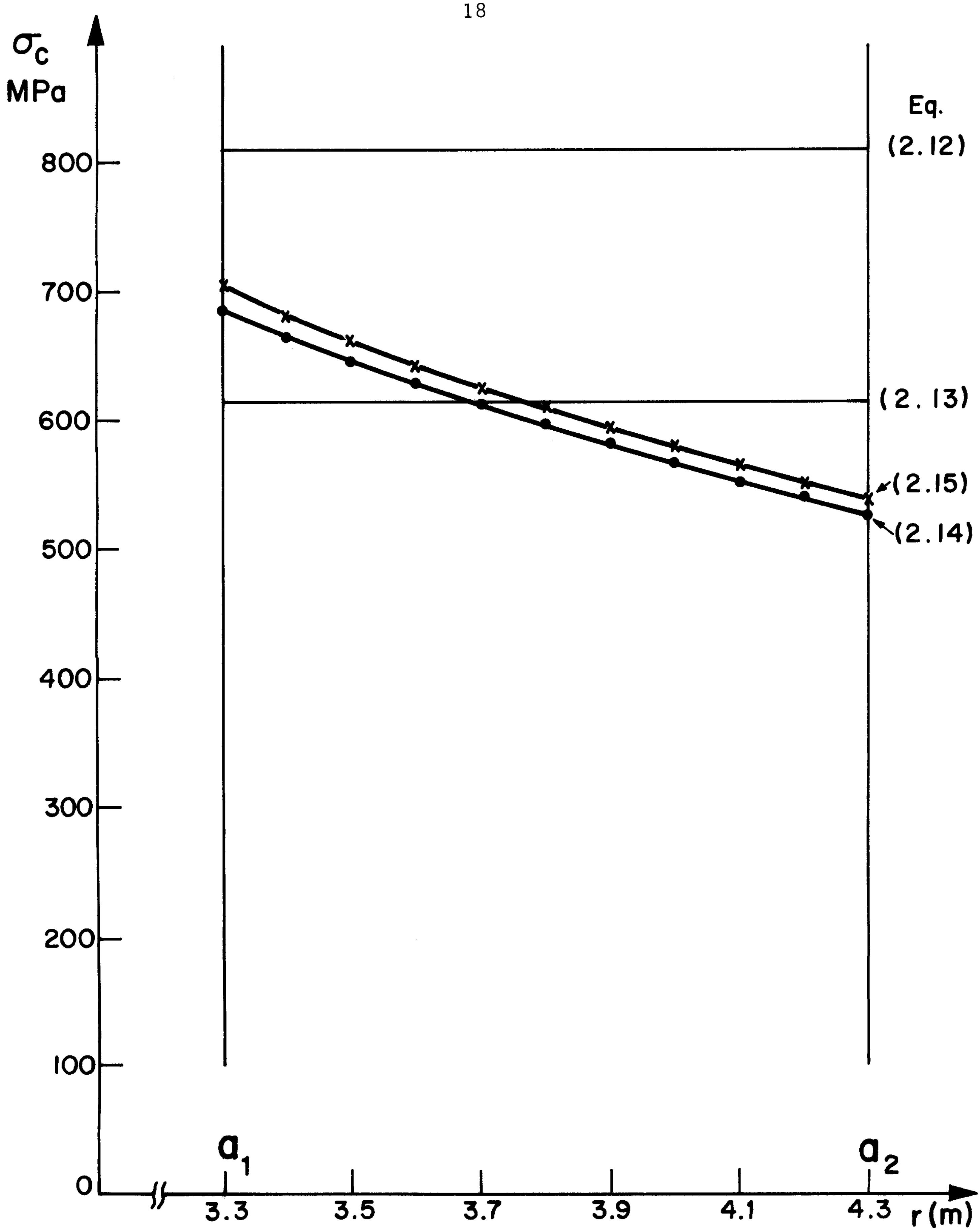


FIG. 2. STRESS DISTRIBUTION

$$I_u = 0.3826 \times 10^{17} \frac{A^2 s}{m^4} .$$

These correspond to a maximum temperature of about 60 K and 40 K, respectively. The time constant τ of a coil is 72 s and the constant discharge resistor has to be 0.1654 ohm.

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