Barrier Cell Sheath Formation

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BARRIER CELL SHEATH FORMATION

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ABSTRACT

The solution for electrostatic potential within a simply modeled tandem mirror thermal barrier is seen to exhibit a sheath at each edge of the cell. The formation of the sheath requires ion collisionality and the analysis assumes that the collisional trapping rate into the barrier is considerably slower than the barrier pump rate.

I. Introduction

It has been recently shown[1] that a significant improvement in tandem mirror performance can be realized by locating a potential depression in a region between the central cell and end plugs. This region, which forms in a dip in the magnetic field and serves to separate central cell and plug electrons has been called a thermal barrier. We have shown in ref. (2) that for a collisionless plasma, obtaining the desired solution to Poisson's equation requires a pre-acceleration of the ion distribution before entering the barrier cell. We will now show, using a heuristic model, that when the pre-acceleration condition is not met a sheath region will form at the barrier cell edge. This sheath formation requires some collisionality of the ions and it serves to provide the ions with the necessary acceleration to form self-consistent potential solutions.

II. Analysis

As in ref. (2) we consider a symmetric dip in magnetic field located between $z = \pm L$ (we allow $L \rightarrow \infty$) characterized by the parameter $\varepsilon = (B_o - B(z))/B_o$ for $B(z)$ the local magnetic field, $B(z) < B_o$ for $|z| < L$ and $B(z) \simeq B_o$ for $|z| \gtrsim L$.

We will characterize the electrons by a Boltzmann relation

$$N_e = N_o \exp(\psi_e), \qquad (1)$$

with $\psi_e = e\phi/T_e$, N_o the central cell density ($N_o = N(|z|) > L$), ϕ the self consistent electrostatic potential and T_e the electron temperature. Two ion species are considered.

1) A streaming half Maxwellian which in a potential and magnetic field depression has a density[2]

$$N_s(z) = \frac{2N_o}{\sqrt{\pi}} \left[e^{-\psi_i} \int_{-\psi_i}^{\infty} d\xi\, \xi^{1/2} e^{-\xi} - \sqrt{\varepsilon}\, e^{-\psi_i/\varepsilon} \int_{-\psi_{i/\varepsilon}}^{\infty} d\xi\, \xi^{1/2} e^{-\xi} \right] \quad (2)$$

for $\psi_i = e\phi/T_i$ and T_i the ion temperature.

2) A trapped ion species is assumed to be present of density

$$N_t(z) = \alpha(N_o e^{-\psi_i} - N_s(\varepsilon,\psi_i)). \quad (3)$$

The parameter α, which must be evaluated, represents the fraction of trapped space that is filled. Clearly $\alpha \leq 1$.

We can now write Poisson's equation in one dimension as

$$\frac{d^2\phi}{dz^2} = -4\pi\,(N_s + N_t - N_e) \quad (4)$$

$$= 4\pi\,[N_e(\psi_e) - (1-\alpha)N_s(\psi_i,\varepsilon) - \alpha N_o e^{-\psi_i}] \; .$$

Since we desire a negative potential solution we require $d^2\phi/dz^2 < 0$ at the cell edge ($z = \pm L$). At the cell edge $\psi_e = \psi_i = \varepsilon = 0$ and and we can expand (4) in small ψ_i, ψ_e, ε which will yield a necessary condition for negative solutions

$$\left(T_i/T_e\right) \left.\frac{dN_e}{d\psi_e}\right|_{\psi_e = 0} = \left.\frac{dN_s}{d\psi_i}\right|_{\psi_i, \varepsilon=0} (1 - \alpha) - \alpha N_o \qquad (5)$$

For $\alpha = o$ we retrieve the condition given in ref (2). Clearly if α is sufficiently close to 1 at the cell edge this criterion is always satisfied.

We will now obtain an approximate form for α, appropriate for a strongly pumped barrier cell. The evaluation of α involves an understanding of the velocity space boundary layer. If we consider velocity space as shown in fig. 1a, we observe that the distribution function contours for a strongly pumped barrier cell wrap around the loss cone boundary. Ions on the inside of the loss cone boundary in the cross-hatched area remain trapped until they are pumped and these trapped ions form the so-called boundary layer. Since in the strongly pumped situation the trapped ions only fill a small region of the available trapped velocity space, this case corresponds to $\alpha \approx 0$. Fig 1b shows the situation as we approach the edge of the cell ($\varepsilon \to 0$) so that the boundary layer fills the trapped velocity space region. This situation then corresponds to $\alpha \approx 1$.

An estimate of the behavior of α can now be determined as follows: Baldwin, Cordey and Watson[3] have shown in a somewhat similar situation that $N_t/N_s \sim O(\lambda^\nu)$ for λ a small parameter relating transit (or bounce) time (τ_t) to collision time (τ_c) and ν is shown to have the value 3/4. In our case transit time corresponds to residence time of an ion before it is pumped (τ_p) and therefore a strong pump implies $\lambda = \tau_p/\tau_c << 1$.

By eq. 3, N_t is bounded and $\mathrm{Max}(N_t/N_s) \to 0$ as $\psi, \varepsilon \to 0$. Assuming $N_t/N_s \sim \lambda^\nu$ the condition which corresponds to the boundary layer filling the trapped region can be gotten by setting $\alpha = 1$ in (3) which yields

$$N_o e^{-\psi} - N_s \approx \lambda^\nu N_s \qquad (6)$$

We can now perform the intergration in (2) in the limit $\psi \to 0$, $\psi/\varepsilon < 1$ to obtain

$$N_s/N_o \approx 1 - \psi - \sqrt{\varepsilon}\,(1 - \psi/\varepsilon). \qquad (7)$$

Combining 6 and 7 yields a critical ε value of $\varepsilon^* \sim \lambda^{2\nu}$ which defines the magnetic field $(B^* = B_o(1 - \varepsilon^*))$ at which the boundary layer fills the trapped velocity space. We will therefore make an ad hoc estimate for α;

$$\alpha = e^{-\varepsilon/\varepsilon^*} \qquad (8)$$

Combining the quasi-neutrality assumption with equations 3 and 8 yields

$$N_e(\psi_e) = N_s(\varepsilon, \psi_i)(1 - e^{-\varepsilon/\varepsilon^*}) + N_o e^{-\psi_i + \varepsilon/\varepsilon^*}. \qquad (9)$$

Fig. 2 indicates a typical numerical solution for the potential obtained from substituting (2) into the quasi-neutrality equation. The potential drops very rapidly between B_o and B^* to form a sheath. (This approach is clearly only valid if the sheath width exceeds the Debye length.)

III. Conclusions

We can now understand the significance of the pre-acceleration condition derived in ref. (2). For a sufficient pre-acceleration a smooth potential depression is exhibited as the plasma flows into the barrier cell, whereas in the absence of a pre-acceleration a sheath region develops at the edge of the field depression which will effectively result in an ion acceleration. The width of this sheath region depends on the collisionality of the ions and the strength of the pump in the approximate relation $\Delta B \sim B_o(\tau_p/\tau_c)^{2\nu}$. As the pump gets stronger ($\tau_p \to o$) this sheath transforms to a step in potential.

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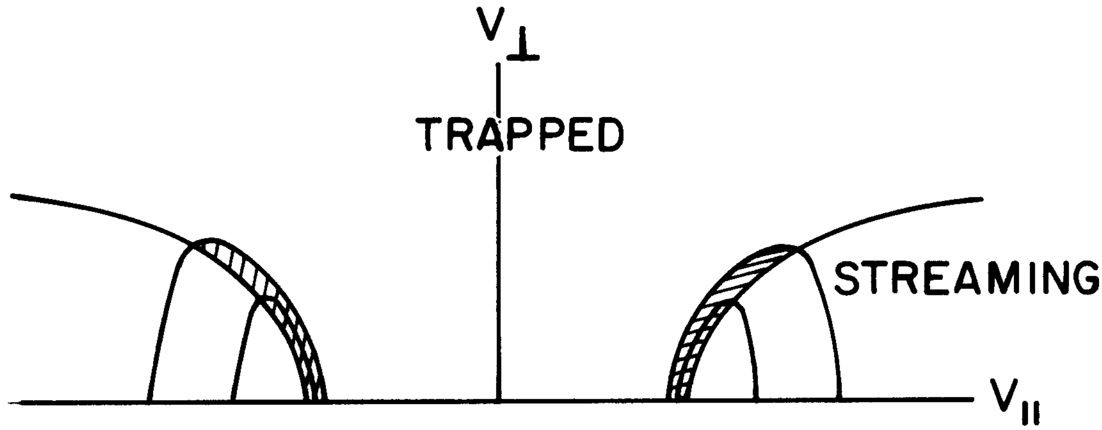
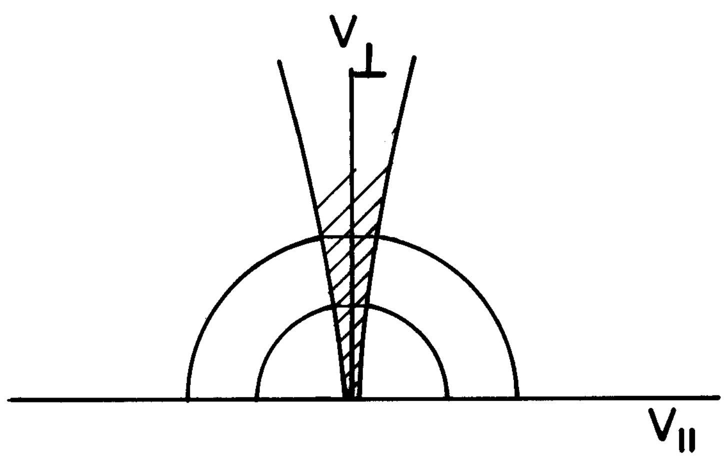
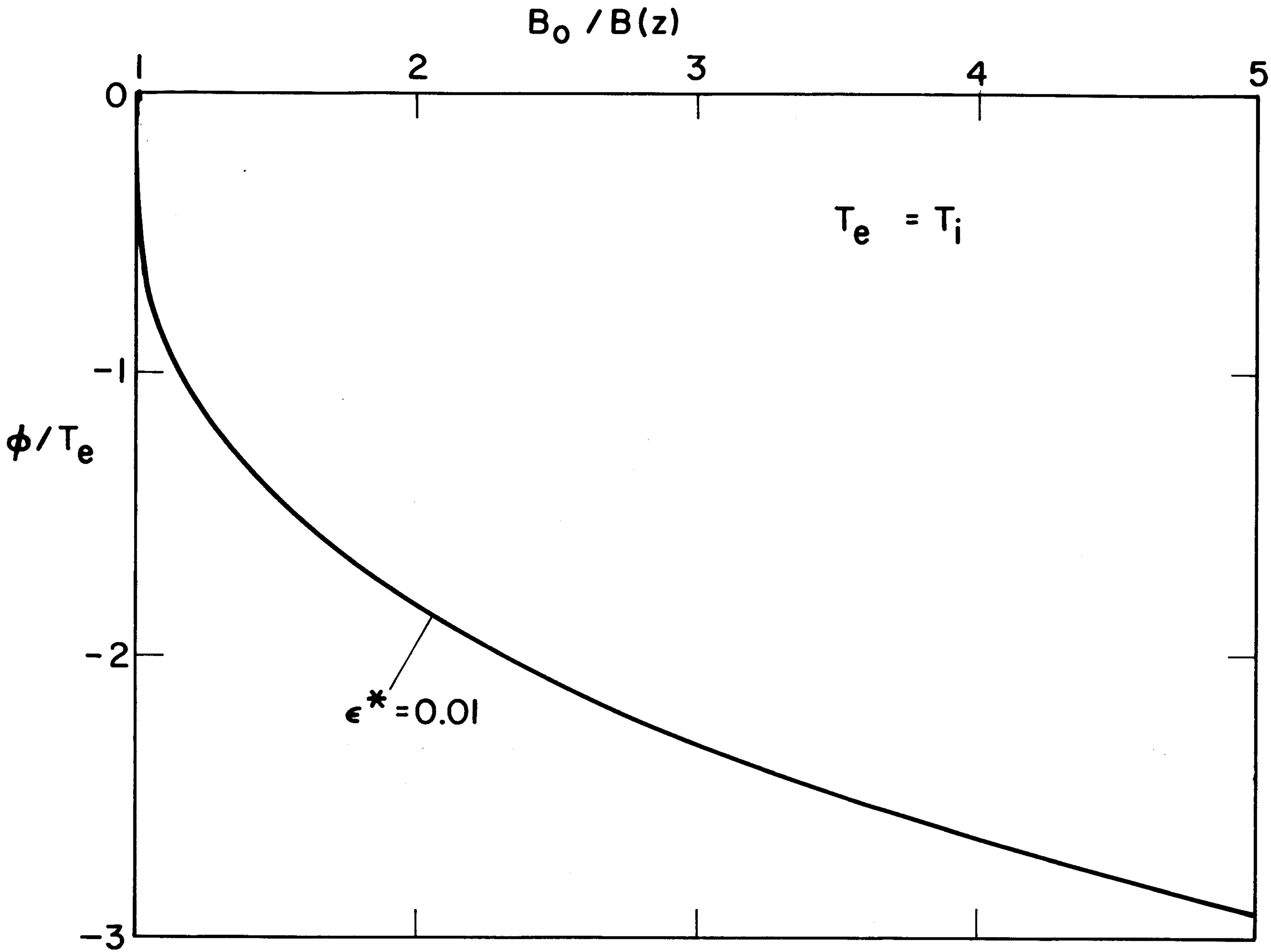


(a)



(b)

1a) Velocity space within the barrier cell. Cross-hatched region is the boundary layer. b) Velocity space diagram near the cell edge where boundary layer overlap occurs.



2. Potential as a function of magnetic field variation within a well pumped barrier cell for $T_e = T_i$ and $\varepsilon^* = .01$.