Power Balance in a Tandem Mirror Reactor With Thermal Barriers

J.F. Santarius and R.W. Conn

January 1980

UWFDM-340

FUSION TECHNOLOGY INSTITUTE

UNIVERSITY OF WISCONSIN

MADISON WISCONSIN

Power Balance in a Tandem Mirror Reactor With Thermal Barriers

J.F. Santarius and R.W. Conn

Fusion Technology Institute
University of Wisconsin
1500 Engineering Drive
Madison, WI 53706

http://fti.neep.wisc.edu

January 1980

UWFDM-340

Power Balance in a Tandem Mirror Reactor With Thermal Barriers

John F. Santarius
R.W. Conn

January 1980

Fusion Engineering Program
Department of Nuclear Engineering
University of Wisconsin
Madison, Wisconsin 53706

UWFDM-340

I. Introduction

The tandem mirror reactor with thermal barriers has great potential for producing economic fusion power. In light of this, the present study examines the parametric dependence of important effects and seeks a preliminary set of fusion reactor design parameters.

The basic tandem mirror, without thermal barriers, was proposed by Fowler and Logan[(1)] and independently by Dimov, et al.[(2)] Because excessive end loss is the chief obstacle to confinement in simple mirror machines, they proposed increasing the potential in a separate end plug cell at each end of a long solenoid, thus electrostatically confining ions. Unfortunately, this entails the use of large amounts of input power,[(3)] and therefore gives a small Q, the ratio of power out to power in.

However, Baldwin and Logan[(4)] have recently proposed adding a thermal barrier between the central cell and the end cell. By creating a potential drop in the barrier region, plug electrons become thermally insulated from central cell electrons. This allows the increase of end plug potential by heating only plug electrons rather than heating all electrons in the machine or increasing density by neutral beam injection as in the old case. The resultant decrease in input power to the plasma allows Q's of fifteen or greater, sufficient for a net power producing machine.

The details of the analysis will be presented in Sec. II. Sec. III contains a brief description of the numerical method involved. The parametric dependence of the equations is discussed in Sec. IV. A set of possible reactor parameters and power flow analysis is given in Sec. V. Goals of central cell length of about 100 meters and Q greater than 15 were achieved. Sec. VI concludes, summarizes the results, and points to future directions.

II. Plasma Model

A zero-dimensional treatment was chosen for this analysis since, while still capable of relatively quick solution, it contains most of the salient features which a tandem mirror reactor plasma will possess. The plasma is treated here as consisting of four components: 1) electrons either trapped in the central cell or passing through both central cell and plug; 2) ions trapped in the central cell; 3) electrons trapped in the plug; and 4) ions trapped in the plug. The electrons of the first component are grouped together because the passing electrons spend most of their time in the central cell and therefore equilibrate in temperature with the central cell electrons. No separate equations are written for particles in the barrier because the final scheme to be used for barrier generation is as yet uncertain and the analysis here attempts to be as general as possible. The effects of the barrier are treated in a reasonable way which will be described later in this section. All units will be in the CGS system except energy which will be in keV and linear distance which will be in meters.

The approximate axial profiles of magnetic field, potential, and density are shown in Fig. 1. Radial dependence is neglected for simplicity. The potential will probably rise much more quickly on the plug side of the barrier than as shown, but the dependence shown should give fair agreement and is much easier to treat. The plug mid-plane radius, r_p, is given by flux conservation

$$r_p = r_c \left[\frac{B_c}{B_p} \left(\frac{1-\beta_c}{1-\beta_p}\right)^{1/2}\right]^{1/2} \qquad (1)$$

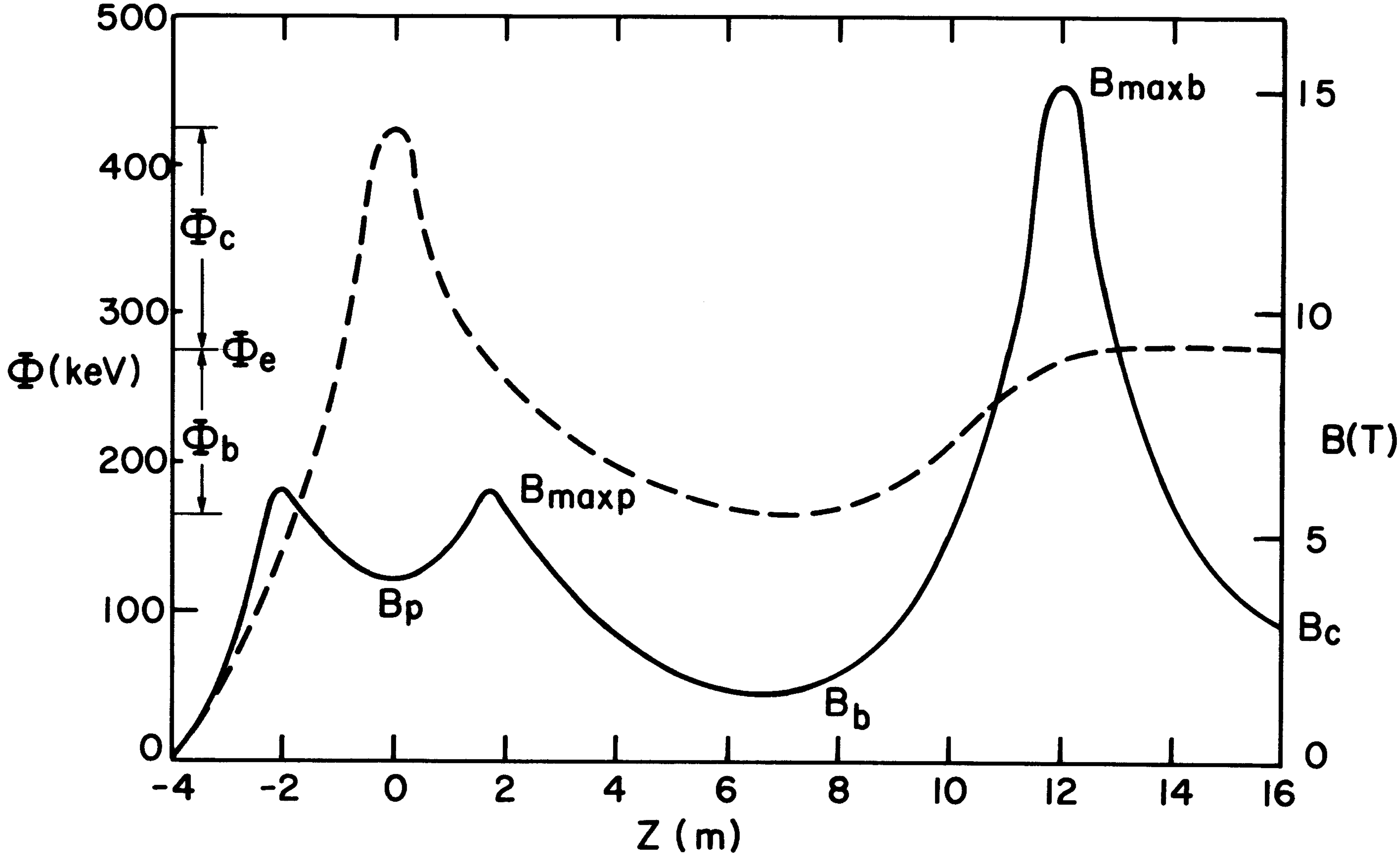
500
400
300
200
100
0
Φ(keV)
Φc
Φe
Φb
-4
-2
0
2
4
6
8
10
12
14
16
Z (m)
B(T)
15
10
5
0
Bmaxb
Bmaxp
Bp
Bb
Bc

Figure 1

where r_c is the central cell radius, β_c is the central cell beta, β_p is the plug beta at the midplane, and beta is the ratio of plasma pressure to magnetic field pressure. Because the plug is assumed to have a minimum-B configuration, the plug volume, V_p, is approximated by a sphere of radius r_p. To find the barrier volume, V_b, flux conservation in a cylindrical barrier with average field given by assuming parabolic dependence of B on z gives

$$r_b = r_c \left[\frac{B_c}{\langle B\rangle_b} \left(\frac{1-\beta_c}{1-\beta_b}\right)^{1/2}\right]^{1/2} \tag{2}$$

where β_b is the barrier beta at the midplane, and $\langle B\rangle_b$ is given by

$$\langle B\rangle_b = \frac{2}{3} B_b + \frac{1}{6} (B_{maxb} + B_{maxp}) \ . \tag{3}$$

V_b is then approximately $\pi r_b^2 L_b$ where L_b is a given length of the barrier.

The densities in the central cell and plug are related to the limits on beta through

$$n_c = \frac{2.5\times10^7 \beta_c B_c^2}{(T_{ic}+T_{ec}+T_\alpha)} \tag{4}$$

and

$$n_p = \frac{2.5\times10^7 \beta_p B_p^2}{(0.9\ E_p+T_{ep})} \tag{5}$$

where the average perpendicular alpha energy is $T_\alpha = 8.3\times10^{10} <\sigma v>_{DT} E_\alpha f_{e\alpha} T_{ec}^{1.5}$, $<\sigma v>_{DT}$ is the fusion reaction rate, E_α is the alpha creation energy, $f_{e\alpha}$ is the fraction of alpha energy deposited into electrons, T_{ec} is the central cell electron temperature, T_{ic} is the central cell ion temperature, T_{ep} is the plug electron temperature, E_p is the plug ion energy, and the factor of 0.9 appears in n_p because neutral beam injection causes most of the plug energy to be perpendicular.(5) The density of electrons in the plug consists of two components: 1) electrons trapped in the plug and 2) electrons passing through the plug, n_{pass}. Flux conservation of electrons passing over the barrier potential Φ_b gives

$$n_{pass} = n_c \frac{B_p}{B_{maxb}} \left[\frac{T_{ec}}{\pi(\Phi_b+\Phi_c)}\right]^{1/2} e^{-\Phi_b/T_{ec}} \quad . \qquad (6)$$

The number of electrons trapped in the plug is then assumed to be $n_p - n_{pass}$.

The density of the barrier will be reduced by whatever pumping process balances ion trapping, but will always be at least equal to the density of central cell ions streaming through the barrier, $n_{pass,i}$, which is given by flux conservation as

$$n_{pass,i} = \frac{n_c}{R_b} \left(\frac{T_{ic}}{\pi\Phi_b + T_{ic}}\right)^{1/2} \qquad (7)$$

where R_b is the barrier mirror ratio, $R_b = B_{maxb}/B_b$. A useful measure of pumping efficacy is g_b, the ratio of total barrier density to streaming density, then

$$n_b = g_b \, n_{pass,i} \quad . \qquad (8)$$

Thus, when $g_b=2$, there are equal numbers of trapped and streaming ions in the barrier.

The barrier enters the physics of the problem in three ways: 1) the potential Φ_b insulates plug and central cell electrons, 2) particles may be lost from the barrier, and 3) the power into the plasma inherent in the pumping process contributes to Q. Various pumping mechanisms are possible and the philosophy of this study is to estimate the pumping power to the correct order, but to concentrate on the remainder of the physics. Once the choice of barrier schemes solidifies, modification of the computer program to examine any specific scheme is straightforward.

For pumping power, then, trapped ions are all assumed to be removed from deep in the well, at energy Φ_b+T_{ic}, and at the rate

$$J_{trap} = \frac{n^2_{pass,i}}{5.5\times10^9 T_{ic}^{1.5}} \left(1 + .55 \frac{B_{maxb}}{\langle B\rangle_b}\right) \qquad (9)$$

which is an approximation to a formula fitted to a Fokker-Planck code.[5] Also, a factor f_{bar} will be included, recognizing that more input power may be necessary than is actually removed from the plasma. This is motivated by results from a model including axial dependence and a specific barrier scheme.[5,6] Thus,

$$P_{pump} = f_{bar}\, J_{trap}\, (\phi_b + T_{ic}) \ . \qquad (10)$$

This somewhat ad hoc approach to the barrier is an attempt to find a reasonable value for the pumping power without analyzing a specific pumping scheme. Indeed, no specific model would exactly fit this analysis.

The barrier potential, Φ_b, is given by flux conservation assuming that the flux at the peak barrier field point is related to n_b by a Boltzmann distribution:

$$\left(\frac{T_{ic}}{\pi\Phi_b+T_{ic}}\right)^{1/2} e^{\Phi_b/T_{ec}} = \frac{R_b}{g_b} . \qquad (11)$$

The potential Φ_c, which confines most of the central cell ions, is given by particle balance between electrons becoming passing electrons by escaping from the plug and the collisional trapping of passing electrons into the plug.(10) An approximate formula is then

$$\Phi_c = \left(\frac{T_{ep}}{T_{ec}} - 1\right) \Phi_b + T_{ep} \, \ell n \left[\frac{n_p}{n_c} \left(\frac{T_{ec}}{T_{ep}}\right)^{\nu_c}\right] . \qquad (12)$$

The case ν_c=0, which is derived by assuming a Boltzmann relation between electrons on either side of the barrier, will be used in most of this analysis. Recent work(11) indicates that ν_c will be close to 0.5; this would significantly degrade performance. However, at the cost of extra ECRH heating of mirror-trapped electrons in the barrier, Φ_b could be raised sufficiently to overcome the deleterious effects of non-zero ν_c.(12)

The potential Φ_e, which confines electrons, follows from particle balance:

$$(1 + f_{is}) \frac{n_c^2}{(n\tau)_{ic}} V_c + \frac{n_c^2}{2} <\sigma v>_{DT} V_c + \frac{n_p^2}{(n\tau)_{ip}} 2V_p + J_{trap} 2V_b = \frac{n_c^2}{(n\tau)_{ec}} V_c \qquad (13)$$

f_{is} is the ratio of auxiliary cold electron current to ion ionization current in the central cell, if present; V_c, V_b, and V_p are the central cell, barrier, and plug volumes; and the $(n\tau)$'s are given as follows. Note that the J_{trap} term implicitly assumes that all of the ions which trap in the barrier are lost. When some pumping is done by neutral beams injected into the barrier loss cone, these ions are not lost and the J_{trap} term is correspondingly reduced.

The Logan-Rensink model is used for $(n\tau)_{ip}$ of ions in the plug.(7)

$$(n\tau)_{ip} = \left[\frac{\Lambda_{iip}(m_D/m_{ip})^{1/2}}{3.9\times10^{12}E_{inj}^{1.5}\,\ell og_{10}R_{eff}} + \frac{\Lambda_{eip}(m_H/m_{ip})}{10^{13}T_{ep}^{1.5}\,\ell n(E_{inj}/E_{out})}\right]^{-1} \tag{14}$$

where m_D is the mass of deuterium, m_H is the mass of hydrogen, m_{ip} is the plug ion mass, Λ_{iip} and Λ_{eip} are Coulomb logarithms, E_{inj} is the neutral beam injection energy, E_{out} is given by

$$E_{out} = \frac{E_{inj}}{1 + \frac{\tau_i}{\tau_{DR}}} + \frac{(\phi_c+\phi_e)}{\left[\frac{R_p \sin^2\theta}{(1-\beta_p)^{1/2}} - 1\right]} \frac{\tau_i/\tau_{DR}}{(1 + \frac{\tau_i}{\tau_{DR}})} \tag{15}$$

where R_p is the plug mirror ratio; θ is the beam injection angle, taken here as $\pi/2$; β_p is the plug beta, and τ_i/τ_{DR}, the ratio of ion collision time to electron drag time, is given by

$$\frac{\tau_i}{\tau_{DR}} = .11 \left(\frac{E_{inj}}{T_{ep}}\right)^{1.5} \frac{\Lambda_{eip}}{\Lambda_{iip}} \frac{\ell og_{10}R_{eff}}{\ell n\left(\frac{E_{inj}}{E_{out}}\right)} \left(\frac{m_D}{m_{ip}}\right)^{1/2} \tag{16}$$

where T_{ep} is the plug electron temperature and

$$R_{eff} = \frac{R_p/(1-\beta_p)^{1/2}}{1 + \frac{(\phi_e+\phi_c)}{E_{inj}}} \sin^2\theta \ . \tag{17}$$

For ions in the central cell, $(n\tau)_{ic}$ is given by Pastukhov's analysis as corrected by Cohen, et al.[8,9]

$$(n\tau)_{ic} = \frac{1}{Z_{ic}(2m_i)^{1/2}\pi e^4\Lambda_i} \frac{G(Z_{ic}R_c)}{I\left(\frac{T_{ic}}{\phi_c}\right)} T_{ic}^{1/2} \phi_c e^{\phi_c/T_{ic}} \tag{18}$$

where m_i is the ion mass, e is the electronic charge, Λ_i is the Coulomb logarithm, Z_i = 0.5 for ions because they scatter only slightly off of electrons, R_c=B_{maxb}/B_c is the central cell mirror ratio,

$$G(x) \equiv \frac{\pi^{1/2}}{2} \left(1 + \frac{1}{x}\right)^{1/2} \ell n\left[\frac{(1+\frac{1}{x})^{1/2}+1}{(1+\frac{1}{x})^{1/2}-1}\right] \tag{19}$$

and

$$I(x) = \frac{1+\frac{x}{2}}{1+\frac{x^2}{4}} \ . \tag{20}$$

For electrons in the central cell, similar analysis gives

$$(n\tau)_{ec} = \frac{1}{Z_e(2m_e)^{1/2}\pi e^4\Lambda_e} \frac{G(Z_eR_c)}{I\left(\frac{T_{ec}}{\Phi_e}\right)} T_{ec}^{1/2} \Phi_e e^{\Phi_e/T_{ec}} \tag{21}$$

where m_e is the electron mass and $Z_e=1$ for electrons because they scatter off both ions and electrons.

Also, the Pastukhov $(n\tau)_{ep}$ for plug electrons which escape over Φ_b and are replaced by passing electrons is:

$$(n\tau)_{ep} = \frac{1}{Z_e(2m_e)^{1/2}\pi e^4 \Lambda_{ep}} \frac{G(Z_eR_p)}{I(\frac{T_{ep}}{\Phi_b+\Phi_c})} T_{ep}^{1/2} (\Phi_b + \Phi_c) e^{\frac{\Phi_b+\Phi_c}{T_{ep}}} \tag{22}$$

where $R_p=B_{maxp}/B_p$ is the plug mirror ratio.

The collisional energy transfer rate from species β to species α is approximated by assuming both species are Maxwellian and using standard formulas.[13]

$$\nu^{\alpha/\beta} = 5.7\text{x}10^{-24} \frac{(m_\alpha m_\beta)^{1/2} Z_\alpha^2 Z_\beta^2 n_\beta \Lambda_{\alpha\beta}}{(m_\alpha T_\beta + m_\beta T_\alpha)^{1.5}} . \tag{23}$$

The assumption of Maxwellians is inaccurate for streaming components, but collisional terms tend to be small, and no serious consequences of this approximation are anticipated. The $\nu^{\alpha/\beta}$'s also incorporate volume ratios when necessary to include the effect of the energy given to passing electrons being spread over the whole central cell.

The plug electron power balance is given by

$$f_{ep} P_{eaux} + (n_p - n_{pass}) \nu^{ep/ip} (E_p - \frac{3}{2} T_{ep}) = \frac{3}{2} (n_p - n_{pass}) \nu^{ep/ec} (T_{ep} - T_{ec})$$
$$+ \frac{(n_p - n_{pass})^2}{(n\tau)_{ep}} (T_{ep} - T_{ec}) + \frac{n_p^2}{(n\tau)_{ip}} (\phi_b + T_{ep}) \tag{24}$$

where P_{eaux} is the amount of auxiliary plug electron heating and f_{ep} is the fraction of P_{eaux} which goes to trapped plug electrons. The second and third terms give the collisional energy transfer rate between trapped plug electrons and plug ions or central cell electrons. The fourth term is the energy transfer occurring when hot plug electrons are detrapped and replaced by cold passing electrons.[4] The last term arises from loss of plug electrons into the central cell to balance the plug ion end loss.

The central cell electron power balance is given by

$$\frac{f_{ec}}{R_v} P_{eaux} + \frac{n_c^2}{4} <\sigma v>_{DT} f_{e\alpha} E_\alpha \left(1 + \frac{E_R}{E_{fus}}\right) + f_{is} \frac{n_c^2}{(n\tau)_{ic}} \Phi_e + n_c \nu^{ec/ip} \left(E_p - \frac{3}{2} T_{ec}\right) + \frac{3}{2} n_c \nu^{ec/ep} (T_{ep} - T_{ec}) + \frac{(n_p - n_{pass})^2}{R_v (n\tau)_{ep}} (T_{ep} - T_{ec})$$
$$+ \frac{n_p^2}{R_v (n\tau)_{ip}} (\Phi_b + T_{ep}) = \left[\frac{n_c^2}{(n\tau)_{ec}} + \frac{n_c^2}{(n\tau)_{xfe}}\right] (\Phi_e + T_{ec}) + \frac{3}{2} n_c \nu^{ec/ic} (T_{ec} - T_{ic}) + \frac{3}{2} J_{trap} \frac{2V_b}{V_c} T_{ec} \qquad (25)$$

where f_{ec} is the fraction of auxiliary electron heating which goes to passing electrons; $R_v \equiv V_c/2V_p$ is the central cell to plug volume ratio; $f_{e\alpha}$ is the fraction of alpha particle energy which goes to electrons, given by[4,14]

$$f_{e\alpha} = 0.882\, e^{-T_{ec}/67.4} . \qquad (26)$$

E_{fus} is the total energy output per fusion, E_R is the average reacting ion energy, taken to be[5]

$$E_R = 45 + \frac{3}{2} T_{ic} \tag{27}$$

and $(n\tau)_{xfe}$ is the $(n\tau)$ for cross-field electron transport, taken to be infinite for the present study. The first term is the auxiliary electron heating; the second is the power from fusions; the third comes from a cold electron current, if present; the fourth, fifth and ninth come from collisional energy transfer between central cell electrons and plug ions, plug electrons, or central cell ions; the sixth is energy transferred by plug electrons detrapping and being replaced by passing electrons; the seventh represents plug electrons lost into the central cell to balance plug ion loss; the eighth is the electron loss from the machine; and the last term represents cold electrons which come in with refueling ions required to replace ions lost from the barrier. Cold electrons injected with the barrier neutral beams are neglected here, but will lower T_{ec} or T_{ep}, depending on where they appear in the barrier. Lowering T_{ec} will allow higher n_c for a given β_c, a good effect, but lowering T_{ep} will require higher P_{eaux}. It will therefore be advantageous to design pump beams to charge exchange mainly on the central cell side of the barrier.

The plug ion power balance is given by

$$\frac{n_p^2}{(n\tau)_{ip}} (E_{inj} - E_{out}) = \frac{n_p^2}{(n\tau)_{ip}} \frac{\langle\sigma v\rangle_{cxp}}{\langle\sigma v\rangle_{ionp}} (E_p - E_{inj}) + n_p \nu^{ip/ep} (E_p - \frac{3}{2} T_{ep})$$

$$+ n_p \nu^{ip/ec} (E_p - \frac{3}{2} T_{ec}) \tag{28}$$

where $<\sigma v>_{cxp}$ is the plug charge exchange rate and $<\sigma v>_{ionp}$ is the plug ionization rate. The first term is the power deposited by neutral beams; the second represents energy lost to charge exchange; and the third and fourth terms give the collisional energy transfer between ions in the plug and plug electrons or passing electrons.

The central cell ion power balance is given by

$$\frac{n_c^2}{4} <\sigma v>_{DT} [f_{i\alpha} E_\alpha (1 + \frac{E_R}{E_{fus}}) - E_R] = \frac{3}{2} n_c \nu^{ic/ec} (T_{ic} - T_{ec}) + \frac{n_c^2}{(n\tau)_{ic}}$$

$$(\Phi_c + T_{ic}) + \frac{n_c^2}{(n\tau)_{xfi}} T_{ic} + \frac{3}{2} [\frac{n_c^2}{(n\tau)_{ic}} + \frac{n_c^2}{(n\tau)_{xfi}} + \frac{n_c^2}{2} <\sigma v>_{DT}] \frac{<\sigma v>_{cxc}}{<\sigma v>_{ionc}} T_{ic} \quad (29)$$

where

$$f_{i\alpha} = 1 - 0.908 e^{-T_{ec}/101.7} \quad (30)$$

is the fraction of alpha particle energy deposited in the ions;[4,14] $(n\tau)_{xfi}$ is the $(n\tau)$ for cross-field ion transport, taken as infinite for the present study; $<\sigma v>_{cxc}$ is the central cell charge exchange rate; and $<\sigma v>_{ionc}$ is the central cell ionization rate. The first term is the power from fusions; the second comes from collisional energy transfer between central cell electrons and ions; the third represents ion energy end loss; the fourth gives cross-field ion energy loss; and the last term is power lost to charge exchange. It has been tacitly assumed, as a first approximation, that fueling ions are injected with an energy which will balance out the energy loss of ions being pumped from the barrier. These terms then cancel in the ion power balance equation and are not shown here.

The ratio of power out to power in is then given by

$$Q = \frac{P_{fus} V_c}{(P_{NB}+P_{eaux})2V_p+P_{pump}2V_b} \tag{31}$$

where

$$P_{NB} = \frac{n_p^2}{(n\tau)_{ip}} E_{inj} , \tag{32}$$

$$P_{fus} = \frac{n_c^2}{4} <\sigma v>_{DT} E_{fus} , \tag{33}$$

and P_{pump} was given in Eq. (10).

Neutron wall loading, Γ_w, is given by

$$\Gamma_w = \frac{0.8 P_{fus} V_c}{2\pi(r_c+2\rho_\alpha)L_c}$$

where ρ_α is the gyroradius of a 3.5 MeV α particle and a distance of $2\rho_\alpha$ is assumed between the central cell plasma and the wall.

III. Method of Solution

The power balance equations are written as rate equations for the various dT/dt's by subtracting loss terms from gain terms. Given a set of initial conditions, the rate equations are then time-stepped until equilibrium is reached. The method is based on that of Shaing, et al.[15] Note that, since equilibrium equations are solved, solutions are technically valid only near equilibrium, and time serves only as an iteration variable.

Magnetic field profile, β's, fusion power P_{fus}, neutron wall loading Γ_w, injection energy, and pumping parameter g_b are held fixed, while densities, radii, central cell length, potentials, T_{ep}, T_{ec}, and E_p are iterated. For a given set of parameters, T_{ic} is held constant. This gives Φ_c through the condition $dT_{ic}/dt=0$; which in turn gives T_{ep} from Eq. (12). P_{eaux} is then chosen such that $dT_{ep}/dt=0$ unless this causes $P_{eaux} < 0$ in which case T_{ep} is allowed to vary.

Generally then, only for T_{ec} and E_p are rate equations solved by the iterative procedure. These are solved in a loop along with the $(n\tau)$'s; then the n's, r's, Φ's, T_{ep} and P_{eaux} are updated and the procedure is repeated. Solution time is generally a few seconds on the MFECC CDC 7600 machine.

Input guesses for the Φ's and T's are required, but the final results are only mildly sensitive to these values.

IV. Results

The dependence of equilibrium conditions on several important parameters has been studied, and a typical set of values is given in Tables I and II. Power parameters given there are calculated using the method outlined in Ref. 16. The efficiencies used were: 67% for the plug neutral beams, 67% for the plug auxiliary heating, 50% for the barrier pumping process, and 60% for the direct convertor. Unless otherwise noted, the graphs presented here use the parameters of Tables I and II as a base case and vary only one input parameter.

There is a strong trade-off between plug neutral beam injection energy and plug auxiliary electron heating. Fig. 2 shows, in particular, a knee between E_{inj} of 400 keV and 600 keV; this knee is present for a wide range of base cases. This accords with Eq. (17), where a knee might be expected for $E_{inj} \sim (\Phi_e + \Phi_c) \sim 422$ keV. In order to minimize the required ECRH plug electron heating while not unduly straining neutral beam technology, E_{inj}=500 keV was chosen for the base case. Parameters other than E_{inj} and

Table I

Model Reactor Power and Machine Parameters

Power Parameters	
Q	17.7
Neutron wall loading	2.5 MW/m^2
Fusion power	3000 MW
Thermal power	3900 MW
Gross electric power	1600 MW
Plug neutral beam electric power	99 MW
Plug RF electric power	54 MW
Barrier pumpout electric power	244 MW
Net electric power	1200 MW
Recirculating power fraction	0.25
Net plant efficiency	0.31
Machine Parameters	
Central cell length	102 m
Central cell wall radius	1.51 m
Central cell magnetic field	3.35 T
Barrier peak magnetic field	15 T
Barrier mirror ratio	10
Plug peak magnetic field	6 T
Plug mirror ratio	1.5

Table II

Model Reactor Plasma Parameters

Plugs	
Neutral beam energy	500 keV
Mean ion energy	990 keV
Beam trapping fraction	0.19
Beam power absorbed	12 MW
RF power absorbed	36 MW
Density	$2.5\times10^{13}\,cm^{-3}$
Electron temperature	192 keV
Potential $\phi_c+\phi_e$	422 keV
Beta	0.64
$(n\tau)_{ip}$	$7.85\times10^{13}\,cm^{-3}\,sec$
Barrier	
Density	$6.4\times10^{12}\,cm^{-3}$
Potential ϕ_b	114 keV
Pumping parameter g_b	2
Pumping power	122 MW
Central Cell	
Density	$1.0\times10^{14}\,cm^{-3}$
Potential ϕ_c	145 keV
Ion temperature	40 keV
Electron temperature	41.5 keV
Beta	0.4
$(n\tau)_{ic}$	$1.95\times10^{15}\,cm^{-3}\,sec$

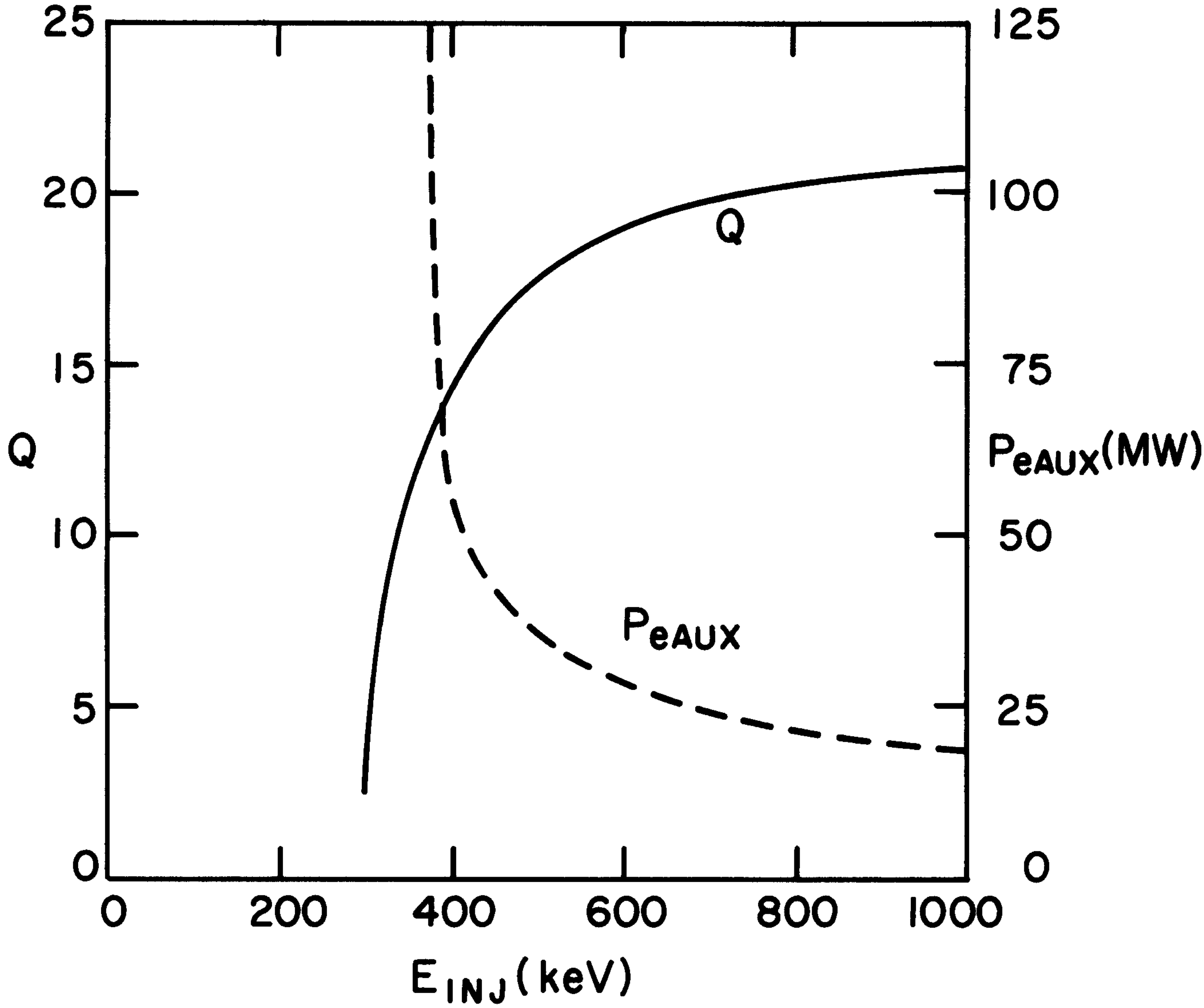
25
20
15
10
5
0
Q
125
100
75
50
25
0
PeAUX(MW)
Q
PeAUX
0
200
400
600
800
1000
EINJ(keV)

Figure 2

P_{eaux} remained approximately constant as E_{inj} varied.

Lowering the neutron wall loading, Γ_w, significantly raises Q, as seen in Fig. 3. However, Fig. 4 shows that this is at the cost of a large increase in central cell length, L_c, although r_c decreases somewhat. When L_c is held constant and P_{fus} is allowed to vary, there is little change in Q as Γ_w varies, as shown in Fig. 3. However, a constant L_c implies higher P_{fus} for higher Γ_w, thus higher input power for a given Q; the dependence of input power on Γ_w is shown in Fig. 5. Constraints on ECRH, neutral beam, and pumping technology plus economic considerations for Q thus set an effective limit on Γ_w. A value of Γ_w=2.5 MW/m^2 was chosen for the Table I and II parameters.

As B_{maxb} increases, Fig. 6 shows that Q increases almost linearly. This agrees with Eqs. (7) and (9) which show that J_{trap} and therefore P_{pump}, which is the major contributor to Q, are approximately inversely proportional to B_{maxb}. Similarly, Q increases as B_b decreases as seen in Fig. 7. Both figures also indicate barrier mirror ratios, R_b.

One of the most important tandem mirror reactor design goals will be to aim for a high β_c. Fig. 8 shows that Q is a strongly increasing function with β_c for either constant P_{fus} or constant L_c. Fig. 9 shows the concomitant inverse dependence of P_{eaux} and P_{pump} on β_c. Present theory points to the ballooning mode as the most likely limiting factor on β_c.

Fig. 10 indicates that there is little dependence of Q on the pumping parameter g_b, the ratio of total barrier ion density to trapped barrier ion density. This is useful in that it allows some leeway in the calculation of g_b, a difficult parameter to predict.

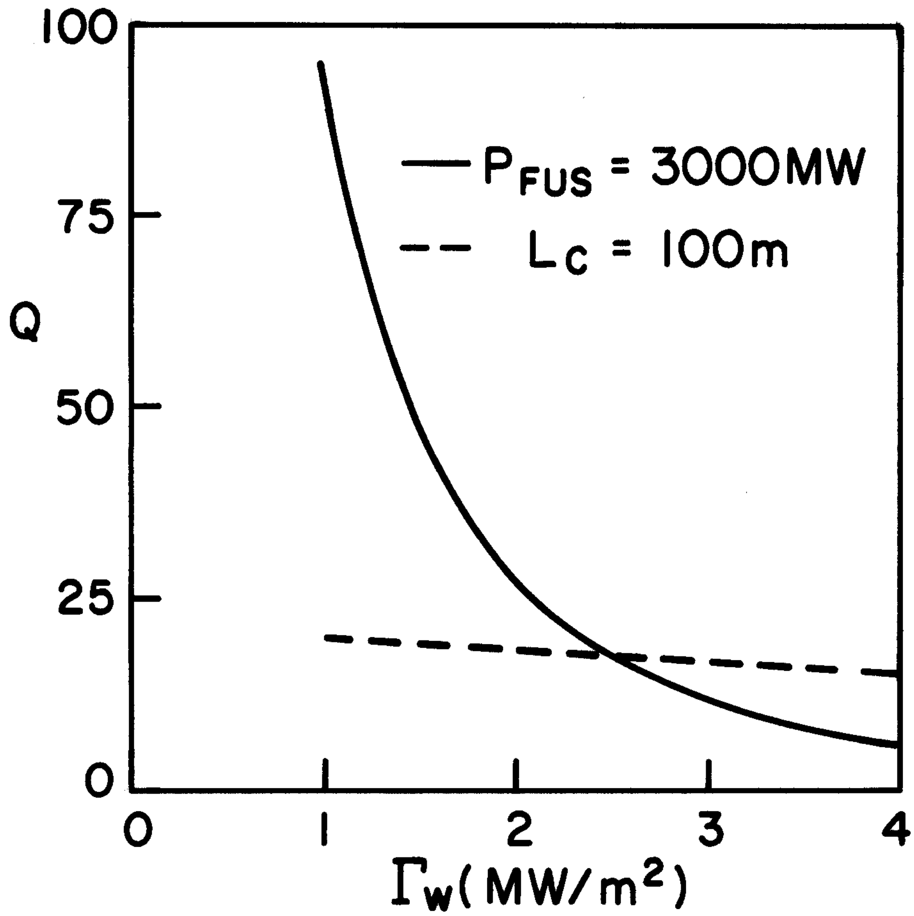

Figure 3

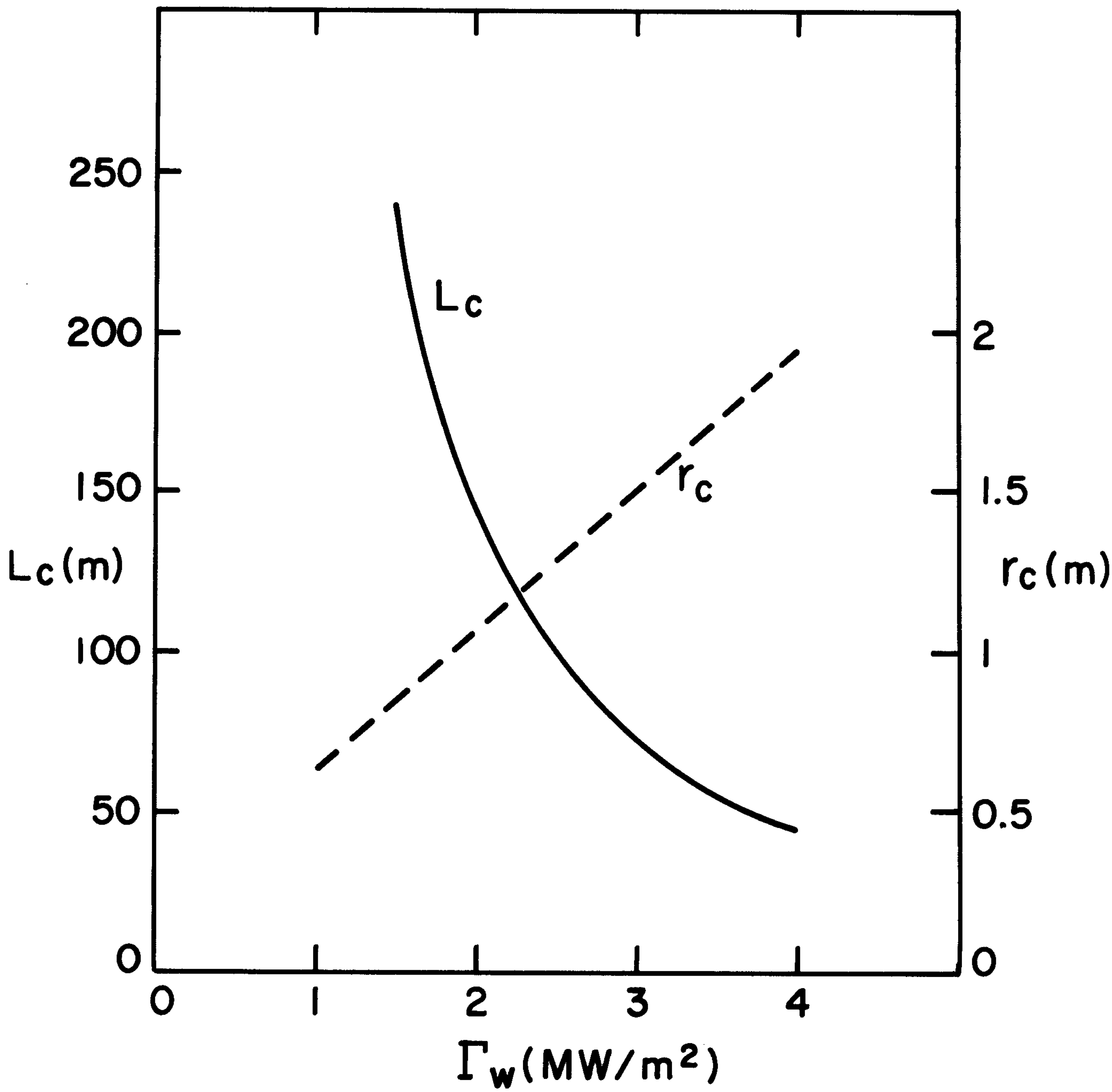

Figure 4

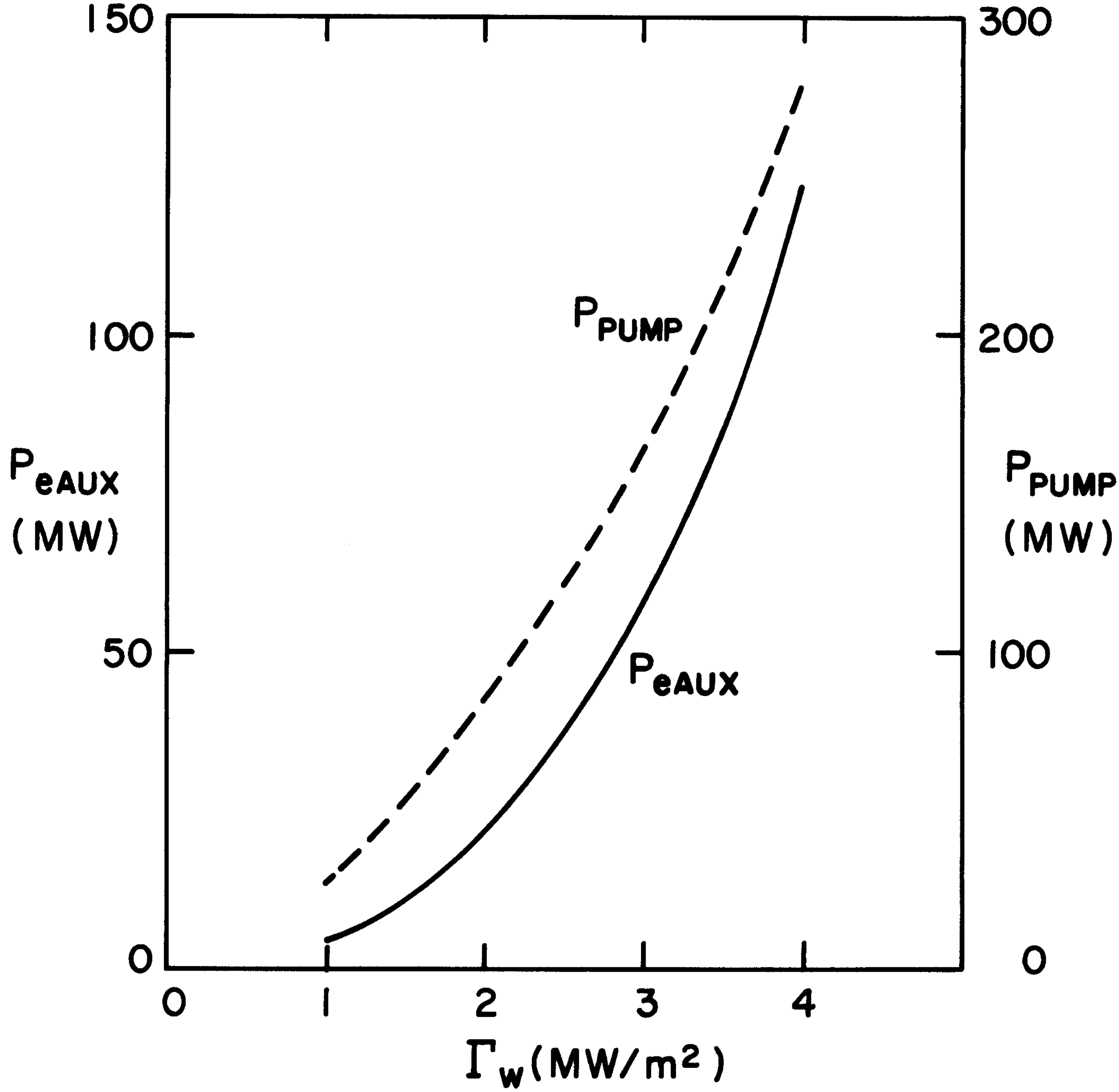
150
100
50
0
300
200
100
0
P_{eAUX} (MW)
P_{PUMP} (MW)
P_{PUMP}
P_{eAUX}
0
1
2
3
4
Γ_w (MW/m²)

Figure 5

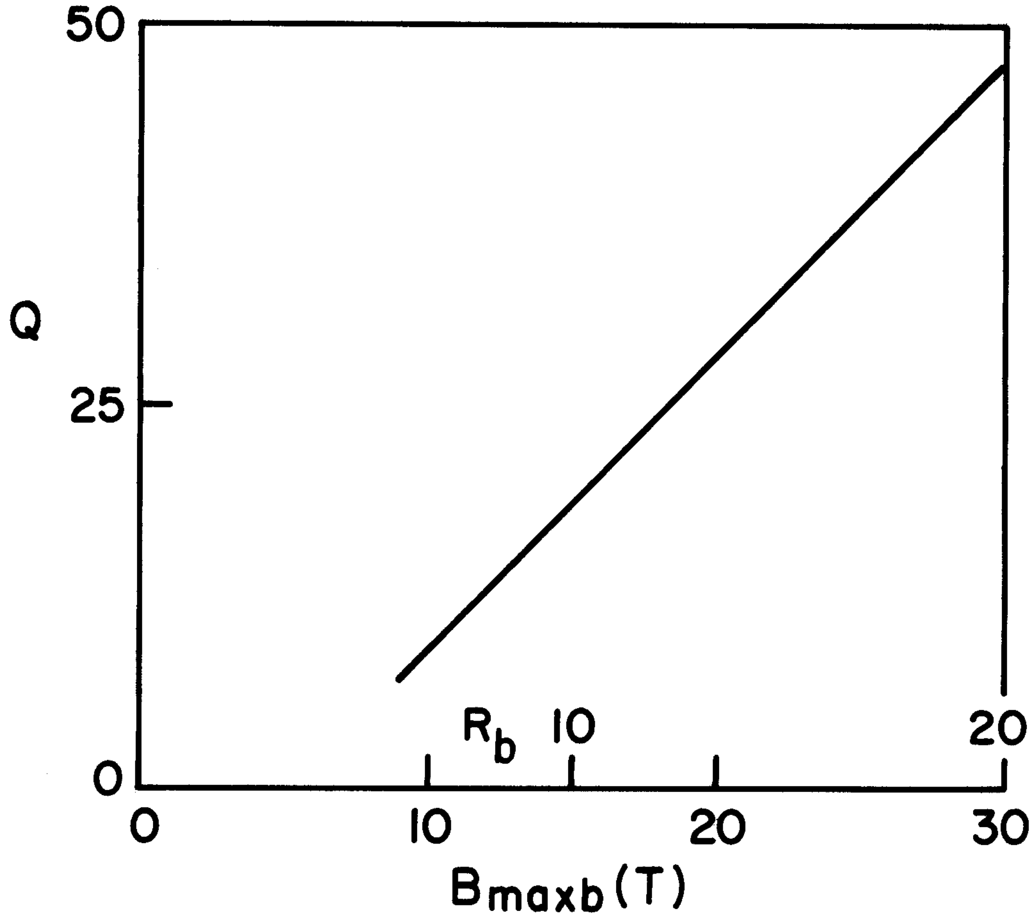

Figure 6

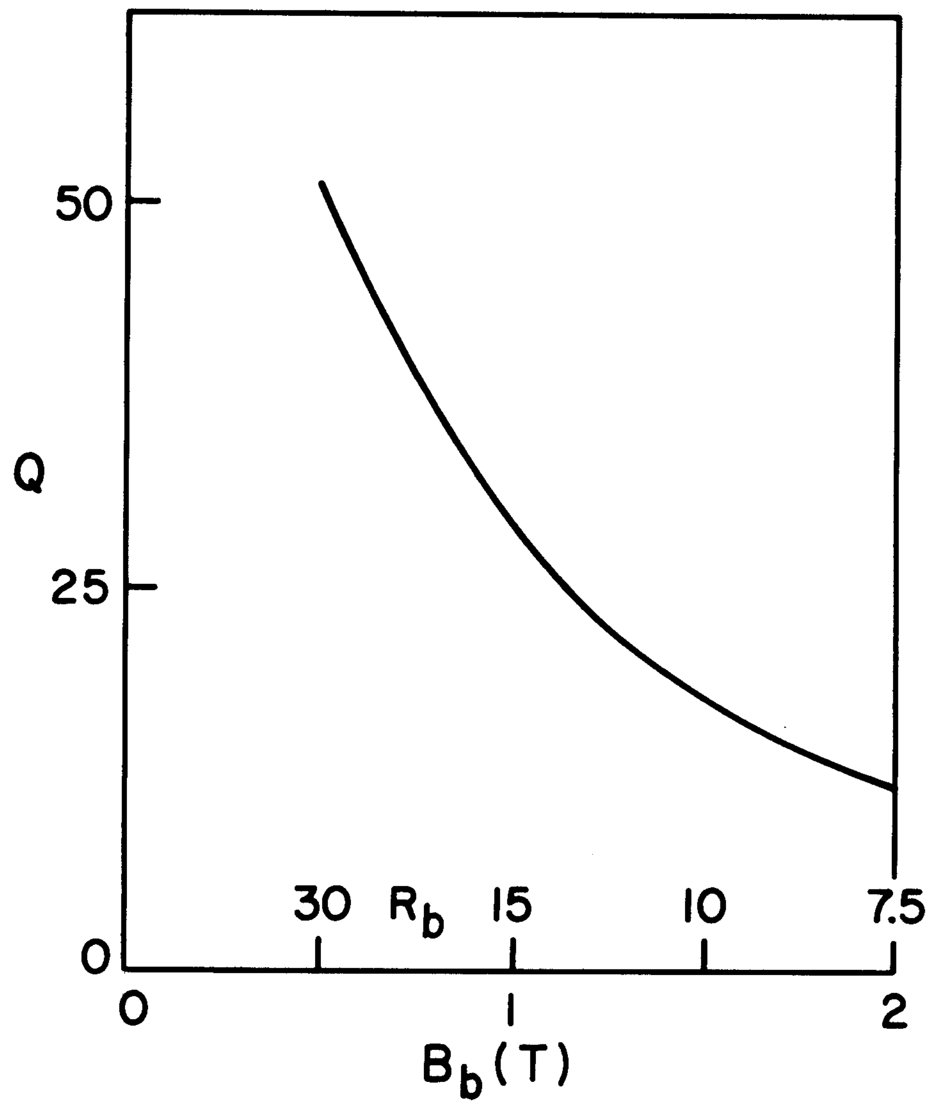

Figure 7

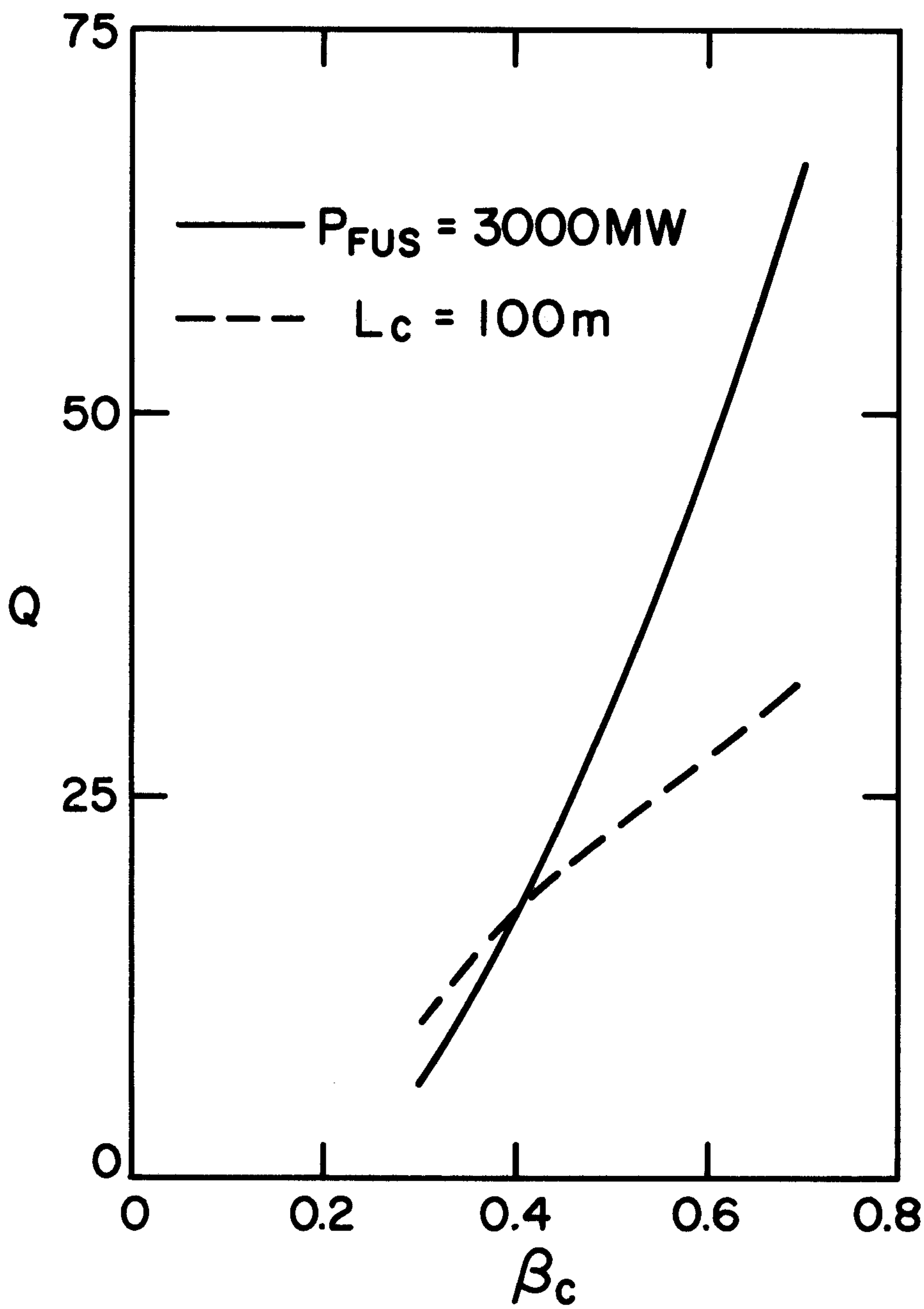

Figure 8

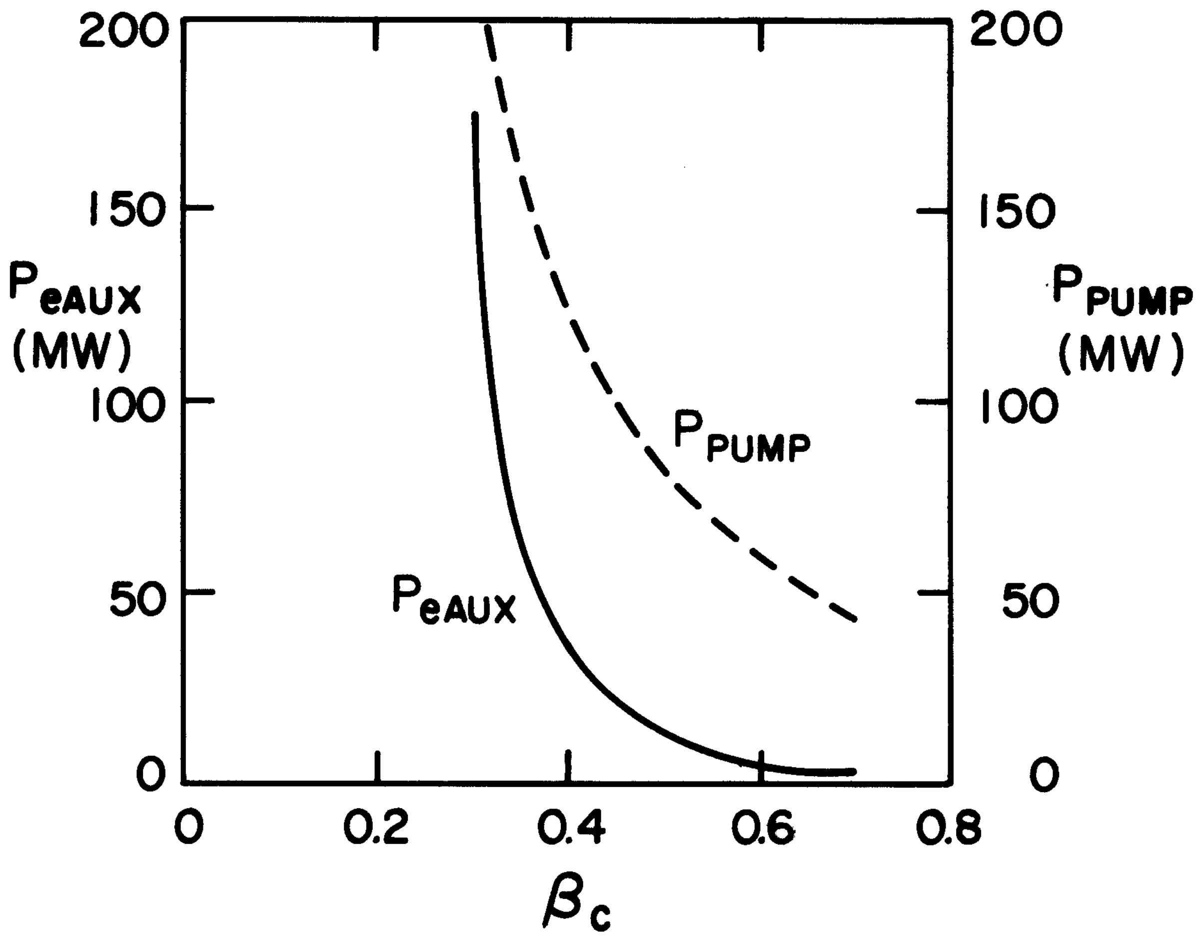
200
150
100
50
0
P_{eAUX} (MW)
200
150
100
50
0
P_{PUMP} (MW)
P_{PUMP}
P_{eAUX}
0
0.2
0.4
0.6
0.8
β_c

Figure 9

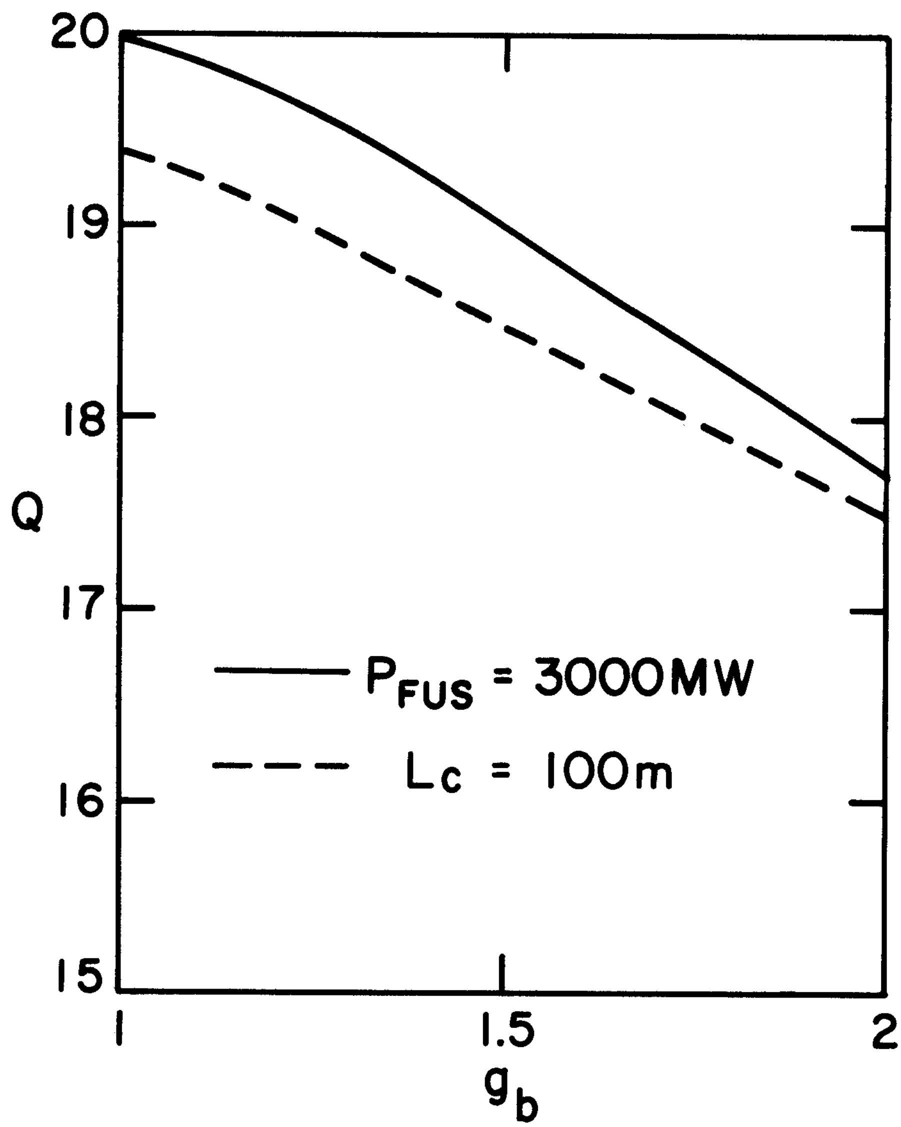

Figure 10

The dependence of Q and L_c is shown as T_{ic} varies in Fig. 11. Some work remains in optimizing Q for a given T_{ic}, but preliminary study indicates that Fig. 11 is reasonably accurate. Because the Q dependence is rather shallow from 25 to 40 keV and L_c drops quickly with rising T_{ic}, a good compromise choice is T_{ic}=40 keV.

V. Model Reactor

Base case parameters are exhibited in Tables I and II, and some considerations for choices made are given in Sec. IV. Although much work remains in both understanding the physics and in developing the required technology, particularly for neutral beams and RF heating, the model is encouraging. Neutron wall loading and Q are high, and plausible assumptions for efficiencies lead to an overall plant efficiency of 31%, yet the central cell length of 100 meters is reasonable.

The model attempts to extrapolate the technology about thirty years, while addressing most pertinent physics questions. The physics of the barrier remains unclear, but is almost certain to affect details of the model. Yet, a fair amount of leeway exists with which to compensate for new results which push the model in a more pessimistic direction.

VI. Conclusions

Power balance equations have been presented for a tandem mirror with an approximate model for the barrier physics. The results are plausible, with dependence in agreement with expectations from physical considerations and values of believable magnitude. The model reactor based on these equations would be a net power producing machine with a Q of 17.7.

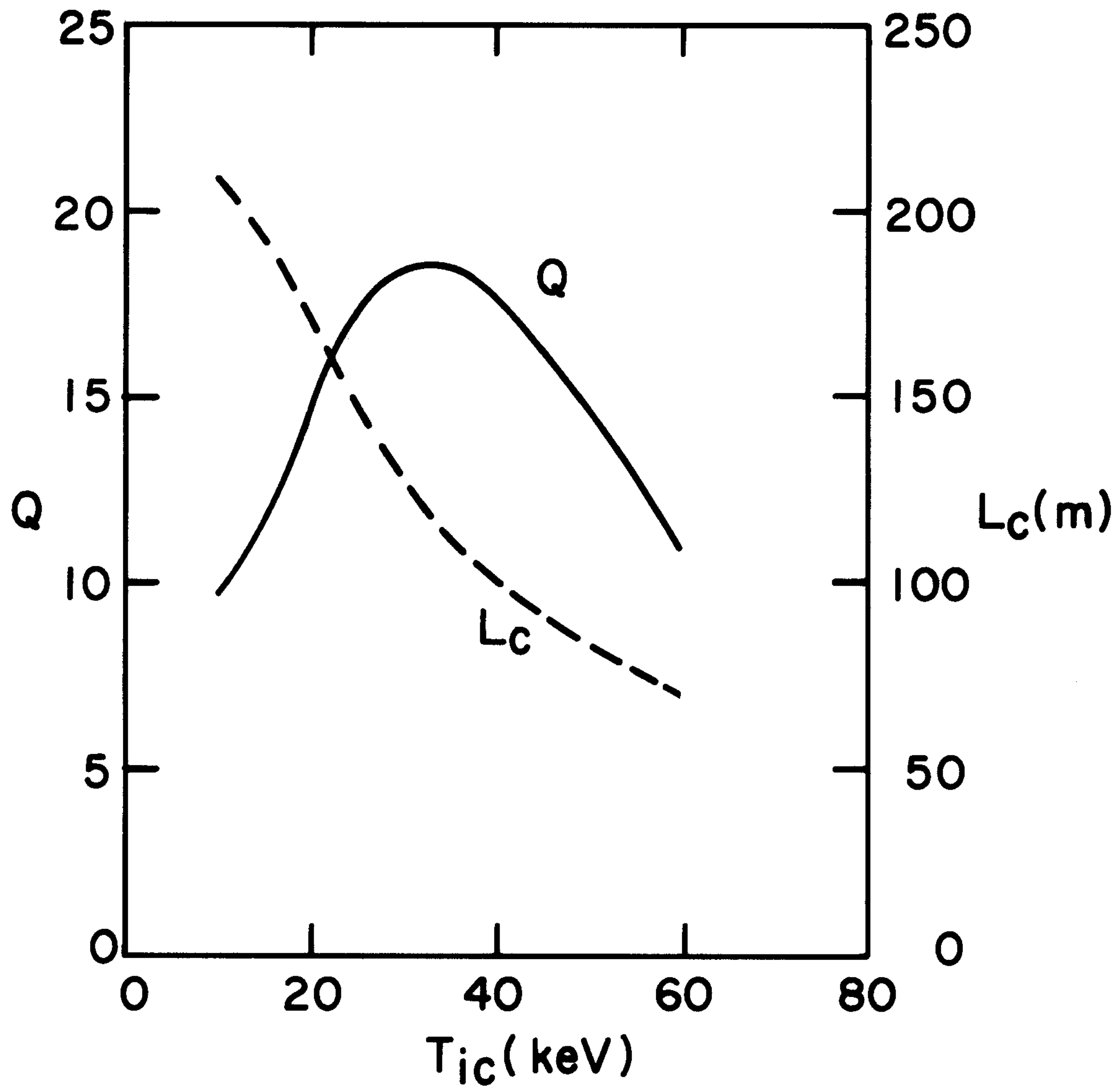

Figure 11

The chief weakness of the analysis, disregarding the inherent problems of a zero-dimensional model, is the treatment of the barrier. Work is in progress to add the capability of allowing arbitrary amounts of RF, neutral beams, and passive pumping to remove ions from the barrier, modelling each effect as accurately as feasible.

Other future directions include further investigation of parameter space, plus the inclusion of the effects of radial transport.

Acknowledgement

Support for this work was provided by the United States Department of Energy.

References

1. T.K. Fowler and B.G. Logan, Comments Pl. Ph 2, 167 (1977).

2. G.I. Dimov, V.V. Zakaidakov, and M.E. Kishinevsky, Fiz Pl 2, 597 (1976).

3. R.W. Moir, W.L. Barr, G.A. Carlson, W.L. Dexter, J.N. Doggett, J.H. Fink, G.W. Hamilton, J.E. Lee, B.G. Logan, W.S. Neef, Jr., M.A. Peterson, and M.E. Rensink, Lawrence Livermore Lab Report UCRL-52302 (1977).

4. D.E. Baldwin and B.G. Logan, Phys. Rev. Lett. 43, 1318 (1979).

5. G.A. Carlson, et al., Lawrence Livermore Lab Report UCRL-52836 (1979).

6. B.M. Boghosian, D.A. Lappa, B.G. Logan, Lawrence Livermore Lab Report UCID-18314 (1979).

7. B.G. Logan and M.E. Rensink, Lawrence Livermore Lab Memo MFE/CPI/78-181.

8. V.P. Pastukhov, Nuc. Fus. 14, 3 (1974).

9. R.H. Cohen, M.E. Rensink, T.A. Cutler, and A.A. Mirin, Nuc. Fus. 18, 1229 (1978).

10. R.H. Cohen, Lawrence Livermore Lab. Memo MFE/TC/79-310 (1979).

11. J.A. Dorning, R.H. Cohen, M.E. Rensink, A.A. Mirin, Bull. APS 24, 1060 (1979).

12. B.G. Logan, private communication.

13. D.L. Book, NRL Plasma Formalary.

14. M.E. Rensink, private communication.

15. K-C Shaing, R.W. Conn, J. Kesner, Univ. of Wis. Report UWFDM-267 (1978) and Bull. APS 23, 882 (1978).

16. R.W. Conn, "Magnetic Fusion Reactors", in Fusion, E. Teller, Editor, Academic Press (1980).