Interaction of Lower Hybrid Wave Fields with Drift-Cyclotron Loss-Cone Instability

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October 1979

UWFDM-319

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Abstract

The dispersion relation for the drift-cyclotron loss-cone mode in the presence of the lower hybrid wave is calculated using both electrostatic and finite β models. It is found that lower hybrid wave fields with frequency ω_o can stabilize the mode if $\omega_{\ell h}< \omega_o< \omega_+$, or $\omega_o< \omega_-< \omega_{\ell h}$, where $\omega_{\ell h}= \omega_{pi}/(1+ \omega_{pe}^2/ \omega_{ce}^2)^{1/2}$, $\omega_{\pm}=\left[\pm A+(A^2+4\omega_{\ell h}^2)^{1/2}\right]/2$, and $A=\omega_{\ell h}^2\varepsilon/\omega_{ci}k$. If plasma β is greater than a critical value β_c, there is another stabilization region, namely, $\omega_o>\delta\omega_{\ell h}$, where δ is a numerical constant. Even though the stabilization effect is small in this region, the lower hybrid wave frequency for electron heating should be in this region to avoid enhancing the particle loss rate.

I. Introduction

Electron heating is indispensable for the plug of a tandem mirror reactor where thermal barriers are used to enhance the electrostatic potential barrier. Various electron heating schemes have been proposed, one of which is lower hybrid wave heating. In this paper, we study the effect of the lower hybrid wave on the drift-cyclotron loss-cone mirror instability, which is driven unstable by the coupling between a positive energy electron drift wave and a negative energy ion Bernstein wave. This mode has been observed in the experiments.[1-3]

The interaction of the lower hybrid wave with microinstabilities has been studied by many authors[4-8] almost always in the electrostatic limit. Here we study both electrostatic and finite β models for the drift-cyclotron loss-cone mode and find that the finite β results are quite different from the electrostatic case, thus showing the importance of including finite β effects in the calculation.

The paper is organized as follows. In Sec. II, we discuss particle orbits and the equilibrium distribution function. In Sec. III, we briefly review the results of the electrostatic model. In Sec. IV, we calculate the dispersion relation for the finite β model. Concluding remarks are given in Sec. V.

II. Equilibrium Distribution

Consider an inhomogeneous plasma in the presence of a uniform steady magnetic field, $\vec{B}_0 = B_0 \hat{e}_z$, where $\hat{e}_z$ is the unit vector in the z direction. The plasma has a density gradient in the x direction. A high frequency oscillating electric field $\vec{E} = \vec{E}_0 \cos \omega_0 t \, \hat{e}_x$ is applied in the x direction with $\omega_{ci} << \omega_0 << \omega_{ce}$, where $\omega_{ci(ce)}$ is the ion (electron) cyclotron frequency. The configuration is shown in Fig. 1.

The equation of motion for the particles can be written as

$$\frac{d\vec{r}}{dt} = \vec{v} \ ,$$

$$\frac{d\vec{v}}{dt} = \frac{e_j}{m_j} (\vec{E}_o \cos \omega_o t + \frac{1}{c} \vec{v} \times \vec{B}_o) \ , \qquad (1)$$

where e_j and m_j are the electric charge and mass of each species j, respectively, and c is the speed of light. The solution of Eq. (1) is

$$v_x = u_x + \frac{e_j}{m_j} \left(\frac{\omega_o E_o \sin \omega_o t}{\omega_o^2 - \omega_{cj}^2} \right) \ ,$$

$$v_y = u_y + \frac{e_j}{m_j} \left(\frac{\omega_{cj} E_o \cos \omega_o t}{\omega_o^2 - \omega_{cj}^2} \right) ,$$

$$v_z = u_{\parallel} \ ,$$

$$x = - \frac{u_y}{\omega_{cj}} - \frac{e_j}{m_j} \left(\frac{E_o \cos \omega_o t}{\omega_o^2 - \omega_{cj}^2} \right) + x_o \ ,$$

$$y = + \frac{u_x}{\omega_{cj}} + \frac{e_j}{m_j} \left(\frac{\omega_{cj}}{\omega_o}\right) \left(\frac{E_o \sin \omega_o t}{\omega_o^2 - \omega_{cj}^2} \right) + y_o \ ,$$

$$z = u_{\parallel} t + z_o \ , \qquad (2)$$

where we have used $u_x = u_{\perp} \cos (\omega_{cj} t + \alpha)$ and $u_y = - u_{\perp} \sin (\omega_{cj} t + \alpha)$. The cyclotron frequency of species j is $\omega_{cj} = e_j B_o / m_j c$, and $u_{\perp}$, $u_{\parallel}$, α, x_o, y_o, and z_o are integration constants.

The equilibrium distribution function F_{oj} must satisfy the Vlasov equation

$$\frac{\partial F_{oj}}{\partial t} + \vec{v} \cdot \frac{\partial F_{oj}}{\partial \vec{r}} + \frac{e_j}{m_j} (\vec{E}_o \cos \omega_o t + \frac{1}{c} \vec{v} \times \vec{B}_o) \cdot \frac{\partial F_{oj}}{\partial \vec{v}} = 0 . \qquad (3)$$

If $F_{oj} = F_{oj}(u_\perp^2, u_\parallel, X)$ with $X = x + (e_j/m_j\omega_o)(\omega_o E_o \cos \omega_o t)/(\omega_o^2 - \omega_{cj}^2)$, then

$$u_x \frac{\partial F_{oj}}{\partial X} + \frac{e_j}{m_j c}(\vec{u} \times \vec{B}_o) \cdot \frac{\partial F_{oj}}{\partial \vec{u}} = 0 . \qquad (4)$$

The approximate solution to Eq. (4) is

$$F_{oj} = f_{oj}(u_\perp^2, u_\parallel) \quad [1 + \bar{\varepsilon}(X + \frac{u_y}{\omega_{cj}})] , \qquad (5)$$

where $\bar{\varepsilon} = \partial \ln F_{oj}/\partial x$. If the excursion distance of the particle due to the external field is small compared with the gyroradius, Eq. (5) can be expressed approximately as

$$F_{oj} = f_{oj}(u_\perp^2, u_\parallel) \quad [1 + \varepsilon(x + \frac{u_y}{\omega_{cj}})] , \qquad (6)$$

where $\varepsilon = d\ln n_o/dx$ [9] is the inverse density gradient scale length.

III. Dispersion Relation for the Drift-Cyclotron Loss-Cone Mode-Electrostatic Model

The general dispersion relation for the electrostatic wave with wave number $\vec{k} = k\hat{e}_y$ in the presence of a lower hybrid wave field with frequency ω_o is

$$\varepsilon_d = \frac{\mu^2}{4}\chi_i\left(\frac{(\chi_e^+-\chi_e)(1+\chi_e+\chi_i^+)}{\varepsilon_e\varepsilon^+} + \frac{(\chi_e^--\chi_e)(1+\chi_e+\chi_i^-)}{\varepsilon_e\varepsilon^-}\right) \quad , \qquad (7)$$

where $\mu = -kcE_o/\omega_o B_o$ is the ratio of the electron excursion distance to the wavelength, $\chi_{e(i)}$ and $\chi_{e(i)}^{\pm}$ are the electron (ion) electric susceptibilities at ω and $\omega \pm \omega_o$, respectively; $\varepsilon_d = 1 + \chi_e + \chi_i$ and $\varepsilon^{\pm} = 1 + \chi_e^{\pm} + \chi_i^{\pm}$; $\varepsilon_e = 1 + \chi_e$.[10] To obtain Eq. (7) the dipole approximation is adopted for the lower hybrid wave. We also assume that $\mu \ll 1$ and only keep terms up to μ^2. The effect of the external wave on ions has been neglected in Eq. (7) since the ion excursion distance is much smaller than that of the electron at $\omega_o \gg \omega_{ci}$.

For the drift-cyclotron loss-cone mode, χ_e and $\chi_e^{\pm}$ can be expressed as

$$\chi_e = \frac{\omega_{pe}^2}{\omega_{ce}^2} - \frac{\omega_{pe}^2}{\omega\omega_{ce}}\frac{\varepsilon}{k} \ ,$$

$$\chi_e^{\pm} = \frac{\omega_{pe}^2}{\omega_{ce}^2} - \frac{\omega_{pe}^2}{(\omega\pm\omega_o)\omega_{ce}}\frac{\varepsilon}{k} \ . \qquad (8)$$

To obtain Eq. (8), we assume $ka_e \ll 1$ and $\omega \ll \omega_{ce}$ where a_e is the electron Larmor radius. Assuming

$$f_{oi} = \left(\frac{1}{\pi v_{oi}^2}\right)\left(\frac{R}{R-1}\right)\left[\exp(-v_\perp^2/v_{oi}^2)-\exp(-Rv_\perp^2/v_{oi}^2)\right]$$

with R the mirror ratio and $v_{oi} = (2T_i/m_i)^{1/2}$, we obtain

$$\chi_i = \frac{D}{k^3a_i^3}\frac{\omega_{pi}^2}{\omega_{ci}^2}\,\Omega\cot\Omega \quad , \qquad (9)$$

where $a_i = (v_{oi}/\omega_{ci})[(R + 1)/R]^{1/2}$, $\Omega = \pi\omega/\omega_{ci}$, and $D = 2(R+1)^{3/2}/[\sqrt{\pi}(R+\sqrt{R})]$.[11]

Since $\omega_o >> \omega_{ci}$, ions can be treated as unmagnetized at frequencies $\omega \pm \omega_o$. Assuming $|\omega/k| >> v_{oi}$, we find

$$\chi_i(\omega >> \omega_{ci}) = - \frac{\omega_{pi}^2}{\omega^2} . \tag{10}$$

Substituting Eqs. (8), (9), and (10) into Eq. (7), we obtain the dispersion relation for the electrostatic drift-cyclotron loss-cone mode in the presence of the lower hybrid wave field.

$$1 + \frac{\omega_{pe}^2}{\omega_{ce}^2} - \frac{\omega_{pe}^2}{\omega\omega_{ce}} \frac{\varepsilon}{k} \left(1 - \frac{\mu^2}{2} \left(1 + \frac{\omega_{pe}^2}{\omega_{ce}^2} - \frac{\omega_{pe}^2}{\omega\omega_{ce}} \frac{\varepsilon}{k} - \frac{\omega_{pi}^2}{\omega_o^2}\right) \times \frac{\left(1 + \frac{\omega_{pe}^2}{\omega_{ce}^2}\right)\left(1 - \frac{\omega_{\ell h}^2}{\omega_o^2}\right)}{\left(1+\frac{\omega_{pe}^2}{\omega_{ce}^2}\right)^2 \left(1-\frac{\omega_{\ell h}^2}{\omega_o^2}\right)^2 - \left(\frac{\omega_{pe}^2}{\omega_o\omega_{ce}} \frac{\varepsilon}{k}\right)^2} \right) + \frac{D}{k^3 a_i^3} \frac{\omega_{pi}^2}{\omega_{ci}^2} \Omega\cot\Omega = 0 , \tag{11}$$

where $\omega_{\ell h}^2 = \omega_{pi}^2/(1+\omega_{pe}^2/\omega_{ce}^2)$.

The dispersion relation of the drift-cyclotron loss-cone mode is modified by the pondermotive force produced due to the beating between the side band waves and the pump wave. From Eq. (11), we see that the pondermotive force "effectively" modifies the density gradient. We can define an effective inverse density gradient scale length ε' as

$$\varepsilon' = C\varepsilon$$

where C is the expression in the braces of Eq. (11). If $C > 1$, i.e., $\varepsilon' > \varepsilon$, the lower hybrid wave fields have a destabilization effect on the drift-cyclotron loss-cone mode. On the other hand, if $C < 1$, i.e., $\varepsilon' < \varepsilon$, the lower hybrid wave fields stabilize the mode. Since the term

$$1 + \frac{\omega_{pe}^2}{\omega_{ce}^2} - \frac{\omega_{pe}^2}{\omega\omega_{ce}}\frac{\varepsilon}{k} - \frac{\omega_{pi}^2}{\omega_o^2} < 0,$$

the factor C can be less than 1 if $\omega_{\ell h} < \omega_o < \omega_+$ or $\omega_o < \omega_- < \omega_{\ell h}$, where $\omega_{\pm} = [\pm A + (A^2 + 4\omega_{\ell h}^2)^{1/2}]/2$, and $A = \omega_{\ell h}^2 \varepsilon/\omega_{ci} k$.

To estimate the field strength required to have a significant effect on the drift-cyclotron loss-cone mode, we calculate the critical plasma radius when $\omega_o \sim \omega_{\ell h}$. The stabilizing effect becomes stronger as ω approaches ω_+. Defining $\Delta = (\omega_o - \omega_{\ell h})/\omega_{\ell h}$ and assuming $2\Delta(1+\omega_{pe}^2/\omega_{ce}^2) << (\omega_{pe}^2/\omega_{\ell h}\omega_{ce})(\varepsilon/k)$, we can simplify Eq. (11) as

$$1 + \chi_e + \chi_i \left(1 - \frac{\mu^2}{2}\frac{2\Delta(\omega_o^2/\omega^2)(1+\omega_{pe}^2/\omega_{ce}^2)}{1+\omega_{pe}^2/\omega_{ce}^2 - (\omega_{pe}^2/\omega\omega_{ce})(\varepsilon/k)}\right) = 0 . \qquad (12)$$

Solving Eq. (12) for εa_i we have

$$\varepsilon a_i = ka_i \frac{\Omega}{\pi} \left(\frac{\omega_{ci}^2}{\omega_{pi}^2} + \frac{m_e}{m_i}\right) + \frac{D}{2\pi} \frac{1}{k^2 a_i^2} \Omega^2 \cot\Omega + \frac{ka_i}{2\pi} \frac{\omega_{ci}^2}{\omega_{pi}^2} \Omega \times \left[\frac{D^2}{k^6 a_i^6} \frac{\omega_{pi}^4}{\omega_{ci}^4} \times \right.$$

$$\left. \Omega^2 \cot^2 \Omega + 4 \frac{\Delta D}{ka_i} \frac{\omega_{pi}^2}{\omega_{ci}^2} \left(1 + \frac{\omega_{pe}^2}{\omega_{ce}^2}\right) \frac{\cot\Omega}{\Omega} \frac{R\pi^2}{R+1} \left(\frac{c}{v_{oi}}\right)^2 \left(\frac{E}{B}\right)^2 \right]^{1/2}. \quad (13)$$

Defining $x = k^3 a_i^3 (\omega_{ci}^2/\omega_{pi}^2 + m_e/m_i)/D$, Eq. (13) can be written in dimensionless form as

$$\varepsilon a_i = (A1)\Omega x^{1/3} + \frac{A1}{2} x^{-2/3}\Omega^2 \cot\Omega + \left(\frac{(A1)^2}{4} x^{-4/3} \Omega^4 \cot^2\Omega + (A1)(A2) x^{1/3} \Omega\cot\Omega\right)^{1/2}, \quad (14)$$

where $A1 = (D^{1/3}/\pi)(\omega_{ci}^2/\omega_{pi}^2 + m_e/m_i)^{2/3}$ and $A2 = \pi\delta D[R/(R+1)]\ (c/v_{oi})^2\ (E_o/B_o)^2$. The wavelength and frequency at marginal stability can be determined by the $\min_x \max_\Omega (\varepsilon a_i)$ processes.[12] We obtain

$$2x = \frac{1}{2}\Omega^2 \csc^2\Omega + \frac{1}{2} \frac{\frac{A1}{2} x^{-2/3} \Omega^4 \cot\Omega \csc^2\Omega + (A2) x \Omega^2 \csc^2\Omega - 2(A2) x \cot\Omega}{\left[\left(\frac{A1}{2}\right)^2 x^{-4/3} \Omega^4 \cot^2\Omega + (A1)(A2) x^{1/3} \Omega\cot\Omega\right]^{1/2}}, \quad (15)$$

and

$$1 = \frac{\Omega}{2\sin 2\Omega} - \frac{\frac{x^{-2/3}}{2}(A1)\Omega^2\cot\Omega - \left(\frac{A1}{8}\right)x^{-2/3}\Omega^3\csc^2\Omega - \left(\frac{A2}{2}\right)\left(\frac{x\Omega}{\sin 2\Omega}\right)}{\left[\left(\frac{A1}{2}\right)^2 x^{-4/3}\Omega^4\cot^2\Omega + (A1)(A2)x^{1/3}\Omega\cot\Omega\right]^{1/2}} \quad . \qquad (16)$$

Equations (14), (15), and (16) are solved numerically and the results are shown in Figs. 2 and 3. In Fig. 2, we compare the Post-Rosenbluth result[12] with our calculation for the case R=1, T_i=20 keV, E_0/B_0 = 0.5%, and $\Delta=3\times10^{-3}$. It is seen that the unstable region is smaller when the pump wave frequency is slightly higher than the lower hybrid wave frequency. From the result shown in Fig. 3 we see that the hotter the plasma, the smaller the stabilization effect. The reason is that the wavelength of the mode at marginal stability is longer for the hotter plasma. The electron excursion length is thus smaller relative to the wavelength and the stabilization effect is smaller.

IV. Dispersion Relation for the Drift-Cyclotron Loss-Cone Mode - Finite β Model

Since finite β has a significant stabilizing effect on the drift-cyclotron loss-cone mode,[11] we now include finite β effects in the derivation of the dispersion relation. For the drift-cyclotron loss-cone mode with $ka_i \gtrsim 1$, $ka_e \ll 1$, $\omega \lesssim \omega_{ci}$. We can treat the electrons electrostatically and the ions electromagnetically.[11,13] Assuming that the electron temperature $T_e \cong 0$, we can neglect the electron ∇B and curvature drifts.

We first calculate the perturbed distribution function f_1 by integrating linearized Vlasov equations along the unperturbed orbit given in Sec. II. The linearized Vlasov equation is

$$\frac{\partial f_j'}{\partial t} + \vec{v} \cdot \frac{\partial f_j'}{\partial \vec{r}} + \frac{e_j}{m_j}\left(E_o \cos \omega_o t + \frac{1}{c}\vec{v} \times \vec{B}_o\right) \cdot \frac{\partial f_j'}{\partial \vec{v}} = -\frac{e_j}{m_j}(\vec{E}' + \frac{1}{c}\vec{v} \times \vec{B}')$$

$$\times \frac{\partial F_{oj}}{\partial \vec{v}} \tag{17}$$

where f_j' is the perturbed distribution of species j, $\vec{E}'$, and $\vec{B}'$ are perturbed electric and magnetic fields. Then,

$$f_j' = -\frac{e_j}{m_j} \int_{-\infty}^{t} (\vec{E}' + \frac{1}{c}\vec{v} \times \vec{B}') \frac{\partial f_{oj}}{\partial \vec{v}} dt' . \tag{18}$$

Assume that every perturbed quantity has the form

$$G_1(\vec{r},t) = \int d\vec{k} \int d\omega \sum_n G_n \exp[i(\vec{k}\cdot\vec{r}-\omega_n t)]$$

where $\omega_n \equiv \omega + n\omega_o$.[7] Then,

$$\sum_n f_{jn} e^{-i\omega_n t} = -\frac{e_j}{m_j} \sum_n \int_{-\infty}^{t} dt' \, (\vec{E}_n' + \frac{1}{c}\vec{v} \times \vec{B}_n') \cdot \frac{\partial F_{oj}}{\partial \vec{v}} \exp\{i[\vec{k}\cdot(\vec{r}'-\vec{r})-\omega_n t']\}$$

$$= -\frac{e_j}{m_j} \sum_n \int_{-\infty}^{t} dt' \Big[2\, E_{nx}' \, u_x' \, (\frac{\partial F_{oj}}{\partial u_\perp^2} + \frac{\epsilon k}{2\omega_n \omega_{cj}} f_{oj}) + 2\, E_{nx}' \frac{k}{\omega_n} \times$$

$$(v_x' \, u_y' \frac{\partial F_{oj}}{\partial u_\perp^2} - v_y' \, u_x' \frac{\partial F_{oj}}{\partial u_\perp^2} + \frac{v_x' \epsilon}{2\omega_{cj}} f_{oj}) + 2E_{ny}' \times$$

$$(u_y' \frac{\partial F_{oj}}{\partial u_\perp^2} + \frac{\epsilon}{2\omega_{cj}} f_{oj}) + 2E_{nz}' \, u_\parallel' \, (\frac{\partial F_{oj}}{\partial u_\parallel^2} + \frac{k\epsilon}{2\omega_n \omega_{cj}} f_{oj})$$

$$+ \frac{2k}{\omega_n} u_y' \quad u_{\parallel}' \; E_{nz}' \left(\frac{\partial F_{oj}}{\partial u_{\perp}^2} - \frac{\partial F_{oj}}{\partial u_{\parallel}^2}\right) - 2E_{nz}' \frac{k}{\omega_n} v_y' \quad u_{\parallel}' \frac{\partial F_{oj}}{\partial u_{\parallel}^2}\Bigg]$$

$$\times \exp\{i[\vec{k}\cdot(\vec{r}'-\vec{r})-\omega_n t']\} \quad , \tag{19}$$

where $\vec{k} = k\hat{e}_y$. To obtain Eq. (19), we have used the Maxwell's equations. After straightforward algebra, we obtain

$$f_{jn} = -\frac{e_j}{m_j}\Bigg(\sum_{\substack{\ell,m\\ p,q}} (i)^{\ell}(-i)^{m+1}\exp[i(\ell+m)\phi]J_{\ell}(\alpha_j)J_m(\alpha_j)J_p(\mu_j)J_q(\mu_j)\ \{u_{\perp}\frac{\partial F_{oj}}{\partial u_{\perp}^2}$$

$$+ \frac{\epsilon k\ f_{oj}}{2\omega_{n+p-q}\omega_{cj}})\ (E_x')_{n+p-q}\left(\frac{e^{i\phi}}{(\ell+1)\omega_{cj}-\omega_{n-q}} + \frac{e^{-i\phi}}{(\ell-1)\omega_{cj}-\omega_{n-q}}\right)$$

$$+ \frac{\mu_j u_{\perp}\omega_o^2}{2\omega_{cj}}\ \frac{\partial F_{oj}}{\partial u_{\perp}^2}\left(\frac{(E_x')_{n+p-q+1}}{\omega_{n+p-q+1}} - \frac{(E_x')_{n+p-q-1}}{\omega_{n+p-q-1}}\right)\left(\frac{e^{i\phi}}{(\ell+1)\omega_{cj}-\omega_{n-q}}\right.$$

$$\left. - \frac{e^{-i\phi}}{(\ell-1)\omega_{cj}-\omega_{n-q}}\right) - \frac{\mu_j u_{\perp}\omega_o}{2}\ \frac{\partial F_{oj}}{\partial u_{\perp}^2}\ \left(\frac{(E_x')_{n+p-q+1}}{\omega_{n+p-q+1}}\right.$$

$$\left. + \frac{(E_x')_{n+p-q-1}}{\omega_{n+p-q-1}}\right)\left(\frac{e^{i\phi}}{(\ell+1)\omega_{cj}-\omega_{n-q}} + \frac{e^{-i\phi}}{(\ell-1)\omega_{cj}-\omega_{n-q}}\right) + \left(\frac{-i}{2}\right)\ \frac{\mu_j\epsilon f_{oj}\omega_o^2}{\omega_{cj}^2}$$

$$\times\left(\frac{(E_x')_{n+p-q+1}/\omega_{n+p-q+1}}{\ell\omega_{cj}-\omega_{n-q}} - \frac{(E_x')_{n+p-q-1}/\omega_{n+p-q-1}}{\ell\omega_{cj}-\omega_{n-q}}\right) + (i)\ (E_y')$$

$$\times \ [u_\perp \frac{\partial F_{oj}}{\partial^2_{u\perp}} (\frac{e^{i\phi}}{(\ell+1)\omega_{cj}-\omega_{n-q}} - \frac{e^{-i\phi}}{(\ell-1)\omega_{cj}-\omega_{n-q}})$$

$$+ \frac{\varepsilon f_{oj}/\omega_{cj}}{i(\ell\omega_{cj}-\omega_{n-q})}]\}\Bigg) + R(E_z') \ . \tag{20}$$

where $R(E_z')$ are terms that are proportional to E_z'. We do not write down all the $R(E_z')$ terms, since they do not contribute to the perturbed current density $(J_x')_n$ and $(J_y')_n$ which are the quantities to be calculated next. The perturbed current density $(J_x')_n$ and $(J_y')_n$ can be calculated from f_n as

$$(J_{x,y}')_n = \sum_j e_j \int (\vec{v}_{x,y})_j \ f_n \ d^3 \ \vec{v} \ .$$

The detailed calculation and complicated expressions for $(J_x')_n$ and $(J_y')_n$ are given in Appendix A. Again, since $|\mu_i| << |\mu_e|$, we neglect the external wave effect on the ions. Thus,

$$(J_x')_n = - \frac{n_e e^2}{m_e} [i \frac{\varepsilon}{k\omega_{ce}} (E_x')_n - \frac{1}{\omega_{ce}} (E_y')_n]$$

$$(J_y')_n = - \frac{n_e e^2}{m_e} \sum_{p,q} J_p(\mu_e) \ J_q(\mu_e) \ \{i \ (\frac{\omega_{n-q}}{\omega_{ce}^2} + \frac{\varepsilon}{k\omega_{ce}}) \ (E_y')_{n+p-q} + (\frac{1}{\omega_{ce}})$$

$$\times \ (E_x')_{n+p-q} + \frac{\mu_e \omega_o}{2\omega_{ce}} [(\frac{1}{\omega_{n-q-1}} - \frac{1}{\omega_{n+p-q-1}}) \ (E_x')_{n+p-q-1}$$

$$+ \left(\frac{1}{\omega_{n-q+1}} - \frac{1}{\omega_{n+p-q+1}}\right) (E_x')_{n+p-q+1}] + i \frac{\mu_e \omega_o}{2\omega_{ce}^2} [(E_y')_{n+p-q-1} +$$

$$(E_y')_{n+p-q+1}] + i \frac{\mu_e \varepsilon \omega_o}{2k\omega_{ce}} \left[\frac{(E_y')_{n+p-q-1}}{\omega_{n-q-1}} + \frac{(E_y')_{n+p-q+1}}{\omega_{n-q+1}}\right] - \frac{\omega_o^2}{\omega_{ce}} \frac{pq(E_x')_{n+p-q}}{\omega_{n-q}\omega_{n+p-q}}\}$$

$$+ \frac{i}{2} \frac{\omega_{pi}^2}{\omega_{ci}} \frac{\omega_n^2}{k^2} \sum_{\ell} \int du_\perp^2 \frac{\partial f_{oi}/\partial u_\perp^2}{\ell - \omega_n/\omega_{ci}} J_\ell^2 (\alpha_i) (E_y')_n . \qquad (21)$$

Fourier analyzing the Maxwell equations, we have

$$\left(1 - \frac{\omega_n^2}{k^2 c^2}\right) (E_x')_n = \frac{4\pi i \omega_n}{k^2 c^2} (J_x')_n ,$$

$$(E_y')_n = - \frac{4\pi i}{\omega_n} (J_y')_n . \qquad (22)$$

Substituting (21) into (22) and assuming $\omega_n^2/k^2c^2 \ll 1$ and $\omega_n \omega_{pe}^2 \varepsilon/k^3 c^2 \omega_{ce} \ll 1$, we obtain a relationship between $(E_x')_n$ and $(E_y')_n$

$$(E_x')_n = i \frac{\omega_n \omega_{pe}^2}{k^2 c^2 \omega_{ce}} (E_y')_n . \qquad (23)$$

Combining Eqs. (21), (22), and (23), we obtain

$$[1 + \chi_i(\omega_n)]\,(E_y')_n = -\sum_{p,q} [J_p\, J_{n+p-q}\, \chi_e\,(\omega_{q-p}) + P\, J_p\, J_{n+p-q}\, \chi_e'\,(\omega_{q-p})$$

$$- (n+p-q)\, J_p\, J_{n+p-q}\, \chi_e'(\omega_n) + p(n+p-q)\, J_p\, J_{n+p-q}\, \hat{\chi}_e(\omega_n,\omega_{q-p})]\,(E_y')_q\,, \qquad (24)$$

where $\chi_e(\omega_{q-p}) = \frac{\omega_{pi}^2}{\omega_{ci}^2}\left(\frac{m_e}{m_i} + \frac{\omega_{pi}^2}{k^2c^2} - \frac{\varepsilon\omega_{ci}}{k\omega_{q-p}}\right)$,

$$\chi_e'(\omega_{q-p}) = \frac{\omega_{pi}^2}{\omega_{ci}^2}\,\frac{\omega_{pi}^2}{k^2c^2}\,\frac{\omega_o}{\omega_{q-p}}\,,$$

$$\hat{\chi}_e(\omega_n,\omega_{q-p}) = \frac{\omega_{pi}^2}{\omega_{ci}^2}\,\frac{\omega_{pi}^2}{k^2c^2}\,\frac{\omega_o^2}{\omega_n\omega_{q-p}}\,,\quad \chi_i(\omega_n) = -2\pi\,\frac{\omega_{pi}^2}{\omega_{ci}}\,\frac{\omega_n}{k^2}\sum_{\ell}\int du_\perp^2\,\frac{\partial f_{oi}/\partial u_\perp^2}{\ell-\omega_n/\omega_{ci}}\,J_\ell^2\,(\alpha_i)\,.$$

If we assume weak coupling, i.e., $\mu_e \ll 1$, we need only consider terms of the form $\chi_{e(i)}(\omega)$, $\chi_{e(i)}^{\pm}$, $\chi_e'(\omega)$, $\chi_e'^{\pm}$, $\hat{\chi}_e(\omega)$, $\hat{\chi}_e^{\pm}$ in Eq. (24). Neglecting all terms of order higher than μ_e^2 and defining $\mu = \mu_e$ we obtain the dispersion relation

$$\varepsilon_d = \frac{\mu^2}{4}\,\chi_i\left(\frac{(\chi_e^+-\chi_e)(1+\chi_e+\chi_i^+)}{\varepsilon_e\varepsilon^+} + \frac{(\chi_e^--\chi_e)(1+\chi_e+\chi_i^-)}{\varepsilon_e\varepsilon^-}\right)\,. \qquad (25)$$

Notice that Eq. (25) has the same form as Eq. (7), except that the definition of χ_e is different. Electron electric susceptibility χ_e now has an extra finite β term, ω_{pi}^2/k^2c^2. Substituting various expressions for $\chi_{e(i)}$ and $\chi_{e(i)}^{\pm}$ into Eq. (25), and using the fact that $1 + \chi_e + \chi_i \simeq O(\mu^2)$, we find

$$\frac{\omega_{ci}^2}{\omega_{pi}^2} + \frac{m_e}{m_i} + \frac{\omega_{pi}^2}{k^2c^2} + \frac{D}{k^3a_i^3}\Omega\cot\Omega - \frac{\varepsilon\omega_{ci}}{k\omega}\left[1 - \frac{\mu^2}{2}\left(\frac{\omega_{ci}^2}{\omega_{pi}^2} + \frac{m_e}{m_i} + \frac{\omega_{pi}^2}{k^2c^2} - \frac{\varepsilon\omega_{ci}}{k\omega} - \frac{\omega_{ci}^2}{\omega_o^2}\right) \frac{\frac{\omega_{ci}^2}{\omega_{pi}^2} + \frac{m_e}{m_i} + \frac{\omega_{pi}^2}{k^2c^2} - \frac{\omega_{ci}^2}{\omega_o^2}}{\left(\frac{\omega_{ci}^2}{\omega_{pi}^2} + \frac{m_e}{m_i} + \frac{\omega_{pi}^2}{k^2c^2} - \frac{\omega_{ci}^2}{\omega_o^2}\right)^2 - \left(\frac{\varepsilon\omega_{ci}}{k\omega_o}\right)^2}\right] = 0 . \qquad (26)$$

We again define $\varepsilon' = C\varepsilon$ where C is the expression in the braces of Eq. (26). The lower hybrid wave fields can stabilize (destabilize) the drift-cyclotron loss-cone mode if $C < (>)\ 1$. For the case

$$\frac{\omega_{ci}^2}{\omega_{pi}^2} + \frac{m_e}{m_i} + \frac{\omega_{pi}^2}{k^2c^2} - \frac{\varepsilon\omega_{ci}}{k\omega} - \frac{\omega_{ci}^2}{\omega_o^2} < 0 ,$$

the factor C can be less than 1 if $\omega_r < \omega_o < \omega_+$, or $\omega_o < \omega_- < \omega_r$, where $\omega_r^2 = \omega_{pi}^2/(1 + \frac{\omega_{pe}^2}{\omega_{ce}^2} + \frac{\omega_{pi}^4}{k^2c^2\omega_{ci}^2})$, $\omega_\pm = [\pm A + (A^2 + 4\omega_R^2)^{1/2}]/2$, and $A = \omega_r^2\ \varepsilon/k\omega_{ci}$. Thus, the stabilization region is shifted toward the low frequency side because of the finite β effect.

However, for the finite β drift-cyclotron loss-cone mode, the electron electric susceptibility χ_e is no longer always negative; it can be positive if $\beta > \beta_c$. The critical β_c is defined as the plasma β at which $\chi_e(\omega,k) = 0$ at the marginally stable case. By setting $\Omega = \pi/2$ at the marginally stable case, we obtain

$$\beta_c = 1.24 \frac{R}{(R + \sqrt{R})^{2/3}} \left(\frac{\omega_{ci}^2}{\omega_{pi}^2} + \frac{m_e}{m_i} \right)^{1/3} . \tag{27}$$

For $\beta > \beta_c$, $\chi_e(\omega,k)$ is positive in the marginally stable case, and the lower hybrid wave fields can stabilize the drift-cyclotron loss-cone mode if $\omega_0 > \delta(\beta)\omega_{\ell h}$. The factor δ as a function of β is plotted in Figs. 4 and 5 for a hydrogen plasma at R = 1 and for T_i = 10 keV and 1 MeV, respectively. We see that the minimum δ is around 1.5 for both cases. However, since

$$\left| \frac{\omega_{ci}^2}{\omega_{pi}^2} + \frac{m_e}{m_i} + \frac{\omega_{pi}^2}{k^2c^2} - \frac{\varepsilon\omega_{ci}}{\omega} - \frac{\omega_{ci}^2}{\omega_0^2} \right| < |\chi_i| ,$$

and $|\chi_i|$ is an order of magnitude smaller than $|\chi_e|$, thus, the factor C is roughly equal to $1 - 0.1\mu^2$. For $\mu^2 \ll 1$, the factor $C \sim 1$; thus, the effect of the lower hybrid wave fields on the drift-cyclotron loss cone mode is negligible.

The stabilization effect of the lower hybrid wave fields has been predicated and proved in Q machine experiments.[6,14] The lower hybrid wave field was excited by a coil around the machine.[14,15] The resonance frequency is around the lower hybrid frequency. By adjusting the resonance frequency, the fluctuations associated with the drift wave instability were suppressed. The qualitative stabilization frequency region is the same as predicated by theory.[6] The stabilization of the drift-cyclotron loss-cone mode by the lower hybrid wave field may also be proved by similar experimental schemes. However, a difficulty may arise due to the fact that the most stabilizing frequency region is lower than the lower hybrid wave frequency. A very high electric field strength (E = 15 kV/cm) is required to improve the critical plasma radius by 25% for $B_0 = 2T$, T_i = 20 keV plasma. The field strength should be lower for lower B_0 field and T_i.

V. Concluding Remarks

One of the most important goals of mirror and tandem mirror research is to achieve classical particle confinement in the minimum B mirror well. Several microinstabilities can exist in the mirror well and might affect particle confinement. The drift-cyclotron loss-cone mode is one such instability.

We studied the effects of the lower hybrid wave fields on the drift-cyclotron loss-cone mode, and found that lower hybrid wave fields can stabilize the mode if $\omega_r < \omega_0 < \omega_+$ and $\omega_0 < \omega_- < \omega_r$. There is also a new stabilization frequency region which does not exist in the low β case. For $\beta > \beta_c$, the lower hybrid wave fields can stabilize the mode if $\omega_0 > \delta\omega_{\ell h}$. However, the stabilization effect is small in this region. Nevertheless, if we want to use lower hybrid waves to heat electrons in the plugs, we still should choose the wave frequency $\omega_0 > \delta\omega_{\ell h}$ in order to avoid enhancing the particle loss rate.

Acknowledgment

This work was supported by the U.S. Department of Energy.

Appendix A

By definition

$$\sum_n \vec{J}_n \exp(-i\omega_n t) = \sum_{n,j} \exp(-i\omega_n t) \quad e_j \int \vec{v}\, f_{nj}\, d^3v$$

$$= - \sum_{n,j} \frac{e_j^2}{m_j} \exp(-i\omega_n t) \quad , \tag{A1}$$

where

$$(\hat{J}_n)_x = \sum_{p,q} \pi\, J_p\,(\mu_j)\, J_q\,(\mu_j) \quad \{[-\, i\, u_\perp^2\, P_{11}(\frac{\partial f_{oj}}{\partial u_\perp^2} + \frac{\varepsilon k f_{oj}}{2\omega_{n+p-q}\omega_{cj}})$$

$$+ i\, \frac{u_\perp^3}{2}\, P_{11e} \frac{\varepsilon}{\omega_{cj}}\, \frac{\partial f_{oj}}{\partial u_\perp^2}]\, (E_x)_{n+p-q} + \Bigg(u_\perp^2\, P_{121}\, \frac{\partial f_{oj}}{\partial u_\perp^2} + P_{122}$$

$$\frac{\varepsilon u_\perp f_{oj}}{2\omega_{cj}} + u_\perp^3\, P_{12e}\, \frac{\varepsilon}{\omega_{cj}}\, \frac{\partial f_{oj}}{\partial u_\perp^2}\Bigg) (E_y)_{n+p-q} + i\, \frac{\mu_j \omega_o^2}{k\omega_{cj}}\, \sin\, \omega_o t$$

$$[-u_\perp\, P_{11P1}\, (\frac{\partial f_{oj}}{\partial u_\perp^2} + \frac{\varepsilon k f_{oj}}{2\omega_{n+p-q}\omega_{cj}}) + u_\perp^2\, P_{11P2}\, \frac{\varepsilon}{\omega_{cj}}\, \frac{\partial f_{oj}}{\partial u_\perp^2}]\, (E_x)_{n+p-q}$$

$$+ \frac{\mu_j \omega_o^2}{k\omega_{cj}}\, \sin\, \omega_o t \Bigg(u_\perp\, P_{12P1}\, \frac{\partial f_{oj}}{\partial u_\perp^2} - u_\perp^2\, P_{12P2}\, \frac{\varepsilon}{\omega_{cj}}\, \frac{\partial f_{oj}}{\partial u_\perp^2}$$

$$+ P_{12P3}\, \frac{\varepsilon}{\omega_{cj}}\, f_{oj}\Bigg) (E_y)_{n+p-q} - i\, \frac{\mu_j \omega_o^2}{2\omega_{cj}} \Bigg(u_\perp^2\, P_{121}\, \frac{\partial f_{oj}}{\partial u_\perp^2} + \frac{\varepsilon}{\omega_{cj}}\, u_\perp$$

$$P_{122}\, f_{oj} - \frac{\varepsilon}{\omega_{cj}}\, u_\perp^3\, P_{12e}\, \frac{\partial f_{oj}}{\partial u_\perp^2}\Bigg) \quad [\frac{(E_x)_{n+p-q+1}}{\omega_{n+p-q+1}} - \frac{(E_x)_{n+p-q-1}}{\omega_{n+p-q-1}}] +$$

$$+ i \frac{\mu_j \omega_o}{2} \left(u_\perp^2 P_{11} \frac{\partial f_{oj}}{\partial u_\perp^2} - u_\perp^3 \frac{\varepsilon}{\omega_{cj}} P_{11e} \frac{\partial f_{oj}}{\partial u_\perp^2} \right) \left(\frac{(E_x)_{n+p-q+1}}{\omega_{n+p-q+1}} \right.$$

$$\left. + \frac{(E_x)_{n+p-q-1}}{\omega_{n+p-q-1}} \right) - i \frac{\mu_j^2 \omega_o^4}{2k\omega_{cj}^2} \sin \omega_o t \left(u_\perp P_{12P1} \frac{\partial f_{oj}}{\partial u_\perp^2} + \frac{\varepsilon}{\omega_{cj}} P_{12P3} f_{oj} \right.$$

$$\left. - \frac{\varepsilon}{\omega_{cj}} u_\perp^2 P_{12P2} \frac{\partial f_{oj}}{\partial u_\perp^2} \right) \left(\frac{(E_x)_{n+p-q+1}}{\omega_{n+p-q+1}} - \frac{(E_x)_{n+p-q-1}}{\omega_{n+p-q-1}} \right)$$

$$- \frac{\mu_j^2 \omega_o^4}{2k\omega_o \omega_{cj}} \sin \omega_o t \left(u_\perp P_{122} \frac{\partial f_{oj}}{\partial u_\perp^2} - \frac{\varepsilon}{\omega_{cj}} u_\perp^2 P_{121} \frac{\partial f_{oj}}{\partial u_\perp^2} \right)$$

$$\left[\frac{(E_x)_{n+p-q+1}}{\omega_{n+p-q+1}} + \frac{(E_x)_{n+p-q-1}}{\omega_{n+p-q-1}} \right] \Big\} ,$$

$$(\hat{J}_n)_y = \sum_{p,q} \pi J_\ell (\mu_j) J_q (\mu_j) \Big\{ \Big[-u_\perp^2 P_{121} \left(\frac{\partial f_{oj}}{\partial u_\perp^2} + \frac{\varepsilon_k f_{oj}}{2\omega_{cj} \cdot \omega_{n+p-q}} \right)$$

$$+ u_\perp^3 \frac{\varepsilon}{\omega_{cj}} P_{21e} \frac{\partial f_{oj}}{\partial u_\perp^2} \Big] (E_x)_{n+p-q} + i \left({}_\perp^2 P_{22} \frac{\partial f_{oj}}{\partial u_\perp^2} + u_\perp P_{221} \right.$$

$$\left. \times \frac{\varepsilon f_{oi}}{\omega_{cj}} + u_\perp^3 P_{22e} \frac{\varepsilon}{\omega_{cj}} \frac{\partial f_{oj}}{\partial u_\perp^2} \right) (E_y)_{n+p-q} + i \frac{\mu_j \omega_o}{k} \cos \omega_o t$$

$$\left[- u_\perp P_{11P1} \left(\frac{\partial f_{oj}}{\partial u_\perp^2} + \frac{\varepsilon k f_{oj}}{2\omega_{cj} \omega_{n+p-q}} \right) + u_\perp^2 P_{11P2} \frac{\varepsilon}{\omega_{cj}} \frac{\partial f_{oj}}{\partial u_\perp^2} \right] .$$

$$(E_x)_{n+p-q} + \frac{\mu_j\omega_o}{k} \cos \omega_o t \left(u_\perp P_{12P1} \frac{\partial f_{oj}}{\partial u_\perp^2} - u_\perp^2 P_{12P2} \frac{\varepsilon}{\omega_{cj}} \right.$$

$$\left. \times \frac{\partial f_{oj}}{\partial u_\perp^2} + P_{12P3} \frac{\varepsilon}{\omega_{cj}} f_{oj} \right) (E_y)_{n+p-q} + i \frac{\mu_j \omega_o^2}{2\omega_{cj}} \left(u_\perp^2 P_{12P2} \frac{\partial f_{oj}}{\partial u_\perp^2} \right.$$

$$\left. - \frac{\varepsilon u_\perp}{\omega_{cj}} P_{221} f_{oj} - u_\perp^3 \frac{\varepsilon}{\omega_{cj}} P_{22e} \frac{\partial f_{oj}}{\partial u_\perp^2} \right) \left(\frac{(E_x)_{n+p-q+1}}{\omega_{n+p-q+1}} - \frac{(E_x)_{n+p-q-1}}{\omega_{n+p-q-1}} \right)$$

$$- i \frac{\mu_j \omega_o}{2} \left(u_\perp^2 P_{11P2} \frac{\partial f_{oj}}{\partial u_\perp^2} + u_\perp^3 \frac{\varepsilon}{\omega_{cj}} P_{21e} \frac{\partial f_{oj}}{\partial u_\perp^2} \right) \left(\frac{(E_x)_{n+p-q+1}}{\omega_{n+p-q+1}} \right.$$

$$\left. + \frac{(E_x)_{n+p-q-1}}{\omega_{n+p-q-1}} \right) - i \frac{\mu_j^2 \omega_o^3}{2k\omega_{cj}} \cos \omega_o t \left(u_\perp P_{12P1} \frac{\partial f_{oj}}{\partial u_\perp^2} + \frac{\varepsilon}{\omega_{cj}} \right.$$

$$\left. P_{12P3} f_{oj} - u_\perp^2 \frac{\varepsilon}{\omega_{cj}} P_{12P2} \frac{\partial f_{oj}}{\partial u_\perp^2} \right) \left(\frac{(E_x)_{n+p-q+1}}{\omega_{n+p-q+1}} - \frac{(E_x)_{n+p-q-1}}{\omega_{n+p-q-1}} \right)$$

$$- \frac{\mu_j^2 \omega_o^2}{2k} \cos \omega_o t \left(u_\perp P_{122} \frac{\partial f_{oj}}{\partial u_\perp^2} - \frac{\varepsilon}{\omega_{cj}} u_\perp^2 P_{121} \frac{\partial f_{oj}}{\partial u_\perp^2} \right)$$

$$\left. \times \left(\frac{(E_x)_{n+p-q+1}}{\omega_{n+p-q+1}} + \frac{(E_x)_{n+p-q-1}}{\omega_{n+p-q-1}} \right) \right\} ,$$

$$\mu_j = \frac{e_j}{m_j} \frac{k\omega_{cj} E_o}{\omega_o(\omega_o^2 - \omega_{cj}^2)} ,$$

$$P_{11} = \sum_{\ell} \frac{4J_{\ell}'^{2}}{\ell\omega_{cj}-\omega_{n-q}} \quad ,$$

$$P_{11e} = -\frac{4}{\alpha_j\omega_{cj}} + \frac{8}{\alpha_j}\frac{\omega_{n-q}}{\omega_{cj}} \sum_{\ell} \frac{(J_{\ell}J_{\ell}'/\alpha_j)-J_{\ell}'^{2}}{\ell\omega_{cj}-\omega_{n-q}} \quad ,$$

$$P_{121} = \frac{4}{\alpha_j}\frac{\omega_{n-q}}{\omega_{cj}} \sum_{\ell} \frac{J_{\ell}J_{\ell}'}{\ell\omega_{cj}-{}_{n-q}} \quad , \quad P_{11P2} = iP_{121} \quad ,$$

$$P_{122} = -\sum_{\ell} \frac{4J_{\ell}J_{\ell}'}{\ell\omega_{cj}-\omega_{n-q}} \quad , \quad P_{11P1} = i\, P_{122} \quad ,$$

$$P_{12e} = -\frac{4}{\alpha_j^2}\left[-\frac{1}{\alpha_j}\frac{\omega_{n-q}}{\omega_{cj}^2} + \frac{\omega_{n-q}^2}{\omega_{cj}^2}\sum_{\ell}\frac{J_{\ell}'J_{\ell}-J_{\ell}^2/\alpha_j}{\ell\omega_{cj}-\omega_{n-q}}\right] \quad ,$$

$$P_{12P1} = -\frac{4i}{\alpha_j}\left(\frac{1}{\omega_{cj}} + \frac{\omega_{n-q}}{\omega_{cj}}\sum_{\ell}\frac{J_{\ell}^2}{\ell\omega_{cj}-\omega_{n-q}}\right) \quad ,$$

$$P_{12P2} = \frac{4i}{\alpha_j^2}\left(\frac{\omega_{n-q}}{\omega_{cj}^2} + \frac{\omega_{n-q}^2}{\omega_{cj}^2}\sum_{\ell}\frac{J_{\ell}^2}{\ell\omega_{cj}-\omega_{n-q}}\right) \quad , \quad P_{22} = i\, P_{12P2} \quad ,$$

$$P_{12P3} = -2i\sum_{\ell}\frac{J_{\ell}^2}{\ell\omega_{cj}-\omega_{n-q}} \quad ,$$

$$P_{21e} = -4\sum_{\ell}\left(\frac{\omega_{n-q}^2}{\alpha_j^2\omega_{cj}^2}J_{\ell}'\,J_{\ell} - J_{\ell}'^{2}/\alpha_j\right)/(\ell\omega_{cj}-\omega_{n-q}) \quad ,$$

$$P_{221} = -\frac{2}{\alpha_j}\left(\frac{1}{\omega_{cj}} + \frac{\omega_{n-q}}{\omega_{cj}}\sum_{\ell}\frac{J_{\ell}^2}{\ell\omega_{cj}-\omega_{n-q}}\right) \quad ,$$

$$P_{22e} = \frac{4}{\alpha_j} [\frac{1}{\omega_{cj}} (\frac{1}{2} + \frac{\omega_{n-q}^2}{\alpha_j \omega_{cj}^2} - \sum_{\ell} \frac{\omega_{n-q}/\alpha_j \omega_{cj}}{\ell\omega_{cj} - \omega_{n-q}} (J_\ell J_\ell' - \frac{\omega_{n-q}^2}{\omega_{cj}^2} J_\ell^2/\alpha_j)] \ ,$$

$$\alpha_j = \frac{ku_\perp}{\omega_{cj}} \ , \ J_\ell' = dJ_\ell/d\alpha_j, \ J_\ell \text{ is the Bessel function of order } \ell \ ,$$

and $\quad f_{oj} = \int f_{oj} \, du_\parallel$.

Since $\omega_o >> \omega_{ci}$, $|\mu_i| << |\mu_e|$, we set $\mu_i = 0$. Assuming $\alpha_i >> 1$, $\alpha_e << 1$, and neglect all the terms of order of $0 \left(\mu_e \frac{\omega_o^2}{\omega_{ce}^2}\right)$, $0 \left(\mu_e \frac{\omega_o}{\omega_{ce}} \frac{\varepsilon}{k}\right)$ or higher, we then obtain Eq. (21) in Sec. IV.

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Figure Captions

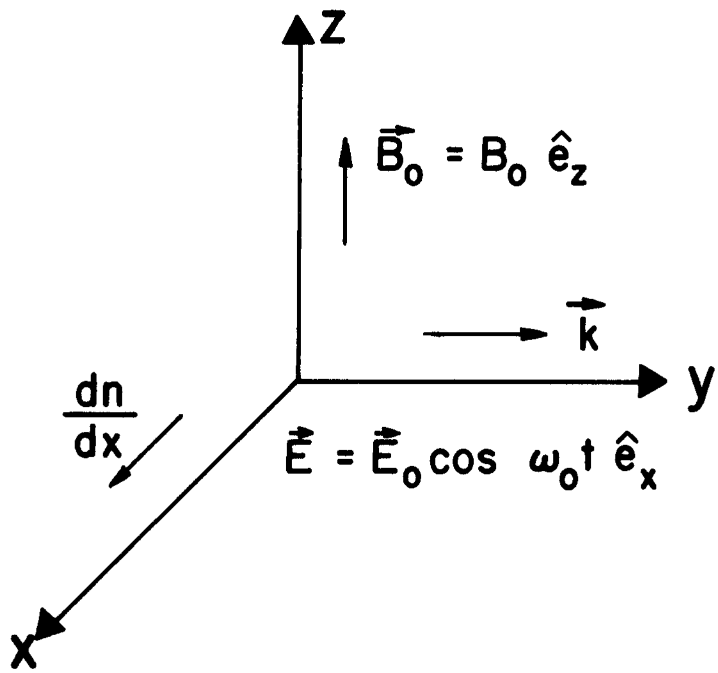
Fig. 1 Configuration of the coordinates.

Fig. 2 Critical characteristic length R_c/a_i $(=1/\varepsilon a_i)$ as a function of density $(\omega_{ci}^2/\omega_{pi}^2)$ for T_i = 20 keV, E_o/B_o = 0.5%, and $\Delta = 3 \times 10^{-3}$ at R = 1. Curve P-R is the result of Post and Rosenbluth at R = 1.

Fig. 3 Critical characteristic length R_c/a_i $(=1/\varepsilon a_i)$ as a function of density $(\omega_{ci}^2/\omega_{pi}^2)$ at R=2, $\Delta = 3 \times 10^{-3}$, and E_o/B_o = 0.5% for T_i = 20 keV and 200 keV, respectively.

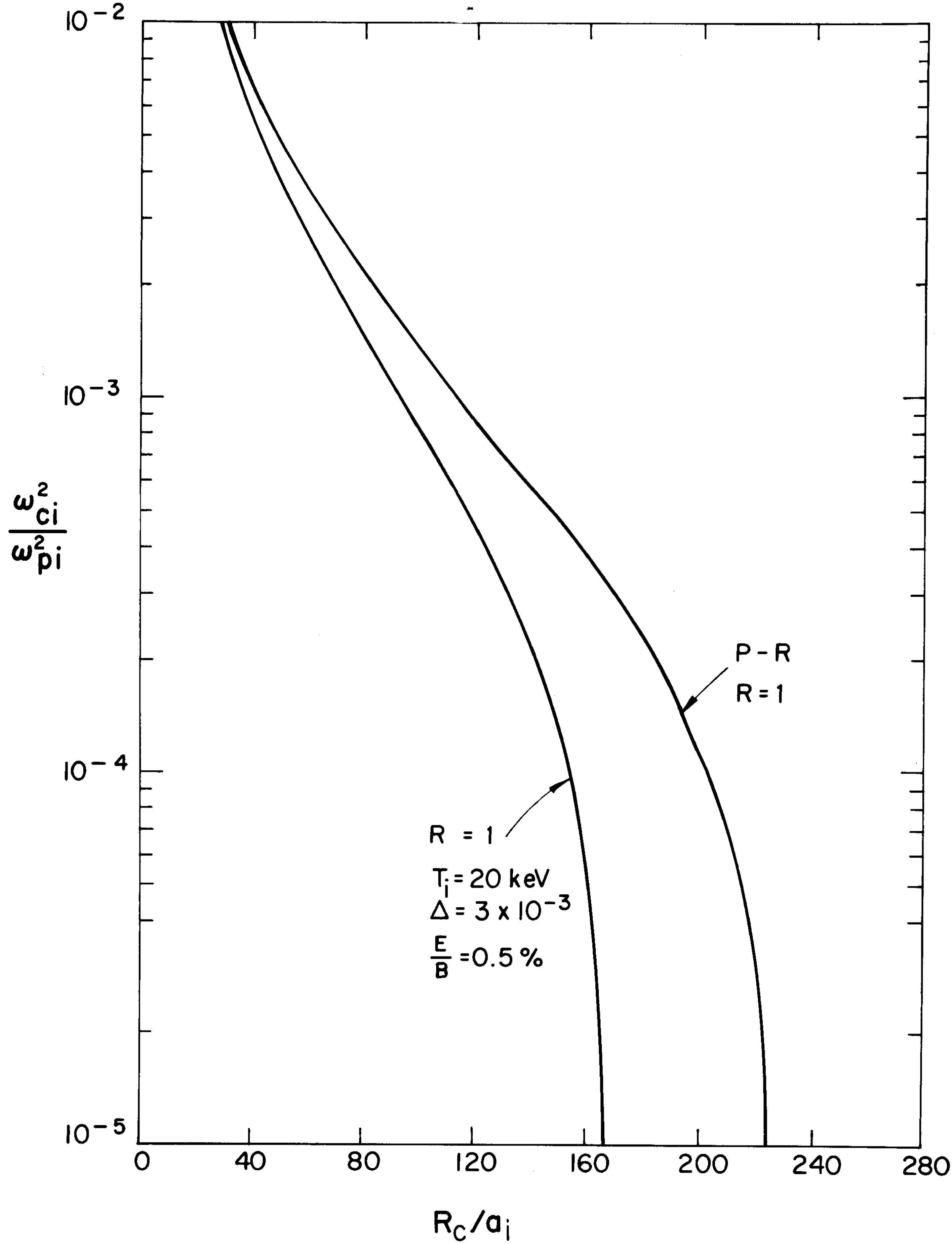
Fig. 4 Factor δ as a function of plasma β at T_i = 10 keV.

Fig. 5 Factor δ as a function of plasma β at T_i = 1 MeV.



Z
$\vec{B}_o = B_o \hat{e}_z$
$\vec{k}$
y
$\frac{dn}{dx}$
$\vec{E} = \vec{E}_o \cos \omega_o t \hat{e}_x$
X

Figure 1



10^-2
10^-3
10^-4
10^-5
ω²ci / ω²pi
P - R
R = 1
R = 1
Ti = 20 keV
Δ = 3 x 10^-3
E/B = 0.5 %
0
40
80
120
160
200
240
280
Rc/ai

Figure 2

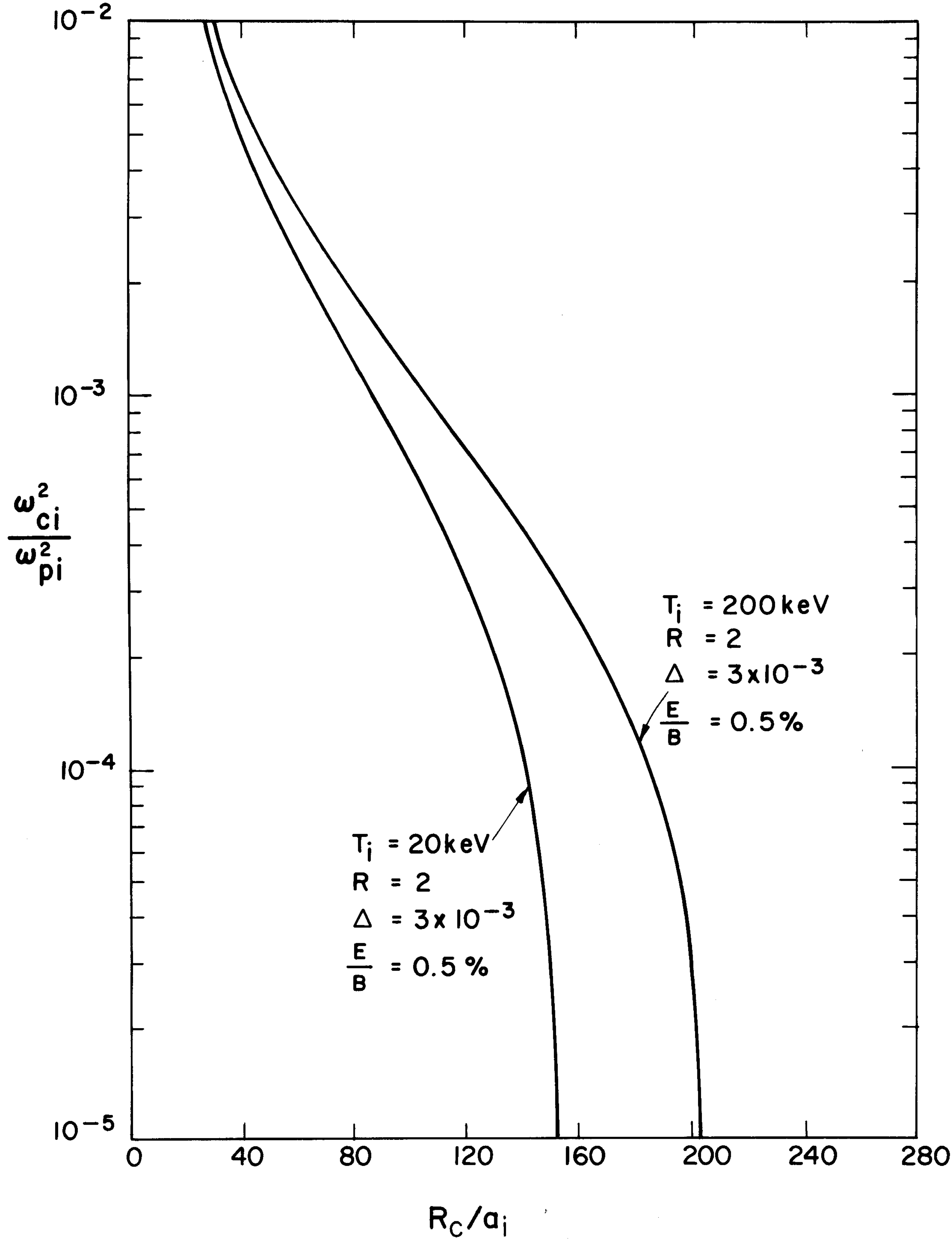
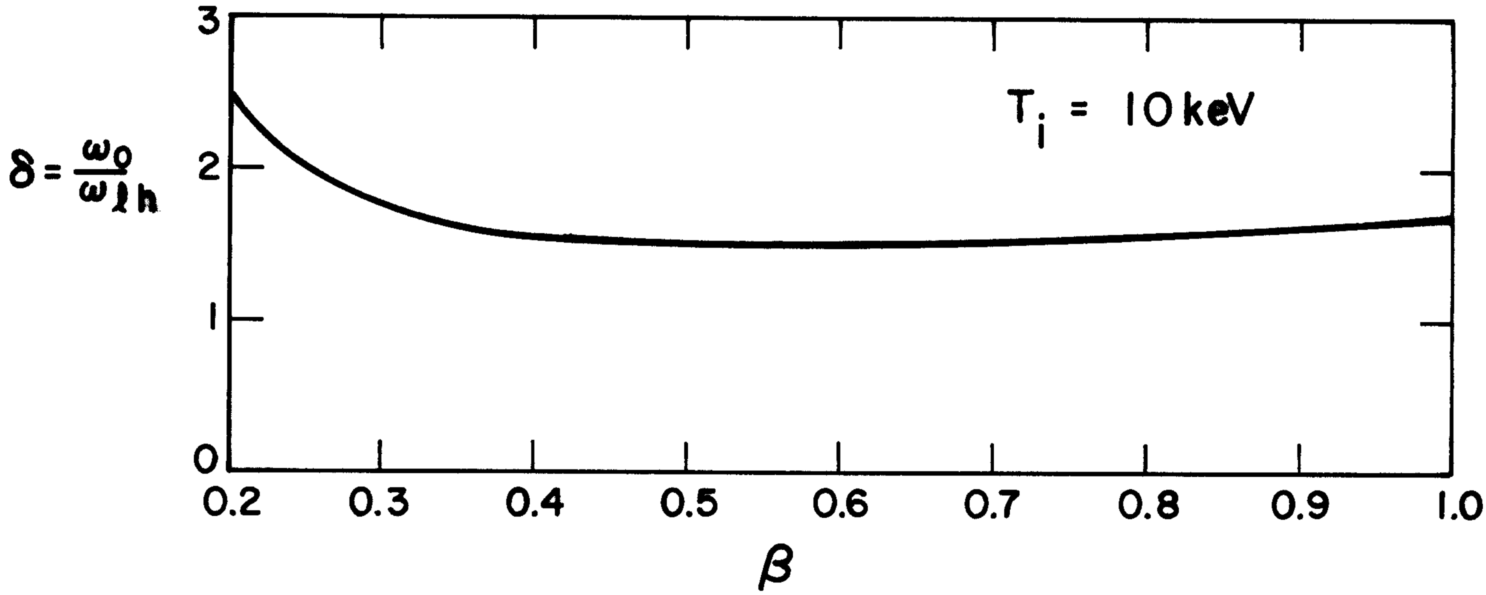

10^{-2}
10^{-3}
10^{-4}
10^{-5}
$\frac{\omega_{ci}^2}{\omega_{pi}^2}$
T_i = 200keV
R = 2
Δ = 3×10^{-3}
$\frac{E}{B}$ = 0.5%
T_i = 20keV
R = 2
Δ = 3×10^{-3}
$\frac{E}{B}$ = 0.5 %
0
40
80
120
160
200
240
280
R_C/a_i

Figure 3



3
2
1
0
$\delta = \frac{\omega_0}{\omega_{\ell h}}$
T_i = 10 keV
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
β

Figure 4

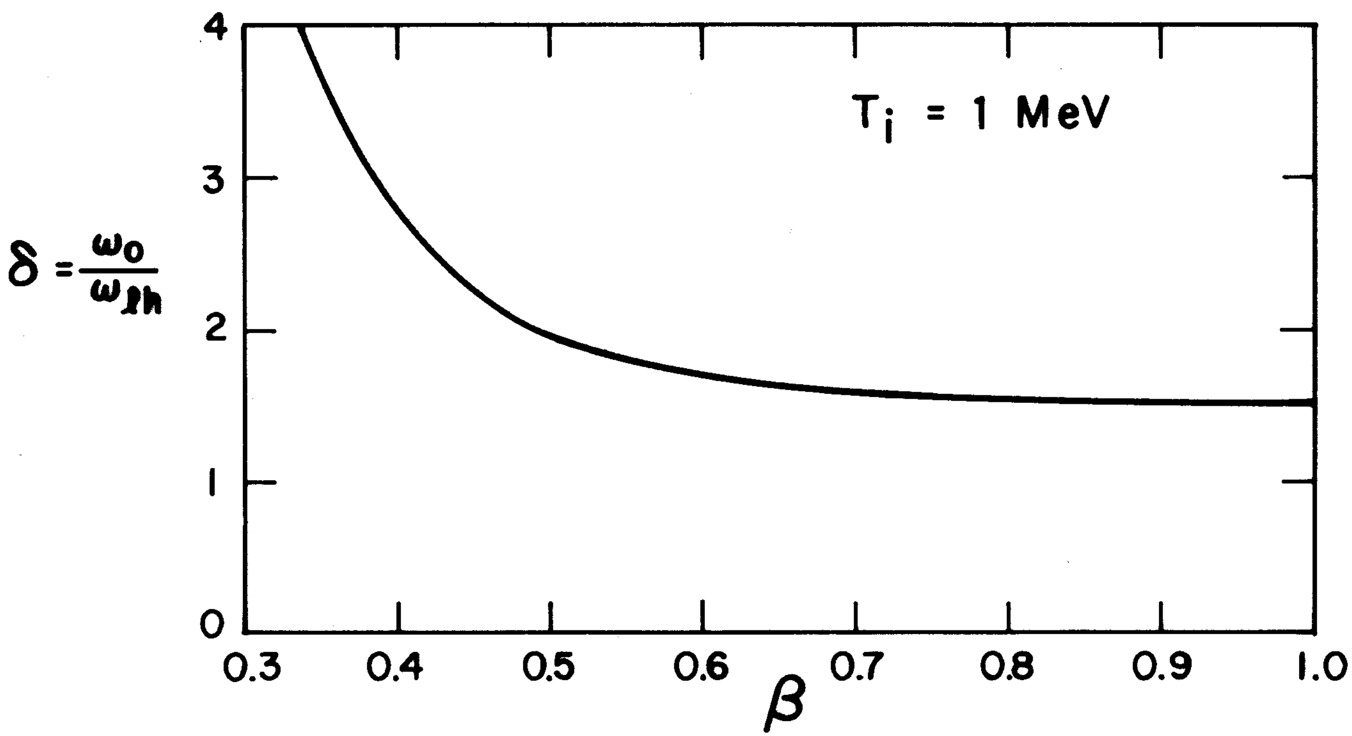


Figure 5