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Abstract

Lorentz ionization of low energy, less than 200 keV, neutral injection beams was studied as they pass through the toroidal magnetic field of a tokamak reactor. An equivalent electric field, $\vec{v}\times\vec{B}$, which these atoms see, was calculated as a function of position along the injection path. By assuming a $1/n^3$ population distribution, the fraction of the beam lost via Lorentz ionization was determined. Neglecting the competing effects of spontaneous decay and inverted cascade, it was found that less than one percent of the beam will be lost through Lorentz ionization.

The requirement of intense magnetic fields in thermonuclear devices forces one to consider the loss, by Lorentz ionization, of particles from the neutral injection beams before they reach the plasma. This ionization is due to the Lorentz force, $e\vec{v} \times \vec{B}$, where e is the electronic charge, $\vec{v}$ is the beam velocity, and $\vec{B}$ is the magnetic field at the point being considered. For our purposes the $\vec{v} \times \vec{B}$ field can be considered as an effective electric field which, if large enough, can ionize highly excited neutral atoms.[1] Any particles ionized in this manner before the beam enters the plasma is considered lost in the divertor field. As an example, a 20keV proton beam moving perpendicular to a uniform 50kG magnetic field sees an equivalent electric field of 0.979×10^5 Volts/cm. This field is sufficient to ionize the ninth and higher excited states of atomic hydrogen. What one actually would like to know is the fraction of the neutral beam which is lost via this mechanism as a function of beam energy. To do this the $\vec{v} \times \vec{B}$ field must be known at each point on the injection path along with the excited state distribution as a function of beam energy. Knowing these quantities, and the field required to ionize the various excited states, the fraction of the beam Lorentz ionized can be found.

First consider the problem of calculating the magnitude of the effective electric field, $|\vec{E}| = |\vec{v} \times \vec{B}|$, assuming a beam of neutral deuterium atoms incident on the toroidal reactor as shown in Figure 1. Injection is tangent to the magnetic axis as shown, with θ the instantaneous angle between the velocity and the magnetic field. r, the distance from the center of the toroid to the point in question, will be the main independent variable. Note that both the magnitude of $\vec{B}$ and the angle between $\vec{B}$ and $\vec{v}$ varies with r, that is as the particle moves along the injection path. The poloidal magnetic field will be neglected and only the effect of the toroidal magnetic field will be considered. The effective electric field can be written as

$$|\vec{E}| = \sqrt{\frac{2E}{m}}\ B \sin\theta \qquad 1)$$

where E is the beam energy, m is the mass of the particles, B is the magnitude of the magnetic field, and θ is defined above. If E is measured in keV, m in amu, and B in kilogauss, this equation becomes

$$|\vec{E}| = 439\sqrt{\frac{E}{m}}\ B \sin\theta \quad \text{Volts/cm} \qquad 2)$$

To find the r dependence of B and sin θ one uses Figure 1. Some important dimensions of the device which will be needed are:

major radius = 12.5 meters

plasma radius = 2.5 meters

first wall radius = 2.5m/.9 = 2.78 meters

blanket and shield thickness = 2.25 meters

Using these dimensions the distance from the center of the toroid, point d in Figure 1, to the outside of the shield region, point a, is 17.53 meters. Defining x as the distance between points b and c in Figure 1, sin θ is

$$\sin\theta = \frac{x}{r} = \frac{\sqrt{r^2 - R^2}}{r} = \sqrt{1-\left(\frac{R}{r}\right)^2} \qquad 3)$$

Combining equations 2 and 3 yields

$$|\vec{E}| = 429\sqrt{\frac{E}{m}}\; B\sqrt{1-\left(\frac{R}{r}\right)^2} \qquad 4)$$

The variation of B with position can be obtained by noting that it is proportional to 1/r as follows

$$RB_o = rB \qquad 5)$$

where B_o is the magnetic field on axis and B is the field at r. Solving this for B and substituting into equation 4 gives

$$|\vec{E}| = 439\sqrt{\frac{E}{m}}\; B_o \frac{R}{r}\sqrt{1-\left(\frac{R}{r}\right)^2} \qquad 6)$$

This equation now specifies the effective electric field, $|\vec{E}|$, as a function of r.

Before solving equation 6 numerically some useful information can be obtained by maximizing $|\vec{E}|$ with respect to the independent variable r. To do this a new variable y, defined as 1/r, is used. Differentiating $|\vec{E}|$ with respect to y and setting the derivative equal to zero gives

$$\frac{\delta|\vec{E}|}{\delta y} = C\sqrt{1-y^2R^2} + 1/2Cy(1-y^2R^2)^{-1/2}(-2yR^2) = 0$$

where C is $439\sqrt{\frac{E}{m}}\, B_oR$. This equation can be simplified to

$$\sqrt{1-y^2R^2} = y^2R^2/\sqrt{1-y^2R^2}$$

the solution of which is $yR = 1/\sqrt{2}$. The result of this simple calculation is that the value of r at which $|\vec{E}|$ is a maximum is

$$r = \sqrt{2}R = 17.59 \text{ meters} \qquad 7)$$

Therefore, for the case of injection tangent to the magnetic axis the maximum $|\vec{E}|$ occurs just outside the shield region. The physical significance of this point of maximum $|\vec{E}|$ is that as the beam enters the device and moves toward point C, the magnitude of B increases while the angle between $\vec{v}$ and $\vec{B}$ decreases. For r large sin θ approaches 1, but B tends to zero making their product go to zero. At point C the value of sin θ is zero yielding $|\vec{E}| = 0$. Somewhere between these two extremes one would expect a maximum, which is the point indicated in equation 7. As this point is approached from outside, the 1/r dependence of B dominates the decreases in sin θ, the net effect being an increase in $|\vec{E}|$. Once the maximum is passed the decrease in sin θ dominates causing $|\vec{E}|$ to decrease. An important point is that the place where the maximum of $|\vec{E}|$ occurs is independent of the beam energy, even though the value of $|\vec{E}|$ at the maximum goes as $\sqrt{E}$. The results of numerical calculations of $|\vec{E}|$ as a function of r are shown in Tables I and II for a D° beam at several energies. Table I is for beam energies of 20keV to 100keV whereas Table II is for energies of 200keV to 1MeV. Z is defined to be

17.53 meters minus the variable x used above. Therefore Z is the distance the beam has penetrated past the edge of the shield region. The values of $|\vec{F}|$ are in units of 10^4 Volts/cm.

Consider now the problem of calculating the fraction of neutrals in each excited state. To do this one can use the theoretical calculations of Butler and May,[2] who calculate the quantity $n^3 I_n/I_i$ for a beam of protons incident on a gas cell of atomic hydrogen. I_i is the proton current incident on the gas cell and I_n is the equivalent current of neutrals particles in the n^{th} excited state exiting from the gas cell (n being the principal quantum number of the state in question). Two problems which arise immediately are the fact that an incident proton beam is used, whereas the case under consideration uses deuterons, and that the gas cell contains atomic hydrogen, while in practice molecular hydrogen is used. As far as the charge exchange process is concerned a 1keV proton is equivalent to a 2keV deuteron since the reaction depends on the relative velocity of the interacting particles. This provides a solution to the first problem. The answer to the second problem is that once the beam energy increases above 10keV differentiation between an atomic gas and a molecular gas is no longer necessary. Therefore the data of Butler and May for atomic hydrogen can be used, in place of molecular hydrogen, for beam energies exceeding 10keV. The higher the beam energy the better this assumption is. Table III shows data from Butler and May for both incident proton and deuteron beams. Figure 2 is a plot of $n^3 I_n/I_i$ versus the incident deuteron energy with the crosses indicating data taken from Butler and May. The circle represent a seventh order polynomial least squares fit of the data. The polynomial used was:

$$n^3 I_n/I_i = C_0+C_1E+C_2E^2+C_3E^3+C_4E^4+C_5E^5+C_6E^6+C_7E^7$$

where $C_0 = -2.9814152$

$C_1 = 2.8000403 \times 10^{-1}$

$C_2 = -7.9591272 \times 10^{-3}$

$C_3 = 1.1208736 \times 10^{-4}$

$C_4 = -8.893974 \times 10^{-7}$

$C_5 = 4.05984598 \times 10^{-9}$

$C_6 = -9.978478 \times 10^{-12}$

$C_7 = 1.0251942 \times 10^{-14}$

E is the beam energy in keV

The fraction of neutral particles produced, which are in the j^{th} and higher excited states, is trivially

$$\text{Fraction} = F_j = \sum_{n=j}^{\infty} I_n/I \qquad 8)$$

where I is the total equivalent neutral current from the charge exchange cell. To get from $n^3 I_n/I_i$, which is given in Figure 2, to I_n/I, which is what is required, the neutralization efficiency is needed. Assuming an equilibrium cell the neutral fraction is

$$F_{o\infty} = I/I_i$$

Combining equations 8 and 9 leads to

$$F_j = \sum_{n=j}^{\infty} \left(\frac{n^3 I_n}{I_i}\right) \frac{1}{F_{o\infty} n^3} \qquad 10)$$

For a given beam energy both $n^3 I_n/I_i$ and $F_{o\infty}$ are constant, by defining the former as f, equation 10 can be cast in the form

$$F_j = \frac{f}{F_{o\infty}} \sum_{n=j}^{\infty} \frac{1}{n^3} \qquad 11)$$

This can be rewritten in a more convenient form as

$$F_j = \frac{f}{F_{o\infty}} \left[\sum_{n=1}^{\infty} \frac{1}{n^3} - \sum_{n=1}^{j} \frac{1}{n^3} \right] \qquad 12)$$

The infinite series is known to be 1.2020569, while the second series can be obtained by summing. Values for $F_{o\infty}$ were obtained from Allison[3] and are plotted in Figure 3 as crosses with the circles representing a sixth order polynomial least squares fit of the crosses. The polynomial used was

$$F_{o\infty} = A_0 + A_1E + A_2E^2 + A_3E^3 + A_4E^4 + A_5E^5 + A_6E^6$$

where $A_0 = 8.71474 \times 10^{-1}$

$A_1 = 4.93177 \times 10^{-3}$

$A_2 = -2.93508 \times 10^{-4}$

$A_3 = 4.46092 \times 10^{-6}$

$A_4 = -3.54641 \times 10^{-8}$

$A_5 = 1.40604 \times 10^{-10}$

$A_6 = -2.16749 \times 10^{-13}$

E is the beam energy in keV

Table IV lists the results of numerical calculations performed using equation 12. Each listing in the table is the percentage of the total neutral beam at the specified energy in the indicated state and all higher states. To get the fraction of the beam in any one excited state requires subtraction of adjacent listings. By using equation 12 to calculate the excited state populations a $1/n^3$ distribution was assumed, n being the principal quantum number of the excited state in question. Such a distribution was used because there is some experimental justification for it,[4] although the exact distribution is not known.

To complete the calculation of the Lorentz ionized fraction of the neutral beam, the electric field necessary to ionize any given excited state must be known. This information can be obtained from Bailey, Hiskes, and Riviere,[5] whose results are shown in Table V. A transition time of 10^{-8} seconds was used. The spread in electric field required for ionization of each energy state is due to the various vibrational and rotational states available to the electron. Values of the fields are in units of 10^4 volts/cm. Combining this table with the results of the effective electric field seen by the beam as it passes through the magnetic field yields information as to the lowest state ionized. The numbers in parentheses opposite the maximum effective electric fields in Tables I and II give the principal quantum number of this state. Knowing this excited state one can find the fraction of the beam ionized using the excited state distribution of Table IV. Table VI then lists the desired results of the Lorentz ionized fraction of the neutral deuterium beam for various energies. Only energies of up to 200 keV have been considered since the charge exchange efficiency becomes unreasonably low, less than 20%, above 200 keV. Assuming the initial ion beam can be made with 50% efficiency, an overall efficiency of less than 10% is found at higher energies. If neutral beams are used for steady state fueling of the reactor, then energies of up to 1 MeV may be required for uniform penetration of the plasma.[6] Such beams cannot be made by the methods used above. Rather one would start with a D^- beam, which can be created at approximately 25% efficiency,[7] accelerate it to the required energies, and then use a gas cell to strip off an electron to make D°. The efficiency of such a stripping process is about 90% at energies of several hundred keV.[8]

In addition to the Lorentz force considered above there are several other effects operating on the beam which change the ionization fractions given in Table VI. First is the fact that as the neutral beam enters the magnetic field some of the electrons may be excited to higher energy states. By such a process electrons initially in bound states too low for Lorentz ionization may be raised to states that can be ionized. A competing effect is that of spontaneous decay which causes excited electrons to drop to lower states. Since the transition time for this decay is on the order of 1μs for the n=9 state and varies as $n^{4.5}$, there can be considerable attenuation of the excited states.[1] For example, the beam may travel up to 10 meters to reach the plasma, and since the velocities are on the order of 5×10^6 meters/sec, the time of flight to the plasma is 2μs. This represents two e-folding times for the n = 9 state. These two competing processes have been neglected in this study, but would have to be considered in an exact calculation. Just the same, compared to the charge exchange and beam formation efficiencies, a Lorentz ionized fraction of several percent is negligible. To increase the Lorentz ionized fraction to several percent, from the calculated tenths of percent, the net effect of the two processes must be equivalent to an order of magnitude increase in the effective electric field. This increase allows ionization of the n = 5 or 6 state depending on the beam energy. Therefore unless the magnetic excitation is a very large effect, the Lorentz ionization mechanism does not seriously affect the beam at energies below 200 keV.

REFERENCES

1. J. R. Hiskes, Nuclear Fusion 2, 38 (1962).
2. S. T. Butler and R. M. May, Phys. Rev. 137A, 10 (1965).
3. S. K. Allison, Revs. Mod. Phys. 30, 1137 (1958).
4. L. D. Stewart, Ph.D. Thesis, University of Wisconsin, Nuclear Engineering Department (1970), p. 155.
5. D. S. Bailey, J. R. Hiskes, and A. C. Riviere, Nuclear Fusion 5, 41 (1965).
6. A. C. Riviere, Nuclear Fusion 11, 363 (1971).
7. A. S. Schlachter, P. J. Bjorkholm, D. H. Loyd, L. W. Anderson, and W. Haeberli, Phys. Rev. 177, 184 (1969).
8. H. K. Forsen, L. D. Stewart, and W. L. Stirling, "Energetic Particle Beams and Fusion Devices," ORNL-TM-3788 (1972).

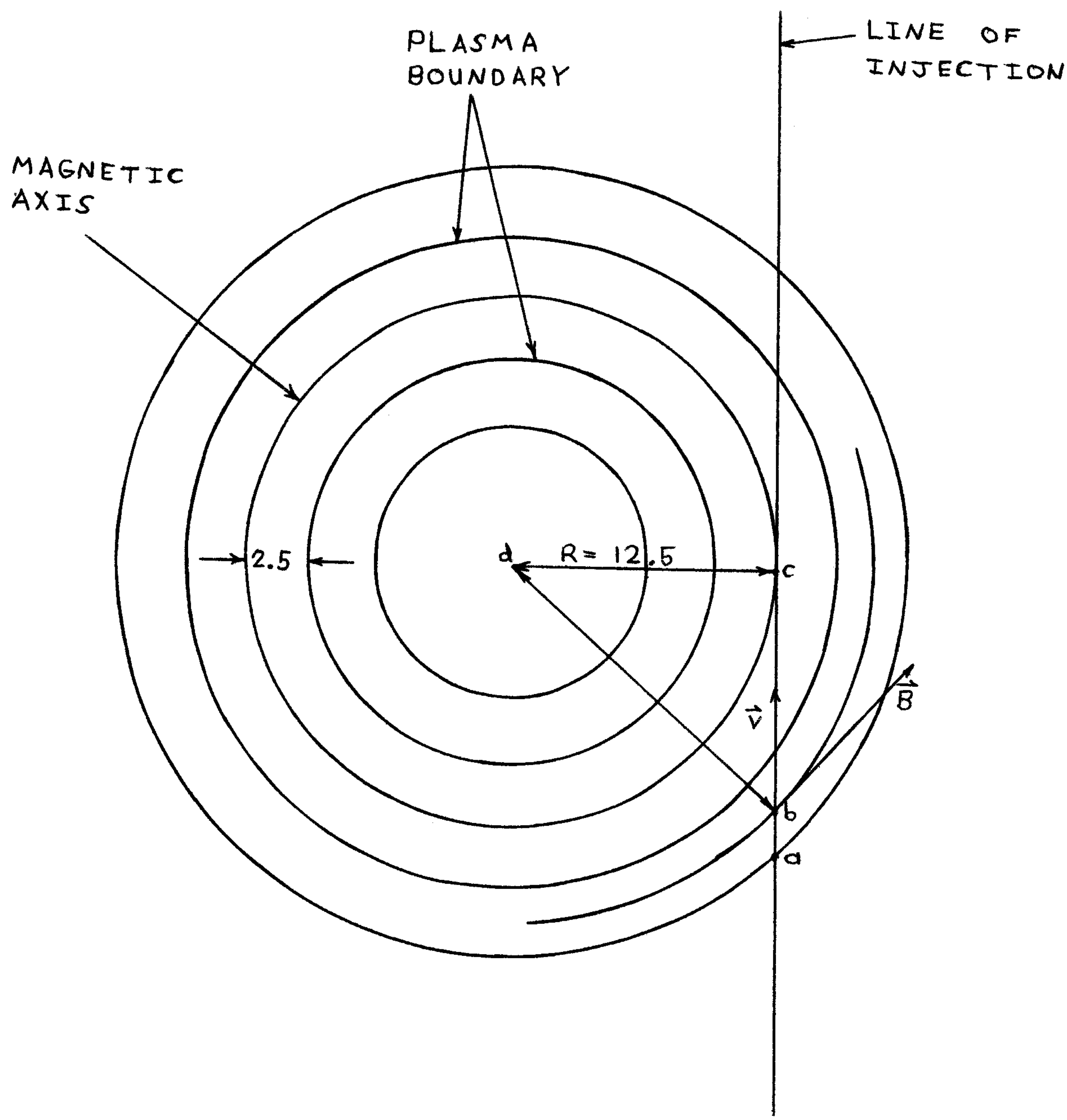
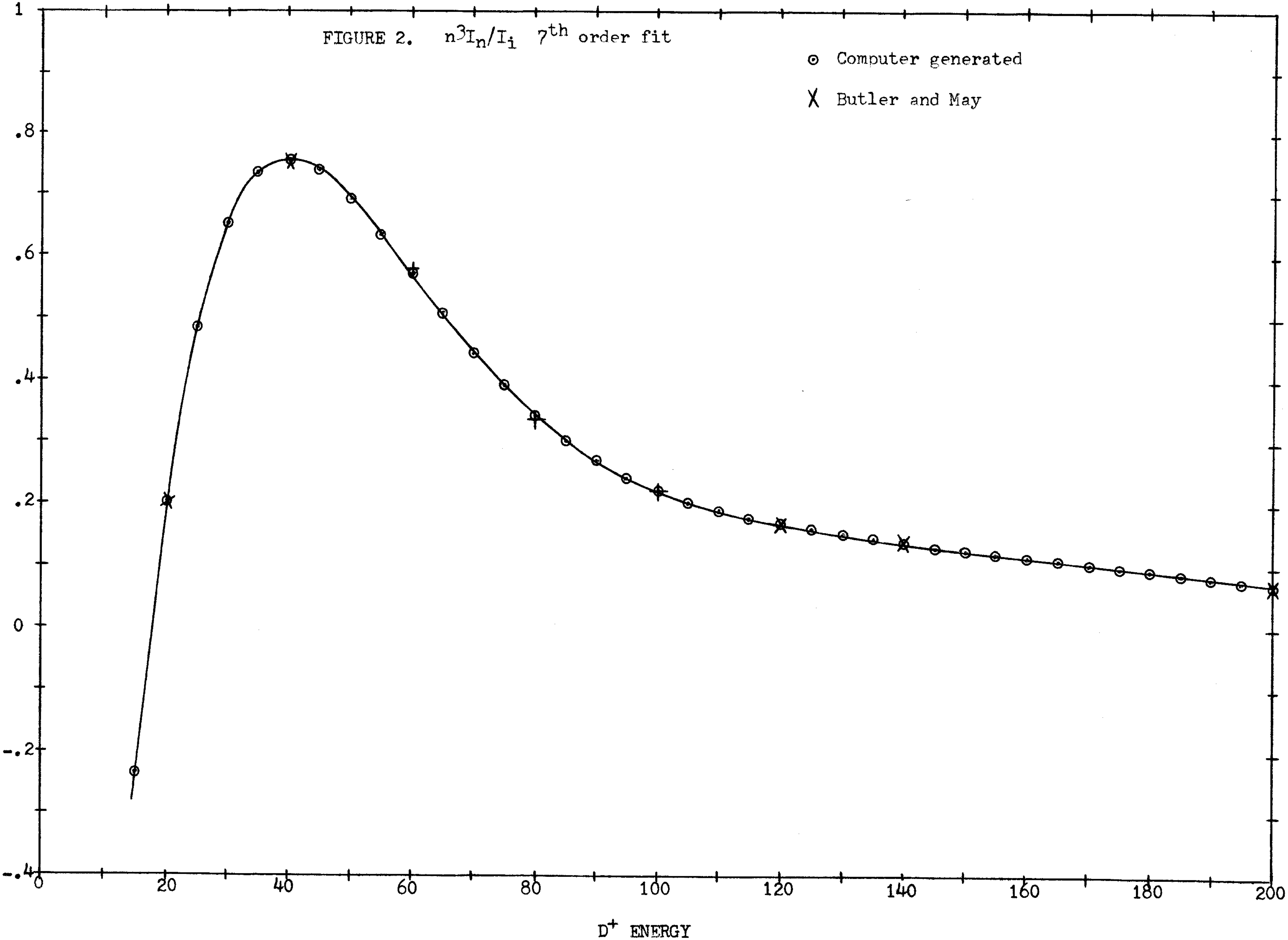

PLASMA
BOUNDARY
LINE OF
INJECTION
MAGNETIC
AXIS
2.5
d
R= 12.5
c
$\vec{v}$
$\vec{B}$
b
a

FIGURE 1

FIGURE 2. $n^3 I_n / I_i$ 7th order fit



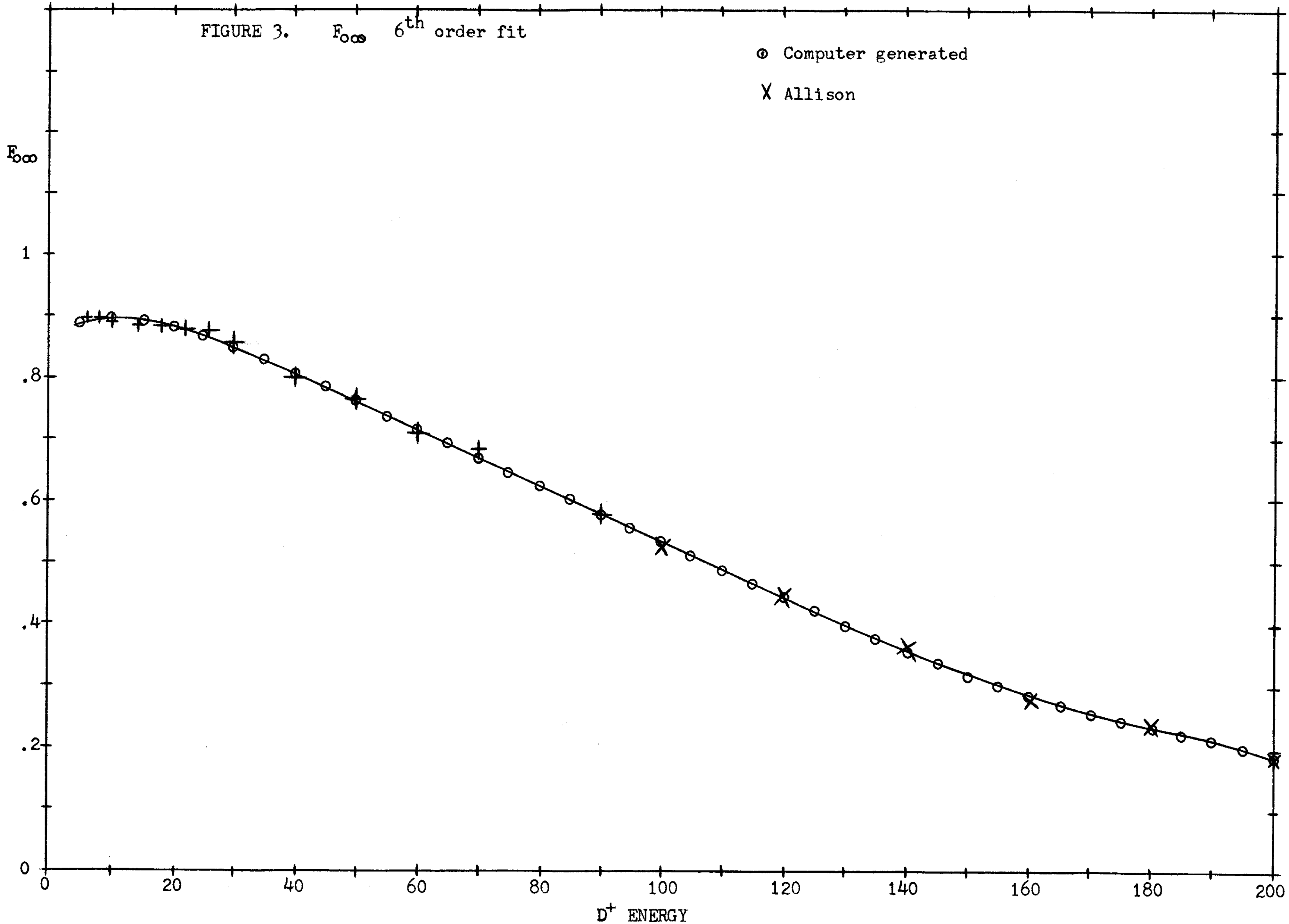


FIGURE 3. $F_{o\infty}$ 6^{th} order fit
⊙ Computer generated
X Allison
$F_{o\infty}$
1
.8
.6
.4
.2
0
0
20
40
60
80
100
120
140
160
180
200
D^+ ENERGY

$|\vec{E}|$ for a D° Beam at Various Energies in Units of 10^4 V/cm

Radius (r)	Z	20keV	40keV	60keV	80keV	100keV
12.5m	12.29m	0 V/cm	0 V/cm	0 V/cm	0 V/cm	0 V/cm
13	8.72	1.87	2.65	3.25	3.75	4.19
13.5	7.19	2.48	3.51	4.30	4.96	5.55
14	5.99	2.85	4.04	4.94	5.71	6.38
14.5	4.94	3.10	4.39	5.37	6.20	6.93
15	4.00	3.27	4.62	5.66	6.54	7.31
15.5	3.13	3.38	4.79	5.86	6.77	7.57
16	2.30	3.46	4.90	6.00	6.92	7.74
16.5	1.52	3.51	4.96	6.08	7.02	7.85
17	0.77	3.54	5.00	6.13	7.07	7.91
17.5	0.04	3.55 (11)	5.02 (10)	6.15 (10)	7.10 (9)	7.93 (9)
18	-0.66	3.55	5.02	6.14	7.09	7.93
18.5	-1.35	3.54	5.00	6.12	7.07	7.91
19	-2.02	3.52	4.97	6.09	7.03	7.86
19.5	-2.68	3.49	4.94	6.05	6.98	7.81
20	-3.32	3.46	4.90	6.00	6.93	7.74

TABLE I

$|\vec{E}|$ for a D° Beam at Various Energies in Units of 10^4 V/cm

Radius (r)	Z	200keV	400keV	600keV	800keV	1MeV
12.5m	12.29m	0 V/cm	0 V/cm	0 V/cm	0 V/cm	0 V/cm
13	8.72	5.93	8.38	10.3	11.9	13.3
13.5	7.19	7.85	11.1	13.6	15.7	17.6
14	5.99	9.02	12.8	15.6	18.1	20.2
14.5	4.94	9.01	13.9	17.0	19.6	21.9
15	4.00	10.3	14.6	18.0	20.7	23.1
15.5	3.13	10.7	15.1	18.5	21.4	23.9
16	2.30	10.9	15.5	19.0	21.9	24.5
16.5	1.52	11.1	15.7	19.2	22.2	24.8
17	0.77	11.2	15.8	19.4	22.4	25.0
17.5	0.04	11.2 (8)	15.9 (8)	19.4 (7)	22.4 (7)	25.1 (7)
18	-0.66	11.2	15.9	19.4	22.4	25.1
18.5	-1.35	11.2	15.8	19.4	22.4	25.0
19	-2.02	11.1	15.7	19.3	22.2	24.9
19.5	-2.68	11.0	15.6	19.1	22.1	24.7
20	-3.32	11.0	15.5	19.0	21.9	24.5

TABLE II

$n^3 I_n / I_i$ for Various Beams and Energies

H+	D+	n^3In/Ii
10keV	20keV	.20
20	40	.75
30	60	.58
40	80	.34
50	100	.22
60	120	.17
70	140	.14
100	200	.07

TABLE III

Excited State Populations in Percentages

Beam Energy (keV)	5	6	7	8	9	10	11
20	.557	.375	.269	.202	.158	.126	.103
30	1.874	1.259	.904	.680	.530	.424	.348
40	2.288	1.538	1.104	.830	.647	.518	.424
50	2.225	1.495	1.731	.807	.629	.504	.413
60	1.949	1.310	.940	.707	.551	.441	.361
70	1.623	1.091	.783	.589	.459	.368	.301
80	1.339	.900	.646	.486	.379	.303	.248
90	1.134	.762	.547	.411	.320	.257	.210
100	1.008	.678	.468	.366	.285	.228	.187
200	.937	.630	.452	.340	.265	.212	.174

TABLE IV

Principal Quantum Number of Excited State	Threshold Electric Field (10^5 V/cm)
5	6.5-9.0
6	3.4-5.4
7	2.0-3.4
8	1.2-2.1
9	.7-1.4
10	.49-.89
11	.36-.61

TABLE V

Beam Energy (keV)	Lowest State Ionized	Percentage of Beam Ionized
20	11	.103%
40	10	.518%
60	10	.441%
80	9	.379%
100	9	.285%
200	8	.340%
400	8	
600	7	
800	7	
1000	7	

TABLE VI