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THE APPLICATION OF UNCERTAINTY ANALYSIS IN CONCEPTUAL FUSION REACTOR DESIGN

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ABSTRACT

We briefly describe the theories of sensitivity and uncertainty analysis and then apply them to a new conceptual tokamak fusion reactor design - NUWMAK. NUWMAK is a moderately sized tokamak fusion reactor of medium toroidal field, high power density and high neutron wall loading and is designed to minimize thermal cycling and provide internal thermal energy storage by use of the eutectic $\text{Li}_{62}\text{Pb}_{38}$ compound both as the breeding and energy storage material. Neutronics calculations are performed in P₃S₄ by DOT and ANISN.

The responses investigated in this study include the tritium breeding ratio, first wall Ti dpa and gas productions, nuclear heating in the blanket, energy leakage to the magnet, and the dpa rate in the superconducting magnet aluminum stabilizer. The sensitivities and uncertainties of these responses are calculated. The cost/benefit feature of proposed integral measurements is also studied through the uncertainty reductions of these responses.

I. INTRODUCTION

The theories of sensitivity and uncertainty analysis have been developed¹⁻¹² in the past decade to deal with the problem of estimation of nuclear response uncertainties, nuclear data adjustments, etc., which then provide vital information for nuclear reactor design. Most of the applications have been in the field of fast breeder reactors. In this study, the methodology of sensitivity and uncertainty analysis will be applied to the field of fusion reactor design.

Basically, sensitivity analysis is developed to answer questions of the relationship between the changes of a design quantity and of the basic data field. The sensitivity coefficient, mathematically the first derivative of the response with respect to the basic cross sections, can be calculated and used quantitatively for indicating the rate of change of a reactor performance parameter with respect to changes in the cross sections. It also points out what cross section data, as a function of nuclide, reaction type, and energy, are important in analyzing a given reactor system. Moreover, it connects the uncertainty of a given design

quantity and the uncertainties of the basic data field through a simple mathematical expression.

Nuclear reactor design calculations have always been limited in their accuracies by both computational methods and uncertainties in nuclear data. After a period of extensive refinements in calculational methods and computer codes, the main uncertainty has been believed to be in the basic nuclear data. The calculation of the uncertainty of a design quantity is made possible by implementing the statistical error propagation model in connection with sensitivity analysis. For the case where the calculated accuracy of certain design quantities are unsatisfactory for meeting the design criteria, one must improve the basic data considerably. The cross section adjustment methodology, by which one utilizes information from integral measurements by fitting the calculated integral quantity to the experimental results, is used as a tool to improve our current knowledge of the basic data. Here, we also consider the differential measurement as a special integral experiment without confusion.

Sometimes our best knowledge about the basic data field, which may include the information from available integral experiments, still leads to an unsatisfactory result for the design performance parameters of concern. Therefore a new measurement should be proposed. In this study the benefit of the proposed measurement is evaluated through the error reduction of the responses. The cost to benefit ratios of the proposed measurements are then obtainable if we know the cost of the experiment as a function of experiment errors. The cost to benefit ratio should serve as the indicator of how well the measurement has to be done for an optimal design. It also gives a reference for the funding agency if several measurements are proposed.

In the following section, the theories of sensitivity and uncertainty analysis are reviewed and a procedure for analyzing the economical aspects of integral experiments is given. Section III gives the calculational results and analyses for a tokamak fusion reactor design as an application of the theories. The conclusions are given in Section IV.

II. THEORY

II-1. Sensitivity Methodology for a Fixed Sources System

In the neutronic and photonic calculations of fusion reactor blanket and shield ~~design~~, we have always encountered transport problems with fixed sources. The main objective in such problems is to calculate a certain result which is a flux-integrated quantity or more simply a response. By using the conventional and convenient operator notation, the forward and adjoint fluxes, $\phi(\xi)$ and $\phi^*(\xi)$, can be calculated from the forward and adjoint Boltzmann equations:

$$L\phi = S \quad (1)$$

and

$$L^\dagger \phi^* = S^*, \quad (2)$$

where L and L^\dagger are the forward and adjoint transport operators, S is the physical source distribution, and S^* is the functional derivative of the result of interest.

In this study we will concentrate on the responses which are linear functionals of the flux $\phi(\xi)$. By using the notation $(,)$ to indicate integration over the phase space, we can define a response, R , by the following expression:

$$R = (\Sigma_R, \phi), \quad (3)$$

where Σ_R is the response function associated with the result of interest.

The variation of the response, δR , due to the variations of the transport operator and of the response function itself, can be easily derived from the well-developed generalized perturbation theory^{1,2} or variational principles^{6,7} in connection with the forward and adjoint fluxes. That is,

$$\delta R = (\delta \Sigma_R, \phi) - (\phi^*, \delta L \phi). \quad (4)$$

The first term on the right-hand side of Eq. (4) could be called the direct effect (or detector term) which measures the effect of the response function itself to the response and is relatively easy to calculate. In contrast, the second term, called the indirect effect (or flux perturbation term) which affects the response through the transport operator, is relatively difficult to calculate due to the complexity of the transport operator and requires some computer programming efforts.

Let us concentrate on a set of multigroup nuclear data $\{\Sigma_i\}$ which is contained in L_i , where L_i is a subset of the transport operator L . The linearity of the transport operator enables us to evaluate the contribution of L_i to the response R and therefore the variation of the response due to the variation of L_i can be expressed as

$$\delta R_i = (\delta \Sigma_R, \phi) - (\phi^*, \delta L_i \phi). \quad (5)$$

From Eq. (5) we can obtain the sensitivity coefficient of R with respect to Σ_i which is defined as

$$S_i = \partial R / \partial \Sigma_i \quad (6)$$

and the relative sensitivity,

$$P_i = (\partial R / R) / (\partial \Sigma_i / \Sigma_i). \quad (7)$$

If the group cross sections are concerned, then, for group i , we have

$$P_i(E) = \left\{ - \iiint d^3r d\Omega \Sigma_i(\underline{r}, E) \phi(\underline{r}, E, \Omega) \phi^*(\underline{r}, E, \Omega) \right. \\ \left. + \iiint d^3r d\Omega d\Omega' dE \phi^*(\underline{r}, E, \Omega) \Sigma_i^S(\underline{r}, E' \rightarrow E, \Omega' \rightarrow \Omega) \phi(\underline{r}, E', \Omega') \right\} / R \quad (8)$$

The first term on the right hand side of Eq. (8) represents the collision term which removes the particle from that point in phase space and the second term is the collective gain term for the particles emerging from such collision at other energies and angles.

The sensitivity profile, a graphical display of $P_i(E)$, can be presented for a series of spatial zones as a function of energy. By looking at these graphs one can immediately determine how sensitive the response is to a particular cross section set in a particular energy range. Therefore, the sensitivities are useful in determining which cross section uncertainties are important in a given system and should serve as a quantitative guide to nuclear data measurements and evaluations.

II-2. Uncertainty Analysis

Assuming that the response R can be related to a set of cross section data $\{\Sigma_i\}$ by a symbolic expression $R = R(\Sigma_i)$ which, in general, is a non-linear function of Σ_i , we linearize the function R and use the definition of the sensitivity coefficients S_i to get the following expression:

$$R - E(R) = \sum_i S_i (\Sigma_i - E(\Sigma_i)), \quad (9)$$

where E denotes the expectation value. Furthermore, we let $D = \{d_{ij}\}$ be the covariance or dispersion matrix for the cross section set $\{\Sigma_i\}$. That is,

$$d_{ij} = E((\Sigma_i - E(\Sigma_i))(\Sigma_j - E(\Sigma_j))). \quad (10)$$

Following the general derivation of statistical error propagation, the variance of R , $V(R)$, can be obtained as

$$V(R) = \sum_{i,j} S_i S_j d_{ij}. \quad (11)$$

The uncertainty or standard deviation of R defined as $\Delta R = \sqrt{V(R)}$, has the following matrix expression:

$$(\Delta R)^2 = SDS^t, \quad (12)$$

where, $S = (S_1, S_2, \dots, S_k)$ is a row matrix representing the sensitivities of R with respect to a set of cross sections. S^t is the transpose of S , and k is the total number of cross sections of interest. A common

practice in the uncertainty calculations is that the relative sensitivity coefficients are provided from the sensitivity analysis. Therefore a modified mathematical form is implemented, that is,

$$(\Delta R/R)^2 = \hat{P} \hat{D} P^t, \quad (13)$$

where P is the relative sensitivity matrix, and $\hat{D} = \{\hat{d}_{ij}\}$ the relative covariance matrix of the cross section defined as

$$\hat{d}_{ij} = d_{ij} / \Sigma_i \Sigma_j. \quad (14)$$

Eq. (12) mathematically connects the uncertainty of the response and the uncertainties of the cross sections through the associated sensitivity coefficients. It also clearly demonstrates that both the highly sensitive and uncertain cross sections are most responsible for the uncertainty of the response and therefore gives the designer a direction to focus on if the accuracy criteria for some design parameters are not satisfied.

II-3. Statistical Inference in Cross Section Adjustment

Cross section adjustment technique by utilizing the available information from integral experiments to improve our knowledge about the basic data set has been developed extensively 3-5,9-12 over the past decade. Sometimes no statistical justification or physical basis is given for the adjustment procedure or the usefulness of the method is limited by some strict assumptions. A general method based on the Bayesian inference in statistical analysis proposed by Dragt¹² will be outlined here.

Basically the concept of probability distribution could be used to describe our degree of belief that a certain parameter (e.g., cross section) has a certain value or distribution, the so-called prior distribution. After an experiment is carried out, the posterior knowledge about the parameter contained in the posterior distribution should be improved by drawing inferences from the experimental data. As a result, the uncertainty of the basic nuclear data decreases and so does the uncertainty of the design parameter of interest.

The consistent method, proposed by Gandini³, is basically a maximum likelihood estimation (least square method) of the integral quantities and basic differential data constrained by their functional relationship. The approach of this method - sampling theory in statistical analysis - ends up with the same result as the Bayesian approach when the probability distributions are assumed normal. However, Bayesian method is easier to use when dealing with non-normal distributions. It provides a satisfactory way of explicitly introducing and keeping track of assumptions about prior knowledge or ignorance. For instance, after a study of residuals had suggested model (or assumption) inadequacy, it might be desirable to reanalyze the data in relation to a less restrictive model into which the initial model was embedded. If

non-normality was suspected, for example, it might be sensible to postulate that the sample came from a wider class of parent distributions of which the normal was a member. The consequential analysis could be difficult via sampling theory but is readily accomplished in a Bayesian framework¹⁴.

Let us consider a nuclear data field (mostly group cross sections) combined into a column vector Σ of order m . Let us also assume that the prior distribution of Σ is an m -dimensional multi-variate normal distribution with prior mean $\bar{\Sigma}$ and prior covariance matrix D , i.e., $p_0(\Sigma) \sim N_m(\bar{\Sigma}, D)$. The assumption of normality, a common practice to simplify the analysis procedure in scientific investigation, is not absolutely necessary which will be discussed later. Suppose we are interested in n integral quantities, denoted as a column matrix Y , which can be expressed as a function of Σ , i.e., $Y = Y(\Sigma)$. Let \bar{Y} be the calculated values of Y . From sensitivity theory, we have

$$\bar{Y}(\Sigma) = \bar{Y}(\bar{\Sigma}) + S(\Sigma - \bar{\Sigma}), \quad (15)$$

where, $S = \{s_{ij}\}$ is the sensitivity matrix and s_{ij} represents the sensitivity coefficient of the i th integral quantity with respect to the j th cross section.

An important function which plays a very significant role in Bayesian inferences is the likelihood function $L(\Sigma|Y^{ex})$, where Y^{ex} represents the experimental result for Y . It is "the" function through which the experimental data Y^{ex} modifies prior knowledge of Σ ; it can therefore be regarded as representing the information about Σ coming from the data. The likelihood function is formally identical with the conditional probability function of Y^{ex} , given Σ , i.e., $p(Y^{ex}|\Sigma)$. The only difference between these two functions is that in $p(Y^{ex}|\Sigma)$ Y^{ex} represents a vector of variables and Σ a vector of constant parameters, while in $L(\Sigma|Y^{ex})$ Y^{ex} represents a vector of fixed values and Σ a vector of parameters of which optimal estimators are to be determined.

The Bayes' theorem states that

$$p_n(\Sigma|Y^{ex}) = \frac{p_0(\Sigma)L(\Sigma|Y^{ex})}{\int p_0(\Sigma)L(\Sigma|Y^{ex})d\Sigma} \quad (16)$$

where $p_n(\Sigma|Y^{ex})$, or simply $p_n(\Sigma)$ without confusion, is the posterior distribution of the vector Σ after n observations Y^{ex} have been obtained. Again, we assume that the likelihood function is normally distributed with covariance E of order n and mean $Y(\Sigma)$, that is, $L(\Sigma|Y^{ex}) \sim N_n(Y(\Sigma), E)$. Then we have two mathematical expressions:

$$L(\Sigma|Y^{ex}) = \text{const} \cdot \exp\{-1/2[Y^{ex} - \bar{Y}(\Sigma)]^t E^{-1}[Y^{ex} - \bar{Y}(\Sigma)]\} \quad (17)$$

and

$$p_0(\Sigma) = \text{const} \cdot \exp\{-1/2(\Sigma - \bar{\Sigma})^t D^{-1}(\Sigma - \bar{\Sigma})\} \quad (18)$$

By using Eq. (15), we get

$$\begin{aligned} y^{\text{ex}} - \bar{Y}(\Sigma) &= y^{\text{ex}} - \bar{Y}(\bar{\Sigma}) - S(\Sigma - \bar{\Sigma}) \\ &= V - S(\Sigma - \bar{\Sigma}) \end{aligned} \quad (19)$$

where

$$V = y^{\text{ex}} - \bar{Y}(\bar{\Sigma}). \quad (20)$$

It is clear that V represents the differences between the experimental values of Y and the calculated values associated with the prior mean $\bar{\Sigma}$.

The posterior distribution of Σ can be derived by combining Eq. (16), (17), (18) and (19).

$$p_n(\Sigma) = \text{const} \cdot \exp\{-1/2(\Sigma - \bar{\Sigma}')^t D'^{-1}(\Sigma - \bar{\Sigma}')\} \quad (21)$$

where

$$\bar{\Sigma}' = \bar{\Sigma} + (S^t E^{-1} S + D^{-1})^{-1} S^t E^{-1} V \quad (22)$$

$$D'^{-1} = D^{-1} + S^t E^{-1} S \quad (23)$$

Therefore, the posterior distribution of Σ is also normally distributed with posterior mean $\bar{\Sigma}'$ and covariance matrix D' defined by Eq. (22) and (23).

Usually the number of differential data m is much larger than the number of integral quantities n . The matrix D with order $m \times m$ has to be inverted in both Eq. (22) and (23). That would pose a potential problem if m gets too large. A reduction of matrix order is therefore required for practical applications. The final result without showing the tedious matrix manipulations is given as follows:

$$\bar{\Sigma}' - \bar{\Sigma} = D S^t (E + S D S^t)^{-1} V \quad (24)$$

$$D' = D - D S^t (E + S D S^t)^{-1} S D \quad (25)$$

Equations (24) and (25) are the basic formulae employed as the computer algorithm in the cross section adjustment procedure. Notice that the matrix to be inverted, i.e., $E + S D S^t$, is a square matrix of order n which is much smaller than the order m of matrix D .

The characteristic of sequential inferences in Baye's theory provides the possibility of a further matrix reduction in the procedure for cross section adjustments. If Y consists of k statistically independent subsets, Y_1, Y_2, \dots, Y_k , the adjustment can be performed in k steps. First, a new nuclear data set with covariance matrix is obtained by an adjustment by Y_1 . Then this new set is used as the prior data set for an adjustment on the basis of Y_2 . This procedure continues until the final set $\bar{\Sigma}'$ and covariance matrix D' are obtained with the final adjustment on the basis of Y_k . In the extreme case, if

all experiments are uncorrelated, the calculation can be done without any matrix inversion at all.

The assumptions of normality for the prior distribution and likelihood function eventually lead to the result that the posterior distribution is also normally distributed with the posterior mean and covariance matrix defined in Eq. (24) and (25) respectively. If one has a strong feeling that either the prior distribution or the likelihood function should not be normally distributed, the adjustment procedure could be carried out by using the real probability density function for Eq. (16). In that case, the posterior mean and covariance could still be calculated by implementing a computer program and using the relatively complicated posterior probability density function.

II-4. Economical Aspect of Integral Experiments

In practical nuclear reactor design we always want to know the mean values and the accuracies of certain design quantities. It is not the accuracy of the differential data that really concerns us. If the accuracy of a certain response is not able to satisfy our design criteria after using our best knowledge about the differential data, some experimental measurements may have to be carried out in order to reach the design goal. Here we discuss a procedure to preanalyze the economic values of these proposed integral experiments and to provide a quantitative basis for the choice of the proposed experiments.

Following the derivation from the previous section, let us assume that the current estimation (which may include information from the available integral experiments) of a set of differential nuclear data Σ is $\bar{\Sigma}$ with covariance matrix D , and the integral parameter y of the proposed integral experiment has an experimental uncertainty e and the sensitivity vector S with respect to Σ . The adjusted data $\bar{\Sigma}'$ and covariance matrix D' can be obtained from Eq. (24) and (25) or

$$\bar{\Sigma}' - \bar{\Sigma} = DS^t(e^2 + SDS^t)^{-1}v \quad (26)$$

$$D' = D - DS^t(e^2 + SDS^t)^{-1}SD \quad (27)$$

Suppose that R is the design quantity we are interested in and S_R is a row matrix representing the sensitivity of R with respect to Σ . From Eq. (12) we can get the prior and posterior variance of R , i.e.,

$$\begin{aligned} (\Delta R)^2 &= S_R D S_R^t \\ (\Delta R')^2 &= S_R D' S_R^t \\ &= S_R D S_R^t - S_R D S^t (e^2 + SDS^t)^{-1} S D S_R^t \end{aligned} \quad (28)$$

Since $e^2 + SDS^t$ is just a number, Eq. (28) can be rearranged:

$$(\Delta R)^2 - (\Delta R')^2 = \frac{(S_R DS^t)^2}{e^2 + SDS^t} \quad (29)$$

Eq. (29) gives a simple analytical dependence of the error reduction on the accuracy of the new integral experiment in the most general case. It is clear that the more accurate the new integral experiment, the larger the error reduction. In the extreme cases, when e approaches infinity, Eq. (29) is zero, i.e., we cannot get any information from the experiment, which is expected. On the other hand, we expect to get the maximum information which can be extracted from the new experiment when we have the limit of an infinitely small measuring error. The maximum extractable information, or the maximum variance reduction in R , is called the information content¹⁰ (IC) for the given experiment and the response R and can be defined as

$$IC = \frac{(S_R DS^t)^2}{SDS^t} \quad (30)$$

Half of the IC can be gained when $e^2 = SDS^t \equiv e_H^2$. In general, e_H gives a quantitative value of how accurate the experiment should be to obtain some value from it.

Eq. (30) allows one to compare the quantitative error reduction in R (or a set of R) in a nuclear reactor design when several integral measurements are proposed. However, it does not include some very important information, namely, what the experiment will cost and how much the error reduction is worth. A more objective comparison of different experiments therefore requires a full economic analysis of the benefits resulting from the error reduction.

To simplify the problem, we assume that the cost of electricity from a power plant is a function of a set of nuclear responses R_r and therefore a function of errors in R_r , ΔR_r . That is,

$$C = C(\Delta R_r). \quad (31)$$

For a set of changes $\delta(\Delta R_i)$, the cost changes by

$$\delta C = \sum_r \frac{\partial C}{\partial (\Delta R_r)} \delta(\Delta R_r), \quad (32)$$

where $\partial C / \partial (\Delta R_r)$ can be called the cost sensitivity coefficient with respect to the error in the response R_r . The benefit for the integral experiment through error reduction is then

$$C_B = \delta C = \sum \frac{\partial C}{\partial (\Delta R_r)} \delta(\Delta R_r)$$

$$\begin{aligned}
 &= \sum_r \frac{\partial C}{\partial (\Delta R_r)} \sqrt{\Delta R_r^2 - \Delta R_r'^2} \\
 &= \sum_r \frac{\partial C}{\partial (\Delta R_r)} \frac{S_{R_r} DS^t}{(e^t + SDS^t)^{1/2}} \quad (33)
 \end{aligned}$$

The benefit sensitivity vector with respect to the basic nuclear data, B, can be defined to further simplify the problem:

$$B = \sum \frac{\partial C}{\partial (\Delta R_r)} S_{R_r} \quad (34)$$

By assuming C_I to be the cost of the integral experiment we can get a mathematical expression for the cost-to-benefit ratio (CTBR).

$$CTBR = \frac{C_I}{C_B} = \frac{C_I (e^2 + SDS^t)^{1/2}}{BDS^t} \quad (35)$$

The calculated value of CTBR should provide a quantitative basis for comparing different experiments. Usually one should choose the one with the smallest value of CTBR. Since C_I is also a function of e (C_I decreases when e increases) and the sensitivity S is influenced by the materials, geometry, compositions, etc., of the experiment, an optimal value of CTBR could be obtained by properly choosing the experimental error and experimental setups. An optimal integral experiment is therefore attainable through the pre-analysis of the cost-to-benefit ratio even before the experimental facility is built.

III. APPLICATION OF UNCERTAINTY ANALYSIS TO NUWMAK DESIGN

III-1. Neutronics Analysis of NUWMAK

A new conceptual design of a tokamak reactor, called NUWMAK¹⁵, has the characteristics of medium field, high power density, high degree of modularity, and moderate size. This design is especially attractive from the viewpoints of system accessibility, low levels of long term radioactivity and minimum penetrations. The power density (10 MW/m²) and electrical power output (660 MW) are chosen as typical of a full scale reactor operating in a base-loaded mode. The TF coil set is unique in that just eight superconducting coils are used. A set of 16 small water cooled copper trim coils that do not encircle the vacuum chamber correct the field ripple to below 2%. The blanket is designed to minimize thermal cycling, to provide internal energy storage, and to eliminate the need for an intermediate heat exchanger. A lithium-

lead eutectic, $\text{Li}_6\text{Pb}_{38}$, with a melting point of 464°C is used as the tritium breeding and thermal energy storage material. The titanium alloy, Ti-6Al-4V, is used as structural material to maintain a low level of long term radioactivity. Table 1 gives the major features of the NUWMAK design.

<u>Power</u>	
Total Thermal Power	2097 MW _t
Net Electric Power	660 MW _e
<u>Plasma</u>	
Major Radius	5.13 m
Minor Radius	1.13 m
Plasma Height to Width Ratio (b/a)	1.64
Plasma Current	7.2 MA
Toroidal Beta	6%
$n_e T_E$	$2 \times 10^{14} \text{ cm}^{-3}\text{-sec}$
$q(a)$	2.64
<u>Magnet</u>	
On-Axis Toroidal Field	6.05 Tesla
Toroidal Field at NbTi Conductor	11.5 Tesla
Stabilizer	Aluminum
Number of Toroidal Field Coils	8
Number of Cu Trim Coils	16
<u>Blanket</u>	
Structural Material	Titanium Alloy
Coolant	Boiling Water
Breeding Material	$\text{Li}_6\text{Pb}_{38}$
Average Neutron Wall Loading	4.34 MW/m^2

A schematic of the blanket and shield design is shown in Fig. 1. The neutronics analysis makes use of the two-dimensional discrete ordinate transport code DOT¹⁶. The calculation was performed with the P3S4 approximation in r-z geometry. The nuclear cross section library is a 25 neutron and 21 gamma coupled ANISN-formatted library processed from DLC-41B/VITAMIN-c by AMPX¹⁷ modules MALOCS and NITAWL.

The neutronics results¹⁸ are summarized in Table 2. In NUWMAK, the breeding ratio associated with the outer blanket alone is 1.24. However, the Li-Pb zone on the inside has been retained so as to maintain minimum thermal cycling of the structure. The total tritium breeding ratio is 1.54, of which 90% is contributed by ${}^6\text{Li}(n,\alpha)\text{T}$ reactions. This is due to the large $\text{Pb}(n,2n)$ and $\text{W}(n,2n)$ reaction rates, ~ 0.57 per source neutron and the sufficient amount of ${}^6\text{Li}$ in natural

SCHEMATIC OF THE BLANKET AND SHIELD
FOR NUWMAK

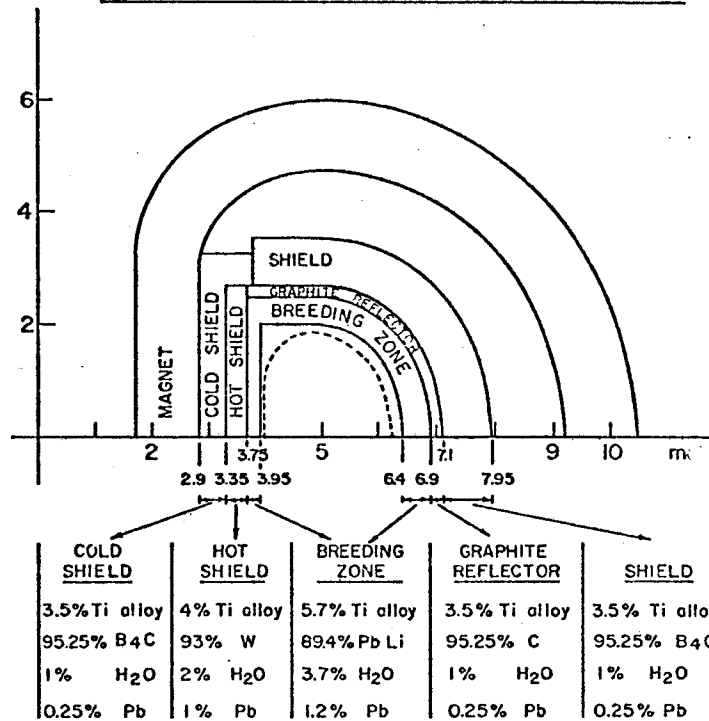


Figure 1

lithium within the lithium lead eutectic. However, the tritium breeding ratio is not very sensitive to the enrichment of ${}^6\text{Li}$ in the total lithium inventory. It shows a maximum value of 1.64 at ~30% ${}^6\text{Li}$ and a value of 1.51 at 90% ${}^6\text{Li}$ enrichment. In the inner shield design, the most restrictive criterion is found to be the resistivity change of the Al stabilizer. The resistivity of the Al at 1.8°K is estimated to increase by 15%/year due to the Al atomic displacement rate (2×10^{-6} dpa/year) and will necessitate periodic annealing approximately every two years. As for the superinsulation, it is found that the dose to mylar would exceed dose limits before plant life. An epoxy based superinsulation whose dose limit is $1 - 5 \times 10^9$ rads is therefore selected. At a dose rate of 3×10^7 rad/yr, the epoxy should last the plant life.

III-2. Sensitivity Analysis

The one-dimensional sensitivity code SWANLAKE has been used to calculate the sensitivity coefficients for the NUWMAK blanket and shield. Forward and adjoint fluxes are calculated by using the one-dimensional discrete ordinate transport code ANISN²⁰ with the P₃S₄ approximation. Cylindrical geometries based on the minor radius are assumed for both inner and outer blanket/shield to simulate the compact-

Table 2
Summary of NUWMAK Neutronics Calculations

<u>Tritium Production</u>	<u>Inner</u>	<u>Outer</u>	<u>Total</u>
${}^6\text{Li}(n,\alpha)\text{T}$	0.2604	1.1249	1.3853
${}^7\text{Li}(n,n'\alpha)\text{T}$	0.0375	0.1192	0.1567
TOTAL	0.2979	1.2441	1.5420
<u>Neutron Multiplication</u>	<u>Inner</u>	<u>Outer</u>	<u>Total</u>
$\text{Pb}(n,2n)$	0.1339	0.4181	0.5520
$\text{W}(n,2n)$	0.0137	----	0.0137
TOTAL	0.1476	0.4181	0.5657
<u>Nuclear Heating (MeV/D-T Neutron)</u>	<u>Inner</u>	<u>Outer</u>	<u>Total</u>
Neutron	2.2665	8.9298	11.1963
Gamma-Ray	2.4143	3.5422	5.9565
TOTAL	4.6808	12.4720	17.1528
Maximum Atomic Displacement Rate in the Aluminum Stabilizer (dpa/year)			2×10^{-6}
Maximum Dose Rate in the Epoxy Based Superinsulators (rad/year)			3×10^7
Maximum Neutron Flux in the NbTi Superconductors ($\text{n}/\text{cm}^2/\text{year}$)			7×10^{15}
Total Nuclear Heating in TF Coils (Watts)			-500

ness of the system. The one-dimensional fluxes are then normalized to the result from DOT and used for the sensitivity analysis. No gamma-ray transport calculations are performed in order to reduce the computing cost. Partial transfer matrices are processed from DLC-41B/VITAMIN-C by AMPX module NITAWL.

In this study we have chosen six key quantities for our sensitivity analysis, namely, the tritium breeding ratio, nuclear heating, first wall dpa and gas production rates, energy leakage to the inner magnet, and dpa rate in the Al stabilizer. A tritium breeding ratio of greater than 1.1 is required as a common practice, otherwise its only impact on the reactor design is through the tritium recovery system. The nuclear heating in the blanket will directly influence the electric power output and the heat transfer design, therefore one needs a more accurate value for this quantity. The first wall lifetime will be strongly affected by the dpa and gas production rates in the first wall

material which could be the determining factor for the choices of the first wall material and wall loading. Since NUWMAK is designed to be compact there is only a 1.05 m thick region for the entire inner blanket and shield, which leads to the use of tungsten as the hot shield material. The responses of dpa rate in Al stabilizer and energy leakage to the inner magnet therefore serve as an indication of the effectiveness of the inner shield design.

Table 3 shows the energy-integrated sensitivities of the above-mentioned six responses to various cross sections of the constituents of the NUWMAK blanket and shield. Caution must be exercised on interpreting these results. The values of Table 3 come directly from SWAN-

Cross Section	R ₁	R ₂	R ₃	R ₄	R ₅	R ₆
Ti total	-0.050	-0.079	-0.016	-0.039	-0.349	-0.334
Pb total	0.137	-0.051	0.120	-0.003	-1.598	-1.238
¹² C total	0.048	-0.024	---	---	-0.633	-0.595
¹⁰ B total	---	-1.795	---	---	-2.106	-2.094
⁶ Li total	-0.865	-0.652	-0.011	---	-0.025	-0.024
⁷ Li total	-0.011	-0.105	---	0.013	-0.319	-0.303
W total					-5.214	-5.282
Pb(n,2n)	0.115	0.006	0.064	---	-1.078	-0.712
inel. level	0.002	-0.025	-0.024	---	-0.146	-0.154
inel. cont.	-0.022	-0.031	-0.002	---	-0.181	-0.127
elastic	0.040	0.001	0.085	---	-0.181	-0.231
⁶ Li(n,α)T	-0.864	-0.644	-0.011	---	-0.001	-0.001
⁷ Li(n,n'α)T	-0.036	-0.047	---	-0.007	-0.132	-0.129
elastic	0.023	-0.042	---	0.018	-0.131	-0.119
Ti(n,2n)	0.011	---	0.004	-0.025	-0.097	-0.084
inel. cont.	-0.028	-0.037	-0.017	-0.028	-0.157	-0.152
elastic	0.007	---	0.014	0.018	-0.052	-0.054
W(n,2n)					-4.123	-4.030
inel. cont.					-0.474	-0.622
inel. level					-0.126	-0.143
elastic					-0.451	-0.449

LAKE calculations which only represent the indirect effect (or flux perturbation term) and do not include the variation of the response function itself (direct effect or detector term). As an example, the

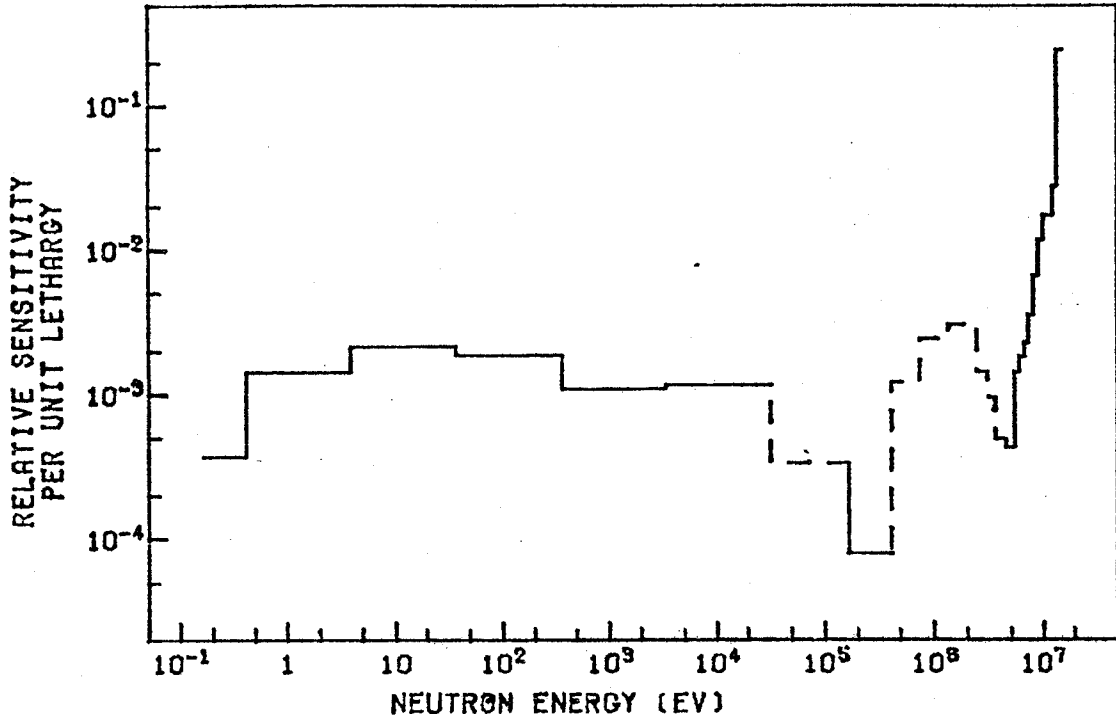
breeding ratio for the outer blanket has **the largest negative sensitivity** with respect to ${}^6\text{Li}$ total cross section, i.e., -0.865 . From the sensitivity theory we would expect a 0.865% decrease in the breeding ratio if the ${}^6\text{Li}$ density increases 1% . This is misleading. Since ${}^6\text{Li}(n,\alpha)\text{T}$ reaction contributes 90% to the total tritium production, the relative sensitivity from direct effect is 0.9 and the net effect is $0.9 - 0.865 = 0.035$. That is, as the tritium breeding decreases 0.865% from the flux perturbation, it also gains 0.9% from the increases of ${}^6\text{Li}(n,\alpha)\text{T}$ macroscopic cross sections. The same trend also happens to the sensitivity of neutron heating with respect to the ${}^6\text{Li}$ total cross section. It has a value of -0.652 from indirect effect and 0.685 from the direct effect and therefore only $0.685 - 0.652$ or 0.033 for the net result.

The characteristic of the breeding ratio being insensitive to most of the materials in the blanket can be explained by the fact that a 50 cm thick breeding zone is more than enough and the tritium production saturates. A result from a variational interpolation²¹ study also shows¹⁸ that the total breeding ratio is about 1.51 at 90% ${}^6\text{Li}$, 1.64 at $\sim 30\%$ ${}^6\text{Li}$ which is a maximum, and 1.54 for natural lithium. Incidentally, the sensitivity of neutron heating, with the fact that the ${}^6\text{Li}(n,\alpha)\text{T}$ reaction contributes most of the neutron heating, has roughly the same behavior as the sensitivity of the breeding ratio, excepting the contribution of boron-10.

Neither the gas production nor dpa rates in the Ti first wall have sensitivities high enough to warrant further investigation. In contrast, the responses in the inner magnet far from the plasma are relatively sensitive to the neutron transport media. The integral sensitivities of the dpa rate in the Al stabilizer are -5.282 , -2.094 , -1.238 , -0.595 and 0.334 to the total cross sections of tungsten, ${}^{10}\text{B}$, lead, graphite and titanium, respectively. Further investigation of the partial cross section sensitivity analysis shows that the $(n,2n)$ cross sections of the tungsten and lead are the dominant reactions. The neutron energy leakage to the inner magnet also shows the same tendency. As the energy dependencies of the sensitivities are concerned, the highest energy group has absolute dominance for these two responses.

From Fig. 2 to Fig. 7, we present some typical graphs, the sensitivity profiles, for a combination of various responses and cross section types. The solid lines represent negative sensitivities, i.e., the response increases as the cross section decreases, and the dashed lines represent positive sensitivities, i.e., the response increases as the cross section increases. Notice that only the flux perturbation terms are presented here. By visualizing these profiles one can have a clear idea for a particular response, in what energy range a particular cross section type would have the greatest impact. Information from the sensitivity profiles would further simplify the procedure in uncertainty analysis by allowing the neglect of the cross section types and energy ranges which have negligible sensitivities.

NUWMAK OUTER BLANKET/SHIELD
RESPONSE -- BREEDING RATIO
CROSS SECTION -- TITANIUM TOTAL



NUWMAK OUTER BLANKET/SHIELD
RESPONSE -- BREEDING RATIO
CROSS SECTION -- LEAD TOTAL

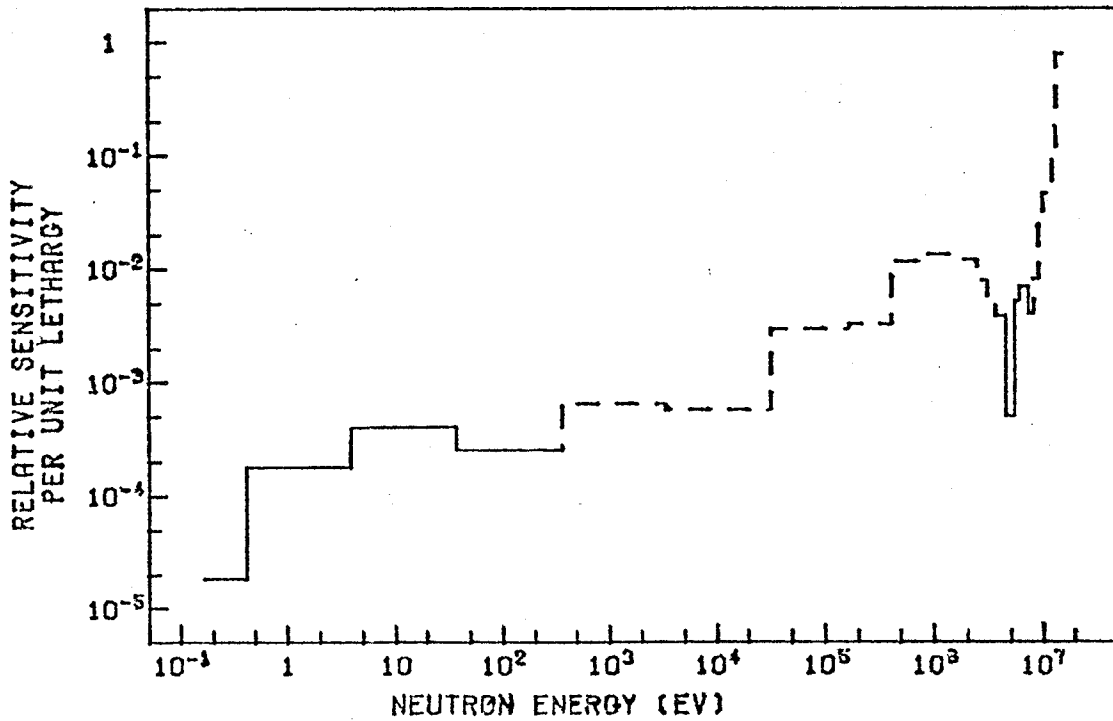
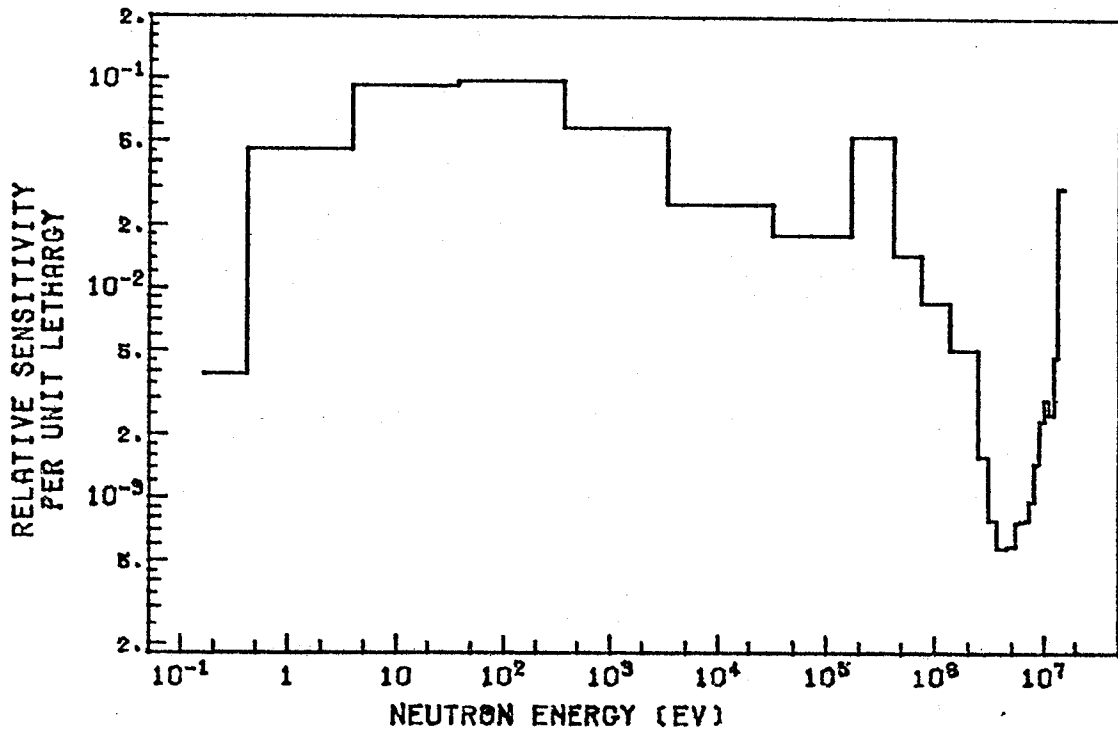


Fig. 2. Sensitivity Profiles of NUWMAK Outer Breeding Ratio to Ti and Pb Total Cross Sections.

NUWMAK OUTER BLANKET/SHIELD
 RESPONSE -- BREEDING RATIO
 CROSS SECTION -- LITHIUM-6 TOTAL



NUWMAK OUTER BLANKET/SHIELD
 RESPONSE -- BREEDING RATIO
 CROSS SECTION -- LITHIUM-7 TOTAL

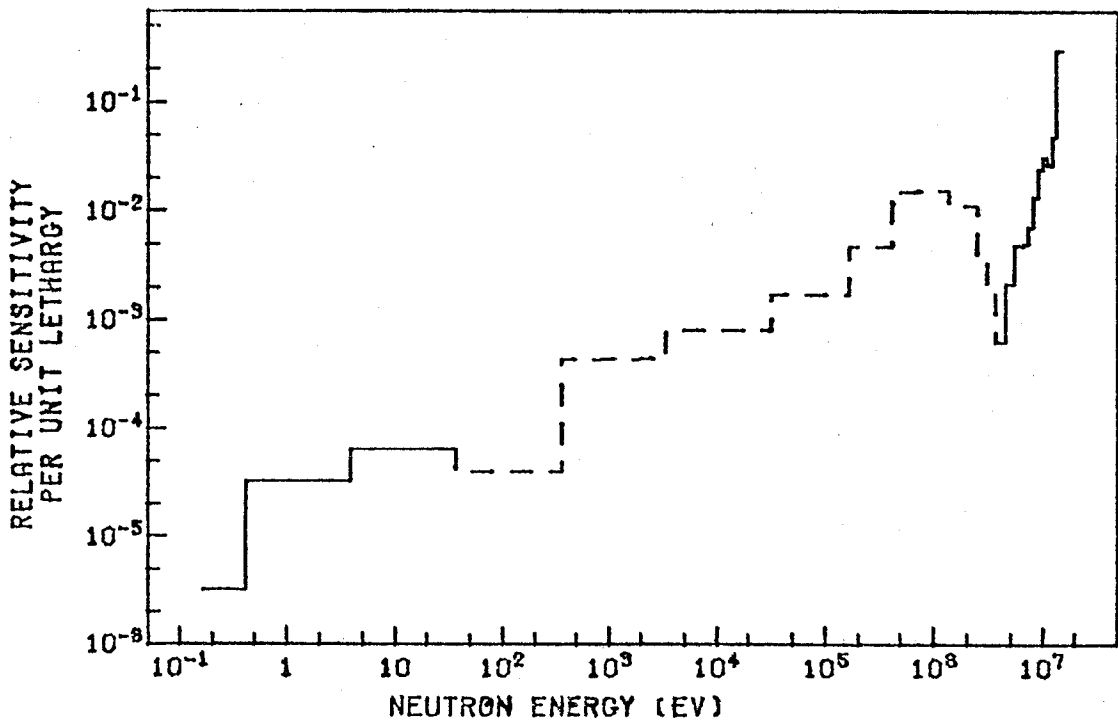
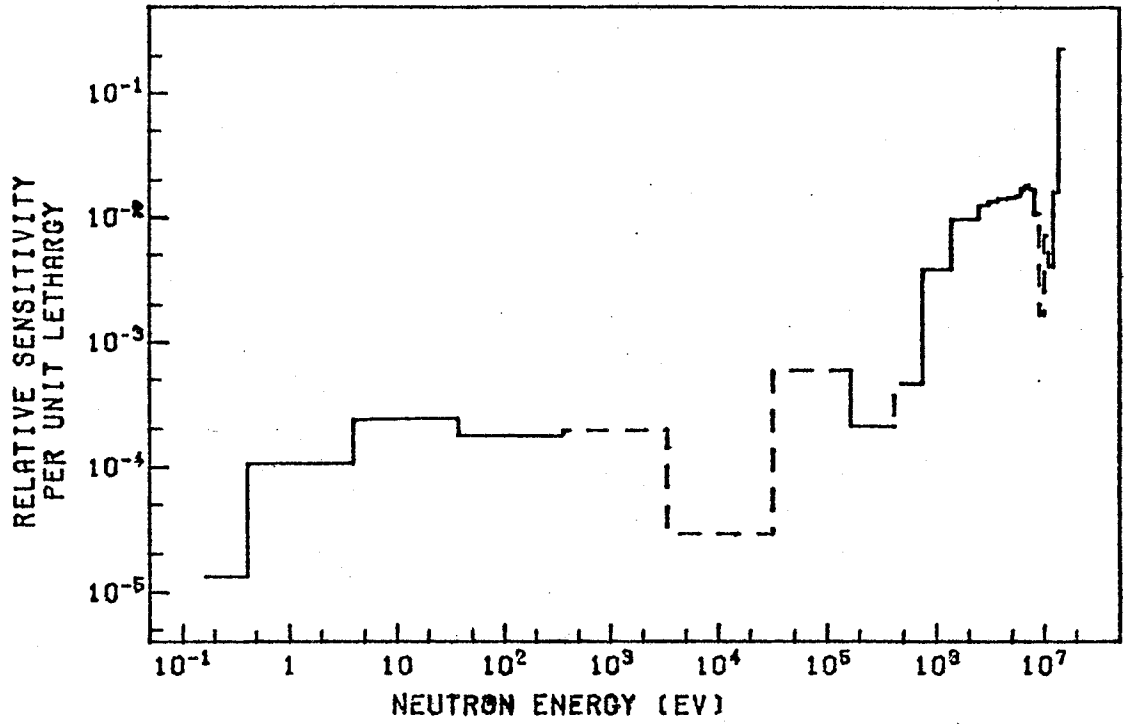


Fig. 3. Sensitivity Profiles of NUWMAK Outer Breeding Ratio to ^6Li and ^7Li Total Cross Sections.

NUWMAK OUTER BLANKET/SHIELD
 RESPONSE -- NEUTRON HEATING
 CROSS SECTION -- LEAD TOTAL



NUWMAK OUTER BLANKET/SHIELD
 RESPONSE -- NEUTRON HEATING
 CROSS SECTION -- GRAPHITE TOTAL

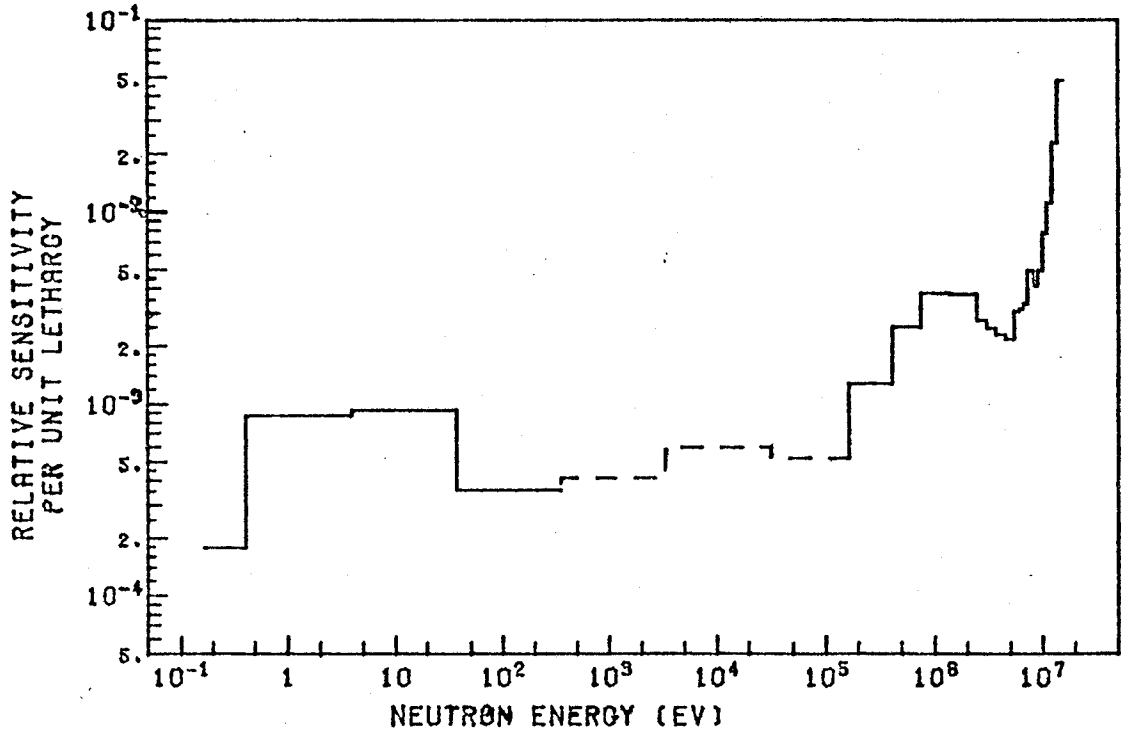
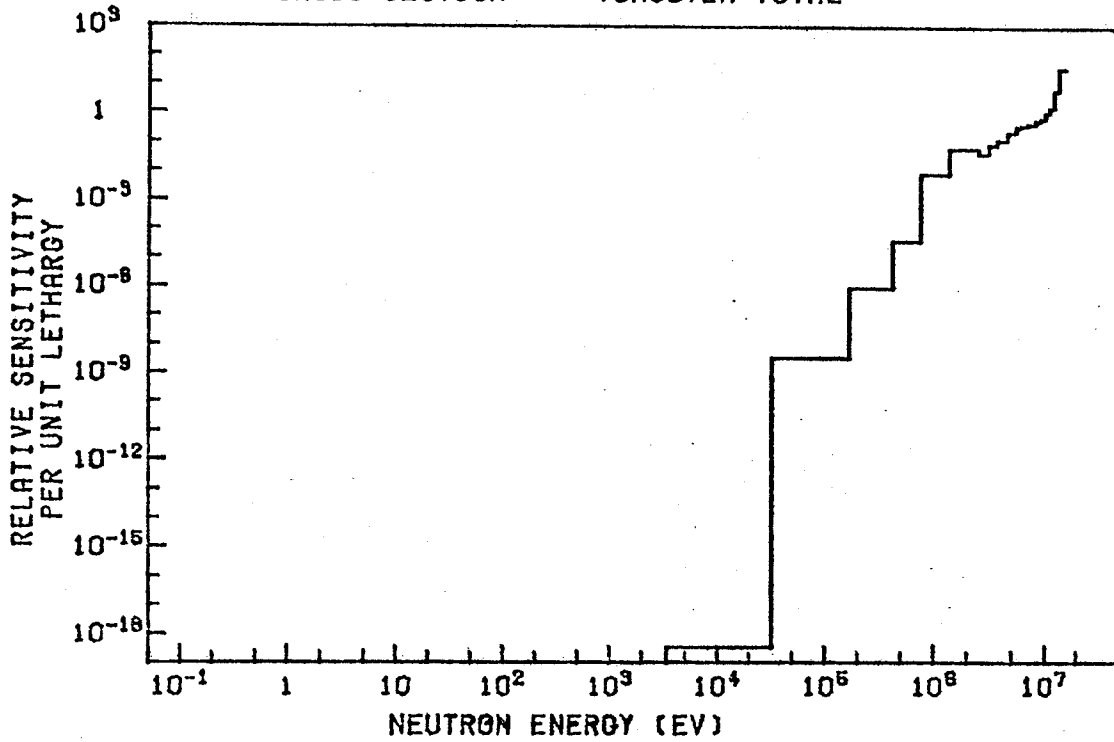


Fig. 4. Sensitivity Profiles of NUKMAK Outer Neutron Heating to Pb and ^{12}C Total Cross Sections.

NUWMAK INNER BLANKET/SHIELD
 RESPONSE -- ALUMINUM DPA IN MAGNET
 CROSS SECTION -- TUNGSTEN TOTAL



NUWMAK INNER BLANKET/SHIELD
 RESPONSE -- ALUMINUM DPA IN MAGNET
 CROSS SECTION -- TITANIUM TOTAL

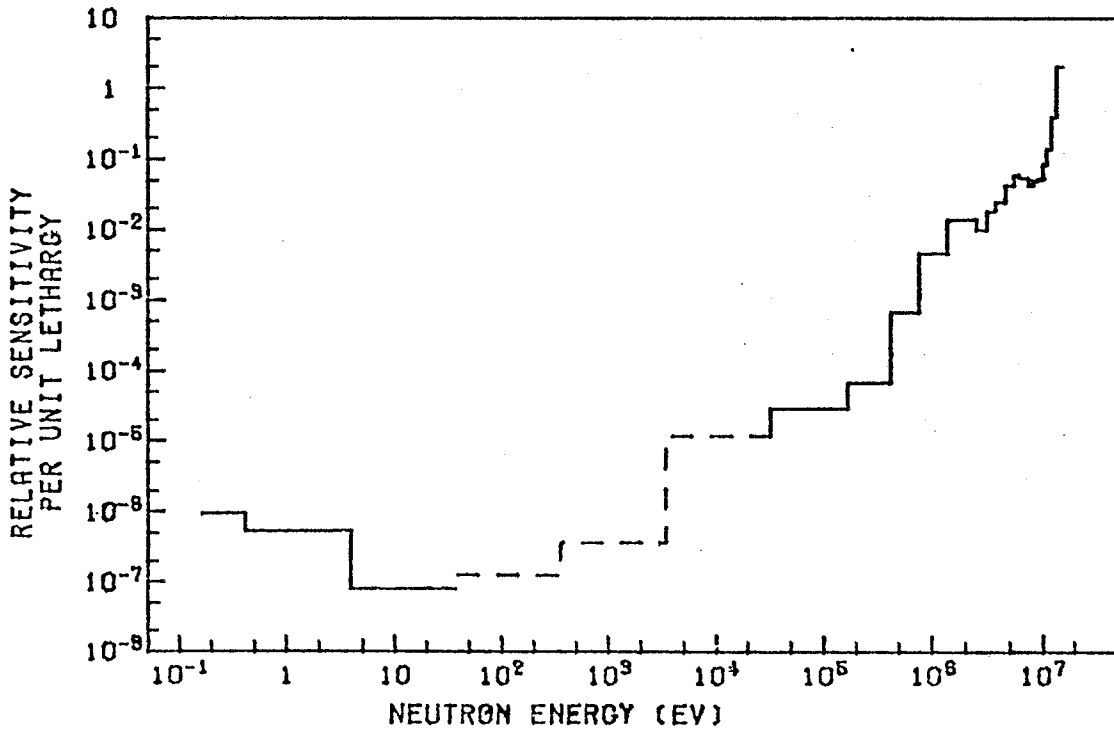


Fig. 5. Sensitivity Profiles of NUKMAK dpa Rate in Al Stabilizer to W and Ti Total Cross Sections.

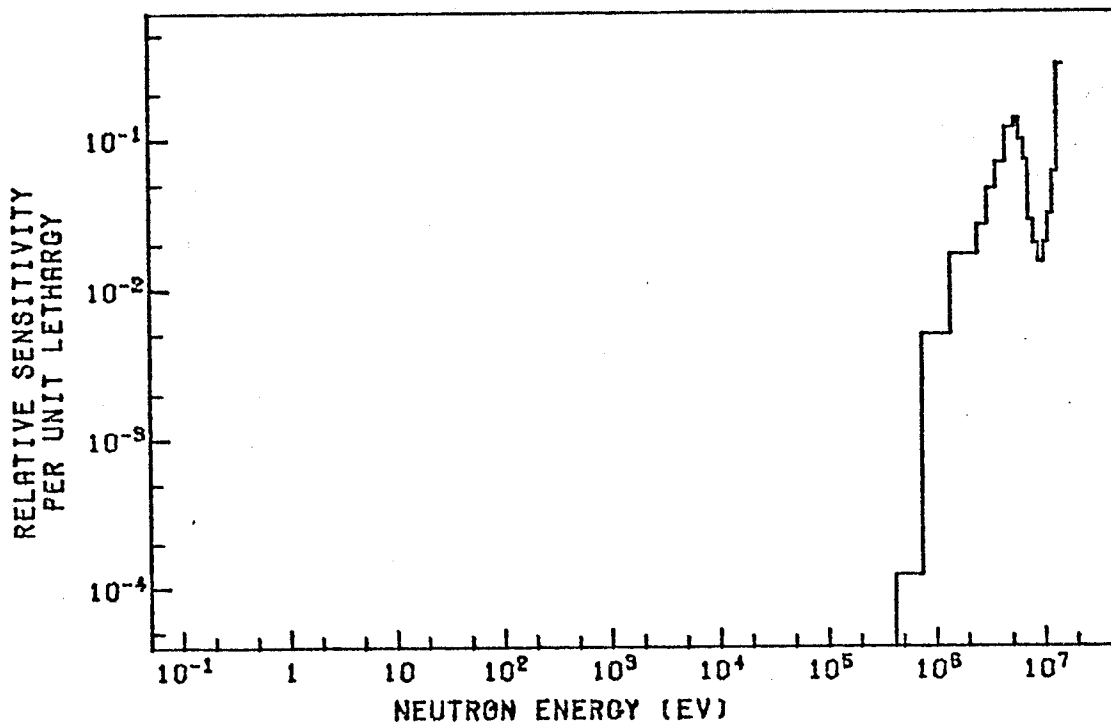
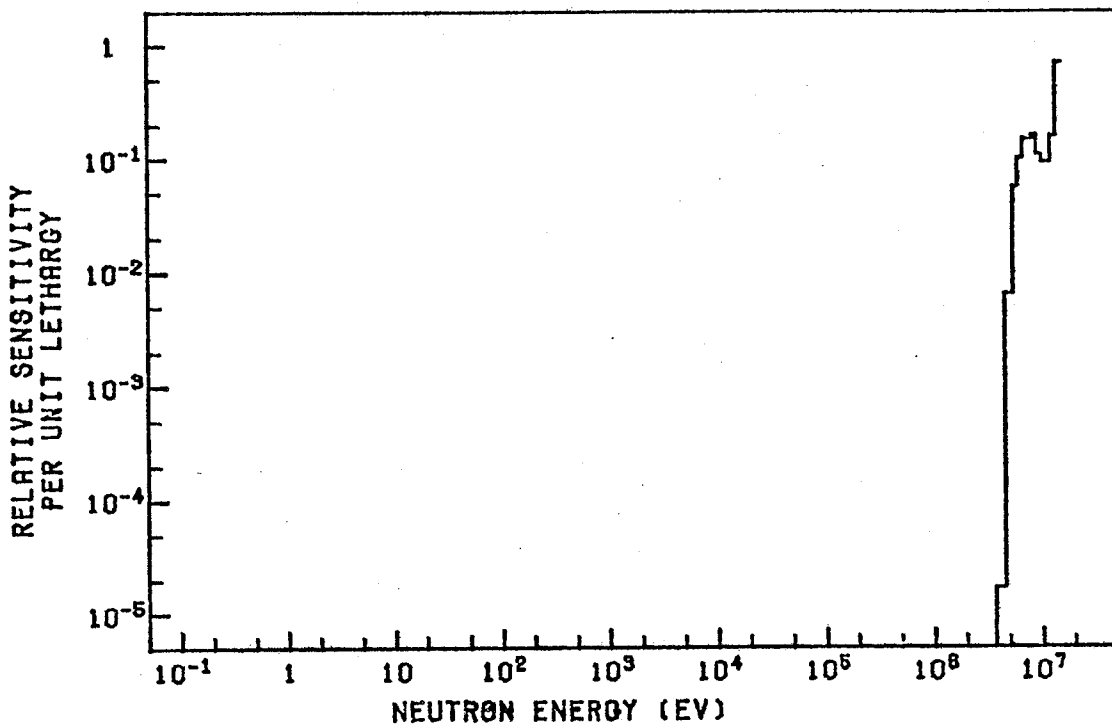


Fig. 6. Sensitivity Profiles of NUWMAK Neutron Energy Leakage to Inner Magnet to Pb Inelastic Continuum and Inelastic Level Cross Sections.

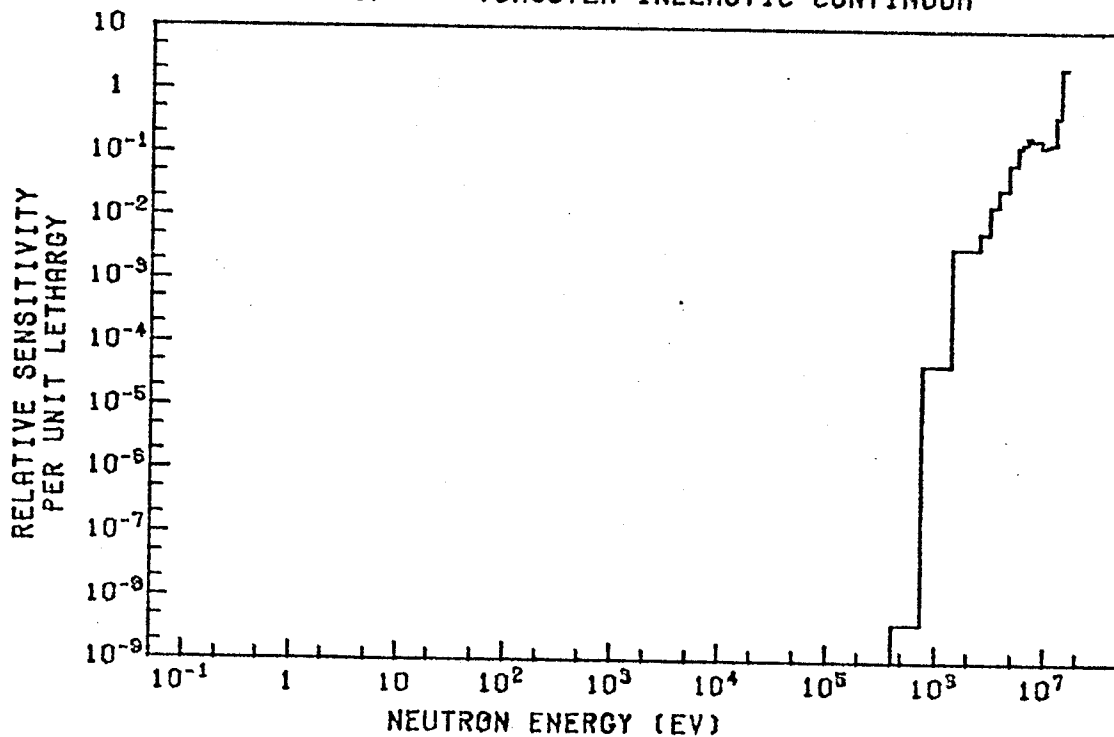
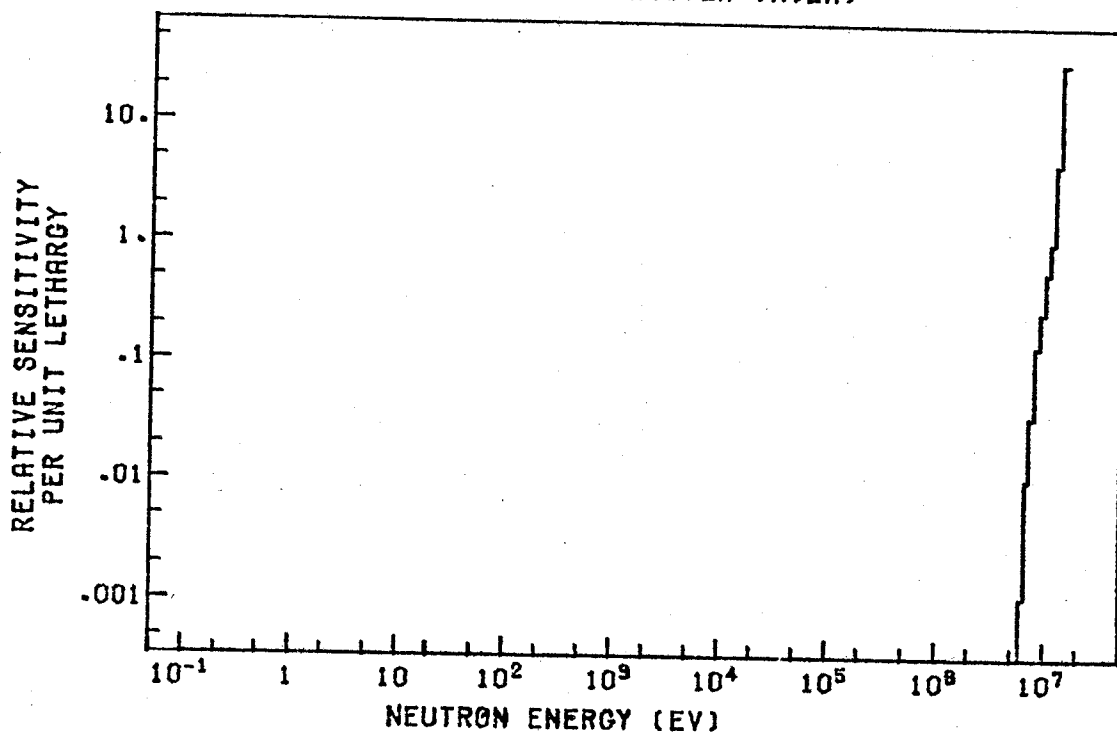


Fig. 7. Sensitivity Profiles of NUWMAK Neutron Energy Leakage to Inner Magnet to W(n,2n) and Inelastic Continuum Cross Sections.

III-3. Uncertainty Analysis

As derived in Section II, Eq. (13) is the basic formula for calculating the uncertainty of a design quantity through its sensitivity coefficients and the covariance matrices of the basic nuclear cross sections. The sensitivity coefficients are obtainable by implementing transport and sensitivity codes. However, the incompleteness of the covariance data seems to cause the most difficulties in the process of uncertainty analysis.

A computer code, called PUFF, is available to process the covariance matrices from error files in the ENDF/B-IV library. In this study, we have modified PUFF to take care of the format changes²² in the pre-preliminary ENDF/B-V file. The following are some major modifications:

- (1) A new subroutine written to read the entire error file to pick up every energy boundary contained in the error file and then to form an error energy grid.
- (2) Subroutine DANNY revised such that it will process the error file in order, i.e., first the sub-subsection of the NI-type for the same reactions, then sub-subsection of the NI-type for different reactions, and finally the sub-subsection of the NC-type.
- (3) Modifications of subroutine PUFF for processing the NI type, LB = 4 and 5.
- (4) The restriction on total number of standard deviations being lifted by storing the standard deviations in a temporary file.
- (5) Adding a new subroutine for plotting the correlation matrices, group cross sections and relative standard deviations in user group structure.

The data covariance matrices of four materials, ${}^6\text{Li}$, Pb, ${}^{10}\text{B}$, and ${}^{12}\text{C}$, have resulted from the preliminary ENDF/B-V files by PUFF processing. Notice that these data have not yet been tested and may contain some errors. Both ${}^6\text{Li}$ and ${}^{10}\text{B}$ only contain error files for the energy regime below 1 MeV. However, it seems to be good enough for NUWMAK application since the responses under investigation in NUWMAK are only sensitive to the low energy absorption cross sections for ${}^6\text{Li}$ and ${}^{10}\text{B}$. As far as the other materials are concerned, there are no error files for Ti, W, ${}^7\text{Li}$, ${}^{10}\text{B}$ in the pre-preliminary version of ENDF/B-V and no Al data are available at the present time.

Table 4 gives a summary of the relative uncertainties for six design quantities in NUWMAK contributed from the uncertainties of total cross sections. Those six quantities are defined in Table 3. The first four responses, which are the breeding ratio, neutron heating, Ti dpa in first wall, and Ti gas production in first wall, all have

Table 4
Relative Uncertainty (%) of the Responses Contributed
From the Uncertainties of Total Cross Sections in NUWMAK

Material	$\Delta R/R$ (%)					
	R_1	R_2	R_3	R_4	R_5	R_6
^6Li	0.72	0.55	----	0.03	----	----
Pb	0.34	0.10	0.02	0.39	5.02	3.89
^{12}C	0.02	0.02	----	----	1.09	1.78
^{10}B	----	0.39	----	----	----	----
^7Li					2.89	2.65
W					42.35	39.86
Ti					2.46	2.14
^{11}B					1.50	1.07
TOTAL	0.80	0.68	0.02	0.39	42.88	40.24
$(\sqrt{\Sigma \Delta R/R})^2$						

uncertainties below 1% which should satisfy the design criteria. The remainders are the neutron energy leakage to the inner magnet and dpa rate in the Al stabilizer which have uncertainties of 42.88% and 40.24% respectively. The results are based on the assumption of 10% uncorrelated uncertainties throughout the entire energy range for the ^7Li , W, Ti, and ^{11}B total cross sections. From Table 4 it is clear that the uncertainties of these two responses are dominated by the tungsten cross sections and could be cut down significantly if considerable efforts were made to improve the accuracy of the 16 MeV tungsten total cross sections.

III-4. Error Reduction

As noted earlier, the uncertainty of the dpa rate in the Al stabilizer for NUWMAK is 40.24% from all the materials and 39.86% from tungsten alone. A major error reduction is possible by simply refining the tungsten 14 MeV cross section. Here we propose a ~~pulse-gate~~ type²³ experiment with a 5 cm thick tungsten sphere surrounding the 14 MeV neutron source. The 14 MeV neutron flux is to be measured outside the sphere such that the 14 MeV total cross section can be derived easily from the flux attenuation.

The response chosen in this analysis is the dpa rate in the Al stabilizer which has a 39.26% uncertainty contributed from the tungsten 14 MeV total cross section alone. Therefore, the calculations will be done just for the first energy group. The sensitivity for the experiment is calculated by SWANLAKE and has a value of -1.1. The design

quantity we are interested in has a sensitivity of -3.93 and the standard deviation of the tungsten cross section is assumed to be 10%. Therefore we have $S_R = -3.93$, $S = -1.1$, $D = (0.1)^2 = 0.01$, and $\Delta R = 0.3926$ (all in relative units). From Eq. (30)

$$IC = \frac{(S_R D S^t)^2}{S D S^t} = S_R^2 D = (\Delta R)^2 = 0.1541,$$

i.e., the information content of the experiment is exactly equal to the variance of the response. This is just a characteristic of one group analysis and is not generally true. By substituting S_R , D , S , and ΔR into Eq. (29), we have

$$(\Delta R')^2 = 1541 - \frac{186461}{e^2 + 121}$$

where e is the experimental error, $\Delta R'$ is the posterior uncertainty of the response, and both e and $\Delta R'$ are in percentage units.

By our definition, $e_H \equiv \sqrt{SDS^t}$ is the experimental error which would reduce the variance of the response by half. In this case, $e_H = 11\%$, which indicates that if our proposed experiment were to be performed with 11% error the uncertainty of the AI dpa would be cut from 39% to 28%. The following gives a one-to-one correspondence between the experimental error and the reduced response uncertainty.

e (%)	0	2	4	6	8	10	11	∞
ΔR (%)	0	7.0	13.4	18.8	23.1	26.4	27.8	39.3

Since a full-scale economic analysis for the NUWMAK design is not available, the economic analysis of the error reduction from the proposed experiment will not be presented here.

IV. CONCLUSIONS

The theories of sensitivity and uncertainty analysis for fixed source problems have been outlined and their application to a new conceptual tokamak fusion reactor, NUWMAK, is studied. Several conclusions can be drawn from this study.

- (1) Sensitivity calculations show that the breeding ratio and neutron heating in the outer blanket of the NUWMAK design are most sensitive to the ${}^6\text{Li}$ and Pb cross sections with integral sensitivities of -0.865 and -0.652 for ${}^6\text{Li}$ and 0.137 and -0.051 for Pb respectively. However, due to the

relatively thick breeding zone design, these sensitivities are comparatively small. A detailed partial cross section analysis also shows that $Pb(n,2n)$ and ${}^6Li(n,\alpha)T$ cross sections are the most dominant in this respect.

- (2) First wall dpa and the gas production rate in the Ti alloy have small sensitivities with respect to all materials, while the neutron energy leakage to the inner magnet and the dpa rate in the Al stabilizer are ~~sensitive~~ with their high sensitivities to most of the materials in the blanket and shield, especially tungsten. Sensitivity profiles indicate the importance of the 14 MeV tungsten cross section as far as these two responses are concerned.
- (3) Detailed calculations from uncertainty analysis conclude that the uncertainties of the breeding ratio, neutron heating, Ti dpa, and first wall gas production are all below 1% which should satisfy the design criteria. However, the relatively high uncertainties of neutron energy to the magnets and dpa rate in the Al stabilizer, 43% and 40% respectively, suggest that a refinement in the tungsten 14 MeV cross section may be worthwhile.
- (4) For a pulsed sphere type tungsten cross section measurement, we have found that with an experimental error of 11% and 6%, the uncertainty of the Al dpa rate could be cut from 39% to 28% or 19% respectively. However, the economic value of the error reduction requires a full scale economic analysis and is not presented here. Nevertheless, a study of the NUWMAK design indicates the reactor costs are quite sensitive to the thickness of the tungsten zone which will in turn place a relatively high value on an accurate knowledge of the tungsten cross section. Since the pulsed sphere experiments are of reasonable cost, it is almost a certainty that the complete analysis would show a favorable cost benefit ratio for such an experiment if the NUWMAK design is the basis.

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