A Separation Method for the Transport Equation and Sensitivity Theory in Fusion-Fission Hybrid Analysis

M. Youssef

February 1979

UWFDM-289

FUSION TECHNOLOGY INSTITUTE

UNIVERSITY OF WISCONSIN

MADISON WISCONSIN

A Separation Method for the Transport Equation and Sensitivity Theory in Fusion-Fission Hybrid Analysis

M. Youssef

Fusion Technology Institute
University of Wisconsin
1500 Engineering Drive
Madison, WI 53706

http://fti.neep.wisc.edu

February 1979

UWFDM-289

"LEGAL NOTICE"

"This work was prepared by the University of Wisconsin as an account of work sponsored by the Electric Power Research Institute, Inc. ("EPRI"). Neither EPRI, members of EPRI, the University of Wisconsin, nor any person acting on behalf of either:

"a. Makes any warranty or representation, express or implied, with respect to the accuracy, completeness, or usefulness of the information contained in this report, or that the use of any information, apparatus, method, or process disclosed in this report may not infringe privately owned rights; or

"b. Assumes any liabilities with respect to the use of, or for damages resulting from the use of, any information, apparatus, method or process disclosed in this report."

A SEPARATION METHOD FOR THE TRANSPORT EQUATION AND SENSITIVITY THEORY IN FUSION-FISSION HYBRID ANALYSIS

M.Z. Youssef
R.W. Conn

February 1979

Fusion Research Program
Nuclear Engineering Department
University of Wisconsin
Madison WI 53706 U.S.A.

UWFDM-289

Abstract

A computational technique which can be used to solve the transport equation in a fusion-fission hybrid system is described. This system is characterized by a highly energetic external neutron source and fissionable materials in the fusion blanket. The transport equation is separated into two parts. A large number of energy groups and high order angular scattering are included in a discrete ordinate S_N method to describe accurately the first generation neutrons produced by the fusion source. Fission is treated only as part of the total capture cross section. A fission source for the second part is generated and the subsequent neutron spectrum associated with fission-produced neutrons is described using a few group method with low order scattering. The total integrated parameters of interest are the summation of the contributions from both parts.

A reduction in the computational cost up to 50% is obtained when the separation technique is applied to the particular fusion-hybrid system, SOLASE-H, with an insignificant error (< 1%) in the integrated design parameters. The sensitivity theory used to evaluate the relative sensitivity coefficient of the total design parateter, R, to perturbation in the system is used to demonstrate that the use of a low order scattering description when solving the second part of the problem will result in an insignificant error in the fissile fuel production rate where a defined adjoint flux to the first part and the adjoint flux of the second part are used. As shown in this study, approximate estimation of the variation of R with time, as a result of burning the bred fissile fuel, can be accounted for using the forward flux of the first part and the adjoint flux of the second part at the beginning of life.

Contents

I. Introduction

The distribution of neutrons in nuclear reactors in space, energy and time is the governing favor in the behavior and performance of these reactors. These reactors can be categorized into three major classes: the purely fusion,[1-3] the purely fission[4] and the hybrid reactors.[5-7] The distribution of the neutrons in these reactors can be, in principle, obtained by solving the transport equation.[8-10] Many techniques have been used to approximate this solution depending on the system complexity, neutron source nature, and the degree of accuracy required for different design parameters of interest. Among these techniques are: The Monte Carlo[11-13] technique, the multi-group P_N[14] method and the multi-group discrete[10] ordinates S_N technique.

The linearity of the transport equation suggests the possibility of utilizing the superposition principle to obtain the final solution. As will be explained in the following section, the transport equation can be separated into two parts where the solution of each part is summed to obtain the total solution for the system. This proposed technique has the advantages of using certain parts of the solution to describe a more complex situation (e.g., high anisotropy in the system) where a higher degree of accuracy is required. In this respect, more sophisticated techniques can be utilized, e.g., Monte Carlo, higher order P_N or discrete ordinate method. The solution to this part can then be used to generate a source term to the second part where less restricted approximations can be used

to solve this part, e.g., lesser numbers of energy groups, lower orders of scattering, lesser angular quadrature, and possibly different neutron cross-section sets. A related research work in connection with analyzing the sensitivity of a hybrid-system parameter to nuclear data uncertainties has been performed in a joint effort using a Monte Carlo technique in the high energy range.[15] In this study we used the discrete ordinates method to describe both parts.

The value of any linear design parameter (response), R, is obtained by adding its value, evaluated from the first part, to the corresponding value for the second part. As will be shown for the application of this computational technique, the contribution to R from the first part dominates in the particular system used in this study. Using less restricted approximations to evaluate the contribution from the second part results in an insignificant error in R and utilizing this technique gives a noticeable reduction in the computational effort and time needed to solve the problem.

In this study, an expression to evaluate the sensitivity coefficient, P, defined as the percentage change in R due to alteration in the system, is developed. Such perturbation may result from atomic density changes, etc. In evaluating P, the solutions to both parts and the coupling between them are used. This sensitivity theory has been particularly developed to analyze the effect on R when different numbers of Legendre terms are used to express the scattering in different materials present in the system. This is

useful in identifying which material in particular will have more impact on R when using low orders of scattering. As such, the concept of the adjoint flux is utilized. A relation between a defined adjoint flux for the first part and the adjoint flux for the second part is given. As an alternate method, these adjoint fluxes can be utilized to evaluate R. We find this latter technique particularly useful for burnup calculations.

As an application, the technique is used to solve the neutron transport equation in the laser fusion-fission hybrid reactor, SOLASE-H.[16-18] This system is characterized by a high energy neutron source (14.1 MeV) confined in a vacuum zone surrounded by the blanket which contains a fertile material. P_3-S_4 approximation and 25-neutron energy groups are used to solve the first part which describes the high anisotropy of the system due to the highly energetic neutron source while P_1S_2 approximation and 16-neutron energy groups are used to solve the second part. In solving the first part, no fissioning is considered in the fertile material which acts only as an absorber. The solution to this part is then used to generate a fission source to the second part. The two solutions are used to evaluate the integrated result, R, which is the fissile fuel production rate in this case.

II. The Theory

II.1. Separation of the Transport Equation Into Two Parts

The time-independent neutron transport equation and the time-independent adjoint equation for both perturbed and unperturbed systems can be written as:

$$L\ \Phi = S(\bar{r},E,\bar{\Omega}) \tag{1}$$

$$L^*\ \Phi^* = \Sigma_r\ (r,E) \tag{2}$$

where

$$L\ \Phi = \bar{\Omega} \cdot \nabla\ \Phi\ \ (\bar{r},E,\Omega) + \Sigma(\bar{r},E)\ \Phi\ (\bar{r},E,\bar{\Omega}) - \int dE'd\bar{\Omega}'\ \Sigma f(\bar{r},E' \rightarrow E,\bar{\Omega}' \rightarrow \Omega) \tag{3}$$

$$L^*\ \Phi^* = -\ \bar{\Omega} \cdot \nabla\ \Phi^*\ \ (\bar{r},E,\bar{\Omega}) + \Sigma(\bar{r},E)\ \Phi^*\ (\bar{r},E,\bar{\Omega}) - \int dE'd\bar{\Omega}'\ \Sigma f(\bar{r},E \rightarrow E',\bar{\Omega} \rightarrow \bar{\Omega}') \tag{4}$$

where $\Sigma(\bar{r},E)$ = total macroscopic cross-section at position r and energy E.

$f(\bar{r},E \rightarrow E',\bar{\Omega} \rightarrow \bar{\Omega}')$ $\equiv$ the differential scattering function. It gives the probability of a neutron of energy E' at position $\bar{r}$ with a direction Ω' to reappear with energy E with a direction Ω at position $\bar{r}$ upon encountering a collision

and

$\Phi\ (\bar{r},E,\bar{\Omega}), \Phi^*\ (\bar{r},E,\bar{\Omega})$ $\equiv$ the forward and adjoint angular fluxes of the system at $\bar{r}$,E and $\bar{\Omega}$.

$S(\bar{r},E,\bar{\Omega})$ ≡ the external neutron source per unit volume per unit time emitted at r with energy dE about E and direction $d\bar{\Omega}$ about $\bar{\Omega}$.

$\Sigma_r(\bar{r},E)$ ≡ the source for the adjoint flux.

We note that

$$\Sigma(\bar{r},E')f(\bar{r},\bar{\Omega}',E'\rightarrow\bar{\Omega},E) = \sum_x \sigma_x(\bar{r},E')\, f_x\,(\bar{r},\Omega',E'\rightarrow\Omega,E)$$

where the sum over 'x' includes elastic and inelastic scattering with their corresponding f's normalized to units; fissioning with f normalized to $\nu(E)$, the (n,2n) reaction, with f normalized to 2, and so on. The integrated result (response functional), R, is given[21] by

$$R = \langle \sigma_r , \Phi \rangle = \langle \Phi^* , S \rangle \qquad (5)$$

where the notation $\langle\ ,\ \rangle$ means integration over all the phase space.

The operator L is divided into two parts, i.e.

$$\hat{L} = \hat{H} - \hat{F} \qquad (6)$$

where

$$\hat{H} \equiv \bar{\Omega} \cdot \nabla\Phi + \sum_{jx}\Sigma \Sigma_{xj} - \sum_j \sum_{x\neq f} \int dE'd\bar{\Omega}'\ \Sigma_{xj}\, f_{xj}\,(\bar{r},E',\bar{\Omega}'\rightarrow E,\bar{\Omega}) \qquad (7\text{-a})$$

and

$$\hat{F} = \sum_{j=fiss} \sum_{x=f} \int dE'd\bar{\Omega}'\ \Sigma_{xj}\, f_{xj}\,(\bar{r},E',\bar{\Omega}'\rightarrow E,\bar{\Omega}) \qquad (7\text{-b})$$

where j denotes the elements present in the system at $\bar{r}$, x is the reaction type, f_f is the scattering probability for fission normalized to $\bar{\nu}$ and 'fiss' denotes the fissionable material.

If Φ_1 denotes the solution to the equation

$$\hat{H}\ \Phi_1 = S \tag{8}$$

where S is the external fusion source and if Φ_2 denotes the solution to the equation

$$(\hat{H} - \hat{F})\ \Phi_2 = \hat{L}\ \Phi_2 = \hat{F}\ \Phi_1 \equiv S_f \tag{9}$$

where S_f is the fission source, then, upon adding Eq. (8) and Eq. (9) we find

$$(\hat{H} - \hat{F})\ \Phi_1 + (\hat{H} - \hat{F})\ \Phi_2 = S$$

which can be written as

$$\hat{L}\ \Phi = S\ .$$

This is the original transport equation for the system with

$$\Phi = \Phi_1 + \Phi_2\ . \tag{10}$$

Equations (8) and (9) give the solution for the two parts of the total solution mentioned in the introduction.

The integrated result, R, can be described by two parts, i.e.

$$R = R_1 + R_2 \tag{11}$$

where

$$R_1 = \langle \Sigma_r\ ,\ \Phi_1 \rangle \tag{11-a}$$

and $$R_2 = \langle \Sigma_r , \Phi_2 \rangle \qquad (11\text{-b})$$

II.2. Evaluating the Integrated Result, R, Using the Adjoint Fluxes of the Two Parts

We define the adjoint flux of the first part described by Eq. (8) as the solution to the equation

$$\hat{H}^* \Phi_1^* = \Sigma_r \quad . \qquad (12)$$

The corresponding adjoint equation to the second part with the fission operator included is

$$\hat{L} \Phi_2^* = \Sigma_r \quad . \qquad (13)$$

The adjoint flux to the second part, Φ_2^*, is the adjoint flux to the total system. This becomes clear upon comparing Eq. (13) and Eq. (2), i.e.,

$$\Phi_2^* = \overset{*}{\Phi} \quad . \qquad (14)$$

When multiplying Eq. (8) by Φ_1^* and Eq. (12) by Φ_1 followed by an integration over all phase space $\vec{\xi} \equiv (\bar{r},E,\bar{\Omega})$ and subtracting the two results we can establish the relation

$$\langle \Phi_1^*, S \rangle = \langle \Sigma_r , \Phi_1 \rangle$$

with $$R_1 = \langle \Phi_1^* , S \rangle \quad . \qquad (15)$$

Similarly, if we multiply Eq. (9) by Φ_2^* and Eq. (13) by Φ_2 and integrating over the phase space $\bar{\xi}$ and upon subtraction we get

$$\langle \Phi_2^* , F\Phi_1 \rangle = \langle \Sigma_r , \Phi_2^* \rangle \tag{16}$$

with

$$R_2 = \langle \Phi_2^* , F\Phi_1 \rangle \tag{17}$$

With the adjoint fluxes defined as importance functions[19] and with $\Phi_2^* = \Phi^*$, therefore, Eq. (17) gives the contribution of the fission source, constructed from the solution to the first part, to the total result R. In other words, Eq. (17) can be used to evaluate the importance of the fission source in contributing to the final result R. If there is more than one fissionable material in the system (fiss > 1), then we have

$$R_2 = \sum_{j=fiss} R_{2j} = \sum_{j=fiss} \langle \Phi_2^* , F_j \Phi_1 \rangle \tag{18}$$

and the contribution to the final result R from a particular fissionable material is:

$$R_{2j} = \langle \Phi_2^* , F_j \Phi_1 \rangle \tag{19}$$

where j = fiss. The total result R is then given by

$$R = R_1 + \sum_{j=fiss} R_{2j} . \tag{20}$$

From Eq. (15) and Eq. (17), and since $\Phi_2^* = \Phi^*$, we have

$$R = R_1 + R_2 = \langle \Phi_2^* , S \rangle$$

which can be written as

$$\langle \Phi_2^* , F\Phi_1 \rangle + \langle \Phi_1^* , S \rangle = \langle \Phi_2^* , S \rangle \quad . \qquad (21)$$

Eq. (21) establishes a relation between the adjoint fluxes of the two parts using the fission and the external sources. A mixed technique can be used to evaluate R. In this respect, Φ_1 is used to evaluate R_1 using Eq. (11-a) and Φ_2^* is used to evaluate R_2 using Eq. (17).

II.3. Testing the Separation Technique, Cross Section Sensitivity Analysis

As mentioned before, when applying the proposed separation technique to a fusion-fission hybrid system, the discrete ordinates S_N method is used. Higher orders of scattering and a larger number of energy groups are used when solving for Φ_1 while low orders of scattering and a smaller number of groups are used when solving for Φ_2.

To demonstrate that using lower orders of scattering in the second part has no severe effect on the accuracy of the total result, R, the concept of the relative sensitivity coefficient is [20-23] used. In this regard, the first order perturbation theory is utilized to investigate the effect of considering a different number of terms in expressing scattering in different materials in the system on the pertinent results R_1 and R_2. In particular, this procedure enables us to identify which element has more impact on R when lower orders of scattering are considered. In the following, the sensitivity coefficient, P, is derived for both parts. These

coefficients are used to perform the investigation mentioned above. An expression to evaluate the total relative sensitivity coefficient is derived using the solutions to both parts. This relation can in general be utilized in connection with the proposed separation method.

II.3-1. The Relative Sensitivity Coefficient, P

From Eq. (1) and (2) and for the unperturbed system, we have

$$L_u^u \Phi^u = S \tag{22}$$

$$L_u^* \Phi^{*u} = \Sigma_{ru} \ . \tag{23}$$

The corresponding equations for the perturbed system are

$$L_p \Phi^p = S \tag{24}$$

$$L_p^* \Phi^{*p} = \Sigma_{rp} \ . \tag{25}$$

Subtracting (25) from Eq. (23), we get

$$L_p^* \Phi^{*p} - L_u^* \Phi^{*u} = \Sigma_{rp} - \Sigma_{ru} \ . \tag{26}$$

Subtracting $L_p^* \Phi^{*u}$ from both sides, we obtain

$$L_p^* (\Phi^{*p} - \Phi^{*u}) = (L_u^* - L_p^*) \Phi^{*u} + \Sigma_{rp} - \Sigma_{ru} \ . \tag{27}$$

Multiply both sides by Φ^p and integrate over the phase space to obtain for the left-hand side, L.H.S.

$$\langle \Phi^p, L_p^* \Phi_p^* \rangle - \langle \Phi^p, L_p^* \Phi^* u \rangle = \langle \Sigma_{rp}, \Phi^p \rangle - \langle L_p \Phi^p, \Phi^* u \rangle$$

$$= \langle \Sigma_{rp}, \Phi^p \rangle - \langle \Phi^* u, S \rangle$$

$$= \langle \Sigma_{rp}, \Phi^p \rangle - \langle \Sigma_{ru}, \Phi^u \rangle$$

$$\text{i.e. L.H.S.} = \delta R \qquad (28)$$

where δR is the change in R resulting from a change in Σ_r, and in Φ. Therefore, Eq. (27), upon multiplying by Φ^p, reduces to

$$\delta R = \langle \Phi_p, -\delta L^* \Phi^* u \rangle + \langle \delta \Sigma_r, \Phi^p \rangle \qquad (29)$$

where

$$\delta L^* = L_p^* - L_u^* \qquad (30\text{-a})$$

and

$$\delta \Sigma_r = \Sigma_{rp} - \Sigma_{ru} \; . \qquad (30\text{-b})$$

Equation (29) is exact and is valid for any variation in the operator δL or the response cross section $\delta \Sigma_r$. δL^* in Eq. (29) can be written in terms of any type of cross section variation $\delta \Sigma_{xj}$ for element j as follows:

$$\delta L^* = \sum_j \sum_x \delta \Sigma_{xj} - \sum_j \sum_{x \neq f} \int dE' d\bar{\Omega}' \delta \{ \Sigma_{xj} f_{xj} (\bar{r}, E, \bar{\Omega} \rightarrow E', \bar{\Omega}') \}$$

$$+ \sum_{j=fiss} \sum_{x=f} \int dE' d\bar{\Omega} \delta \{ \Sigma_{xj} f_{xj} (\bar{r}, E, \bar{\Omega} \rightarrow E', \bar{\Omega}') \} \qquad (31)$$

where the variation in the fission operator δF is written explicitly and the first two terms represent δH. Let us assume that the perturbed differential cross section $\Sigma_{pxj} f_{xj}$ is proportional to the corresponding unperturbed value in all regions of phase space where perturbation takes place, i.e.

$$\Sigma_{pxj}\, f_{xj}(\bar{r},E,\bar{\Omega}\to E',\bar{\Omega}') = c\,\Sigma_{uxj}\, f_{xj}\,(\bar{r},E,\bar{\Omega}'\to E',\bar{\Omega}')\ , \qquad (32)$$

with

$$\Sigma_{pxj}(\bar{r},E) = c\ \Sigma_{uxj}(\bar{r},E) \qquad (33)$$

where c is a constant which is independent r,E,E',$\bar{\Omega}$,j and $\bar{\Omega}'$ in all regions in the phase space where $c\neq 1$ and $\delta c=c-1$.

Then it is clear from Eqs. (32) and (33) that

$$\delta c = \frac{\delta\{\Sigma_{xj}f_{xj}(\bar{r},E,\bar{\Omega}\to E',\Omega')\}}{\Sigma_{uxj}f_{xj}\,(\bar{r},E,\bar{\Omega}\to E',\bar{\Omega}')} = \frac{\Sigma_{pxj}f_{xj}(\bar{r},E\ ,\bar{\Omega}\to E',\bar{\Omega}')-\Sigma_{uxj}f_{xj}(\bar{r},E,\bar{\Omega}\to E',\bar{\Omega}')}{\Sigma_{uxj}f_{xj}(r,E,\bar{\Omega}\to E',\bar{\Omega}')} \qquad (34)$$

$$\delta c = \frac{\delta\Sigma_{xj}}{\Sigma_{uxj}} = \frac{\Sigma_{pxj}(\bar{r},E)-\Sigma_{uxj}(\bar{r},E)}{\Sigma_{uxj}}\ . \qquad (35)$$

Therefore, Eq. (29) becomes

$$\delta R = \sum_j \sum_x \delta R_{xj} = \delta c\ \{\sum_j \sum_x - \langle\Phi^p,\ L^*_{xj}\ \Phi^{*u}\rangle + \langle\Sigma_{ru},\ \Phi^p\rangle\} \qquad (36)$$

It is understood that integration in Eq. (36) is to be performed only over the perturbed regions of phase space where $\delta c \neq 0$. Within the context of the linear perturbation theory, ϕ^p is replaced by Φ^u in Eq. (36) if the variation δc is small. In this case δR is linearly proportional to δc, where

$$\delta R = \delta c\ \{\sum_{jx} - \langle\Phi^u,L^*_{xj}\ \Phi^{*u}\rangle + \langle\Sigma_{ru},\ \Phi^u\rangle\}\ . \qquad (37)$$

Then, the total relative sensitivity profile[20-22], P, is defined as

$$P = \sum_{j,x} P_{jx} = \frac{\delta R/R}{\delta c} = \frac{1}{R} \{ \sum_{j} \sum_{x} - \langle \Phi^{u}, L^{*}_{xj} \Phi^{*u} \rangle + \langle \Sigma_{ru}, \Phi^{u} \rangle \} \quad (38)$$

which gives the percentage change in the final result R for a unit percentage value of δc. In Eq. (38), the summation over the element j and type of reaction x is written explicitly to show that it is possible to evaluate the percentage change in R due to a particular reaction x and for a particular element j. Furthermore, it is possible to assign different values for δc in different energy and spatial regions. This is done in the context of the uncertainty analysis of cross section data.[23]

Writing Eq. (38) explicitly for a particular element, and a particular reaction type, x, we get

$$P_{jx} = \frac{1}{R} \{ \int_{E} dE \int_{r} d\bar{r}\, \Sigma_{xj}(\bar{r},E) \, [-\phi^{ou}(\bar{r},E)\phi^{o*u}(\bar{r},E) + \sum_{\ell=0}^{\infty}$$

$$\frac{2\ell+1}{4\pi} \int_{E'} dE' \phi^{\ell u}(\bar{r},E) \, f^{\ell}_{xj}(\bar{r},E \to E') \, \phi^{*\ell u}(\bar{r},E)]$$

$$+ \int_{E} dE \int d\bar{r}\, \Sigma_{ru}(\bar{r},E) \, \phi^{ou}(\bar{r},E)\} \quad (38\text{-a})$$

for $x \neq f$, $x=r$, and all j and

$$P_{jx} = \frac{1}{R} \{ \int_{E} dE \int d\bar{r}\, \Sigma_{xj}(\bar{r},E) \, [-\phi^{ou}(\bar{r},E) \, \phi^{o*u}(\bar{r},E) + \int_{E'}$$

$$\phi^{ou} \bar{\nu}(E) \chi(E') \, \phi^{*ou}(\bar{r},E') \, dE']$$

$$+ \int_{E} dE \int d\bar{r}\, \Sigma_{ru}(\bar{r},E) \, \phi^{ou}(\bar{r},E)\} \quad (38\text{-b})$$

for x=f, x=r and j=fiss, where the integral over the solid angle $\bar{\Omega}$ is carried out by assuming azimuthal symmetry and expanding the forward and adjoint fluxes in Legendre polynomials with coefficients defined as

$$\phi^{\ell u} = \int \Phi^{u}(\bar{r},E,\bar{\Omega})\ P_{\ell}(\mu)\ d\bar{\Omega} \qquad \text{(a)}$$

$$\phi^{*\ell u} = \int \Phi^{*u}(\bar{r},E,\bar{\Omega})\ P_{\ell}(\mu)\ d\bar{\Omega} \qquad \text{(b)}$$

$$f^{\ell}_{xj} = \int f_{xj}\ (\bar{r},E\rightarrow E',\Omega\rightarrow\Omega') \qquad \text{(c)}$$

$$P_{\ell}\ (\mu_o)\ d\mu_o \qquad (39)$$

where $\mu = \bar{r} \cdot \bar{\Omega}$, $\mu_o = \bar{\Omega} \cdot \bar{\Omega}'$, P_{ℓ} is the Legendre polynomial of order ℓ, $f^{\ell}_{xj}(r,E\rightarrow E')$ is the ℓ'th Legendre moment of the normalized secondary energy-angle distribution function for element j and reaction x, and $\chi(E')$ is the probability that neutrons after encountering fission will appear with energy E'. It is to be understood that if x = absorption then f^{ℓ}'s are zero. If $x\neq r$ the last term in Eqs. (38) is eliminated.

The sensitivity of the result R to the number of terms considered in the expansion of the transfer probability distribution function $f_{xj}(\bar{r},E,\Omega\rightarrow E',\Omega')$ can be evaluated for a particular element j and a particular type of reaction x by truncating the legendre expansion after ℓ=N. If the scattering is highly anisotropic, a higher number of terms are required.

II.3-2. The Relative Sensitivity Coefficient of the First Part, $P^{(1)}$

The relative sensitivity coefficient, $P^{(1)}_{jx}$, for the first part of the problem, associated with fusion source neutrons due to perturbation

in cross section type $x \neq f$, $x = r$ and element j is obtained from Eqs. (38) with $\phi_1^{\ell u}$ and $\phi_1^{*\ell u}$ replacing $\phi^{\ell u}$ and $\phi^{*\ell u}$, respectively, where $\phi_1^{\ell u}$ and $\phi_1^{*\ell}$ are given by Eq. (39) with Φ_1^u and Φ_1^{*u} replacing Φ and Φ^{*u}. For $x \neq r$ the last term in Eqs. (38) is excluded. R_1 replaces R in Eq. (38).

If $x = f=r$ and $j=fiss$, then $P_{jf}^{(1)}$ is given by

$$P_{jf}^{(1)} = \frac{1}{R_1} \{ \int_E dE \int d\bar{r}\ \Sigma_{fj}(\bar{r},E) \left[-\phi_1^o(\bar{r},E)\ \phi_1^{*o}(r,E)\right] + \int_E dE \int d\bar{r}\ \Sigma_r(\bar{r},E)\ \phi_1^{*o}(\bar{r},E)\} \qquad (39\text{-a})$$

which is always negative if $x \neq r$. One notices that although fissioning is not included when solving for Φ_1 but perturbing the fission cross section is reflected on the total absorption cross section.

II.3-3. The Relative Sensitivity Coefficient of the Second Part, $P^{(2)}$

The relative sensitivity coefficient, $P_{jx}^{(2)}$, for the second part is more difficult to evaluate because of the coupling between the first and second part through the fission source, $F\Phi_1$. Rewriting the forward and the adjoint equations for the perturbed and the unperturbed system, we get

$$L_u\ \Phi_2^u = F_u\ \Phi_1^u \qquad (40)$$

$$L_u^*\ \Phi_2^{*u} = \Sigma_{ru} \qquad (41)$$

$$L_p\ \Phi_2^p = F_p\ \Phi_1^p \qquad (42)$$

$$L_p^*\ \Phi_2^{*p} = \Sigma_{rp} \quad . \qquad (43)$$

Subtracting (43) from (41) we get

$$L_p^* \Phi_2^{*p} - L_u^* \Phi_2^u = \Sigma_{rp} - \Sigma_{ru}$$

Subtracting $L_p^* \Phi_2^{*u}$ from both sides we get

$$L_p^* (\Phi_2^{*p} - \Phi_2^{*u}) = (L_u^* - L_p^*) \Phi_2^{*u} + \Sigma_{rp} - \Sigma_{ru} . \quad (44)$$

Multiplying by Φ_2^p and integrating over phase space we get for the left-hand side, L.H.S.,

$$\text{L.H.S.} = \langle \Phi_2^p, \Sigma_{rp} \rangle - \langle \Phi_2^{*u}, F_p \Phi_1^p \rangle \quad (45)$$

where Eq. (43) and Eq. (42) have been used. If we consider

$$F_p = F_u + \delta F \quad (46)$$

$$\Phi_1^p = \Phi_1^u + \delta\Phi_1 \quad (47)$$

we get

$$\text{L.H.S.} = \delta R_2 - \delta R_{coup} \quad (48)$$

where

$$\delta R_2 = R_2^p - R_2^u = \langle \Sigma_{rp}, \Phi_2^p \rangle - \langle \Sigma_{ru} \Phi_2^u \rangle \quad (49)$$

$$\delta R_{coup} = \langle \Phi_2^{*u}, \delta F \Phi_1^u \rangle + \langle \Phi_2^{*u}, (F_u + \delta F) \delta\Phi_1 \rangle \quad (50)$$

and the relation

$$R_2^u = \langle \Phi_2^{*u}, F_u \Phi_1^u \rangle = \langle \Sigma_{ru}, \Phi_2^u \rangle \quad (51)$$

has been used. The δR_{coup} is the variation in the coupling term between the first and the second part. The R.H.S. of Eq. (44) becomes, after multiplying by ϕ_2^p,

$$R.H.S. = \langle \Phi_2^p, -\delta L^* \Phi_2^{*u} \rangle + \langle \delta\Sigma_r, \Phi_2^p \rangle \qquad (52)$$

where δL^* and $\delta\Sigma_r$ are given by Eq. (30). Equating Eq. (48) and Eq. (52) we find

$$\delta R_2 = \delta R_{coup} + \langle \Phi_2^p, -\delta L^* \Phi_2^u \rangle + \langle \delta\Sigma_r, \Phi_2^p \rangle \ . \qquad (53)$$

As was done in connection with Eq. (36), we consider

$$\delta L^* = \delta c \ L_u^* \qquad (54)$$

and

$$\delta\Sigma_r = \delta c \ \Sigma_{ru} \ . \qquad (55)$$

The sensitivity coefficient, $P^{(2)}$, is now written as

$$P^{(2)} = \frac{\delta R_2/R_2}{\delta c} = P^{(2)}_{coup} + P^{(2)}_{uncoup} \qquad (56)$$

where

$$P^{(2)}_{coup} = \frac{\delta R_{coup}/R_2}{\delta c} \ . \qquad (57)$$

$$P^{(2)}_{uncoup} = \frac{1}{R_2} \{ \langle \Phi_2 - L_u^* \Phi_2^* \rangle + \langle \Sigma_r, \Phi_2 \rangle \} \ , \qquad (58)$$

and δR_{coup} is given by Eq. (50). In Eq. (58) we consider $\Phi_2^p \to \Phi^u$ and dropped the subscript u for simplicity. For a particular reaction $x \neq f$, $x = r$ and for all j we have

$$P^{(2)}_{jx} = P^{(2)}_{jx,coup} + P^{(2)}_{jx,uncoup} \tag{59}$$

where

$$P^{(2)}_{jx,uncoup} = \frac{1}{R_2}\{\int_E dE \int d\bar{r}\ \Sigma_{xj}(\bar{r},E)\ [\phi_2^o(r,E)\ \phi_2^{*o}(\bar{r},E) +$$

$$\sum_{\ell=0}^{\infty} \frac{2\ell+1}{4\pi} \int_{E'} dE'\ \phi_2^{\ell}(\bar{r},E)\ f_{xj}^{\ell}(\bar{r},E\rightarrow E')\ \phi_2^{*\ell}(\bar{r},E')]$$

$$+ \int_E dE \int d\bar{r}\ \Sigma_r\ \phi_2^o(\bar{r},E)\} \tag{60}$$

and $$P^{(2)}_{jx,coup} = \frac{\delta R_{jx,coup}/R_2}{\delta c} = \frac{1}{R_2 \delta c} \sum_{j'=fiss} \langle \Phi_2^*,\ F_{j'}\ \delta\Phi_1 \rangle\ . \tag{61}$$

In Eq. (61), $\delta R_{jx,coup}$ is obtained from Eq. (50) with $\delta F=0$ since $x \neq f$. The summation over j' is taken for all the fissionable materials in the fusion blanket.

For x=f, j=fiss and x=r we have

$$P^{(2)}_{jf,uncoup} = \frac{1}{R_2}\{\int_E dE \int d\bar{r} \int \Sigma_{fj}(\bar{r},E)\ [\phi_2^o(\bar{r},E)\ \phi_2^{*o}(\bar{r},E)$$

$$+ \int_E dE'\ \phi_2^o(\bar{r},E)\ \bar{\nu}(E)\ \chi(E')\ \phi_2^{*o}(\bar{r},E')]$$

$$+ \int dE \int d\bar{r}\ \Sigma_r(\bar{r},E)\ \phi_2^o(\bar{r},E)\} \tag{62}$$

and

$$P^{(2)}_{jf,coup} = \frac{\delta R_{jf,coup}/R_2}{\delta c} = \frac{1}{R_2} \langle \Phi_2^*,\ F_j\ \Phi_1^p \rangle + \frac{1}{R_2 \delta c} \sum_{j'=fiss}$$

$$\langle \Phi_2^*,\ F_{j'}\ \delta\Phi_1 \rangle \tag{63}$$

where Eq. (47) and Eq. (50) have been used and we considered

$$\delta F_j = \delta c F_j \ . \tag{64}$$

The total sensitivity coefficient, P, is then given by

$$P = P^{(1)} + P^{(2)} \tag{65}$$

where

$$P^{(1)} = \sum_{j,x} P^{(1)}_{jx} \tag{66}$$

and

$$P^{(2)} = \sum_{j,x} (P^{(2)}_{jx,coup} + P^{(2)}_{jx,uncoup}) \ . \tag{67}$$

It is clear that the evaluation of $P^{(2)}_{jx,coup}$ requires direct calculation to evaluate Φ^p_1 and $\delta\Phi_1$. Using $\Phi^p_1 \rightarrow \Phi^u_1$ and $\delta\Phi_1=0$ leads to an error in evaluating $P^{(2)}_{jx,coup}$ as will be shown later. However, the evaluation of $P^{(2)}_{jx,uncoup}$ is straightforward.

II.3-4. Alternative Procedure to Evaluate the Relative Sensitivity Coefficient, P

From Eq. (29) we have

$$\delta R = <\phi^{*u} - \delta L \Phi^{p}> + <\delta \Sigma_{r}, \Phi^{p}> \tag{68}$$

but

$$\phi^{p} = \Phi_{1}^{p} + \Phi_{2}^{p} , \tag{69}$$

and with $\Phi^{*u} = \Phi_{2}^{*u}$ we get

$$\delta R = <\Phi_{2}^{*u}, - \delta L(\phi_{1}^{p} + \Phi_{2}^{p})> + <\delta \Sigma_{r}, (\Phi_{1}^{p} + \Phi_{2}^{p})> . \tag{70}$$

With $\delta L = \delta H - \delta F$ and using Eq. (30) to express δL and δH we get for the total sensitivity coefficient, P:

$$P = \frac{\partial R/R}{\partial c} = \frac{1}{R} \left(<\phi_{2}^{*u}, - H_{u} \Phi_{1}^{P}> + \frac{1}{\delta c} <\Phi_{2}^{*u}, \delta F_{u} \phi_{1}^{P}> + <\Phi_{2}^{*u}, - L_{u} \Phi_{2}^{P}>\right.$$

$$\left. + <\Sigma_{ru}, \phi_{1}^{P}> + <\Sigma_{ru}, \Phi_{2}^{P}>\right) \tag{71}$$

which is exact. With $P \rightarrow u$ and dropping u for simplicity, we get

$$P = \frac{1}{R_1 + R_2} \left(<\Phi_{2}^{*}, - H\Phi_{1}> + \frac{1}{\partial c} <\Phi_{u}^{*}, \delta F_{u} \Phi_{1}> + <\Sigma_{ru}, \Phi_{1}> + R_2 P_{uncoup}^{(2)}\right) \tag{72}$$

where $P_{uncoup}^{(2)}$ is given by Eq. (58). One notices that the adjoint flux of the second part (and the system), Φ_{2}^{*}, is used, rather than Φ_{1}^{*}, as the weighting function in the first two terms of Eq. (72). Equation (72) avoids direct evaluation of Φ_{1}^{P} and $\delta\phi_{1}$ used in evaluating $P_{jx,coup}^{(2)}$, when P is evaluated from Eq. (57). If Eq. (57) is to be used, this direct evaluation can also be avoided if we notice that

$$\delta R = \delta R_1 + \delta R_2 \tag{73}$$

where δR_1 is given by

$$\delta R_1 = \langle \phi_1^P, -\delta H^* \, {}^*\phi_1^u \rangle + \langle \delta \Sigma_r, \phi_1^P \rangle \ . \tag{74}$$

δR_2 is given by Eq. (49). Equating the R.H.S. of Eq. (73) to the R.H.S. of Eq. (70) and with some manipulation we get

$$\langle (\Phi_2^{*u} - \phi_1^{*u}), -\delta H \, \Phi_1^P \rangle = \langle \Phi_2^{*u}, F_u \, \delta \, \phi_1 \rangle \tag{75}$$

which is exact. With $P \to u$ and dropping u for simplicity, we get

$$\langle (\Phi_2^* - \phi_1^*), -\delta H \, \Phi_1 \rangle = \langle \Phi_2^* , F \, \delta \, \phi_1 \rangle \tag{76}$$

i.e. weighting the perturbed operator H with the difference between the adjoint fluxes ϕ_1^* and Φ_2^* gives the coupling term which is needed in direct evaluation.

In the following, the application of the method of separation discussed in this section to a hybrid system including fissionable material is given. The sensitivity theory cited here is used to demonstrate the effectiveness of this method.

III. Application of the Proposed Methodology

III.1. Results for Different Separation Schemes

The method of separation has been applied to the Laser Driven Fusion-Fission Hybrid Reactor SOLASE-H.[16-17] A one-dimensional spherical geometry scheme has been adopted in carrying out the calculation with the neutron transport code "ANISN"[24] using discrete ordinate S_N method.

The schematic diagram describing the blanket is shown in Fig. (1). The fuel zone consists of ThO_2 fuel assemblies to breed U-233 from the Th(n,γ) reaction. Besides the (n ,$\nu\sigma_f$) reaction in Th to multiply neutrons in the blanket, a Pb front zone has been utilized to enhance the neutron population through (n,2n) reactions. This will increase the uranium breeding ratio (UBR)[25] and the tritium breeding ratio (TBR).[25] Breeding tritium takes place in 3 zones (zone #4, #8 and #10). The D-T neutron source (14.1 MeV neutron) is localized in a zone of 0.5 cm radius at the center and a vacuum cavity of 499.5 cm thickness separates the D-T pellets from the blanket. A reflector zone of Pb+C is located behind the fuel zone to reflect neutrons, hence, increase the value of UBR and TBR.

First, the equation describing the system, Eq. (1), has been solved without performing the separation technique (once-through calculation) using 25 neutron groups and P_3S_4 approximation; 25-g (P_3S_4). The separation method is then applied using 25-g (P_3S_4) in solving the first and second part (Eq. (8) and Eq. (9), respectively). The results of these calculations are shown on Table (1) where summations of integrated parameters from both parts are given. These summations are to be compared with the once-through results. As shown, the results of the once-through calculations and the separation method calculations are the same in both cases, except for the round off error results from preparing the fission source data to the second part. The UBR and the TBR are in agreement in both cases within - 0.41% and + 0.61%, respectively. One should notice the neutron balance represented by the sum of absorption and leakage and the sum of the external source (n,2n), (n,3n) and (n,$\nu\sigma_f$) reactions. As shown in

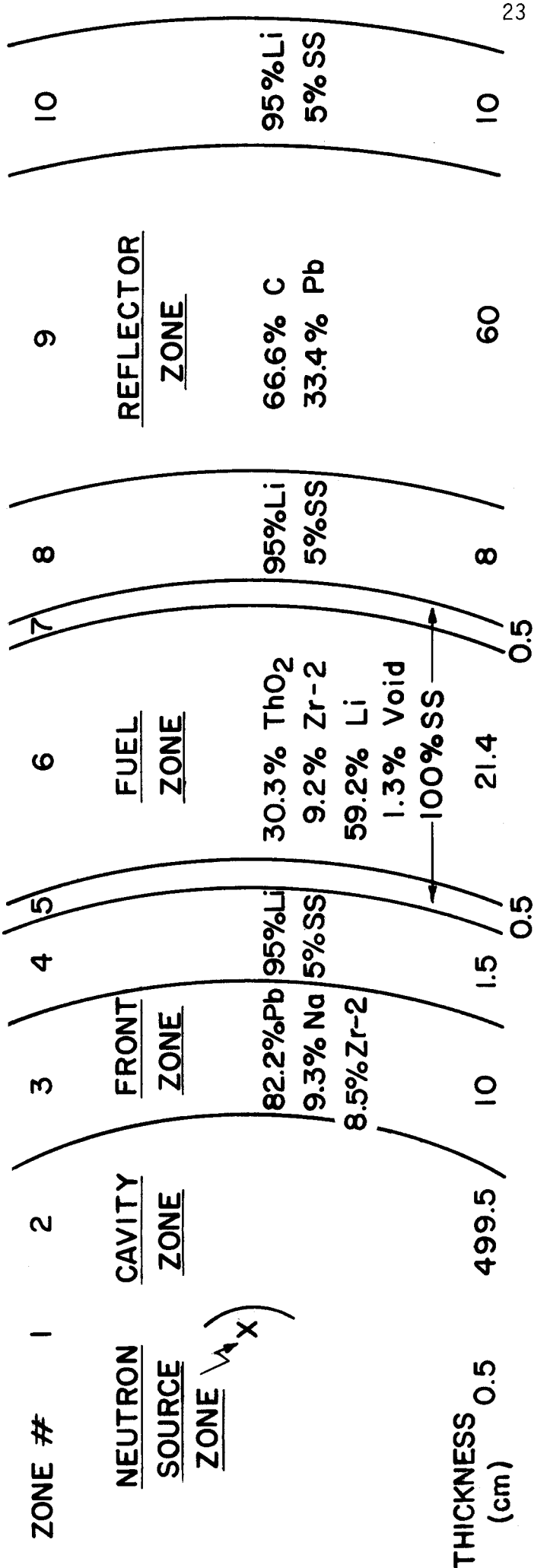

Fig. (1) SCHEMATIC REPRESENTATION IN SPHERICAL GEOMETRY OF THE SOLASE-H BLANKET

Table (1)[+] Once Through and Separation Method Results Using the Same Approximation; 25-g(P_3S_4)

Parameters \ Case	1st Part (Φ_1) 25-g (P_3S_4) Col. #1	2nd Part (Φ_2) 25-g (P_3S_4) 2	Sum Col. 1+2 3	Once-Through 25-g (P_3S_4) 4
Leakage	7.6992-3	3.1777-4	8.0169-3	7.9788-3
Absorption	1.7019	7.9716-2	1.7816	1.7817
Sum (neutron population)	1.7096	8.0034-2	1.7896	1.7896
(n,2n)+(n,3n)	7.0960-1	1.6360-4	7.0976-1	7.0976-1
(n,$\nu\sigma_f$)	0(7.8043-2)	1.8274-3	7.9869-2	7.9869-2
External Source	1	7.8043-2	1	1
Sum[source+(n,2n) (n,3n)+(n,$\nu\sigma_f$)]	1.7096	8.0034-2	1.7896	1.7896
TBR(Li-6) Zone 4	9.8813-2	3.7917-3	1.0260-1	1.0420-1
Zone 8	4.0139-1	2.1769-2	4.2316-1	4.1606-1
Zone 10	7.2363-2	4.0052-3	7.6368-2	7.8075-2
System	5.7257-1	2.9566-2	6.0214-1	5.9833-1
TBR (Li-7) Zone 4	1.2975-2	4.8124-5	1.3023-2	1.3023-2
Zone 8	1.3864-2	1.0359-4	1.3967-2	1.3967-2
Zone 10	9.1570-5	1.9094-7	9.1760-5	9.2546-5
System	2.6929-2	1.5191-4	2.7081-2	2.7082-2
Total TBR	5.9949-1	2.9618-3	6.2922-1	6.2541-1
UBR(Th(n,γ))	8.8858-1	4.1281-2	9.2986-1	9.3386-1

+ All the results are given per D-T neutron

Table (1), the external source to the 2nd part is the fission source evaluated from ϕ_1 . In this blanket, most of the tritium is due to $^6Li(n,\alpha)$ reactions particularly in zone 8 where thermalized neutrons are reflected back from the Pb+C zone. We have used the same order of approximations [25-g (P_3S_4)] in both parts to show that most contributions to the total results are due to the first part where the highly energetic external neutron source is considered. This shows the necessity of performing the first part calculation with more details if better results are required.

To compare different levels of approximations within each part of the separation technique, we have also analyzed the same blanket using a 46-g (P_3S_4) analysis of the first part (related to the external source neutrons) and 25-g (P_3S_4), 25-g (P_1S_2), and 16-g (P_1S_2) approximations for the second part (related to subsequent fission neutron generations). The results are presented in Table (2).

Comparing the first column in Table (1) and Table (2), we find that the tritium production rate from $^6Li(n,\alpha)$ reactions has increased from 0.57 to 0.62 ($\sim$ 8%) when 46 energy groups are used in solving the first part. This increase is due to the use of a finer group structure in the low energy range where this reaction rate is high. Because of the competition between fissile fuel and tritium production, the UBR is reduced from 0.886 to 0.822 ($\sim$ 8%). The fission source to the second part also decreases to 0.0777 compared to 0.078.

When the second part is evaluated using 25-g(P_3S_4) approximation, all the integrated results are slightly smaller than the corresponding values if 25-g (P_3S_4) approximation is used in solving the first part.

Table (2)+ Various Integrated Parameters for Different Approximations Used in the Separation Method

Parameter \ Case	1st Part (ϕ_1) 46-g (P_3S_4)	2nd Part (ϕ_2) 25-g (P_3S_4)	Sum Col. 1+2	2nd Part (Φ_2) 25-g (P_1S_2)	Sum Col. 1+4	2nd Part (ϕ_2) 16-g (P_1S_2)	Sum Col. 1+6
	Col. #1	2	3	4	5	6	7
Leakage	6.9956-3	2.9946-4	7.2201-3	2.6235-4	7.1829-3	3.0732-4	7.2279-3
Absorption	1.7031	7.7091-2	1.1802	7.7052-2	1.7801	8.0453-2	1.7835
Sum (neutron population)	1.7100	7.7390-2	1.7874	7.7314-2	1.7873	8.0760-2	1.7908
(n,2n),(n,3n)	7.0960-1	1.7866-4	7.0978-1	1.8043-4	7.0978-1	2.7582-3	7.1236-1
$n,\nu\sigma_f$	0	1.6872-3	7.8391-2	1.6144-3	7.8316-2	2.2789-3	7.8981-2
External Source	1	7.6702-2	1	7.6702-2	1	7.6702-2	1
Source+(n,2n) (n,3n)+$(n,\nu\sigma_f)$	1.7096	7.8570-2	1.7882	7.9496-2	1.7881	8.1739-2	1.7913
TBR(Li-6) Zone 4	8.8043-2	3.3683-3	9.1411-2	3.6640-3	9.1707-2	3.6551-3	9.1698-2
Zone 8	4.4648-1	2.0988-2	4.6747-1	2.1419-2	4.6789-1	2.2652-2	4.6913-1
Zone 10	8.3050-2	3.8787-3	8.6878-2	3.8061-3	8.6856-2	4.2487-3	8.7298-2
System	6.1757-1	2.8499-2	6.4607	2.8889-2	6.4646	3.0556-2	6.4813-1
TBR(Li-7) Zone 4	1.2957-2	4.6910-5	1.3004-8	4.8064-5	1.3005-2	1.4328-4	1.3100-2
Zone 8	1.3849-2	1.0140-4	1.3950-2	1.1927-4	1.3968-2	4.2475-4	1.4274-2
Zone 10	9.1695-5	1.9206-7	9.1887-5	5.2957-8	9.1747-5	4.7326-7	9.2168-5
System	2.6897-2	1.4850-4	9.7046-2	1.6739-4	2.7064-2	5.6850-4	2.7466-2
Total TBR	6.4447-1	2.8647-2	6.7312-1	2.9057-2	6.7353-1	3.1124-2	6.7559-1
UBR (Th(n,γ))	8.2224-1	4.0070-2	8.6231-1	3.9806-2	8.6204-1	3.9652-2	8.6189-1

+ All the results are given per D-T neutron. Calculations are done using the one-dimensional code, ANISN.

This is partly due to the overestimated value of the fission source from the first part evaluated with 25-g(P_3S_4) approximation. Consequently, the TBR is larger and the UBR is lower.

When solving the second part with 25-g(P_1S_2), i.e. low order of scattering, the leakage in this part has decreased compared to the corresponding value with 25-(P_3S_4) approximation. This is because we considered more neutrons to be scattered back when encountering collisions and theseget a chance to reenter the system. Due to this less restrictive treatment for anisotropy the TBR has increased and consequently, the UBR has decreased (compare cols. 4 and 2, Table (2)).

The results show slight decrease in UBR and slight increases in TBR when the second part is solved with 16-g(P_1S_2) approximation (compare cols. 6 and 2, Table (3)). Neutron leakage, total absorption and total neutron population are larger compared to the 25-g(P_3S_4) case. The differences in the 16-g(P_1S_2) evaluation compared to the 25-(P_3S_4) evaluation aremore pronounced, although small, than the case when 25-g(P_1S_2) and 25-g(P_3S_4) evaluations are compared. In this blanket, the results are more sensitive to the number of neutron energy groups than the order of scattering.

In general, the results show slight decreases in the UBR and slight increases in the TBR when the second part of the problem is solved using lower order P_L-S_N and smaller numbers of groups. The maximum error in the contribution of fission produced neutrons to all reaction rates and to leakage is about 9%, but the largest error in the total reaction rates or the leakage is less than 1%. Thus, the separation method is clearly advantageous to use with a low order treatment of the second and subsequent neutron generations (all fission produced neutrons).

III.2. Evaluation of the Total Result Using the Adjoint Fluxes

One of the total integrated reaction rates of interest in a hybrid blanket is the fissile fuel production rate. In the blanket outlined on Fig. (1), this is the total number of U-233 atoms produced from Th(n,γ) reactions per D-T neutron, UBR. Rather than using the forward flux as in the previous section, UBR can be evaluated using the adjoint fluxes evaluated for the separated problem as given by Eq. (15) and Eq. (17). In Table (3), we give these evaluations with the corresponding percentage differences. From this table, using the adjoint fluxes underestimates R_1 and R_2 and consequently R. This is more pronounced in the first part where the external source (14.1 MeV) is localized. However, the difference between forward and adjoint calculations for the second part is not significant since the fission source is distributed throughout the fuel zone in the blanket and with average energy much lower than the energy of the external source. Since $R_2 << R_1$ in the blanket studied, the adjoint calculation with the lower number of groups and scattering order can be used to evaluate R_2 while the forward calculation with larger group number and order of scattering can be used to evaluate R_1. This will add more reduction in the calculational cost since evaluating Φ_2^* is less expensive than evaluating Φ_2 as will be shown in Section III.5.

III.3. The Sensitivity Analysis

In this section, we discuss the application of sensitivity analysis to a hybrid system and we do so by applying the theory to demonstrate the insensitivity of the main results to the scattering order and number of groups used for the second and subsequent neutron generations (the second part of the flux evaluated in the separation procedure).

Table (3) Evaluation of the Uranium Breeding Ratio, R = UBR,
With the Forward and Adjoint Fluxes of the Two Parts

	R_1			R_2		$R = R_1 + R_2$	
Expression Used	46-g (P_3S_4)	25-g (P_3S_4)	Expression Used	46-25$^+$ P_3S_4-P_3S_4	25-25$^+$ P_3S_4-P_3S_4	46-25$^+$ P_3S_4-P_3S_4	25-25$^+$ P_3S_4-P_3S_4
$\langle\Sigma_{Th}(n,\gamma),\Phi_1\rangle$	0.8222	0.8886	$\langle\Sigma_{Th}(n,\gamma),\Phi_2\rangle$	0.0401	0.0413	0.8623	0.9299
$\langle\Phi_1^*,S\rangle$	0.7999	0.8593	$\langle\Phi_2^*,F\Phi_1\rangle$	0.0399	0.0411	0.8398	0.9004
% Difference*	-2.7	-3.3	% Difference	-0.49	-0.48	-3.47	-3.17

+ $\left.\begin{matrix}46\text{-}25\\ P_3S_4\text{-}P_3S_4\end{matrix}\right]$ means the first part is evaluated with 46-g (P_3S_4) approximation and the second part is evaluated with 25-g(P_3S_4)

* $\dfrac{\langle\Phi_1^*,S\rangle-\langle\Sigma_{Th}(n,\gamma),\Phi_1\rangle}{\langle\sigma_{Th}(n,\gamma),\Phi_1\rangle} \times 100$

To identify which elements have a large sensitivity coefficient for fissile fuel production, the sensitivity coefficient, $P_{jx}^{(1)}$ is evaluated for each element present in the blanket using the 25-g (P_3S_4) approximation. The cross section type perturbed is the total cross section. This perturbation is considered to occur by zone as well as the entire blanket. Eq. (38), with Φ_1 and Φ_1^*, is used to evaluate $P_{jx}^{(1)}$ and four terms are considered in expressing scattering. The last term in Eq. (38) is excluded. R_1, as given by Table (1), is 0.8858 in this case. It turns out that Pb and ^6Li have the highest values of $P_{jx}^{(1)}$ (excluding Th).

The uncoupling sensitivity coefficient, $P_{uncoup}^{(2)}$, given by Eq. (60) (excluding the last term), has also been evaluated with 25-g (P_3S_4) approximation with R_2=0.0413. These two coefficients are given in columns 3 and 4, respectively, in Tables (4) and (5) for Pb and ^6Li. The sensitivity coefficient, P, using the once-through (no separation) calculation using 25-g(P_3S_4) is calculated and tabulated in column 7 using Eq. (38) with R=0.933855.

The predicted percentage change in R_1 and R_2 compared to P_3 calculations, when different numbers of terms are considered in expressing scattering, have been evaluated for $P_{jx}^{(1)}$ and $P_{jx,uncoup}^{(2)}$. The entries P_0, P_1 and P_2 in Table (4) and Table (5) are these predictions where one term, two terms and three terms are considered for scattering. The entry denoted "sens." is the corresponding sensitivity coefficient when P_3 approximation is used.

One notices from these tables that the predicted percentage change in P_3 calculation for both $P_{jx}^{(1)}$ and $P_{jx,uncoup}^{(2)}$ increases when lower numbers of terms are retained in the scattering cross section-Legendre

Table (4) Relative Sensitivity Coefficient of Pb for Pb(n,tot) Cross Section and the Predicted Percentage Changes From P_3 Calculation. 1% Perturbation is Assumed in P(n,tot) Cross Section.

Zone	Predicted % Change From P_3	$P^{(1)}$ $\frac{\langle \phi_1^*, -\delta H \phi_1 \rangle}{R_1 \delta c}$ 25-g(P_3S_4)	$P^{(2)}_{uncoup}$ $\frac{\langle \phi_2^*, -\delta L \phi_2 \rangle}{R_2 \delta c}$ 25-g(P_3S_4)	$\frac{\langle \phi_2^*, -\delta H \phi_1 \rangle}{R \delta c}$ 25-g(P_3S_4)	P Eq. (72)	P Once Through 25-g(P_3S_4)	S_{uncoup} 25-g (P_3S_4)	$P^{(1)}$ 46-g(P_3S_4)	$P^{(2)}_{uncoup}$ 25-g (P_3S_4)	S_{uncoup} 46→25 (P_3S_4)
3	P_o	6.58	-2.92-2	5.19	5.19	5.19	6.26	6.55	-2.82-2	6.24
	P_1	2.17	1.27-2	1.77	1.77	1.77	2.07	2.15	1.22-2	2.05
	P_2	-2.83-1	1.04-3	-2.01-1	-2.01-1	-2.01-1	-2.70-1	-2.84-1	9.48-4	-2.71-1
	Sens.	2.05-1	1.97-3	1.65-1	1.65-1	1.65-1	1.96-1	2.08-1	1.81-3	1.98-1
9	P_o	3.80-1	2.49-1	3.87-1	3.98-1	3.98-1	3.74-1	3.80-1	2.39-1	3.80-1
	P_1	-1.62-2	-6.11-3	-2.196-2	-2.22-2	-2.22-2	-1.56-1	-1.43-2	-5.86-3	-1.39-2
	P_2	3.94-4	-2.05-4	8.21-4	8.12-4	8.12-4	3.67-4	3.93-4	-2.01-4	3.65-4
	Sens.	2.38-2	2.29-2	2.26-2	2.36-2	2.36-2	2.37-2	2.82-2	2.27-2	2.79-2
System	P_o	6.96	2.20-1	5.57	5.58	5.58	6.66	6.93	2.12-1	6.62
	P_1	2.15	6.59-3	1.745	1.75	1.75	2.05	2.13	6.29-3	2.03
	P_2	-2.82-1	8.38-4	-1.99-1	-1.99-1	1.99-1	-2.69-1	-2.84-1	7.47-4	2.71-1
	Sens.	2.29-1	2.49-2	1.87-1	1.88-1	1.88-1	2.19-1	2.36-1	2.45-2	2.26-1

Table (5) Relative Sensitivity Coefficient of L_i-6 for L_i-6(n,tot) Cross Section and the Predicted Percentage Changes From P_3 Calculation

Zone	Predicted % Change From P_3	$\underline{P}^{(1)}$ 25-g(P_3S_4)	$P^{(2)}_{uncoup}$ 25-g (P_3S_4)	$\frac{\langle\phi_2^*-\delta H\phi_1\rangle}{R\delta c}$	P Eq. 72	P Once-Through 25-g (P_3S_4)	S_{uncoup} 25-g (P_3S_4)	$P^{(1)}$ 46-g P_3S_4	$P^{(2)}_{uncoup}$ 25-g P_3S_4	S_{uncoup} 46→25 (P_3S_4)
4	P_6	3.80-2	-6.85-4	3.01-2	3.00-2	3.00-2	3.61-2	3.76-2	-6.78-4	3.43-2
	P_1	-1.67-4	3.22-6	6.64-4	6.64-4	6.64-4	-1.59-4	-2.39-4	2.19-6	-2.28-4
	P_2	-2.52-3	1.29-5	-1.67-3	-1.67-3	-1.67-3	-2.39-3	-2.54-3	1.16-5	-2.39-3
	Sens.	-6.96-2	-5.85-2	-6.72-2	-7.00-2	-7.09-2	-6.88-2	-6.68-2	-5.85-2	-6.64-2
g	P_o	3.27-2	2.50-2	3.62-2	3.74-2	3.74-2	3.16-2	2.96-2	2.46-2	2.94-2
	P_1	-1.82-3	-4.20-4	-2.33-3	-2.35-3	-2.35-3	-1.75-3	-1.59-3	-4.18-4	-1.54-3
	P_2	-4.39-5	3.57-5	-3.22-4	-3.21-4	-3.21-4	-4.02-5	-2.69-5	3.58-5	-2.39-5
	Sens.	-8.98-2	-1.06-1	-8.51-2	-8.92-2	-8.91-2	-9.01-2	-1.09-1	-1.06-1	-1.09-1
10	P_o	1.12-5	3.39-6	1.07-5	1.09-5	1.09-5	1.08-5	1.03-5	3.26-6	9.97-6
	P_1	-1.95-6	-1.99-7	-1.88-6	-1.89-6	-1.89-6	-1.86-6	-2.04-6	-1.96-7	-1.95-6
	P_2	1.44-7	7.08-9	1.40-7	1.41-7	1.41-7	1.37-7	1.59-7	7.27-9	1.52-7
	Sens.	-7.05-7	-5.90-7	-7.03-7	-7.27-7	-7.26-7	-6.97-7	-3.37-7	-5.71-7	-3.48-7
System	P_o	7.06-2	2.43-2	6.63-2	6.74-2	6.74-2	6.83-2	6.73-2	2.39-2	6.53-2
	P_1	-1.99-3	-4.17-4	-1.67-3	-1.69-3	-1.69-3	-1.91-3	-1.84-3	-4.16-4	-1.77-3
	P_2	-2.57-3	4.86-5	-1.99-3	-1.99-3	-1.99-3	-2.44-3	-2.57-3	4.74-5	-2.45-3
	Sens.	-1.59-1	-1.65-1	-1.52-1	-1.60-1	-1.60-1	-1.59-1	-1.76-1	-1.64-1	-1.75-1

expansion and that these predictions are always much less in the second part than the corresponding values in the first part. This is true for all zones. This shows that if lower orders of scattering are used in the second part, the contribution R_2 to the total R will not significantly change. This is due to the fact that the fission source to the second part is isotropic in nature, and hence, using higher orders for scattering is not needed.

Evaluations of $P_{jx}^{(1)}$ with 46-g (P_3S_4) in the first part and $P_{jx,uncoup}^{(2)}$ with 25-g(P_3S_4) in the second part are also included in Tables (4) and (5). The general conclusions mentioned above are clear in this case.

III.4. Evaluation of the Total Relative Sensitivity Coefficient

One notices from Tables (4) and (5) that the summation, $S_{jx,uncoup}$, given by

$$S_{jx,uncoup} = \frac{R_1 P_{jx}^{(1)}}{R} + \frac{R_2 P_{jx,uncoup}^{(2)}}{R} \qquad (77)$$

and introduced in column 8 is not the same as the total sensitivity coefficient, P_{jx}, given in column 7. The difference is the contribution to P_{jx} from the coupling between the two parts of solution. As mentioned in Sec. II.3-3, this coupling coefficient requires direct evaluation of $\delta\phi$, which can be avoided if Eq. (72) is used. The first term in this equation has been calculated using the 25-g (P_3S_4) approximation and the result is given in column 5 in Table (4) and table (5). The result of calculating Eq. (72) is introduced in column 6 of these tables. The second and third terms in Eq. (72) are excluded since $\delta F=0$ and $x \neq \Sigma_{Th}(n,\gamma)$.

The value of P_{jx} evaluated from this equation and the corresponding value evaluated from the once-through calculation should give the same results as shown in Table (4) and Table (5). The contribution to P_{jx} from the coupling term, denoted $D_{jx,coup}$, is the difference between the entries in column (6) and column (8). It can be shown that $D_{jx,coup}$ is given by

$$D_{jx,coup} = \frac{1}{R\delta c} \langle \phi_2^*, F_{jx}\, \delta\phi_1 \rangle \ . \qquad (78)$$

Direct evaluation of $D_{jx,coup}$ has been performed for Pb assuming 1% change in the total cross section in different zones as well as in the total blanket. Adding $D_{jx,coup}$ to $S_{jx,uncoup}$ gives P_{jx} which should equal to P_{jx} evaluated from Eq. (72). The results of these calcuations are given in Table (6). It is clear, then, that evaluation of the sensitivity coefficient using Eq. (72) is recommended when the separation method is used since direct evaluation for the perturbed system is avoided.

III.5. Cost Reduction Using the Separation Method

The costs of the several cases presented in Tables (1) and (2) have been estimated based on the CPU time for overnight runs using the UNIVAC 1110 computer of the University of Wisconsin. These costs are presented in Table (7). A reduction of ~ 46% in the cost is obtained if the first part is evaluated with 46-g (P_3S_4) and the second part evaluated with 25-g (P_1S_2) compared to a once-through 46-g (P_3S_4) run. The associated error in R (the UBR in this case) is ~ 0.03%. The percentage error in R for some of the different separation schemes shown in Table (1), Table (2) and Table (7) are given in Table (8) with the corresponding reduction in the cost.

Table (6) Relative Sensitivity Coefficient for Pb With Pb(n,tot) Cross Section Increased 1% by Zone and System. 25-g (P_3S_4) Calculations for the First and the Second Part Are Used

Zone	$P^{(1)}$ $\frac{\langle \phi_1^*, -\delta H \phi_1 \rangle}{R_1 \delta c}$	$P^{(2)}_{uncoup}$ $\frac{\langle \phi_2^*, \delta L \phi_2 \rangle}{R_2 \delta c}$	$\frac{\langle \phi_2^*, -\delta H \phi_1 \rangle}{R \delta c}$	P Eq. (72)	S_{uncoup}	D_{coup}	$D_{coup} + S_{coup}$
3	2.05-1	1.97-3	1.65-1	1.65-1	1.96-1	-3.10-2	1.65-1
9	2.38-2	2.29-2	2.26-2	2.36-2	2.37-2	-1.00-4	2.36-2
System	2.29-1	2.49-2	1.87-1	1.88-1	2.19-1	-3.10-2	1.88-1

Table (7) Cost of the Computations in Dollars Using the Separation Method for Different Cases Based on the CPU Time of UNIVAC 1110 Machine (Overnight Rates)

Function Evaluation	First-Part ϕ_1 46-g P_3S_4	Second Part ϕ_2 46-g P_3S_4	$C_s^{(*)}$ 46-46 P_3S_4-P_3S_4	Second Part 25-g P_3S_4	$C_s^{(*)}$ 46-25 P_3S_4-P_3S_4	Second Part 25-g P_1S_2	$C_s^{(*)}$ 46-25 P_3S_4-P_1S_2	Second Part 16-g P_3S_4	$C_s^{(*)}$ 46-16 P_3S_4-P_3S_4	Second Part 16-g P_1S_2	$C_s^{(*)}$ 46-16 P_3S_4-P_1S_2	Total Once-Through 46-g P_3S_4
ϕ	18.96	27.90	46.86	10.48	29.44	6.40	25.36	8.66	27.62	5.29	24.25	47
ϕ^*	16.63			12.60								
%R(+)					37%		46%		41%		48%	

	First-Part 25-g $P_3^4{}_3$	Second Part 25-g P_3S_4	$C_s^{(*)}$ 25-25 P_3S_4-P_3S_4	Total Once-Through 25-g P_3S_4
ϕ	8.83	13.00	21.83	~ 22.3
ϕ^*	7.83	12.6		12.60

(*) C_s: total cost with method of separation
46-46 / P_3S_4-P_3S_4] means that the first part is evaluated with 46-g(P_3S_4) and the 2nd part with 46-g(P_3S_4)

(+) %R: percentage of reduction in cost compared to the 46-g(P_3S_4) once through run

Table (8) Percentage Error in the U-233 Breeding Ratio for Different Methods of Separation and the Corresponding Reduction in Cost(*)

	46-25 P_3S_4-P_3S_4	46-25 P_3S_4-P_1S_2	46-16 P_3S_4-P_1S_2
% Error in R	-0.0001%	-0.03%	-0.05%
% Reduction in Cost	37%	46%	48%

* Based on R(UBR) ≡ 0.8623 for 46-g (P_3S_4) once-through run

From Table (8), it is clear that the error in R is significantly small compared to the gain obtained by reducing the computational cost of the different approximations used in the method of separation.

III.6. Application of the Separation Method to Burnup Calculations in Hybrid Blankets

Due to fissioning of the bred fissile fuel in a hybrid blanket, the fissile fuel production rate, UBR, will change with time. The value of R with time can be approximated from the expression

$$UBR(t) \cong UBR_1^o + \sum_{j=fiss} \langle \Phi_2^{*o}, F_j, (t)\Phi_1^o \rangle \qquad (79)$$

where

$$UBR_1^o = \langle \Sigma_{Th}(n,\gamma), \phi_1^o \rangle \ . \qquad (80)$$

In the above equations, the forward flux, ϕ_1^o, at the beginning of life is used to evaluate UBR_1^o which is assumed constant. The adjoint flux of the second part (and the system) at the beginning of life is used to evaluate the contribution to UBR from all fissile materials present in the hybrid blanket at the subsequent time. Knowledge of the variation of the atomic densities of these fissionable materials is required. Eq. (19) can be used to identify which fissile element contributes most to UBR as a function of time.

The results of these calculations for the blanket used in this study is presented in Table (9). UBR is the total uranium breeding ratio at time t, and UBR_1 and UBR_2 are the corresponding contributions to UBR from the first and second part of the separated solution for the neutron flux.

Table (9) Value of UBR vs. Time Using the Separation Method and Eq. (79)

Time	$UBR_1^{o(*)}$	$UBR_{2j}^{(+)}$ (j=U-233)	$UBR_{2j}^{(+)}$ (j=Th-233)	Total UBR_2	Total UBR	UBR From Direct Calculations$^{(**)}$	% Error
t = 0	0.8886	0	0.0413	0.0413	0.9299	0.9299	0
t = 0.7 yr	0.8886	0.0701	0.0406	0.1107	0.9673	0.9673	+3.31
t = 1.4 yr	0.8886	0.1374	0.0401	0.1775	1.0026	1.0026	+6.32

(*) From Eq. (80) 25g-(P_3S_4)

(+) From Eq. (19) 25g-(P_3S_4)

(**) From 25-g (P_3S_4). The variation in ϕ_1 and ϕ_2 with time is considered.

we see that the contribution to UBR_2 comes primarily from fissioning in U-233 and this increases with time as the U-233 builds up. However, the contribution from Th-232 to UBR_2 decreases slightly with time. In fact, it has been shown from the neutronic study using direction calculations[17] that the fission rate in Th is almost constant. The value of $Th(n,\nu\sigma_f)$ reactions per D-T neutron is 0.0799, 0.0822 and 0.0848 at t=0, t=0.7 and t=1.4 yr, respectively. The corresponding values for U-233 fission rates per fusion event are 0.0, 0.1474 and 0.3048, for t=0, 0.2 yr and 1.4 yr, respectively.

IV. Conclusions

The method of separation discussed in this study can be applied to fusion-fission hybrid systems with a substantial reduction in the computational cost (up to 48%) and small errors in the predicted integrated results (< 1%). The method in discrete ordinates is similar to the procedure used by Kotov and et al.[15] where the fusion neutron behavior (the first part) is treated using Monte Carlo techniques. Indeed, any transport method can be used. Here we use high order scattering and discrete ordinates for the zeroth generation (source neutrons) and low order S_N (or diffusion theory) for subsequent neutron generations due to fission.

The theoretical model developed here to evaluate the sensitivity coefficients for both parts of the solution and for the system as a whole can be used to demonstrate the applicability of using lower orders of scattering when solving the second part. In particular, it can be used to identify which element in the system has the greatest impact on the total result R (e.g. fissile fuel production). The sensitivity technique can, of course, be applied to any parameter.

The adjoint fluxes of the two parts can be used to evaluate a particular reaction rate. In a hybrid when the reaction rate is the uranium breeding ratio, we have shown that the variation of UBR due to fissioning in the bred fissile fuel can be accounted for using the beginning of life values of the adjoint flux of the second part and the forward flux of the first part as an approximating function. An error of the order of 6% in R after 1.4 yr of operation is obtained for the hybrid blanket[17] used in this study.

References

1. "Two Component Torus (TCT), Joint Conceptual Design Study", Vols. I-III, Westinghouse Corporation, Pittsburgh, PA (June 1974).

2. "UWMAK-III, A Noncircular Tokamak Power Reactor Design", UWFDM-150, Fusion Feasibility Study Group, Fusion Technology Program, The University of Wisconsin (July 1976).

3. R.W. Conn et al., "SOLASE, A Conceptual Laser Fusion Reactor Design", UWFDM-220, Fusion Research Program, Nuclear Engineering Department, The University of Wisconsin (Dec. 1977).

4. M.M. El-Wakil, "Nuclear Energy Conversion", International Textbook Co. (1971).

5. D.J. Jassby, "Beam Driven Tokamak Fusion-Fission Hybrid", DCTR Fusion-Fission Energy Systems Review Meeting, Dec. 3-4, 1974, Germantown, MD, ERDA-r (1974).

6. "Proceedings of US-USSR Symposium on Fusion-Fission Reactors", Hosted by Lawrence Livermore Laboratory, July 13-16 (1976), CONF-760733, July (1976).

7. "Proceedings of the Secon d Fusion-Fission Energy Systems Review Meeting", U.S. Dept. of Energy, Nov. 2-3 (1977), Washington, D.C., Vol. I-II, CONF-771155, July (1978).

8. Weinberg, A.M. and Wigner, E.P., "The Physical Theory of Neutron Chain Reactors", Univ. of Chicago Press (1958).

9. Davison, B., "Neutron Transport Theory", Oxford University Press (1957).

10. Bell, G.S. and Glasstone, S., "Nuclear Reactor Theory", Van Nostrand Reinhold Co. (1970).

11. Goertzel, G. and M.H. Kalos, "Monte Carlo Methods in Transport Problems", in Prog. Nucl. Energy, Series I, Vol. II, Pergamon Press (1958).

12. Cashwell, E.D. and Everett, C.J., "The Monte Carlo Method for Random Walk Problems", Pergamon Press (1959).

13. Spanier, J. and Celbard, E.M., "The Monte Carlo Principles and Neutron Transport Problems", Addison-Wesley Publishing Co. (1969).

14. Butler, M.K. and Cook, J.M., "Computing Methods in Reactor Physics" (1968).

15. Kotov, V.V., Maynard, C.W., Markovskii, D.V., and Shatolov, G.E., "Analysis of the Sensitivity of Hybrid Reactors to Nuclear Data", I.V. Kurchatov Institute of Atomic Energy, USSR, IAE-2817 (1977).

16. Conn, R.W., Abdel-Khalik, S.I., Moses, G.A., and Youssef, M.Z., "The SOLASE-H Laser Fusion Hybrid", Trans. Am. Nucl. Soc., 30 (1978), p. 58.

17. Youssef, M.Z., Conn, R.W., and Moses, G.A., "Blanket Neutronics Studies for the SOLASE-H Hybrid Reactor", a paper to be published in the ANS annual meeting, June 3-8, 1979, Atlanta, GA (1979).

18. Ragheb, M.H., Youssef, M.Z., Abdel-Khalik, S.I., and Maynard, C.W., "Three-Dimensional Lattice Calculations for a Laser-Fusion Fissile-Enrichment Fuel Factory", Trans. Am. Nucl. Soc. 30 (1978), p. 59. Also, UWFDM-263 and 264, The University of Wisconsin, October (1978).

19. Lewis, J., "Importance - The Adjoint Function," Chapter 2, Pergamon Press, New York (1965).

20. Bartine, D.E., Oblow, E.M., and Mynatt, F.R., "Radiation-Transport Cross-Section Sensitivity Analysis - A General Approach Illustrated for a Thermonuclear Source in Air", Nucl. Sci. Eng. 55, 147-167 (1974).

21. Oblow, E.M., "General Sensitivity Theory for Radiation Transport", ORNL-TM-4110, Oak Ridge National Lab. (1973).

22. Bartine, D.E., Mynatt, F.R., and Oblow, E.M., "SWANLAKE, A Computer Code Utilizing ANISN Transport Calculations for Cross-Section Sensitivity Analysis", ORNL-TM-3809, Oak Ridge National Lab. (1973).

23. Weisbin, C.R., Marable, J.H., Lucius, J.L., Oblow, E.M., Mynatt, F.R., Peelle, R.W., and Perey, F.G., "Application of 'FROSS' Sensitivity and Uncertainty Methodology to Fast Reactor Benchmark Analysis", ORNL/TM-5563, Oak Ridge National Lab. (1976).

24. Engle, W.W., Jr., "A User's Manual for ANISN, RISC-CCC-82, Oak Ridge National Lab. (1967).

25. UBR ≡ Number of ^{233}U Atoms Produced in the Blanket Per D-T Neutron, TBR ≡ Number of Tritium Atoms Produced in the Blanket Per D-T Neutron.