On the Boundary Conditions for Temperatures and Densities in a Divertorless Tokamak

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1. Introduction

In a divertorless tokamak the slope of the density and temperature profiles at the plasma edge are determined by the particle and energy transport processes in the interface or blanket zone defined by the (toroidal or poloidal) limiter. We present a very simple two-dimensional model that describes this transport and yields values for the profile slopes which can be used as boundary conditions in a one-dimensional transport model, e.g., the codes belonging to the WHIST- or CRIST-families.

The geometry we use is explained in Fig. 1. It is obvious that a different transport model will have to be used depending on how the mean free path for ions in the toroidal direction compares to half the toroidal circumference.

The dominant collision mechanism contributing to an appreciable change in ion momentum is charge exchange with energetic neutrals. This occurs with the frequency $n_o\langle\sigma v\rangle_{cx}$ where n_o is the neutral density, and $\langle\sigma v\rangle_{cx}$ is the charge exchange rate coefficient. Hence, the mean free path in the z-direction $\lambda_\parallel$ is:

$$\lambda_\parallel = \frac{v_{th}^i}{n_o\langle\sigma v\rangle_{cx}} \quad , \qquad (1)$$

v_{th}^i being the thermal velocity of the ions defined as $(2\ kT/m)^{1/2}$. $\langle\sigma v\rangle_{cx}$ is known as a polynomial[1] and in the aforementioned codes is numerically calculated by the subroutine SIGMAV. With $T_i = T_o = 10$ eV, $\langle\sigma v\rangle_{cx}$ is 1.5×10^{-8} cm^3 sec^{-1} and $\lambda_\parallel = 1.85 \times 10^{14}/n_o$.

The average distance to the limiter is πR (half the toroidal circumference). For NUWMAK this is 1.6×10^3 cm, and thus the longitudinal ion transport will be collisionless except for extremely high neutral densities (e.g. during gas puffing).

2. Collisionless Parallel Transport ($\lambda_{\parallel} > \pi R$)

In this case the ions can be assumed to stream freely towards the limiter and their parallel flux is independent of z: $\partial\Gamma_{\parallel}/\partial z = 0$. In the blanket we have

$$\nabla \cdot \Gamma = \frac{1}{r}\frac{\partial}{\partial r}(r\Gamma_{\perp}) = \text{sources - sinks} .$$

There are no ion sources in the blanket, but per cm^3 $\Gamma_{\parallel}/2\pi R$ particles are lost. Hence, the continuity equation in the blanket becomes

$$\frac{1}{r}\frac{\partial}{\partial r}(r\Gamma_{\perp}) = -\frac{\Gamma_{\parallel}}{2\pi R} \qquad (3)$$

where $\Gamma_{\parallel}$ is given by

$$\Gamma_{\parallel} = n_o \int_{-\infty}^{+\infty} dv_x \int_{-\infty}^{+\infty} dv_y \int_{0}^{\infty} dv_z\, v_z\, f(v) = \langle n_o v \rangle = n_o\, v_{th} \ . \qquad (4)$$

But also $\Gamma_{\parallel} = n_i \frac{\pi R}{\tau_{\parallel}}$,

where $\tau_{\parallel}$ is the time needed to cover the distance πR. Hence,

$$\tau_{\parallel} = \pi R/v_{th} \ . \qquad (6)$$

The continuity equation now becomes

$$\frac{1}{r}\frac{\partial}{\partial r}(r\Gamma_{\perp}) = -\frac{n_i}{2\tau_{\parallel}} \ . \qquad (7)$$

We assume Bohm diffusion in the blanket. If the plasma edge can be kept at a sufficiently low temperature (e.g. by maintaining a halo of radiating impurity ions around the plasma) the T_e - and hence D_B - depends only weakly on the radial position in the blanket compared to $n_i(r)$, and, using Fick's law, the continuity equation reduces further to

$$\frac{\partial}{\partial r}\left(r\,\frac{\partial n_i}{\partial r}\right) = \frac{n_i}{2\tau_\parallel}\,\frac{r}{D_B} \tag{8}$$

where $D_B = 6.25 \times 10^6$ T/B cm^2/sec with T in eV and the magnetic field B in Gauss. This expression can be written out as

$$r\,\frac{\partial^2 n_i}{\partial r^2} + \frac{\partial n_i}{\partial r} - \frac{n_i r}{2\tau_\parallel D_B} = 0\ . \tag{9}$$

The solution of this ordinary differential equation is a zeroth order modified Bessel function. The requirement $\lim_{r \to \infty} n_i(r) = 0$ leaves only

$$n_i(r) \sim K_o\left(\frac{r}{\sqrt{2\tau_\parallel D_B}}\right) \quad \text{for } r \geq a\ .$$

Matching $n_i(r)$ at the interface $r = a$, i.e. requiring $n_i(a)$ to have a prescribed value yields

$$n_i(r) = \frac{n_i(a)}{K_o\left(\frac{a}{\sqrt{2\tau_\parallel D_B}}\right)}\ K_o\left(\frac{r}{\sqrt{2\tau_\parallel D_B}}\right)\ . \tag{10}$$

Using $K_o'(z) = -\,K_1(z)$ we find

$$\frac{dn_i}{dr} = -\,\frac{n_i(a)}{K_o\left(\frac{a}{\sqrt{2\tau_\parallel D_B}}\right)}\ K_1\left(\frac{r}{\sqrt{2\tau_\parallel D_B}}\right)\ \frac{1}{\sqrt{2\tau_\parallel D_B}}\ . \tag{11}$$

Evaluating this for $r = a$:

$$\frac{1}{n_i}\,\frac{dn_i}{dr}\Big|_{r=a} = \frac{-1}{\sqrt{2\tau_\parallel D_B}}\ \frac{K_1\left(\frac{a}{\sqrt{2\tau_\parallel D_B}}\right)}{K_o\left(\frac{a}{\sqrt{2\tau_\parallel D_B}}\right)}\ . \tag{12}$$

For large arguments, $K_1(z) \simeq K_0(z)$ and

$$\frac{1}{n_i}\frac{\partial n_i}{\partial r}\Big|_{r=a} = \frac{-1}{\sqrt{2\tau_{\parallel} D_B}} \quad . \tag{13}$$

If we now define the characteristic length for the ion density by:

$$\lambda_{n_i} = \left(- \frac{1}{n_i}\frac{d_{ni}}{dr}\Big|_{r=a}\right)^{-1} \tag{14}$$

we find:

$$\lambda_{n_i} = \sqrt{2\tau_{\parallel} D_B} \quad . \tag{15}$$

A similar reasoning can be set up for the parallel energy transport: $\partial Q_{\parallel}/\partial z = 0$. For the average parallel particle transport we have

$$\langle\Gamma_{\parallel}\rangle = \int_{-\infty}^{+\infty} dv_x \int_{-\infty}^{+\infty} dv_y \int_{0}^{+\infty} dv_z\, v_z\, f(v) = \frac{1}{\sqrt{2\pi}}\, n_i \left(\frac{kT}{m_i}\right)^{1/2} \quad . \tag{16}$$

If we assume that the energy transport in **the parallel direction is entirely** convective (adiabatic process), then

$$\langle Q_{\parallel}\rangle = \int_{-\infty}^{+\infty} dv_x \int_{-\infty}^{+\infty} dv_y \int_{0}^{\infty} dv_z \frac{mv^2}{2} v_z\, f(v) = \sqrt{\frac{2}{\pi}}\; m_i\, n_i \left(\frac{kT_i}{m_i}\right)^{3/2} \tag{17}$$

i.e.

$$Q_{\parallel} = 2kT_i\, \Gamma_{\parallel} \quad . \tag{18}$$

Hence,

$$Q_{\parallel} = 2kT_i\, n_i \frac{\pi R}{\tau_{\parallel}} \tag{19}$$

and

$$\frac{1}{r}\frac{\partial}{\partial r}(rQ_{\perp}) = - \frac{n_i\, kT_i}{\tau_{\parallel}} \, . \tag{20}$$

Writing $Q_{\perp}$ out yields a non-linear differential equation for which no easy solution seems to exist. Hence, we proceed by matching the energy flux at the interface (as opposed to matching the temperatures). If no particles or energy reach the wall, the eqs. (7) and (20) can readily be integrated to yield

$$Q_{\perp}(a) = \frac{1}{a} \int_a^{a+d} \frac{n_i \; kT_i}{\tau_{\parallel}} \, rdr \tag{21}$$

and

$$\Gamma_{\perp}(a) = \frac{1}{a} \int_a^{a+d} \frac{n_i}{2\tau_{\parallel}} \, rdr \; . \tag{22}$$

If $T_i(r)$ can be regarded as constant for $a < r < a + d$, then

$$Q_{\perp}(a) = 2 \; kT_i \; \Gamma_{\perp}(a) \tag{23}$$

and, from

$$\Gamma_{\perp}(a) = - D \frac{\partial n_i}{\partial r} \Big|_{r=a} \; ,$$

$$Q_{\perp}(a) = - 2 \; kT_i \; D \frac{\partial n_i}{\partial r} \Big|_{r=a} \; . \tag{24}$$

Continuity of the energy flux requires to lowest order

$$Q_{\perp}(a) = - n\chi \frac{\partial kT_i}{\partial r} - \frac{3}{2} kT_i \; D \frac{\partial n_i}{\partial r} \; . \tag{25}$$

Hence,

$$n_i \; \chi \frac{\partial kT_i}{\partial r} + \frac{3}{2} kT_i \; D \frac{\partial n_i}{\partial r} = 2 \; kT_i \; D \frac{\partial n_i}{\partial r} \tag{26}$$

or

$$\frac{1}{T_i} \frac{\partial T_i}{\partial r} = \frac{1}{2} \frac{D}{\chi} \frac{1}{n_i} \frac{\partial n_i}{\partial r} \tag{27}$$

i.e.

$$\frac{1}{T_i} \frac{\partial T_i}{\partial r} = - \frac{D}{2\chi} \frac{1}{\sqrt{2\tau_{\parallel} D_B}} \; . \tag{28}$$

Thus, the characteristic length for the ion temperature is given by

$$\lambda_{T_i} = \frac{2\chi}{D} \sqrt{2\tau_{\parallel} D_B} \; . \tag{29}$$

3. Collisional Parallel Transport ($\lambda_{\parallel} < \pi R$)

In this case the parallel particle transport depends on the density gradient in the z-direction:

$$\Gamma_{\parallel} = - D_{\parallel} \frac{\partial n_i}{\partial z} \quad . \tag{30}$$

We still assume Bohm diffusion in the radial direction:

$$\Gamma_{\perp} = - D_B \frac{\partial n_i}{\partial r} \quad . \tag{31}$$

The continuity equation now is $\nabla \cdot \vec{\Gamma} = 0$ because there are neither sinks nor sources. Written out this becomes

$$\frac{1}{r}\frac{\partial}{\partial r}(r\,\Gamma_{\perp}) = - \frac{\partial \Gamma_{\parallel}}{\partial z} \tag{32}$$

or

$$\frac{1}{r}\frac{\partial}{\partial r}(r\,\Gamma_{\perp}) = \frac{\partial}{\partial z}\left(D_{\parallel}\frac{\partial n_i}{\partial z}\right) \simeq D_{\parallel}\frac{\partial^2 n_i}{\partial z^2} \simeq - D_{\parallel}\frac{n}{\lambda_{\parallel}^2} \tag{33}$$

where $D_{\parallel} = \frac{1}{3}\frac{\lambda_{\parallel}^2}{\tau_{\parallel}}$ and hence $\frac{D_{\parallel}}{\lambda_{\parallel}^2} = \frac{1}{3\tau_{\parallel}}$. Writing the continuity equation out yields

$$\frac{\partial n_i}{\partial r} + r\,\frac{\partial^2 n_i}{\partial r^2} - \frac{n_i r}{3\tau_{\parallel} D_B} = 0 \ . \tag{34}$$

The solution of this ordinary differential equation again is a zeroth order modified Bessel function $K_o\left(\frac{r}{\sqrt{3\tau_{\parallel} D_B}}\right)$. In a similar way as described in the previous section we find

$$\frac{1}{n_i}\frac{\partial n_i(r)}{\partial r} = \frac{-1}{\sqrt{3\tau_{\parallel}D_B}}\frac{K_o(\ldots)}{K_o(\ldots)} = \frac{-1}{\sqrt{3\tau_{\parallel}D_B}} \qquad (35)$$

and

$$\lambda_{n_i} = \sqrt{3\tau_{\parallel}D_B} \qquad (36)$$

i.e. only a factor $\sqrt{3/2}$ larger than in the collisionless case. Similarly, we find

$$\lambda_{T_i} = \frac{2\chi}{D_\perp}\sqrt{2\tau_{\parallel}D_B} \quad .$$

<u>4. Width of the Limiter (d):</u>

The fraction of the density at the edge $n_i(a)$ is

$$f = \frac{K_o\left(\frac{a+d}{\sqrt{2\tau_{\parallel}D_B}}\right)}{K_o\left(\frac{a}{\sqrt{2\tau_{\parallel}D_B}}\right)} \quad .$$

At 1 eV the factor $\sqrt{2\tau_{\parallel}D_B} = 1.16$

and

$$f \simeq e^{\frac{d}{\sqrt{2\tau_{\parallel}D_B}}} \sqrt{\frac{a}{a+d}} \quad .$$

With d = 10 cm, $f \simeq 2 \times 10^{-4}$ which gives a sufficient protection of the first wall.

<u>5. Boundary Conditions for the Electron Temperature</u>

The electron energy transport towards the limiter is higher than for the ions because the sheath potential around the limiter causes an enhanced transport of the more energetic particles. We express this by[2]

$$Q^e_{\parallel} = 2kT_e\,\gamma_e\,\Gamma^i_{\parallel} \qquad (37)$$

with

$$\gamma_e = 1 + \ell n \left(\frac{m_i T_e}{m_e T_i}\right)^{1/4} . \quad (38)$$

This holds as long as $T_i \gtrsim T_e$. If $T_i < T_e$, then $\gamma_e = 1$ and $Q_{\parallel}^e = 2kT_i\ \Gamma_{\parallel}^i$. (38a)

If the parallel transport is collisionless, we have in analogy to eq. 18

$$Q_{\parallel}^e = 2kT_e\ \gamma_e\ \Gamma_{\parallel}^i . \quad (39)$$

Using eqs. (5-6) to eliminate the parallel ion flux we get

$$Q_{\parallel}^e = 2kT_e\ \gamma_e\ n_i \frac{\pi R}{\tau_{\parallel}} \quad (40)$$

as compared to eq. (19)

$$Q_{\parallel}^i = 2kT_i\ n_i \frac{\pi R}{\tau_{\parallel}} .$$

This shows that if $T_e \simeq T_i$, an expression for the characteristic length of the electron temperature can be obtained by replacing $\tau_{\parallel}/\gamma_e$ in the corresponding formula for the ion temperature:

$$\lambda_{T_e} = \frac{2X}{D} \sqrt{\frac{2\tau_{\parallel} D_B}{\gamma_e}} \quad (41)$$

If the parallel transport is collisional we simply substitute $D_{\parallel}$ by $\gamma_e D_{\parallel}$ (which is equivalent to $\tau_{\parallel} \rightarrow \tau_{\parallel}/\gamma_e$) and obtain:

$$\lambda_{T_i} = \frac{2X}{D_{\perp}} \sqrt{\frac{2\tau_{\parallel} D_B}{\gamma_e}} . \quad (42)$$

References

1. Freeman, R.L. and Jones, E.M., Atomic Collision Processes in Plasma Physics Experiments, Report CLM-R137, Culham 1974.

2. Mense, A.T., Ph.D. Thesis, U. of Wisconsin (1977).

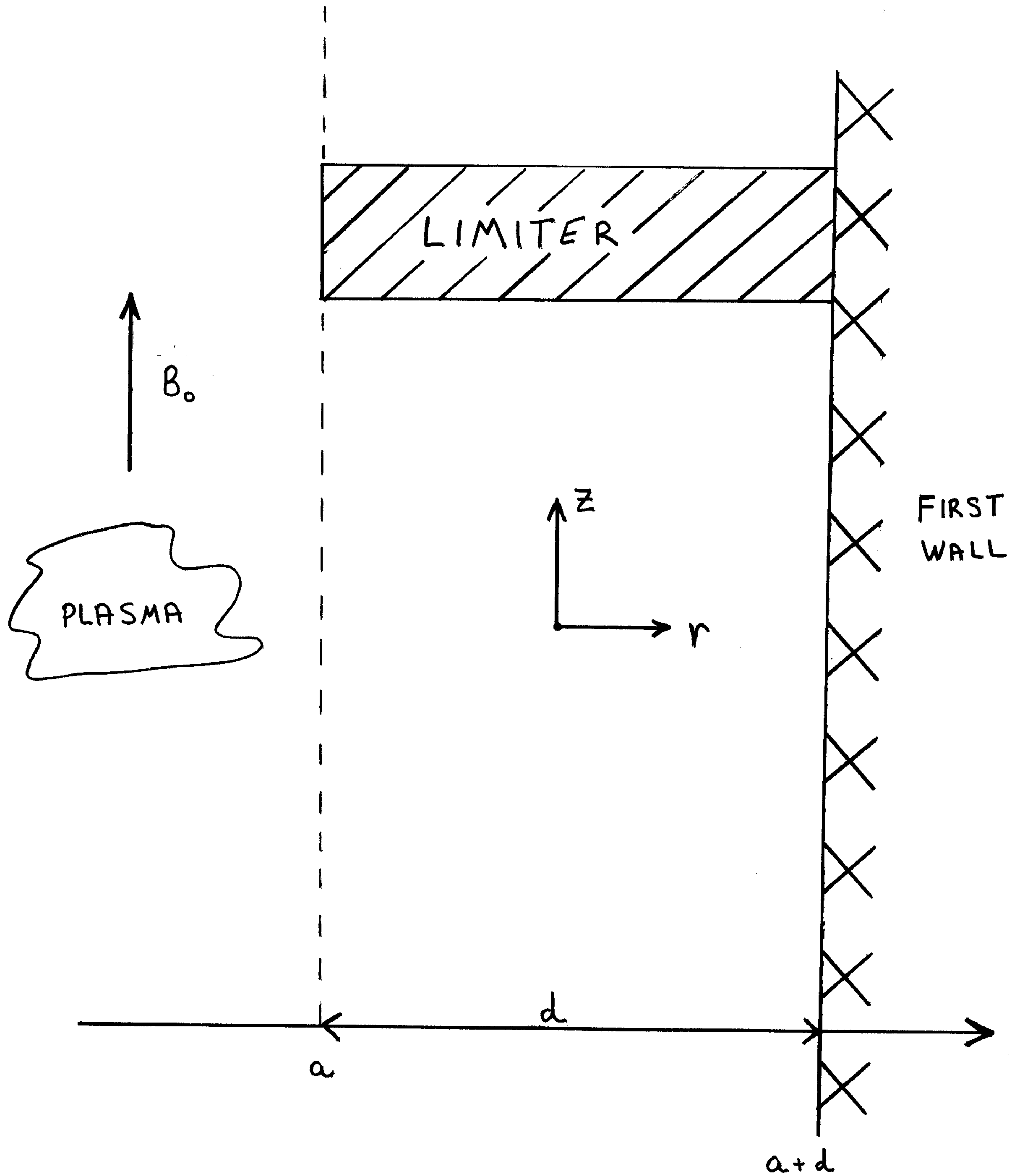

Fig. 1: Geometry of the interface area