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May 1979

UWFDM-269

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ABSTRACT

An analytic formula is derived for nonadiabatic jumps in the magnetic moment $v_\perp^2 / B$ for proton orbits in a parabolic magnetic field resembling the high-β field in the neutral sheet. The results show that only orbits with large pitch angle ($\alpha > 77°$) are adiabatic and predict a neutral sheet thickness close to 1.5 R_e. These orbits are shown to possess a near-integral resonance between their longitudinal (bounce) and transverse (gyrational) motion.

INTRODUCTION

Parabolic fields are frequently used in modelling the earth's magnetotail, and more recently in studying tearing modes in magnetic islands for fusion plasmas [Irby, Drake and Griem, 1978]. Such fields are of interest in geophysics primarily for their property of drift-free charged particle motion, as shown by Stern and Palmadesso [1975] for adiabatic orbits, and by Pellat and Schmidt [1978] for general (nonadiabatic) orbits. Stern and Palmadesso also speculate that a significant fraction of the proton orbits in the earth's plasma sheet may be nonadiabatic. A knowledge of adiabaticity is essential in estimating the degree of particle trapping [Taylor and Hastie, 1971] and for classifying orbits near the neutral sheet [Wagner, Kan and Akasofu, 1979].

In this paper we derive an analytic formula for nonadiabatic jumps in the magnetic moment $\mu_0 = v_\perp^2/B$ as a charged particle crosses the symmetry plane of a static parabolic magnetic field. Electric fields are assumed to be negligible. The treatment is similar to that employed by the author in calculating $\Delta\mu$ for cusped fields [Howard, 1971] (to be referred to as I) and for the dipole field [in Taylor and Hastie, 1971]. A large field gradient is often erroneously assumed to "cause" adiabaticity to break down. The parabolic field affords a striking refutation of this notion, since ∇B vanishes on the midplane! As we shall see, the fundamental adiabaticity parameter is a_0/R_0, where a_0 is the gyroradius formed from the total velocity and R_0 is the radius of curvature, both taken at the midplane crossing. In addition, the pitch angle will be seen to play an important role in determining adiabaticity, a fact hitherto unrecognized in magnetospheric orbital calculations.

The results show that typical (5 keV) proton orbits in the neutral sheet are highly nonadiabatic unless the midplane pitch angle α_0 exceeds 77°. Since nonadiabatic orbits suffer severe pitch angle scattering on each pass through the midplane, these particles will be rapidly transported away from the midplane. The large-α_0 particles that remain are well confined in the z-direction and in fact form the neutral sheet. The predicted neutral sheet thickness of about 1.3 R_e is consistent with observation.

PARTICLE MOTION IN DRIFT-FREE FIELDS

Following Pellat and Schmidt [1978] we consider the general two-dimensional field

$$\vec{B} = \hat{x}f'(z) + \hat{z}B_o \tag{1}$$

given by the vector potential

$$A_y = B_o x - f(z). \tag{2}$$

Figure 1 depicts this field for the parabolic case $f(z) = bz^2$. Symmetry about the midplane requires that $f'(0) = 0$, so that the field minimum is B_o and occurs at $z = 0$. Note that while $\nabla\cdot\vec{B}$ necessarily vanishes, a current is required to sustain this field;

$$\nabla\times\vec{B} = f''(z)\hat{y} \tag{3}$$

and consequently a scalar potential does not exist.

The Hamiltonian is

$$H = \frac{1}{2m}\left[P_x^2 + (P_y-\psi)^2 + P_z^2\right], \tag{4}$$

where we have introduced the stream function $\psi = \frac{e}{c}A_y$. Since y is cyclic, P_y is conserved and the y motion is given by

$$\dot{y} = \frac{1}{m}(P_y-\psi(x,z)). \tag{5}$$

The x and z motions are given by Hamilton's equations

$$\dot{P}_x = m\ddot{x} = m\Omega\dot{y} \tag{6}$$

$$\dot{P}_z = m\ddot{z} = -ef'(z)\dot{y}, \tag{7}$$

where $\Omega \equiv eB_0/mc$. Equation (6) integrates immediately to

$$\dot{x} = \Omega y + \dot{x}_0, \tag{8}$$

where we have taken $y_0 = 0$ with no loss in generality. Energy conservation requires $|\dot{x}|$, $|\dot{y}| \leq v$, so that (5) and (8) lead to the following bounds on the possible motions;

$$|P_y - \psi(x,z)| \leq mv \tag{9}$$

$$|\Omega y + \dot{x}_0| \leq v \ . \tag{10}$$

Equation (10), first derived somewhat differently by Pellat and Schmidt [1978] establishes the drift-free property of (1) for arbitrary orbits, regardless of adiabaticity. Equation (9) says that each orbit is sandwiched between two flux surfaces, a familiar result for two-dimensional static fields [Schmidt, 1966]. Note that (8) implies the existence of a new constant of the motion

$$\tilde{P}_x \equiv P_x - m\Omega y, \tag{11}$$

a consequence of the translational symmetry of the field.

The equations of motion (5-6) may be written

$$\ddot{x} + \Omega^2(x-\bar{x}) = \frac{\Omega^2}{B_0} f(z) \tag{12}$$

$$\ddot{y} + \Omega^2(y-\bar{y}) = \frac{\Omega}{B_0} f'(z)\dot{z} \tag{13}$$

with $\bar{x} \equiv \dot{y}_o/\Omega$ and $\bar{y} \equiv -\dot{x}_o/\Omega$. Thus, all midplane orbits are perfect circles centered at $(\bar{x},\bar{y})$;

$$(x-\bar{x})^2 + (y-\bar{y})^2 = (x_o-\bar{x})^2 + \dot{x}_o^2/\Omega^2. \tag{14}$$

It is also of interest to calculate the longitudinal bounce frequency for a particle mirroring in a parabolic field. The bounce time (there and back) is

$$\tau_b = 2\int_{-\ell_{max}}^{\ell_{max}} \frac{d\ell}{v_{\parallel}} = \frac{4}{B_o v}\int_0^{z_{max}} \frac{B dz}{(1-B/B_{max})^{1/2}} . \tag{15}$$

Taking $B = B_o \cosh u$ this becomes

$$\tau_b = \frac{4R_o}{v}\int_0^{\cosh^{-1}M} \frac{\cosh^2 u\, du}{(1-\frac{1}{M}\cosh u)^{1/2}} . \tag{16}$$

or

$$\tau_b = \frac{8R_o}{3v}\left[2M^2\left(\frac{M+1}{M}\right)^{1/2} E(k) - (2M-1)\left(\frac{M}{M+1}\right)^{1/2} K(k)\right], \tag{17}$$

where $K(k)$ and $E(k)$ are complete elliptic integrals and

$$k = \left(\frac{M-1}{M+1}\right)^{1/2} \tag{18}$$

and M is the orbital mirror ratio.

For orbits close to the midplane we define $\delta = M - 1 << 1$ and expand

$$E(k) \approx \frac{\pi}{2}\,(1-\frac{1}{4}k^2) \approx \frac{\pi}{2}\,(1-\frac{1}{8}\delta)$$
$$K(k) \approx \frac{\pi}{2}\,(1+\frac{1}{4}k^2) \approx \frac{\pi}{2}\,(1+\frac{1}{8}\delta). \tag{19}$$

Using these expansions in (17) we then have

$$\tau_b \approx \frac{2\sqrt{2}\,\pi R_o}{v}\,(1+\frac{11}{8}\delta). \tag{20}$$

When $\delta \to 0$ ($\alpha_0 \to 90°$) the particle undergoes simple harmonic motion about the midplane of period

$$\tau_{b_o} = \frac{2\sqrt{2}\,\pi R_o}{v}\;. \tag{21}$$

This result may be derived independently from the equation of motion

$$m\ddot{\ell} = -\tilde{\mu}\nabla_{\parallel} B \approx -\tilde{\mu}(\frac{\partial^2 B}{\partial \ell^2})_o\;\ell\;, \tag{22}$$

where $\tilde{\mu} = mv_{\perp}^2/2B \approx mv^2/2B$. Since $(\partial^2 B/\partial \ell^2)_o = B_o/R_o^2$, we have

$$\ddot{\ell} + \frac{v^2}{2R_o^2}\,\ell = 0\;. \tag{23}$$

Thus, $\Omega_o = v/\sqrt{2}\,R_o$, which leads directly to (21).

ADIABATICITY

The validity of the guiding center theory hinges upon the constancy of the magnetic moment, the first term of the adiabatic invariant series, $\mu = \mu_o + \varepsilon\mu_1 + \varepsilon^2\mu_2 + \cdots$. For a mirroring orbit such as that depicted in Figure 1, the nonadiabatic jump in μ between reflection points is given by the real part of [Howard, 1971]

$$\Delta\mu = -2v\mu_o^{1/2} \int_C \frac{e^{i\phi}}{RB^{1/2}} d\ell, \tag{24}$$

where $\varepsilon = mc/e$ and R is the radius of curvature, the integral being taken along the zero order guiding center field line. The gyration phase is

$$\phi = \phi_o - \frac{1}{\varepsilon} \int_0^\ell B \frac{d\ell}{v_\parallel} . \tag{25}$$

In many cases of interest, the integral in (24) may be evaluated asymptotically by expanding the integrand about a point (or points) in the complex ℓ-plane where B vanishes. For the general drift-free field (1) it is convenient to transform to z as variable of integration. The field curvature is then

$$\frac{1}{R} = \frac{\frac{d}{dx}(B_x/B_z)}{(1+B_x^2/B_z^2)^{3/2}} = \frac{B_o^2}{B^3} f''(z). \tag{26}$$

Alternatively, we have from (3)

$$\frac{1}{R} = \frac{1}{B}\left(\frac{4\pi}{c} J_y + \frac{\partial B}{\partial n}\right), \tag{27}$$

where $\partial B/\partial n$ is the normal derivative. Note that on the midplane $\partial B/\partial n \to 0$, while R_o remains finite. Using (26) in (24) we have

$$\Delta\mu = - 2vB_o\mu_o^{1/2} \int \frac{f''(z)}{B^{5/2}} e^{i\phi} dz . \tag{28}$$

To proceed further, the function f(z) must be specified.

Following Pellat and Schmidt [1979] we introduce the parabolic field by setting $f(z) = bz^2$ in (2). This gives

$$B^2 = B_o^2 + 4b^2z^2, \tag{29}$$

$$\frac{1}{R} = \frac{2bB_o^2}{B^3} , \tag{30}$$

and therefore $b = B_o/2R_o$. Thus, (29) and (30) take the simpler forms

$$B^2 = B_o^2 (1+z^2/R_o^2) \tag{31}$$

$$R = (\frac{B}{B_o})^3 R_o , \tag{32}$$

where R_o is the radius of curvature at the midplane crossing. Thus,

$$\Delta\mu = \frac{-2v\mu_o^{1/2}B_o^2}{R_o} \int_C \frac{e^{i\phi}dz}{B^{5/2}} , \tag{33}$$

with

$$\phi = \phi_o - \frac{1}{\varepsilon B_o} \int_0^z \frac{B^2 dz}{v_{\parallel}} . \tag{34}$$

The function B(z) vanishes when

$$\tilde{z} = \pm iR_o . \tag{35}$$

Choosing the zero on the negative imaginary axis, we deform the contour as in I, retaining only that portion along the branch cut from $-i\infty$ to $\tilde{z}$ and back around $\tilde{z}$. Note that since a scalar potential does not exist in this case, the general formula (11) given in I does not apply.

Near $\tilde{z}$ we have

$$B \approx \frac{2^{1/2}B_o}{R_o^{1/2}} \exp(-i\pi/4)(z-\tilde{z})^{1/2} \tag{36}$$

$$\phi \approx \phi_o - \frac{1}{\varepsilon}\int_0^{\tilde{z}} \frac{B^2 dz}{B_o v_\parallel} + \frac{iB_o}{\varepsilon v R_o}(z-\tilde{z})^2 . \tag{37}$$

Substituting (36) and (37) in (33) and evaluating the integral by the method of steepest descent as in I we obtain, finally,

$$\frac{\Delta\mu}{\mu_o} = AM^{1/2}\lambda^{-1/8} \exp[-F(M)/\lambda] \sin\phi_o , \tag{38}$$

where

$$A = \frac{\pi}{2^{1/4}\Gamma(9/8)} , \tag{39}$$

$$\lambda = \frac{\varepsilon v}{R_o B_o} , \tag{40}$$

$$F(M) = \int_0^{\pi/2} \frac{\cos^3\theta \, d\theta}{(1-M^{-1}\cos\theta)^{1/2}} , \tag{41}$$

and where M is the orbital mirror ratio $M = B_{max}/B_o$, which is related to the pitch angle at the midplane via

$$\text{cosec}\,\alpha_o = M^{1/2}. \tag{42}$$

It is instructive to relate this to the maximum excursion in z, using (31),

$$z_{max} = R_o(M^2-1)^{1/2} = R_o \cot\alpha_o\,. \tag{43}$$

As Figure 2 shows, the function F(M) is rapidly changing for $M \lesssim 4$, and slowly changing thereafter, tending to 2/3 as $M \to \infty$. A series in inverse powers of M is readily obtained. Note that the dimensionless parameter $\lambda = a_o/R_o$, where a_o is the gyroradius formed from the total velocity. To gain the full accuracy from (38), $\Delta\mu$ is to be taken between turning points, whereas μ_o must be evaluated at the first order midplane guiding center. Figure 3 plots $\Delta\mu/\mu$ maximized over the gyrophase ϕ_o vs. $1/\lambda$ for M = 2, 5 and 10 (α_o = 45°, 26.57°, and 18.43°). Also shown is the limiting curve for M = 1.05, roughly corresponding to the adiabaticity limit of 5 keV protons, as described in the next section.

APPLICATION TO THE MAGNETOTAIL

The plasma sheet is thought to be [Roederer, 1974] about 6 R_e in thickness, extending from about 10 R_e to well beyond the orbit of the moon. The proton density is nearly constant ($\approx 0.5/cm^3$) out to 1 - 2 R_e on either side of the midplane, gradually falling off to a very small value at about $z = 3\ R_e$. We shall allude to the inner region as the "neutral sheet." The proton energy spectrum near the neutral sheet at $X \equiv -X_{sm} = 20\ R_e$ has been measured by the Vela satellites to be approximately a 5 keV Maxwellian [Bame et al., 1967]. There is apparently no reliable data on the X-dependence of E_p [Frank, 1970]; we shall take $E_p = 5$ keV along the midnight meridian of the neutral sheet in estimating the midplane gyroradius.

More accurate and complete data is available on the magnetic field near the neutral sheet. The field component normal to the midplane is slowly changing in the range $20\ R_e < X < 70\ R_e$, following the same inverse power law as the total field just outside the plasma sheet, B_z, $B \propto X^{-0.3}$ [Behannon, 1970]. It should be noted that these measurements are extremely difficult due to the motion ("flapping") of the magnetotail itself. At $X = 20\ R_e$, where $B_z = B_o \approx 4\gamma$, the gyroradius for 5 keV protons is $a_o = \varepsilon v/B_o = 0.40\ R_e$. Using the above scaling, we have

$$a_o = 0.40 \left(\frac{X}{20}\right)^{0.3} R_e . \tag{44}$$

From (27) the midplane radius of curvature is given by

$$\frac{1}{R_o} = \frac{4\pi}{c}\frac{J_y}{B_o} , \tag{45}$$

while (3) implies

$$\frac{4\pi}{c} J_y = \left(\frac{\partial B_x}{\partial z}\right)_o \quad . \tag{46}$$

Therefore

$$\frac{1}{R_o} = \frac{1}{B_o} \left(\frac{\partial B_x}{\partial z}\right)_o \quad . \tag{47}$$

Analyzing Explorer 34 data at $X \approx 30\ R_e$, Bowling and Wolf [1970] find

$$B_x \approx 1.25\ B_{x_s} \tanh(z/z_s), \tag{48}$$

where z_s is the plasma sheet half-thickness and B_{x_s} the corresponding horizontal field. Assuming that (48) holds for all X of interest, we obtain

$$\left(\frac{\partial B_x}{\partial z}\right)_o \approx \frac{1.25\ B_{x_s}}{z_s} \quad . \tag{49}$$

Note that if the field were truly parabolic, one would have $(\partial B_x/\partial z)_o = B_{x_s}/z_s$. This close agreement and the measured small gradient of B_z along z strongly support the parabolic field model, at least in the vicinity of the neutral sheet. Taking $B_{x_s} = 20\gamma$ and $z_s = 3\ R_e$ near $X = 20\ R_e$ leads to

$$R_o = 0.8\ \frac{B_o}{B_{x_s}}\ z_s = 0.48\ R_e. \tag{50}$$

Thus, R_o scales as z_s, which changes only slightly in the range of interest.

We are now in a position to estimate adiabaticity in the neutral sheet.

Equations (44) and (50) combine to give

$$\lambda = a_o/R_o = 0.83 \left(\frac{X}{20}\right)^{0.3} , \qquad (51)$$

so that λ varies from 0.83 to 0.95 between X = 20 and 70 R_e. Figure 3 predicts changes in μ well in excess of 100% for small pitch angle particles throughout the neutral sheet, unless the proton spectrum softens considerably at larger X.

The consequences of this high degree of nonadiabaticity are profound. If $\Delta\mu \gtrsim 100\%$, the reflection points will wander wildly and at random, depending on the midplane phase ϕ_o, which may be assumed stochastic. At the same time, the constancy of P_y confines the orbit between two flux surfaces. This process leads to rapid transverse diffusion of an initially isotropic plasma away from the midplane of the neutral sheet. On the other hand, Figure 3 shows that particles with sufficiently large pitch angle (M approaching unity) will be adiabatically confined to the vicinity of the midplane. Let us somewhat arbitrarily take $\Delta\mu/\mu$ = 20% as the transition from adiabatic to nonadiabatic motion. For 5 keV protons at 20 R_e, (51) gives λ = 0.83. From (38) the corresponding mirror ratio for $\Delta\mu/\mu$ = 20% is about M = 1.05 (α_o = 77.4°), which by (43) gives a maximum z-value of 0.65 R_e. Protons within this narrow band will be adiabatically confined for relatively long times. We conclude that most of the 5 keV protons within the neutral sheet, roughly bounded by ± z = 1 R_e are adiabatic. Further, their adiabaticity demands pitch angles in excess of 77°; a 5 keV proton having a smaller α_o would be rapidly lost due to nonadiabatic reflection point excursions. Thus, the plasma pressure may be expected to be highly anisotropic, with $P_\perp \gg P_\parallel$, where $P_\parallel$ is measured parallel to B_z. One might further speculate that the fairly sharply defined adiabatic limit determines the width of the neutral sheet. The validity of this conjecture could be examined by a numerical simulation of the evolution of an initially isotropic plasma. The 500 eV electrons are, of course, highly adiabatic for all pitch angles and would be expected to follow any ejected protons in order to maintain charge neutrality.

Finally we point out an interesting connection between adiabaticity and resonance between the longitudinal and orbital frequencies. The gyroperiod at $X = 20\ R_e$ is

$$\tau_g = \frac{2\pi\varepsilon}{B_o} = 15.7\,\text{s} . \tag{52}$$

(This is much less than the ion-ion 90° scattering time $\tau_{ii} = 3.4 \times 10^{11}$ s, so that the plasma is extremely collisionless.) Combining (21) and (52), we obtain

$$\frac{\tau_{b_o}}{\tau_g} = \sqrt{2}\,\frac{R_o B_o}{\varepsilon v} = \frac{\sqrt{2}}{\lambda} . \tag{53}$$

Thus, adiabatic motion ($\lambda \ll 1$) occurs if and only if $\tau_{b_o} \gg \tau_g$, unless α_o is close to 90°. In the case of neutral sheet orbits, α_o is unusually large. Fig. 3 suggests that adiabaticity in the neutral sheet breaks down at about $\lambda = \sqrt{2}/2$, for which (53) gives $\tau_{b_o} = 2\ \tau_g$. The consequences of this resonance might be pursued using the methods of Chirikov [1971].

DISCUSSION

We have explored the general characteristics of proton orbits in the plasma sheet of the earth's magnetotail, using a simple parabolic model field. We consider here possible generalizations of this work. One might suppose the new constant $\tilde{P}_x$ to be useful in constructing a Vlasov equilibrium of the plasma sheet along the lines of the Harris [1962] equilibrium for $B_z = 0$. However, it is easily seen that the field (1) does not correspond to a Vlasov equilibrium, since B_z crosses with J_y to require a gradient of density in the x-direction, which in turn implies a gradient in B_x. Thus, while $\tilde{P}_x$ may be considered constant over one gyroradius, it changes substantially over the several radii required in a kinetic description.

It would be interesting to carry out the calculation of $\Delta\mu$ for the Harris-type field

$$\vec{B} = \hat{x} B_{x_s} \coth 1 \tanh(z/z_s) + \hat{z} B_o \tag{54}$$

used recently by Wagner, Kan and Akasofu [1979] in classifying plasma sheet trajectories. Since $|B|$ tends to a constant as $z \to \infty$, this field possesses a genuine loss cone, whereas the parabolic field reflects all particles. This calculation is in progress.

ACKNOWLEDGEMENT

Support for this work was provided by the U.S. Department of Energy.

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FIGURE CAPTIONS

Fig. 1. Parabolic magnetic field. The slightly nonadiabatic orbit shown is constrained by the conservation of P_y to lie between two flux surfaces labelled ψ_1 and ψ_2.

Fig. 2. The function F(M), illustrating the dependence of the exponent of $\Delta\mu/\mu$ on pitch angle (cosec $\alpha_0 = M^{1/2}$). Note that $F(M) \rightarrow 2/3$ as $M \rightarrow \infty$ ($\alpha_0 \rightarrow 0$).

Fig. 3. Maximum $\Delta\mu/\mu_0$ vs. a_0/R_0, where a_0 is the gyroradius formed from the total velocity. The curve for M = 1.05 corresponds to $\Delta\mu/\mu_0 \approx 20\%$ and is taken to define the thickness of the neutral sheet.

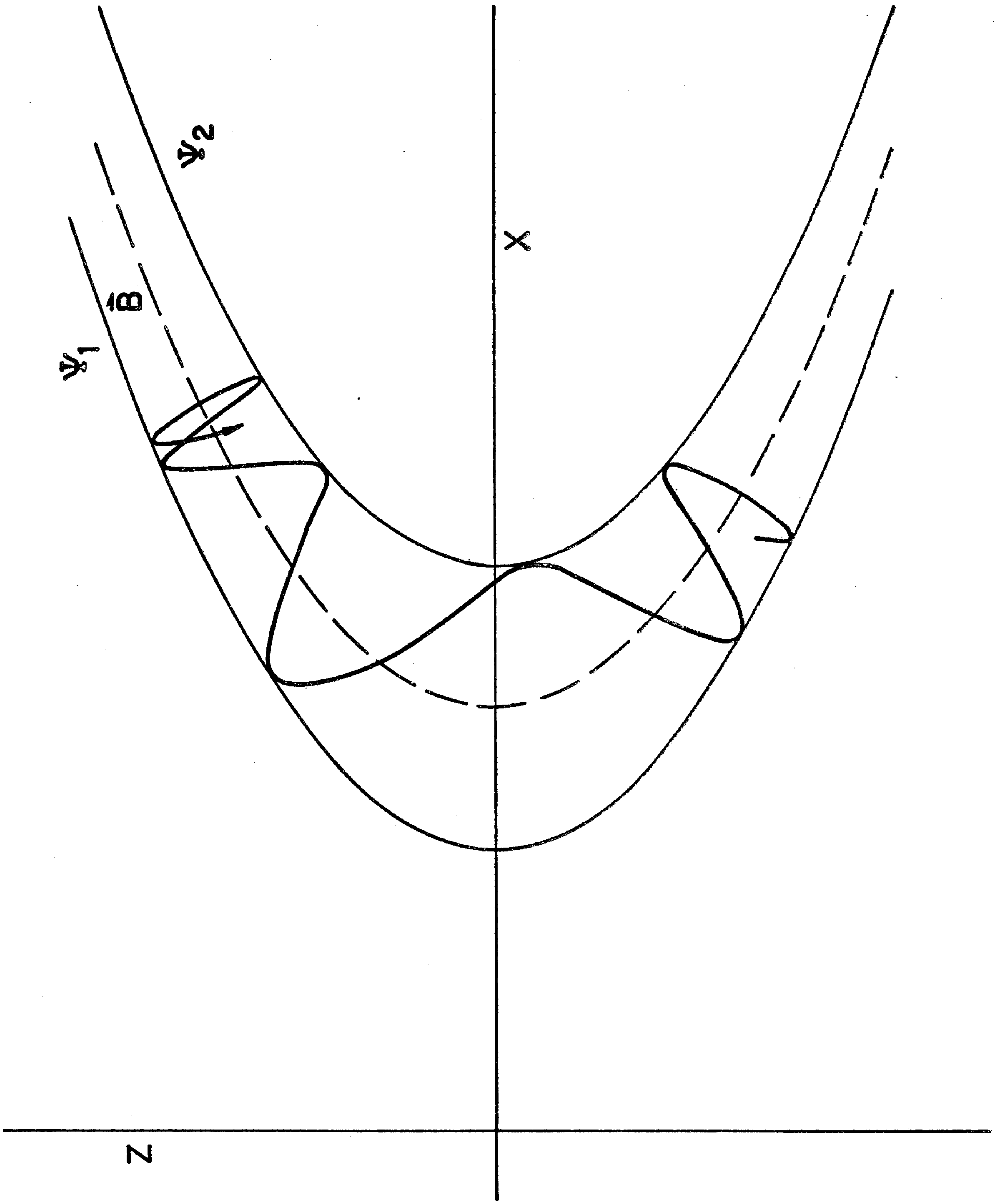
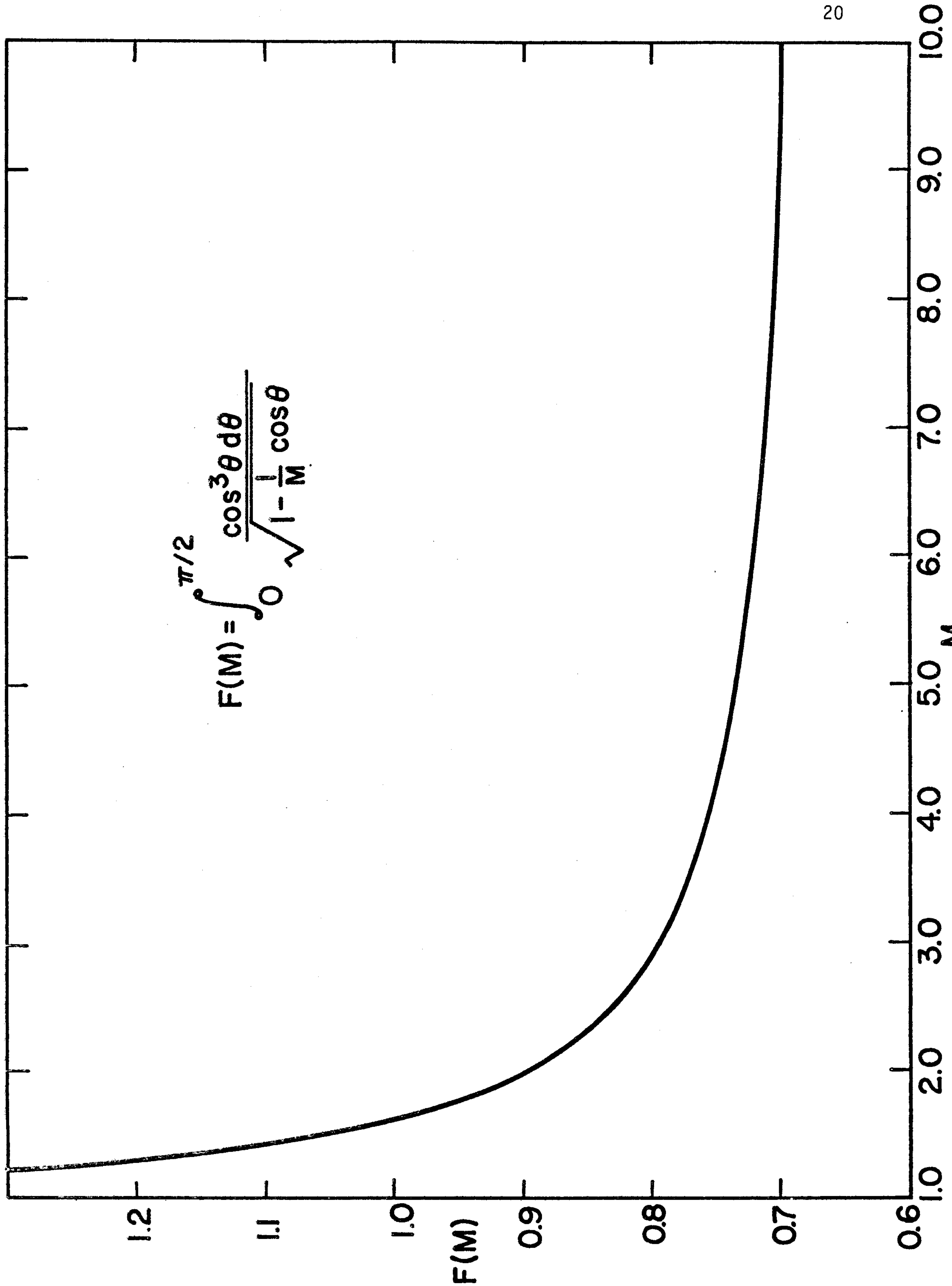


Fig. 1



$F(M) = \int_0^{\pi/2} \frac{\cos^3\theta \, d\theta}{\sqrt{1 - \frac{1}{M}\cos\theta}}$
F(M)
1.2
1.1
1.0
0.9
0.8
0.7
0.6
M
1.0
2.0
3.0
4.0
5.0
6.0
7.0
8.0
9.0
10.0

Fig. 2

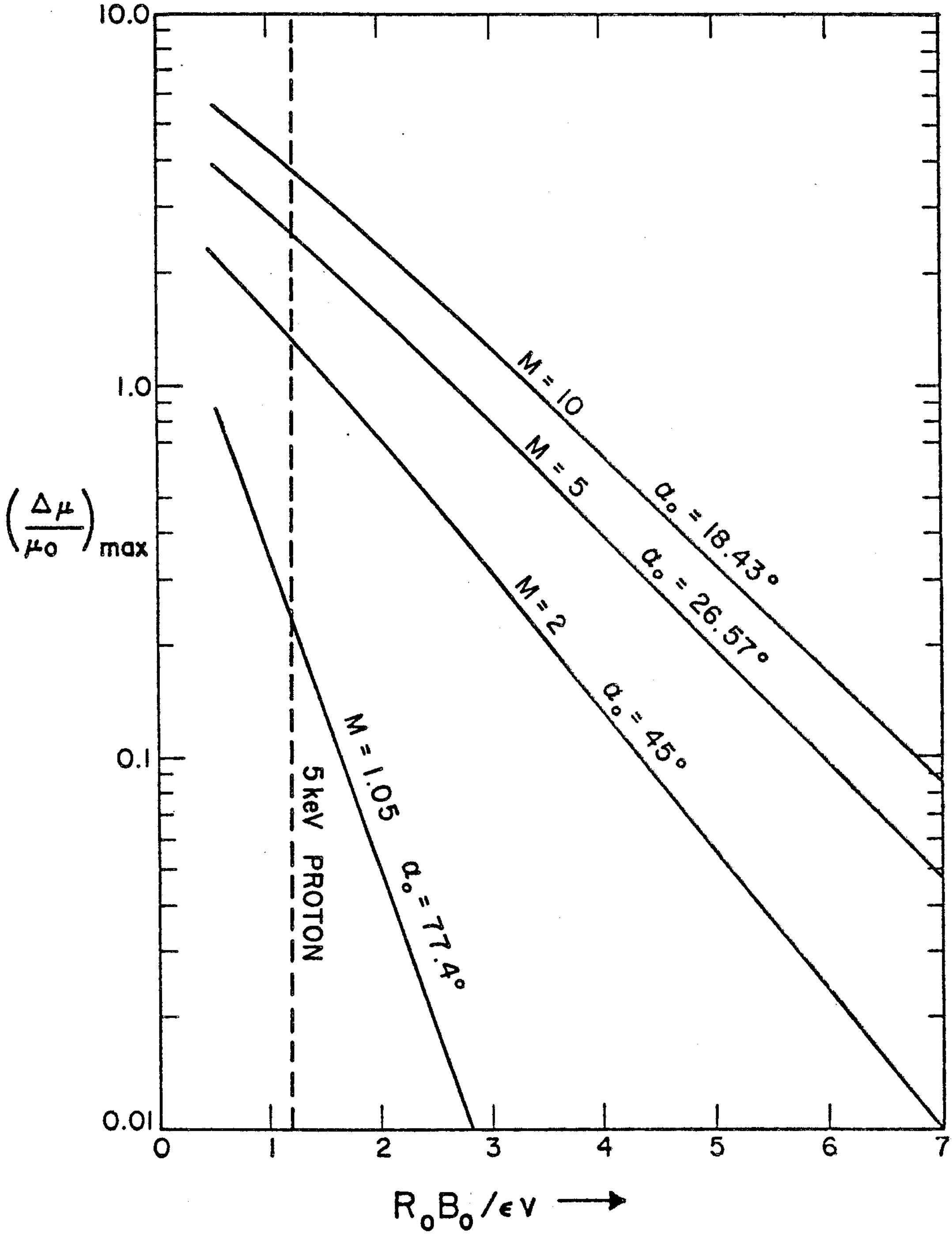

10.0
1.0
0.1
0.01
$\left(\frac{\Delta\mu}{\mu_0}\right)_{max}$
0
1
2
3
4
5
6
7
$R_0B_0/\epsilon V$
M = 10
α_0 = 18.43°
M = 5
α_0 = 26.57°
M = 2
α_0 = 45°
M = 1.05
α_0 = 77.4°
5 keV PROTON

Fig. 3