



Parametric Studies of Tandem Mirror Reactors

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Abstract*

To increase net power plant efficiency and reduce the recirculating power fraction, a tandem mirror reactor should have $Q \sim 10$. With this in mind, we solve a set of space and time independent power balance equations including quasi-neutrality requirement for electrons, plug ions, and central cell ions. We investigate the effect of finite radial confinement time on Q , and the impact of plug diameter to length ratio (plug aspect ratio) on the physical dimensions and neutron wall loading of the machine. The preliminary study shows that a $Q = 10$, $3000 \text{ MW}_{\text{th}}$ reactor is possible with an injected deuterium beam energy of 1.5 MeV, and a maximum plug magnetic field of 16.5 T. When the plug aspect ratio is 1, the central cell length is about 200 m and the neutron wall loading is 0.9 MW/m^2 . If a fatter plug plasma is feasible, e.g. with an aspect ratio of 2, then the solenoid length can be reduced to about 100 m and the neutron wall loading can be increased by 20%. In either case, at $Q = 10$, the recirculating power fraction is reduced to 20%, compared to $\sim 36\%$ at $Q = 5$.

*Bull. Am. Phys. Soc. 23 (Sept. 1978) 882.

I. Ion Power Balance Equation in the Plugs

The equation for the high energy neutral beam sustained plugs is

$$\frac{n_p^2}{(n\tau)_p} E_{inj} + \frac{n_p^2}{(n\tau)_p} \frac{\langle\sigma v\rangle_{cx}}{\langle\sigma v\rangle_{ion}} (E_{inj} - E_p) = \frac{n_p^2 (E_p - \frac{3}{2} T_e)}{(n\tau)_{drag}} + \frac{n_p^2}{(n\tau)_p} E_{out} \quad (1)$$

where n_p = peak plug density (cm^{-3})

E_{inj} = injected beam energy (keV)

E_p = average ion energy in the plugs (keV)

$\langle\sigma v\rangle_{cx}, \langle\sigma v\rangle_{ion}$ = $\langle\sigma v\rangle$ for charge exchange and ionization reaction (cm^3/sec).

$(n\tau)_{drag} = 1.0 \times 10^{13} M T_e^{3/2} / Z^2 \ln \Lambda_{ei}$ = electron drag time for ions (sec/cm^3).

E_{out} = average energy carried by ions escaped from the plugs (keV).

$(n\tau)_p$ = $(n\tau)$ product for plugs (sec/cm^3).

For the analytic forms for $(n\tau)_p$ and E_{out} we use Logan's⁽²⁾ expression shown below:

$$(n\tau)_p = \left\{ \left[\left(\frac{C_1}{\ln \Lambda_{ii}} \right) \left(\frac{M_1}{M_D} \right)^{1/2} E_{inj}^{3/2} \log_{10}(R_{eff}) \right]^{-1} + \left[\left(\frac{2 \times 10^3}{\ln \Lambda_{ei}} \right) \left(\frac{M_i}{M_D} \right) T_e^{3/2} \times \right. \right. \\ \left. \left. \ln \left(\frac{E_{inj}}{E_{out}} \right) \right]^{-1} \right\}^{-1} \quad (2)$$

and

$$E_{\text{out}} = \frac{E_{\text{inj}}}{1 + (\tau_{ii}/\tau_{\text{drag}})} + \frac{E_c (\tau_{ii}/\tau_{\text{drag}})}{1 + (\tau_{ii}/\tau_{\text{drag}})} \quad (3)$$

$$\text{where } \tau_{ii}/\tau_{\text{drag}} = C_2 \left(\frac{E_{\text{inj}}}{T_e}\right)^{3/2} \frac{\log_{10}(R_{\text{eff}})}{\log_{10}(E_{\text{inj}}/E_{\text{out}})} \left(\frac{\ln \Lambda_{ei}}{\ln \Lambda_{ii}}\right) \sqrt{\frac{M_D}{M_i}} \quad (4)$$

$$\ln \Lambda_{ei} = \ln (5.7 \times 10^{13} T_e / \sqrt{n_p})$$

$$\ln \Lambda_{ii} = \ln (1.4 \times 10^{15} \sqrt{T_e E_p / n_p})$$

$$M_i = \text{plug ion mass (a.m.u.)}$$

$$M_D = \text{mass of deuterium (a.m.u.)}$$

$$E_c = \phi_p / (R_m \sin^2 \theta_{\text{inj}} - 1)$$

$$R_{\text{eff}} = R_m \sin^2 \theta_{\text{inj}} / [1 + (\phi_p / E_{\text{inj}})]$$

$$R_m = \text{mirror ratio for plugs}$$

$$\phi_p = \text{ambipolar potential in plugs}$$

$$\theta_{\text{inj}} = \text{beam injection angle}$$

C_1 and C_2 are two constants which give the best fit to Fokker-Planck results. Their values are 3.9×10^{12} and 0.11 respectively. The accuracy of eqs. (7) and (8) compared to Fokker-Planck results is generally about $\pm 5\%$.⁽²⁾

II. Electron Power Balance Equation

$$\begin{aligned} & \frac{n_p^2 (E_p - 3/2 T_e)}{(n\tau)_{\text{drag}}} + \frac{1}{4} n_c^2 \langle \sigma v \rangle_f E_\alpha f_e R_v + P_{\text{eaux}} R_v \\ = & \frac{3}{2} \frac{n_c^2 (T_e - T_c)}{(n\tau)_{\text{drag}}} R_v + (\phi_e + T_e) \left[\frac{n_c^2}{(n\tau)_c} + \frac{n_c^2}{(n\tau)_{\text{xf}}} \right] R_v + (\phi_e + \phi_c + T_e) \frac{n_p^2}{(n\tau)_p} \quad (5) \end{aligned}$$

where

$$E_\alpha = \alpha \text{ particle energy from D-T fusion reaction (keV)}$$

$$\langle \sigma v \rangle_f = \langle \sigma v \rangle \text{ for D-T fusion reaction}$$

$$P_{\text{eaux}} = \text{electron direct heating power per unit volume (keV/cm}^3\text{-sec)}$$

$$R_v = \text{volume of the central cell/volume of two plugs} = V_c/V_p$$

$$(n\tau)_c = \frac{2.5 \times 10^{11} T_c^{3/2} M^{1/2}}{Z^4 \ln \Lambda_{ij}} \times \frac{(R_c + 1) \ln(2R_c + 1)}{2R_c} \left(\frac{\phi_c}{T_c}\right) \exp\left(\frac{\phi_c}{T_c}\right)$$

$(n\tau)_{xf}$ = $(n\tau)$ product for cross field diffusion

$f_e = 1.5 - 0.29 \ln(T_e)$ = the fraction of α power going to electrons

R_c = ratio of magnetic field at the midplane of the plug to that of the central cell

In eq. (5), the effect of cross field diffusion is taken into account through the term $(n\tau)_{xf}$. For perfect radial confinement $(n\tau)_{xf}$ is infinite.

III. Ion Power Balance in the Central Cell

$$\frac{3}{2} \frac{n_c^2 (T_e - T_c)}{(n\tau)_{\text{drag}}} + \frac{1}{4} n_c^2 \langle \sigma v \rangle_f E_\alpha (1 - f_e)$$

$$= [\phi_c + T_c (1 + \frac{3}{2} \frac{\langle \sigma v \rangle_{cx}}{\langle \sigma v \rangle_{ion}})] \frac{n_c^2}{(n\tau)_c} + [T_c (1 + \frac{3}{2} \frac{\langle \sigma v \rangle_\alpha}{\langle \sigma v \rangle_{ion}})] \frac{n_c^2}{(n\tau)_{xf}} \quad (6)$$

IV. Quasineutrality Requirement

The ambipolar potential, ϕ_e , and thus ϕ_p are determined by the quasineutrality requirement, i.e., the ion loss rate is equal to the electron loss rate. This can be expressed as

$$\frac{2(n_c^2 R_v + n_p^2)}{G(R)(n\tau)_{ee} \frac{\phi_e}{T_e} \exp\left(\frac{\phi_e}{T_e}\right)} = \frac{n_c^2}{(n\tau)_c} R_v + \frac{n_p^2}{(n\tau)_p} \quad (7)$$

V. Results

Neglecting alpha buildup and radial losses in the central cell, we find that Q values of order 10 are feasible for a reactor of central cell length

between 100 and 200 meters. The end plugs will require high beam energies and fields, requiring high technology development, whereas the central cell will only require fields in the 1.5 T range. A tandem mirror with $Q \sim 10$ is seen to have the same net thermal efficiency (η_{net}) as a tokamak with $Q \sim 60$.

Power flow diagram in a tandem mirror reactor is shown in Fig. 1. We are assuming direct conversion of all of the charged products that leave the system. In Fig. 2, we compare the Q for a reactor without a direct convertor (such as a tokamak) with a tandem mirror having a direct conversion efficiency of 60%. As a result of this assumption, a mirror Q of ~ 10 is equivalent to a tokamak Q of ~ 60 .

The results of calculations for tandem mirrors with deuterium plugs are shown in Figs. 3-7. A typical set of parameters for a $Q=10$ tandem mirror reactor is shown in Table I. By injecting into field ripple exterior to the plugs so as to create a trapped density at the same level as the central cell, the ambipolar hole within the plug can be reduced and Q increased.⁽³⁾ This exterior cell which serves to reduce the plug ambipolar potential drop we call the "A" cell.

We have furthermore looked at the effect of varying the plug aspect ratio (plug to length ratio) on the physical dimensions of the reactor and on the neutron wall loading. These results are shown in Table 2. One sees that for the same volume ratio and power output, the length of the central cell is much shorter and both the power density and neutron wall loading are increased.

Acknowledgement

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References

1. B. G. Logan, University of California, Lawrence Livermore Lab. Report UCID-17349, Appendix B.
2. B. G. Logan and M. E. Rensink, University of California, Lawrence Livermore Laboratory, MFE Memo MFE/CP1/78-181.
3. B. G. Logan, et al., Seventh International Conference on Plasma Physics and Controlled Nuclear Fusion Research, Innsbruck, Austria, 1978, IAEA-CN-37-R-3.

Table 1
Typical Parameters for a Q=10 Tandem Mirror Reactor

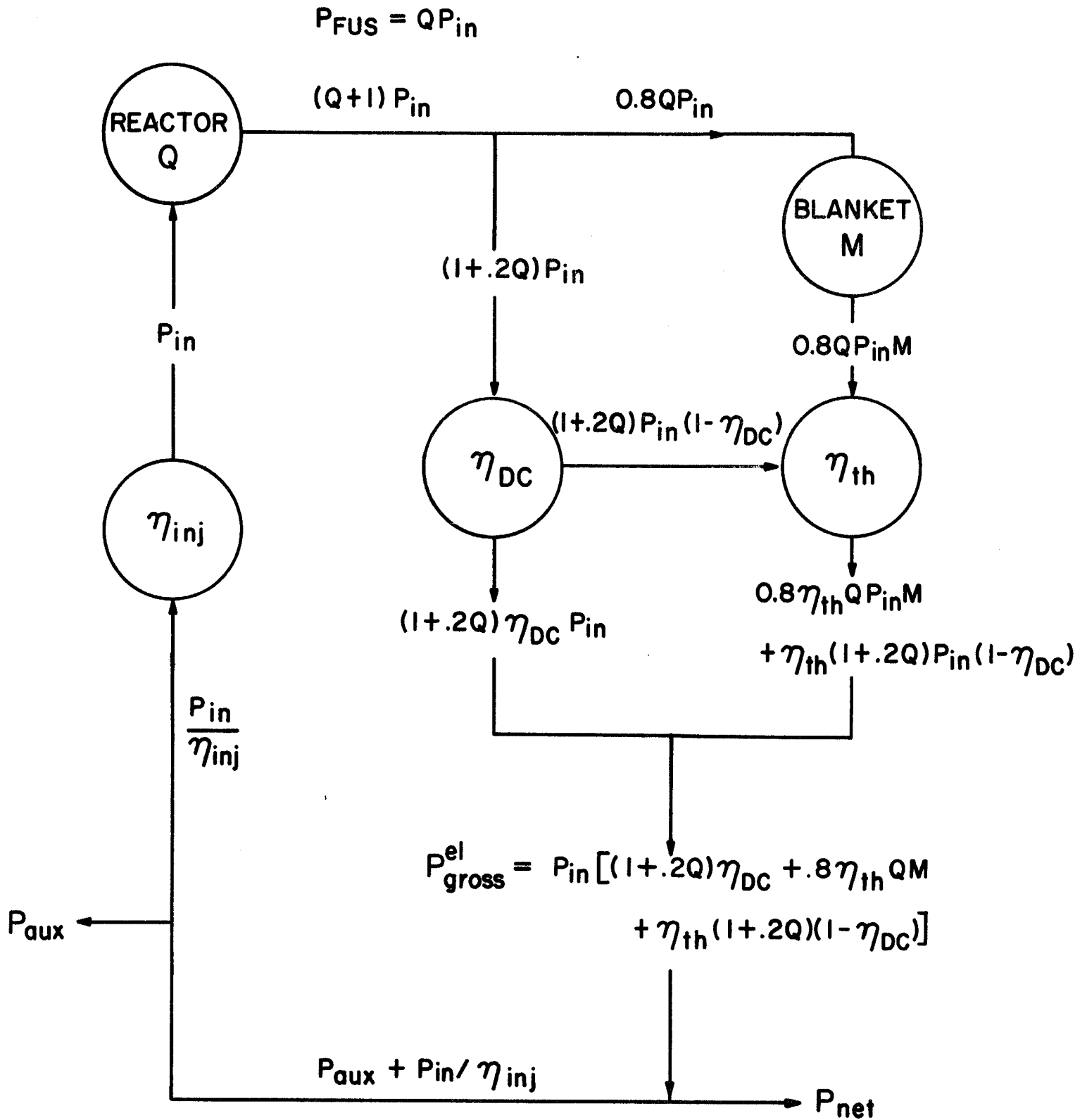
	<u>Without "A" Cell</u>	<u>With "A" Cell</u>
T_e (keV)	45	37
T_c (keV)	40	30
n_c (cm^{-3})	4.1×10^{13}	5×10^{13}
n_p (cm^{-3})	4.1×10^{14}	5×10^{14}
E_p (keV)	955	690
β_p	1	0.94
β_c	0.9	0.92
$(n\tau)_p$ ($\text{cm}^{-3}\text{-s}$)	3.6×10^{14}	2.5×10^{14}
$(n\tau)_c$ ($\text{cm}^{-3}\text{-s}$)	7.4×10^{14}	8.5×10^{14}
P_{th} (MW)	3000	3000
P_{th}/V (MW/m^3)	0.91	1.2
P_n (MW/m^2)	0.88	1 (50 cm away from the plasma surface)
P_{beam} (MW)	300	300
P_{RF}	--	--
E_{inj} (keV)	1500	900
Q	10	10
R_c (m)	2.4	2.35
L_c (m)	184	150
R_p (m)	0.7	0.68
Aspect Ratio	1	1.2
B_{max} (T)	16.5	14
R_p (Vac)	1.4	1.27
B_c (T)	1.45	1.4

Table 2

The Effects of Aspect Ratio on Physical Dimensions
And Neutron Wall Loadings for the Case Without "A" Cell

Aspect Ratio	1	2	3
Plug Radius r_p (m)	0.7	0.9	1
Central Cell Radius r_c (m)	2.4	3	3.5
Central Cell Length L_c (m)	184	115	87
Neutron Wall Loading (MW/m^2)	0.9	1.1	1.3

SAMPLE POWER FLOW DIAGRAM FOR FUSION REACTORS

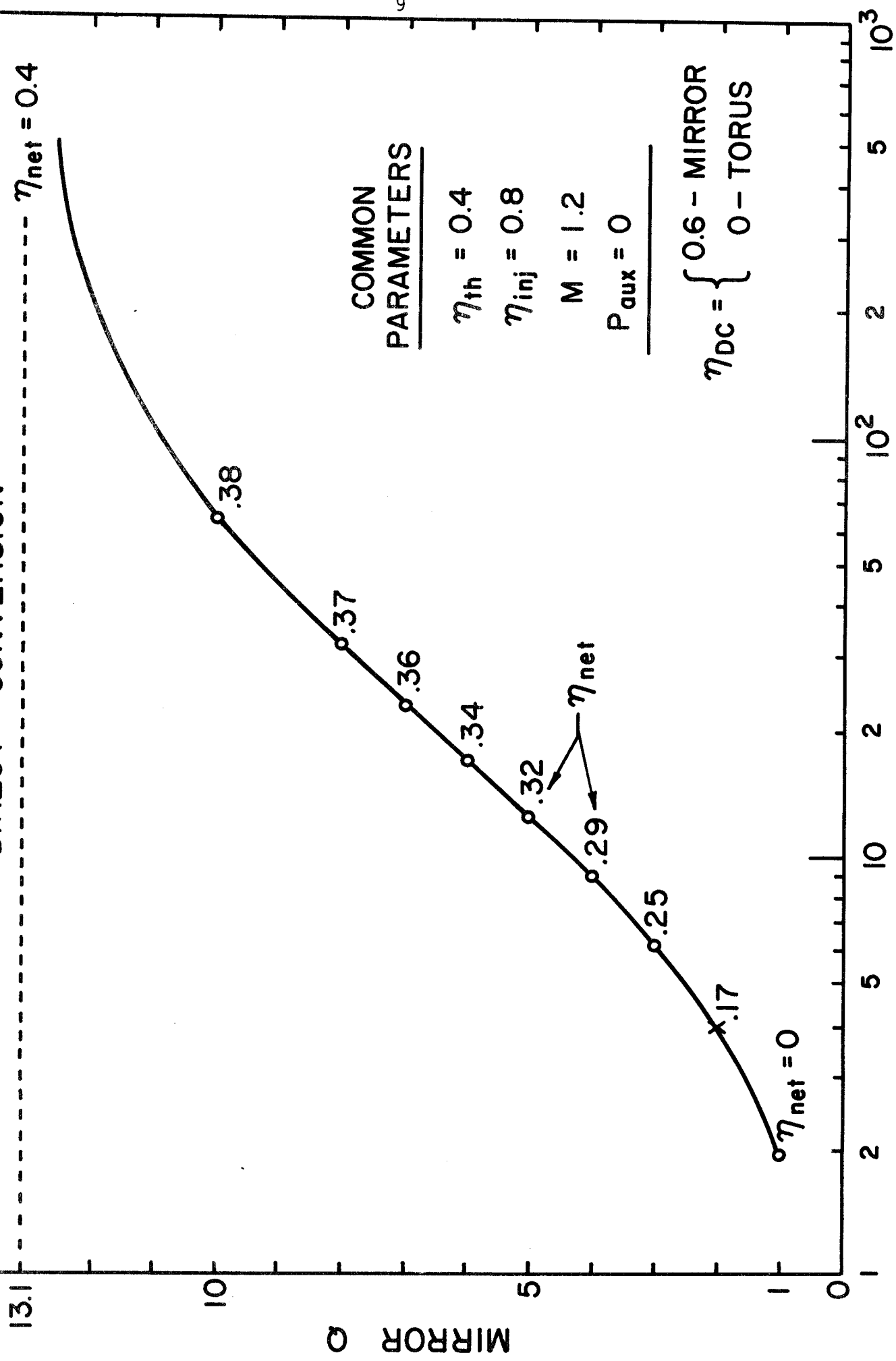


$$\eta_{net} = \frac{P_{net}}{(1+.2Q)P_{in} + 0.8QP_{in}M} = \frac{P_{net}}{P_{in}(1+.2Q + .8QM)}$$

FIGURE 1

DOMAIN INACCESSIBLE TO DEVICES WITHOUT
DIRECT CONVERSION

$\eta_{net} = 0.4$



TOKAMAK Q FIGURE 2

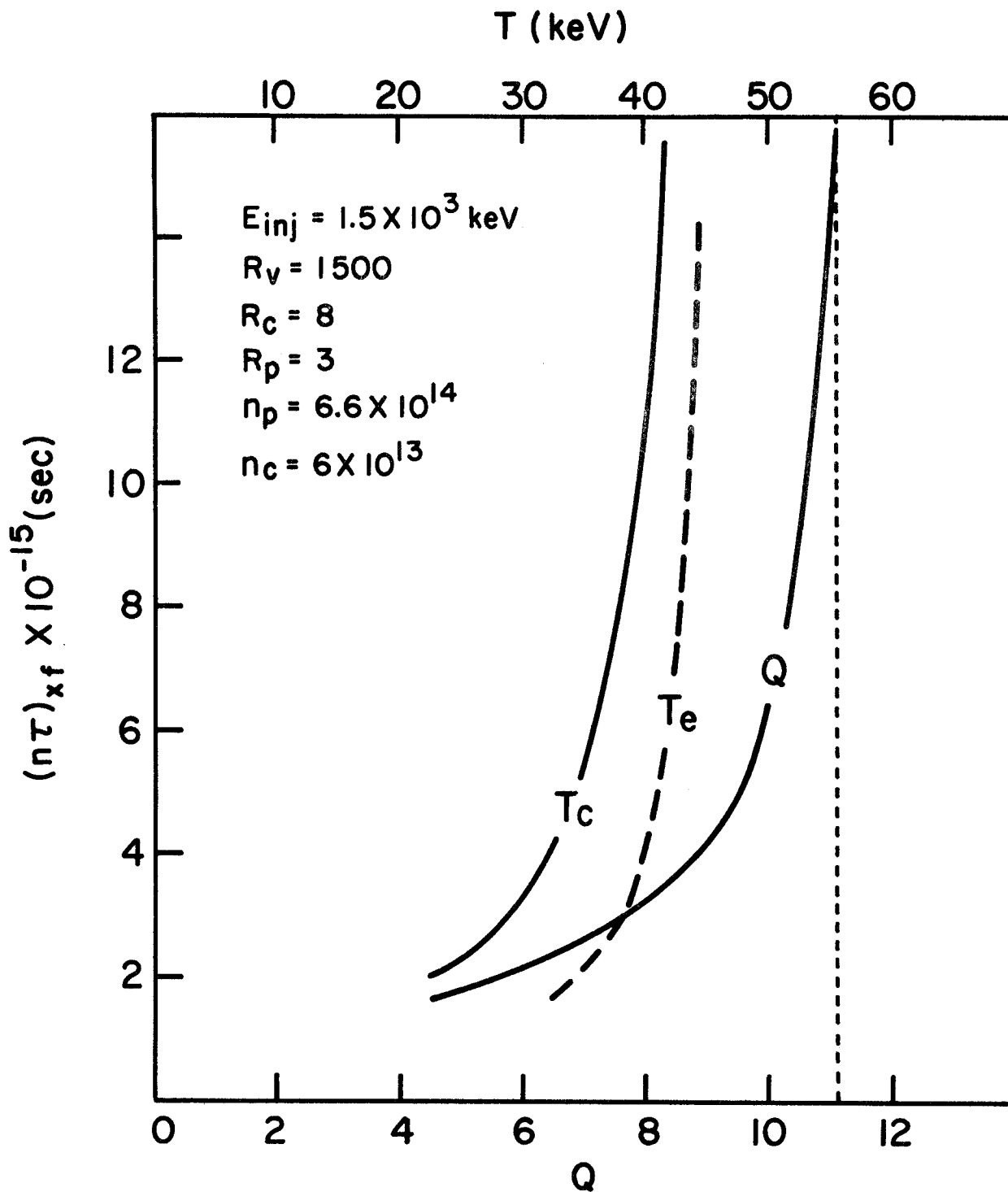


FIGURE 3

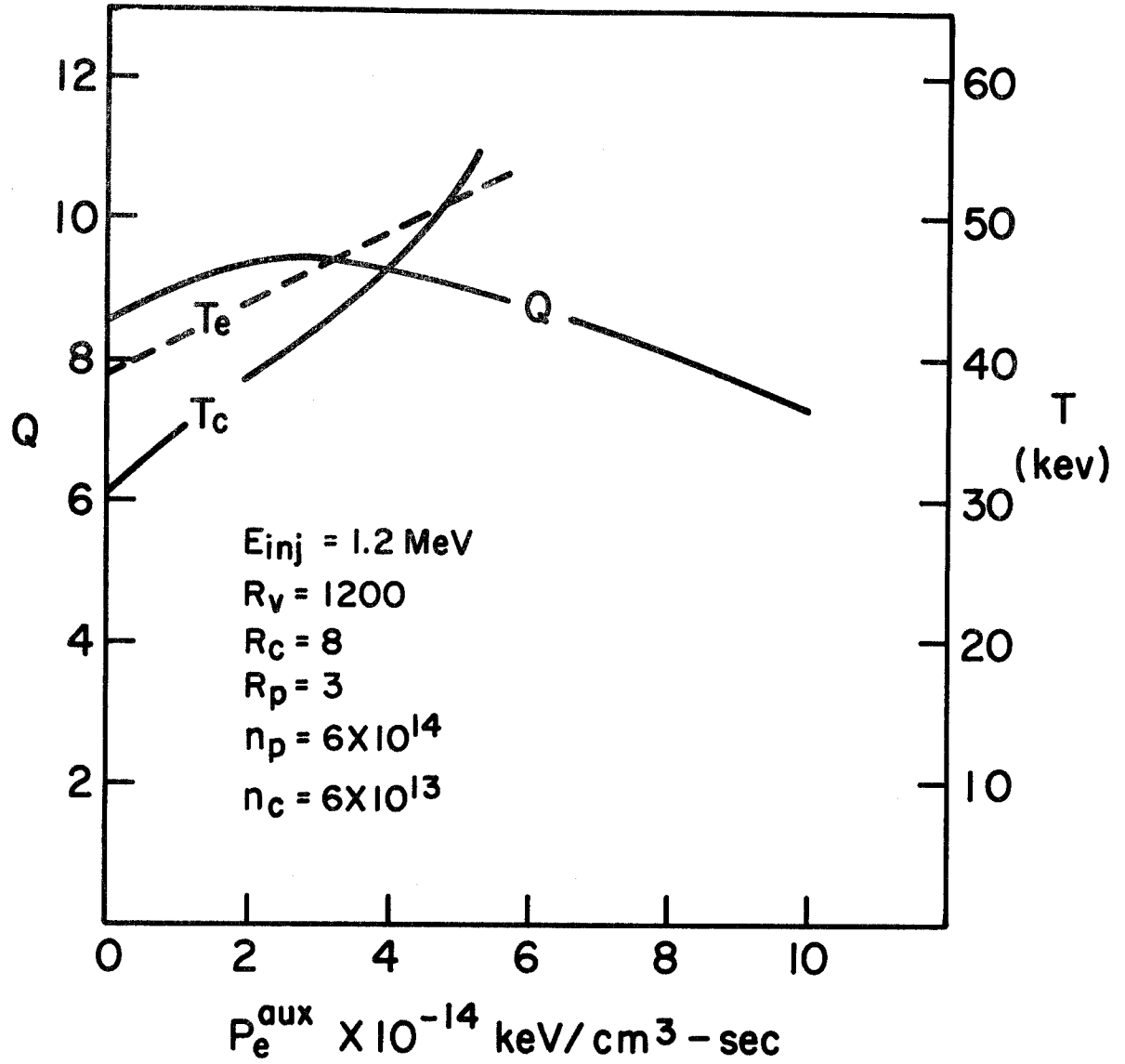


FIGURE 4

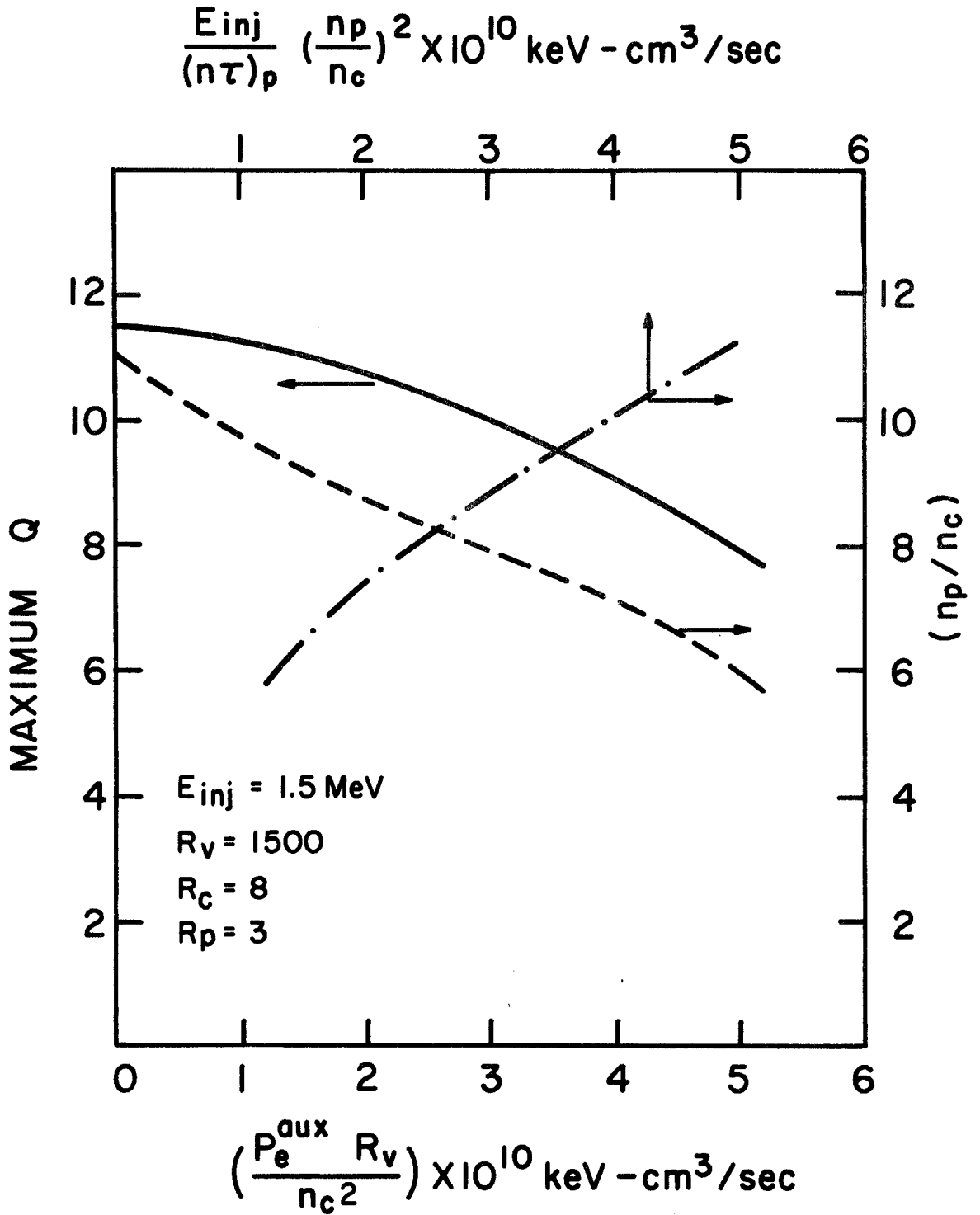


FIGURE 5

Q VERSUS VOLUME RATIO

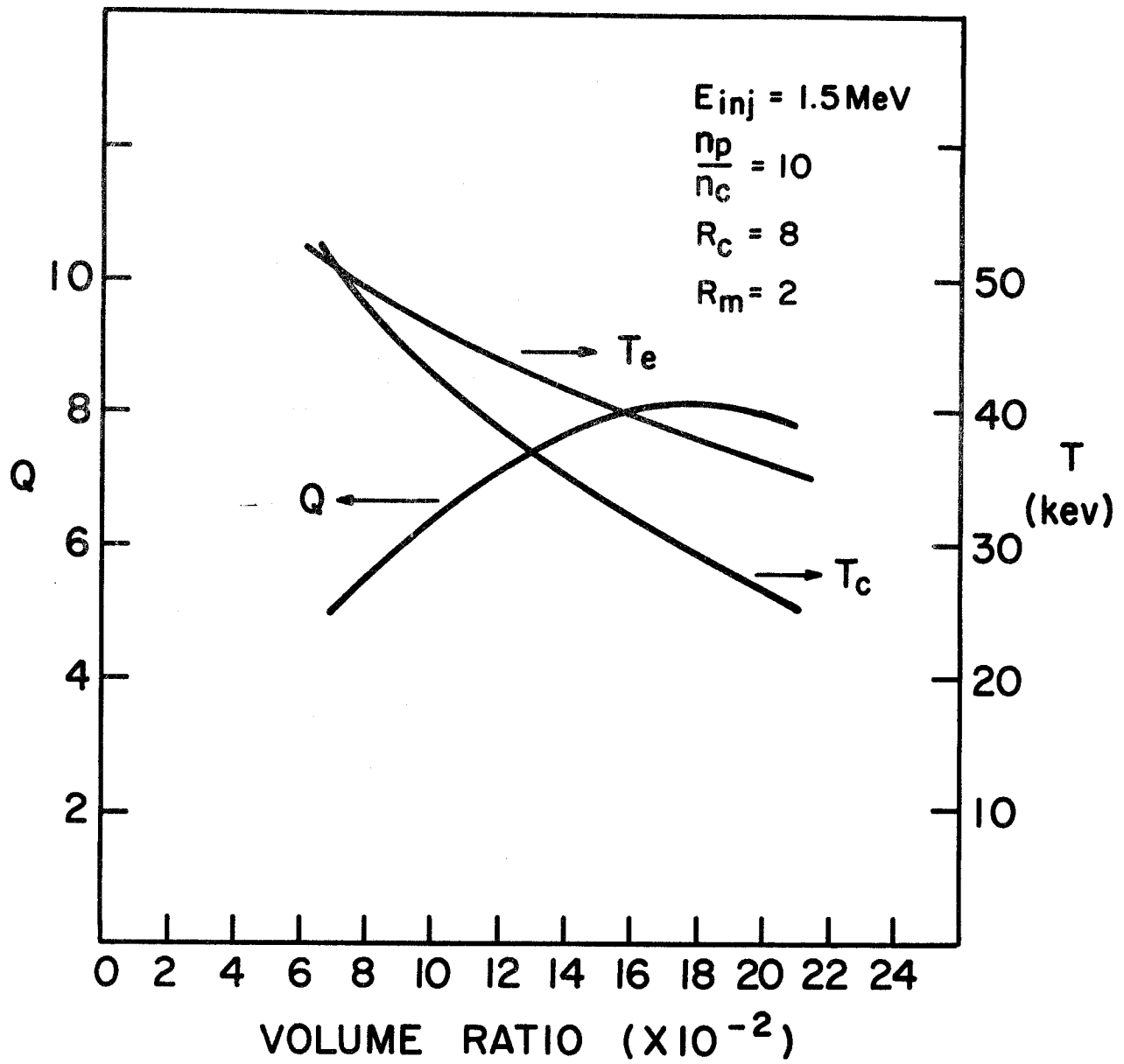


FIGURE 6

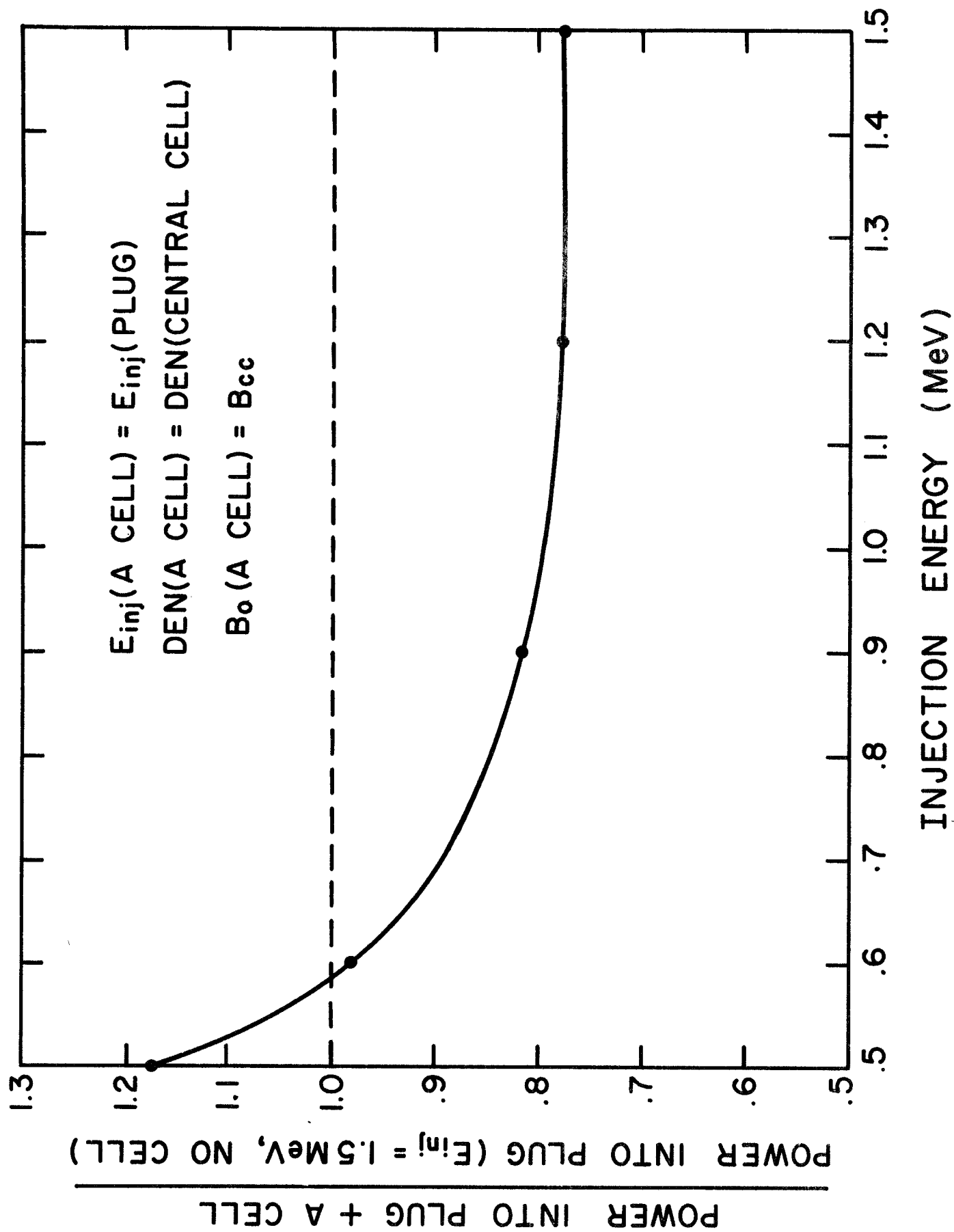


FIGURE 7