Monte Carlo Statistical Weighting Methods for External-Source-Driven Multiplying Systems

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October 1978

UWFDM-265

Proc. of the ANS Topical Mtg. on Computational Methods in Nuclear Engr., April 1979, Williamsburg, VA, 11, 7-11 (1979).

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Magdi M.H. Ragheb
and
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Abstract

Multiplication Weighting, Generalized Secondary Weighting, and Secondary Weighting Monte Carlo Methods are suggested for the treatment of external-source-driven multiplying systems. Such problems arise in the study of fusion-fission hybrid blankets. For these cases, they offer advantages compared to the commonly used Absorption Weighting method. Implementation for the MORSE and KENO Multigroup Monte Carlo Codes has been carried out. Treatment of a fusion-fission Fissile Enrichment Fuel Factory (FEFF) blanket and other problems were undertaken to test the validity and capabilities of the suggested methods. Fissile and Fusile breeding estimates compare satisfactorily to Discrete Ordinates results for one-dimensional problems.

Acknowledgement

This work was partly supported by the Electric Power Research Institute (EPRI). Useful discussions with the first author about an earlier version of the present research with G.E. Whitesides and T. Hoffman, from Oak Ridge National Laboratory, are acknowledged. The manuscript preparation by Miss Gail Herrington is greatly appreciated.

1. Introduction

Monte Carlo statistical weighting methods particularly suited for source-driven multiplying systems are investigated and compared to Analog Monte Carlo and to the commonly used Absorption Weighting method. The latter is currently used in most Monte Carlo codes for external-source nonmultiplying media, and for internal-fission-source multiplying media. In the former, it is basically applied to shielding problems, and in the latter, to criticality problems. Current studies of fusion-fission hybrids, where an external source (fusion plasma) is coupled to a blanket containing fissionable and fertile materials, suggested the present study. Methods of treating such systems were sought, with the requirements of adequately treating both the external fusion and internal fission sources, as well as being readily adaptable, with minimum modifications, to existing Monte Carlo codes. Statistical weighting methods which depend upon secondary particle weighting are suggested. They were implemented in existing Monte Carlo Codes and their validity and capabilities checked against Discrete Ordinates calculations for the blanket of a Fusion-Enrichment-Fuel-Factory (FEFF).[1,2] In the suggested weighting methods, at each collision site, in the fissioning medium, the particle statistical weight is adjusted as described in section 2, rather than by the non-absorption probability. The particle is allowed to scatter or fission according to a prescribed probability. If scattering is sampled, the particle is collided and transported according to the scattering group-to-group transfer and direction probabilities. If fission is sampled, a fission particle is started from the colliding site. Its direction is sampled isotropically, and its energy sampled from the fission spectrum. The particles are followed in the usual way until they leak from the system, or reach a specified weight cut-off and are killed by Russian Roulette. There is no need to consider successive particle generations as in the Absorption Weighting method. The external source is sampled more adequately. The need for the storage of the secondary particle parameters and to follow them in later stages in the simulation is eliminated. These properties make the exposed methods particularly attractive for studies of fusion-fission hybrids and accelerator breeding. In these cases integrated values generated by both the external and fissioning sources are required, such as breeding ratios and heating rates, and an estimate of criticality, for which successive particle generations are necessary, is not needed.

2. Methods Exposition

When the neutron transport equation in multiplying media is to be solved by use of Monte Carlo, the most obvious approach is the analog simulation of the equation, choosing events according to the underlying physical probabilities. Even in the absence of fission this leads to a slow statistical convergence of the results. With fission, the problem would be totally untenable.[6] To avoid some of the difficulties, the statistical simulation is modified to allow only secondary producing reactions with the effect of capture being accounted for by modifying the weight associated with the particle being followed. The weight change is determined by requiring that the weighted expectation of the total number of secondary particles emerging from a collision is unchanged. The probabilities of different secondary producing reactions is arbitrary as far as the theoretical results are concerned, but the rate of convergence will vary with the choices made. They must, however, add to unity. With such a method, a particle history would never terminate and it must be terminated by leakage or by some weight related procedure such as Russian Roulette.

Since many data libraries include in the matrices for energy and angle change, the contributions from neutron producing reactions such as (n, 2n) and (n,3n), the number of secondaries from scattering is not one, but some value ν_s slightly greater than one and is given by:

$$\nu_s = \frac{\Sigma_s+2\Sigma(n,2n)+3\Sigma(n,3n)}{\Sigma_s+\Sigma(n,2n)+\Sigma(n,3n)} = \frac{\Sigma_s+2\Sigma(n,2n)+3\Sigma(n,3n)}{\Sigma_s'} \qquad (1)$$

This allows all secondary producing reactions to be treated as either scattering or fission.

The quantity $\nu_s \frac{\Sigma_s'}{\Sigma_t}$ is what is referred to in current Monte Carlo Multigroup Codes[4,5] as the "nonabsorption probability". This is a misnomer since it is a probability only in the absence of the (n,2n) and (n,3n) processes.

Referring now to Table I, the analog procedure is characterized by the physical probabilities of various reactions with the statistical weight of a particle kept at its original weight and the track continued until terminated by leakage or capture. All branches of multiple particle reactions are followed. The combining of all non-fission particle producing reactions into a generalized scattering reaction and elimination of capture from consideration leads to the Generalized Secondary Weighting Method where scattering occurs with probability P_s and fission with probability P_f subject only to

$$P_s + P_f = 1 \; . \qquad (2)$$

The weighting preserves the total expected number of secondary particles, but in general does not conserve the number of reactions of a given type.

This general form can be specialized to yield several interesting cases including the commonly used Absorption Weighting Method. First, if P_s is chosen as the probability of scattering, given only scattering and fission are allowed, the Secondary Weighting Method results. This creates the correct number of non-capture reactions of each type and the correct number of secondary particles of each type. Another choice is to choose the scattering and fission probabilities on the basis of the number of secondary particles expected from each reaction. This is referred to as the Multiplication Weighting Method since all particle weights are changed by the expected particle multiplication in the collision. Finally, by choosing unit scattering probability, the so-called Absorption Weighting Method is obtained but must be coupled to additional procedures. The weight change is the number of secondary scattered particles per collision, which in the case ν_s is one (when n,2n and n,3n reactions are not considered), is the non-absorption probability and accounts for the name. The weight of the particle is also changed by $\nu_f \frac{\Sigma_f}{\Sigma_t}$, the number of fission secondaries per collision, and this result is stored.

The further procedures necessary in the Absorption Weighting Method may best be understood by considering the neutron transport equation in multiplying media which can be written formally as

$$\psi = (H + F)\, \psi + S \qquad (3)$$

where H is the transport operator without the fission source operator F, S is an independent source distribution, and ψ the angular flux distribution. The Absorption Weighting Method proceeds essentially by solution of

$$\psi_1 = H\,\psi_1 + S \tag{4}$$

while storing data on F ψ_1. This is the purpose of the stored weights $\frac{\nu_f \Sigma_f}{\Sigma_t} w_{n-1}$ mentioned earlier. After sufficient data has been accumulated, a second generation is studied by a similar simulation of

$$\psi_i = H\,\psi_i + F\,\psi_{i-1}\ ,\ i = 2 \tag{5}$$

and the process continued for i = 3, 4, ... until either a converged solution

$$\psi = \sum_{i=1}^{n} \psi_i \tag{6}$$

is obtained or in criticality problems, the ratio

$$K = F\,\psi_n / F\,\psi_{n-1} \tag{7}$$

is stable and the multiplication K obtained. While this procedure is well suited to criticality problems, it is not well adapted to systems dominated by the source S with value of $K \ll 1$. Moreover, implementing Absorption Weighting, one encounters many practical difficulties related to the control of the number of secondaries, their normalization, storage and sampling[4,5,7,8] that are necessary for accounting to the multiplication portion from generation to generation. Our earlier methods are preferable in such cases.

In all cases, the total expected weight ($\bar{w}_n$) of particles emerging from a collision is preserved. Using the probabilities of reactions and the associated modified weights for the different processes from Table I, for the analog process, multiplying the events probabilities by their associated particle weights, and summing:

$$\bar{w}_n = w_{n-1} \left(\frac{\Sigma_s}{\Sigma_t} \cdot 1 + \frac{\Sigma_f}{\Sigma_t} \cdot \nu_f + \frac{\Sigma_c}{\Sigma_t} \cdot 0 + \frac{\Sigma_{(n,2n)}}{\Sigma_t} \cdot 2 + \frac{\Sigma_{(n,3n)}}{\Sigma_t} \cdot 3\right) .$$

Using Eqns. 1 and 2, for the Generalized Secondary Weighting, it is

$$\bar{w}_n = w_{n-1} \left[P_s\left(\nu_s \cdot \frac{\Sigma_s'}{\Sigma_t} \cdot \frac{1}{P_s}\right) + P_f \left(\nu_f \cdot \frac{\Sigma_f}{\Sigma_t} \cdot \frac{1}{P_f}\right)\right] .$$

For the Secondary-Weighting,

$$\bar{w}_n = w_{n-1} \left[\left(\frac{\Sigma_s}{\Sigma_s+\Sigma_f}\right)\left\{\left(\frac{\nu_s\Sigma_s'}{\Sigma_s}\right) \frac{\Sigma_s+\Sigma_f}{\Sigma_t}\right\} + \left(\frac{\Sigma_f}{\Sigma_s+\Sigma_f}\right) \cdot \left\{\nu_f \left(\frac{\Sigma_s+\Sigma_f}{\Sigma_t}\right)\right\}\right] ,$$

and for Multiplication-Weighting:

$$\bar{w}_n = w_{n-1} \left(\frac{\nu_s \Sigma_s'}{\nu_s \Sigma_s' + \nu_f \Sigma_f} \cdot \frac{\nu_s \Sigma_s' + \nu_f \Sigma_f}{\Sigma_t} + \frac{\nu_f \Sigma_f}{\nu_s \Sigma_s' + \nu_f \Sigma_f} \cdot \frac{\nu_s \Sigma_s' + \nu_f \Sigma_f}{\Sigma_t}\right) .$$

Since the quantities $\nu_s \Sigma_s'$ and $\nu_f \Sigma_f$ are readily available in scattering matrices for multigroup cross sections, multiplication weighting is easy to implement. Secondary weighting would require further cross section handling since it requires the knowledge of not only $\nu_s \Sigma_s'$, but also of Σ_f and Σ_s on an individual basis.

3. Demonstrative Examples

The blanket for a fusion-fission hybrid reactor consisting of ThO_2 assemblies to be enriched in ^{233}U to produce assemblies suitable as fuel for an LWR is considered. Such a system is denoted as a Fissile-Enrichment-Fuel-Factory (FEFF). A one-dimensional model of the reactor is shown in Fig. 1. A point 14 MeV neutron source is placed at the center of the cavity and models that caused by laser-driven microexplosions of D-T fuel. The material compositions are shown in Fig. 1, and the atomic densities of the material mixes are shown in Table II. The cross section data used are reported in detail in another study.(2)

The suggested methods were implemented in the MORSE Multigroup Monte Carlo Code.(4,5) Minor modifications were necessary to implement them. Table III shows the results obtained by application of the Absorption-Weighting method for cases of 200 and 800 starting particles. Results of Th(n,γ) reactions and tritium breeding from ^6Li and ^7Li are compared to those obtained by discrete ordinates using the ANISN Code in the S_4P_3 approximation.(3)

Table III shows the results of application of Absorption Weighting, compared to discrete ordinates. No estimates of standard deviations are shown, since one single experiment in each case was considered, with its generated secondaries, and the code considered (MORSE) estimates variances according to the experiment or batch concept.(4,5)

Table IV shows results of application of the Generalized Secondary Weighting Method for different values of P_s and P_f in Table I. As shown in Table VII, use of $P_s = P_f = 0.5$, which amounts to following scattered particles 50% of the time, and fission particles 50% of the time, leads to an unnecessarily large computation time. The reason is that this is not the actual proportion of fission and scattering reactions in the system. The case with $P_f = 0.1$ leads to results closer to the discrete ordinates answers, at about half the expense of the previous case. This shows how important it is to properly apportion the fissions and scatterings in the system under consideration. The Generalized Secondary Weighting Method with $P_f = 0.1$ is applied to the problem at hand for different sample sizes, and the results are compared to discrete ordinates in Table V.

The results for Multiplication Weighting are shown in Table VI. Satisfactory agreement is obtained, even for small numbers of particle histories. For this reason, Multiplication Weighting was adopted for further three-dimensional studies for fusion-fission reactor blankets.[1,2] It is interesting to report that the discrete ordinates calculation cost was $52 compared to $11 for Monte Carlo with 200 histories, which makes Monte Carlo attractive for scoping studies using moderate samples for fusion-fission blankets. G.E. Whitesides[9] kindly implemented an earlier version of our suggested Secondary Weighting method in the KENO criticality code during a summer stay of the first author at ORNL. For testing the validity of the suggested method, two[7-8] problems were considered for which either an exact solution exists, or another known solution can be obtained. The first problem consisted of a cylinder of highly enriched uranium metal whose radius is 2.0 cm and with a length of 10.775 cm. A fundamental mode of source neutrons from a normal KENO run using Absorption-Weighting was used as a fixed source. The KENO computed K_{eff} at the fundamental mode distribution was: 0.345 ± 0.004. If the fundamental mode is used as a fixed source, the ratio of final source to initial source is expected to be:

$$K_{eff} + K_{eff}^2 + K_{eff}^3 + \ldots = \frac{K_{eff}}{1-K_{eff}} = 0.527 \;.$$

The computed value using Secondary Weighting with the modified KENO Code was 0.520 ± 0.021, a good agreement.

The other problem consisted of a sphere of ^{235}U metal with a radius of 2.0 cm with a point fission source at its center. The computed total source/initial source ratio using Secondary-Weighting was 0.408 ± 0.012. This compares to a value of 0.409 computed by discrete ordinates using the ANISN Code.(3)

4. Conclusions and Recommendations

New Statistical Weighting Monte Carlo Methods for the treatment of external source-driven multiplying media are presented. These are easily adaptable to existing Monte Carlo Codes with minor modifications, for the treatment of fusion-fission hybrids and accelerator breeding systems. The Multiplication Weighting Method is recommended over a Generalized Secondary Weighting Method and a Secondary Weighting Method suggested for the treatment of such systems. Compared to the currently used Absorption-Weighting method, these methods offer potential advantages in terms of ease of implementation in existing codes, elimination of the need to consider successive particle generations and the treatment of excessively large numbers of generated secondary particles. Validity of the suggested methods was tested by implementation in two existing Monte Carlo Codes. One-dimensional problems of a hybrid reactor blanket, and a highly enriched cylinder and sphere were considered with comparison to discrete ordinates results. Further assessment of the capabilities of the suggested methods is being carried out by application to three-dimensional complex geometry cell calculations for hybrid blanket designs.[1,2]

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List of Tables

Table I Relationship of Statistical Weighting Methods to Analog Monte Carlo

Monte Carlo Method	Probabilities of Reactions	Statistical Weight Adjustment at n-th Collision	Comments
Analog	Scattering: Σ_s/Σ_t †	$w_n = w_{n-1}$	Follow 1 scattered particle of weight w_n.
	Capture : Σ_c/Σ_t	$w_n = 0.0$	Cease following particle history.
	Fission : Σ_f/Σ_t	$w_n = w_{n-1}$	Follow ν_f fission particles of weight w_n.
	(n,2n) : $\Sigma(n,2n)/\Sigma_t$	$w_n = w_{n-1}$	Follow 2 scattered particles of weight w_n.
	(n,3n) : $\Sigma(n,3n)/\Sigma_t$	$w_n = w_{n-1}$	Follow 3 scattered particles of weight w_n.
Generalized Secondary Weighting	Scattering: P_s	$w_n = \nu_s \frac{\Sigma_s'}{\Sigma_t} \cdot \frac{1}{P_s} w_{n-1}$ *	Follow until weight-related truncation.
	Fission : P_f	$w_n = \nu_f \frac{\Sigma_f}{\Sigma_t} \cdot \frac{1}{P_f} w_{n-1}$	Follow until weight-related truncation.
	$(P_s + P_f = 1)$		
Secondary Weighting	Scattering: $\Sigma_s/(\Sigma_s+\Sigma_f)$	$w_n = \nu_s \frac{\Sigma_s'}{\Sigma_s} \frac{(\Sigma_s+\Sigma_f)}{\Sigma_t} w_{n-1}$	Follow until weight-related truncation.
	Fission : $\Sigma_f/(\Sigma_s+\Sigma_f)$	$w_n = \frac{\nu_f (\Sigma_s+\Sigma_f)}{\Sigma_t} w_{n-1}$	Follow until weight-related truncation.
Multiplication Weighting	Scattering: $\frac{\nu_s \Sigma_s'}{\nu_s \Sigma_s' + \nu_f \Sigma_f}$	$w_n = \frac{\nu_s \Sigma_s' + \nu_f \Sigma_f}{\Sigma_t} w_{n-1}$	Follow until weight-related truncation.
	Fission : $\frac{\nu_f \Sigma_f}{\nu_s \Sigma_s' + \nu_f \Sigma_f}$	$w_n = \frac{\nu_s \Sigma_s' + \nu_f \Sigma_f}{\Sigma_t} w_{n-1}$	Follow until weight-related truncation.
Absorption Weighting	Scattering: 1 Fission : 0	$w_n = \nu_s \frac{\Sigma_s'}{\Sigma_t} w_{n-1}$	Follow until weight-related truncation, and store at each collision a weight: $w_n = \nu_f \frac{\Sigma_f}{\Sigma_t} w_{n-1}$ as contribution to generation fission source.

† $\Sigma_t = \Sigma_s + \Sigma_f + \Sigma_c + \Sigma(n,2n) + \Sigma(n,3n)$

* $\nu_s = \frac{\Sigma_s + 2\Sigma(n,2n) + 3\Sigma(n,3n)}{\Sigma_s + \Sigma(n,2n) + \Sigma(n,3n)} = \frac{\Sigma_s + 2\Sigma(n,2n) + 3\Sigma(n,3n)}{\Sigma_s'}$

Table II Elemental Compositions of Material Mixes for Fusion-Fission Reactor Blanket

Material Composition	Elements	Nuclei Densities Nuclei/(barn·cm)
1. Neutron Multiplication Zone Pb + Na Coolant + Zircalloy-2 Structure	Ni	.183-5
	Cr	.330-5
	Fe	.440-5
	Pb	.253-1
	Na	.237-2
	Zr	.360-2
	Sn	.549-4
2. Fusile Breeding Zones Natural Lithium + Stainless Steel Structure	Ni	.469-3
	Cr	.725-3
	Fe	.307-2
	^6Li	.315-2
	^7Li	.393-1
3. Stainless Steel Structure	Ni	.938-2
	Cr	.145-1
	Fe	.614-1
4. Fissile Breeding Zones ThO_2 + Na Coolant + Zircalloy-2 Structure	Ni	.199-5
	Cr	.359-5
	Fe	.478-5
	O	.139-1
	Na	.150-1
	Zr	.392-2
	Sn	.598-4
	Th	.694-2
5. Reflector/Shield Pb+C	C	.535-1
	Pb	.112-1

Table III Fusile and Fissile Breeding in Blanket Regions (Nuclei/Source Neutron). Comparison of Absorption Weighting With Successive Particle Generationsto Discrete Ordinates

Reaction	5"	6	7	8	Total
Th(n,γ)	2.83-1^{+}	2.28-1	1.95-1	2.28-1	9.34-1
	2.99-1*	2.32-1	1.82-1	2.19-1	9.32-1
	2.58-1**	2.07-1	1.83-1	2.20-1	8.68-1

Reaction	3	10	12	Total
^{6}Li(n,α)T	1.04-1	4.16-1	7.67-2	5.97-1
	1.29-1	3.50-1	8.92-2	5.68-1
	1.03-1	4.29-1	9.32-2	6.25-1
^{7}Li(n,n'α)T	1.30-2	1.40-2	7.47-5	2.71-2
	8.41-3	1.71-2	0.0	2.55-2
	1.28-2	2.41-2	0.0	3.69-2
Total Tritium	1.17-1	4.30-1	7.68-2	6.24-1
	1.37-1	3.67-1	8.92-2	5.94-1
	1.16-1	4.53-1	9.32-2	6.60-1

" Region Numbers

\+ P_3S_4 Discrete Ordinates

* Monte Carlo, 200 histories

** Monte Carlo, 800 histories

Table IV Fissile and Fusile Breeding in Blanket Regions (Nuclei/Source Neutron). Comparison of Generalized Secondary Weighting to Discrete Ordinates for Different Event Probabilities (200 histories, 10 experiments)

Reaction	5"	6	7	8	Total
$Th(n,\gamma)$	2.83-1+ 3.26-1+2.66-2* 2.82-1±2.31-2**	2.28-1 2.53-1+2.31-2 2.36-1±1.52-2	1.95-1 2.26-1+2.60-2 1.79-1±1.12-2	2.28-1 2.40-1+1.53-2 2.19-1±1.69-2	9.34-1 1.05+0+4.64-2 9.16-1±3.43-2

Reaction	3	10	12	Total
$^6Li(n,\alpha)T$	1.04-1 1.18-1+9.86-3 1.12-1±8.83-3	4.16-1 5.03-1+4.84-2 4.07-1±2.96-2	7.67-2 1.01-1+2.03-2 5.18-2±1.48-2	5.97-1 7.22-1+5.34-2 5.71-1±3.43-2
$^7Li(n,n'\alpha)T$	1.30-2 5.99-3+3.26-3 1.91-2±6.03-3	1.40-2 1.13-2+5.91-3 1.27-2±5.33-3	7.47-5 0.00+0+0.00+0 0.00+0±0.00+0	2.71-2 1.73-2+6.75-3 3.18-2±8.05-3
Total Tritium	1.17-1 1.24-1+1.04-2 1.31-1±1.07-2	4.30-1 5.14-1+4.88-2 4.20-1±3.01-2	7.68-2 1.01-1+2.03-2 5.18-2±1.48-2	6.24-1 7.39-1+5.38-2 6.03-1±3.52-2

+ P_3S_4 Discrete Ordinates

* Monte Carlo, $P_s = P_f = 0.5$

** Monte Carlo, $P_s = 0.9$, $P_f = 0.1$

" Region Numbers

Table V Fissile and Fusile Breeding in Blanket Regions (Nuclei/Source Neutron). Comparison of Generalized Secondary Weighting to Discrete Ordinates for Different Numbers of Histories ($P_s = 0.9$, $P_f = 0.1$).

Reaction	5"	6	7	8	Total
Th(n,γ)	2.83-1+	2.28-1	1.95-1	2.28-1	9.34-1
	2.82-1+2.31-2*	2.36-1+1.52-2	1.79-1+1.12-2	2.19-1+1.69-2	9.16-1+3.43-2
	2.73-1±9.39-3**	2.16-1±8.89-3	1.91-1±9.39-3	2.27-1±1.04-2	9.07-1±1.91-2

Reaction	3	10	12	Total
^{6}Li(n,α)T	1.04-1	4.16-1	7.67-2	5.97-1
	1.12-1+8.83-3	4.07-1+2.96-2	5.18-2+1.48-2	5.71-1+3.43-2
	9.28-2±6.23-3	3.86-1±2.14-2	7.14-2±9.21-3	5.50-1±2.41-2
^{7}Li(n,n'α)T	1.30-2	1.40-2	7.41-5	2.71-2
	1.91-2+6.03-3	1.27-2+5.33-3	0.00+0+0.00+0	3.18-2+8.05-3
	1.04-2±2.71-3	1.84-2±3.41-3	1.41-4±1.14-4	2.89-2±4.36-3
Total Tritium	1.17-1	4.30-1	7.68-2	6.24-1
	1.31-1+1.07-2	4.20-1+3.01-2	5.18-2+1.48-2	6.03-1+3.52-2
	1.03-1±6.79-3	4.04-1±2.17-2	7.15-2±9.21-3	5.79-1±2.45-2

\+ P_3S_4 Discrete Ordinates

* Monte Carlo, 200 histories, 10 experiments

** Monte Carlo, 800 histories, 20 experiments

" Region Numbers

Table VI Fissile and Fusile Breeding in Blanket Regions (Nuclei/Source Neutron). Comparison of Multiplication Weighting to Discrete Ordinates for Different Numbers of Histories.

Reaction	5"	6	7	8	Total
Th(n,γ)	2.83-1+ 2.72-1+2.71-2* 2.80-1±1.35-2** 2.71-1±6.75-3***	2.28-1 2.10-1+2.54-2 2.28-1±1.06-2 2.17-1±4.50-3	1.95-1 1.65-1+1.24-2 2.01-1±5.38-3 1.94-1±3.47-3	2.28-1 2.28-1+1.37-2 2.26-1±8.39-3 2.25-1±4.97-3	9.34-1 8.75-1+4.15-2 9.35-1±1.98-2 9.07-1±1.01-2

Reaction	3	10	12	Total
^{6}Li(n,α)T	1.04-1 9.82-2+1.33-2 1.05-1±7.42-3 9.96-2±2.58-3	4.16-1 3.96-1+3.98-2 3.89-1±2.01-2 4.16-1±8.66-3	7.67-2 9.32-2+1.63-2 7.76-2±1.09-2 8.25-2±4.62-3	5.97-1 5.87-1+4.50-2 5.72-1±2.40-2 5.98-1±1.01-2
^{7}Li(n,n'α)T	1.30-2 1.10-2+3.87-3 1.55-2±3.09-3 1.32-2±1.27-3	1.40-2 1.44-2+6.50-3 1.69-2±2.70-3 1.66-2±1.76-3	7.47-5 0.00+0+0.00+0 1.82-3±1.82-4 1.39-4±1.39-4	2.71-2 2.54-2+7.56-3 3.26-2±4.11-3 2.93-2±2.17-3
Total	1.17-1 1.09-1+1.39-2 1.21-1±8.04-3 1.13-1±2.88-3	4.30-1 4.10-1+4.03-2 4.06-1±2.03-2 4.32-1±8.84-3	7.68-2 9.32-2+1.63-2 7.78-2±1.09-2 8.26-2±4.62-3	6.24-1 6.12-1+4.56-2 6.05-1±2.43-2 6.27-1±1.03-2

\+ P_3S_4 Discrete Ordinates

* Monte Carlo, 200 histories, 10 experiments

** Monte Carlo, 800 histories, 20 experiments

*** Monte Carlo, 3200 histories, 40 experiments

" Region Numbers

Table VII Computational Statistics for Different Investigated Methods (200 history cases)

	Absorption Weighting	Multiplication Weighting	Generalized Secondary Weighting	
			$P_f = 0.5$[+]	$P_f = 0.1$
Cpu Time	5 min 18.5 sec	5 min 18.0 sec	11 min 4.6 sec	6 min 1.8 sec
Cpu Cost ($)*	5.57	5.56	11.63	6.33
Memory Usage	7.69	7.65	15.89	8.69
Memory Cost	4.83	4.81	10.04	5.47
Total Cost	10.40	10.37	21.67	11.80

* Based on rates for overnight runs on the UW-UNIVAC-1110.

\+ P_f is probability assigned to fission events.

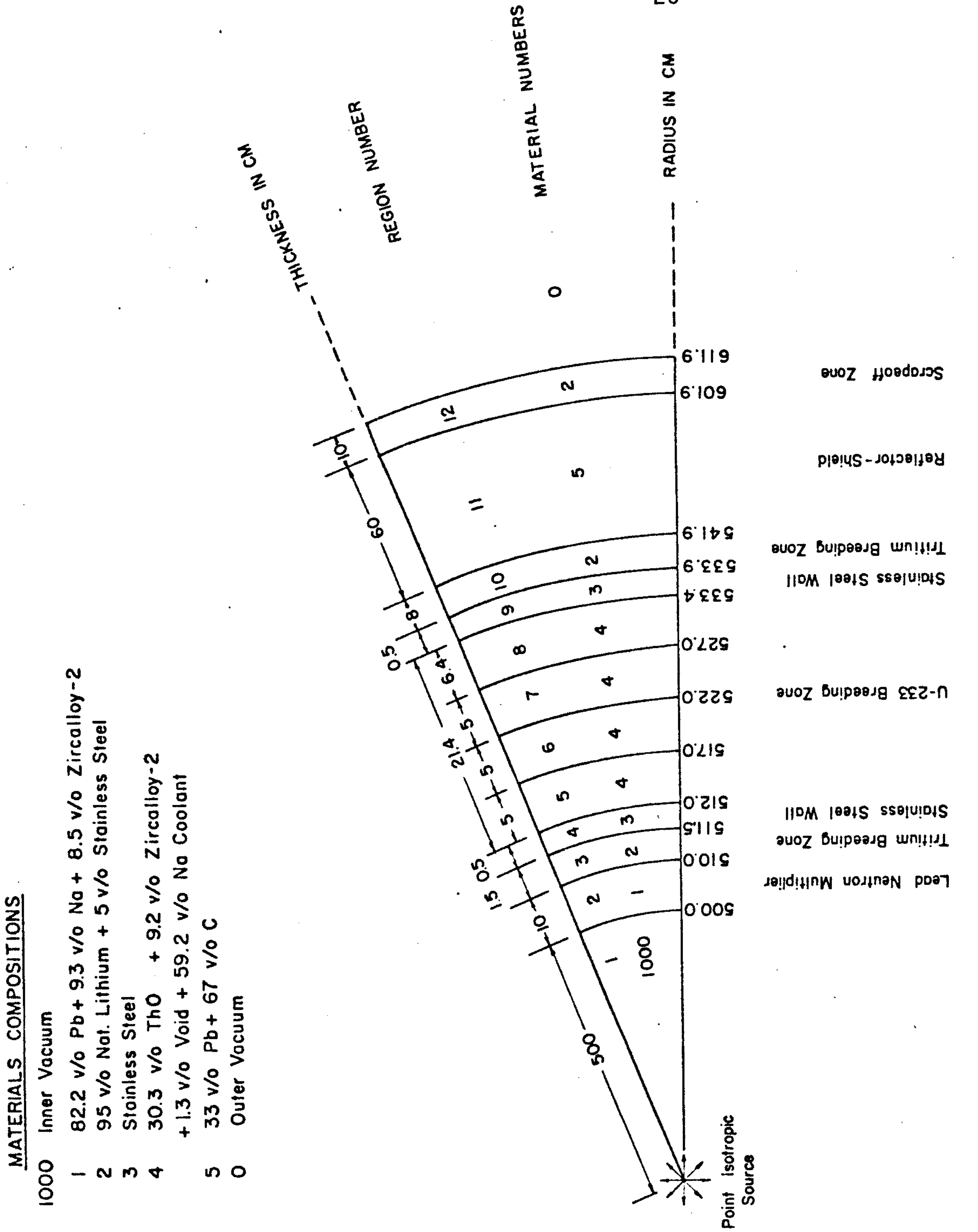

Fig. 1 Spherical Geometrical Model for Hybrid Fusion-Fission Reactor Blanket Design.