Equilibrium Conditions in a Tokamak Device

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by

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EQUILIBRIUM CONDITIONS IN A TOKAMAK DEVICE

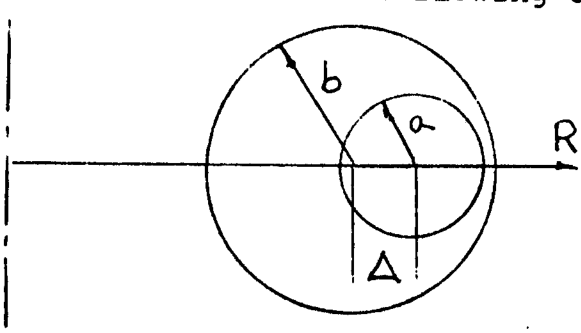
The purpose of this paper is to present the basic principles of this type of equilibrium and the possible solutions available to attain this condition.

We assume that the current already exists in the Tokamak and will ignore the method of creating this current. The basis for this analysis for a plasma is derived from a study of the equation

$$\nabla p = \frac{1}{c}(\vec{J} \times \vec{H}) \qquad (1)$$

By considering a Tokamak we are considering a system with axial symmetry, the plasma being confined by a magnetic field of the current itself B_θ and stabilized by a longitudinal field B_ϕ. If we consider thin plasma columns, then we can make the assumptions $r/R \gg 1$ and develop the forces in a series around r/R. The first term corresponds to a cylindrical approximation $R = \infty$ and we will neglect the higher order terms.

Consider the following coordinate system:



We have let a designate the radius of the plasma, b the radius of the conducting shell, R the major radius of the torus, and Δ the displacement of the column from the center of the shell. For $r > a$, we assume that j = 0 and $\bar{p} = 0$ which is true for a system using a divertor.

To counter the internal forces which tend to increase the radius R of the plasma ring, we use a shell of high electrical conductivity.

The equilibrium is attained by the eddy current created in this shell. A conductor, in this case the plasma, which is displaced from the tube axis will induce current on the nearest side of the tube having the opposite direction of the plasma current. Therefore a force will act on the plasma directed toward the center of the shell.

The following discussion makes the assumption $a \ll b \ll R$ which enables us to see more clearly the physical source of the forces entering into this equilibrium. At first glance we see that we hope to obtain equilibrium in the directions of freedom; 1/ the a direction and 2/ the R direction.

Considering the first, we get

$$\bar{p} + \frac{\overline{B_i^2}}{8\pi} = \frac{B_a^2}{8\pi} + \frac{B_e^2}{8\pi} + p_a \qquad (2)$$

where $\bar{p}$ is the average pressure of the plasma, $\overline{B_i^2}$ the average energy due to the trapped toroidal field, B_a the poloidal field at r=a, B_e the longitudinal field on the surface and p_a the external pressure. For our assumptions of $r > a$, $p_a = 0$ and knowing that $B_a = 2J/ca$, we have

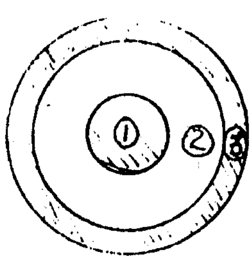
$$\boxed{2\pi a^2 \bar{p} = \frac{J^2}{c^2} + \left(\frac{H_e^2 - \overline{H_i^2}}{4}\right)a^2} \qquad (3)$$

To find the equilibrium condition with respect to R, we must examine the three forces which act to increase R.

1/ The electrodynamical repulsion of the current ring. The force acting on a conductor is always directed in such a way as to increase its own inductance. The force is written

$$F_1 = \frac{J^2}{c^2}\frac{dL}{dR}$$

where L is the inductance and R is the major radius. We therefore need L(R). For a cable wrapped in a conducting shell $1/2\, LI^2 = \Sigma W_i$ in the different regions of the cable. We then get the expression for L of



$$L = 4\pi R\left[\ln\frac{b}{a} + \frac{l_i(a)}{2}\right] \qquad (4)$$

where

$$l_i(a) = \frac{1}{\pi a^2 B_a^2}\int_0^a B_\omega^2\, \rho\, d\rho\, d\omega \qquad (5)$$

Then

$$\vec{F}_1 = \frac{2\pi J^2}{C^2}\left[\ln\frac{b}{a} + \frac{l_i(a)}{2}\right] \qquad (6)$$

2/ The second source of forces is the expansion of the plasma by its own internal pressure and is of the form

$$\vec{F}_2 = 2\pi^2 a^2 \bar{p} \qquad (7)$$

3/ The third source comes from the fact that B_ϕ is not the same at R-a and at R+a which leads to a force in the increasing R direction for $H_a^2 > \overline{H_i^2}$ of

$$\vec{F}_3 = 2\pi^2 a^2 \left(\frac{H_a^2 - \overline{H_i^2}}{8\pi}\right) = \frac{\pi}{4} a^2 \left(H_a^2 - \overline{H_i^2}\right) \qquad (8)$$

To balance these forces we can use two sorces of energy: 1) eddy currents in the shell due to a displacement Δ_o of the current and 2) an externally applied transverse field.

The eddy currents created by the displacement of the plasma current creates an $H_\perp$ which reacts on the plasma given as

$$H_\perp = \frac{2J}{c}\frac{\Delta}{b^2} \qquad (9)$$

Then:

$$\vec{F}_4 = \frac{1}{C}(\vec{J}\times\vec{H})\, 2\pi R = \frac{4\pi J^2}{c^2}\cdot\frac{R\Delta}{b^2} \qquad (10)$$

The externally applied fields apply a force of the same type on the plasma

$$\vec{F}_5 = \frac{J}{c}\cdot H_\perp\, 2\pi R \qquad (11)$$

Let us assume for the moment that the equilibrium is attained without external fields and solely by the eddy currents designating the necessary displacement for this to happen as Δ_o.

$$\Sigma F = 0$$

$$2\pi\frac{J^2}{c^2}\left[\ln\frac{b}{a} + \frac{l_i(a)}{2}\right] + 2\pi^2 a^2 \bar{p} + \frac{\pi}{4} a^2\left(H_a^2 - \overline{H_i^2}\right) - \frac{4\pi J^2}{c^2}\frac{R\Delta_o}{b^2} = 0 \qquad (12)$$

Using relationship #3 and solving for Δ_o, we have:

$$\Delta_o = \frac{b^2 c^2}{4\pi J^2 R}\left\{\frac{2\pi J^2}{c^2}\left[\ln b/a + \frac{l_i(a)-1}{2}\right] + 4\pi^2 a^2 \bar{p}\right\}$$

This expression can be further simplified using the relationships $H_\theta = 2J/ca$ and defining $\beta^* = \Sigma nkT/(H_\theta^2/8\pi)$ we obtain

$$\Delta_o = \frac{b^2}{2R}\left(\ln b/a + \beta^* + \frac{l_i(a) - 1}{2} \right)$$

A more complete and exact calculation of Δ_o done by Shafranov making no assumption on a/b yields

$$\Delta_o = \frac{b^2}{2R}\left(\ln b/a + \left(1 - \frac{a^2}{b^2}\right)\left(\beta^* + \frac{l_i(a) - 1}{2}\right)\right)$$

Now let us consider the more general case including the effect of externally originating fields.

$$\Delta - \Delta_o = - \frac{b\, B_\perp}{B_\theta(b)}$$

or

$$\frac{\Delta - \Delta_o}{b} = - \frac{10\, \phi_\perp^{ext}}{8\pi R I_p} \qquad \text{(engineering units)}$$

POSSIBLE SOLUTIONS

We can see from the above equations that to control the position of the plasma amounts to controlling the transverse flux which passes through the surface of the shell. From the solution of the Cauchy equation $B = \nabla \phi_c$, we find that in order to attain equilibrium we need to contain the energy of the field $B_\perp^\infty$ which is given below.

$$B_\perp^\infty = B_a \times \frac{a}{2R}\left[\log \frac{8R}{a} + \beta^* + \frac{l_i - 3}{2}\right]$$

The fig. 1 presents this situation and we see that for I_p = 400kA R= 98cm., and A = 5, this field is of the order of 1 - 1.6 kg.

The sources of this $B_\perp^\infty$ are varied: Fig. 2

1) the current circulating in the plasma
2) the currents in the liner (vacuum vessel)
3) the currents from exterior windings (compensator)
4) the magnetization currents in the inducing circuit

5) the eddy currents in the shell if there is one.

When we discuss possible systems of equilibrium, we first diferentiate between systems with and without a shell and then discuss the external field of a system with a shell.

For a system without a shell, equilibrium demands that we have a rapidly programmable field that will, at all times, supply $B_{\perp}^{\infty}$.

For systems with a shell we have 4 different values of the external field possible. (Fig. 3 & 4)

a) $\Phi_e = 0$ In this configuration the shell contain all of the plasma field. I_d represents the dipolar currents for equilibrium and $\Sigma I_1 = I_p$. This configuration has the advantage of conserving flux which is necessary for small machines but is a mechanically complicated system.

b) $\Phi_c = \Phi_p^{\infty}$ The shell is the container of only the dipolar currents from the displacement of the current and the external field is supplied independantly. This system is the simplest however the cost would seem prohibitive. The shell must be thick enough to retain the eddy currents suffisant for equilibrium overcoming the shell's resistance. To give examples of the thicknesses needed, 2.5 cm of copper is good for 20msec and 10cm for 100msec.

c) $\Phi_e = \Phi_c = \Phi_p^{\infty} + \Phi_{\perp}^{\infty}$ The shell is this system plays no role except to give stability to the plasma against perturbation of short wavelength.

d) $\Phi_e = \Phi_{\perp}^{\infty}$ This configuration is difficult to acheive.

Let us examine more closely the eddy currents that are created in the shell. Figure 5 gives an idea of the positions and distribution.

Figure 6 brings together the equations that apply to these eddy currents. We have already seen the first equation which relates the displacement Δ to the transversal flux. The eddy currents are calculated from $B_\perp^\infty$ existing inthe region of the shell while $B_\perp^{ext}$ acts on the outer surface. The diagram shows the result of these fields. The total currents circulating apperas from the difference between $B_\perp^\infty$ and $B_\perp^{ext}$. For the particular case of $B_\perp^\infty = B_\perp^{ext}$; the eddy currents are rigorously zero.

The equations describing the evolution takes into account the resistance of the shell and the self inductance of the cuts.

To a $d\emptyset/dt$ corresponds a $d\Delta/dt$. The force which maintains the plasma in position at the center is due to the movement of the plasma. The position Δ of the plasma then becomes a sum of: (Fig. 7)

1/ Δ_o due to the variation of the internal energy of the plasma and can be from 1.65 to 6.5 cm.

2/ Δ_{shell} due to the resistive penetration of the uncut shell. Corresponding to this penetration we have a velocity of the plasma $d\Delta/dt$ depending on $B_\perp^{ext}/B_\perp^\infty$ and $\beta^* + \frac{l_i - 1}{2}$. For $B_\perp^{ext}=0$, $d\Delta/dt = 115 \rightarrow 190$ cm/sec

3/ Δ_{cut} due to the presence of the opening and their own penetration. This penetration is larger than Δ_{shell} and means that with respect to the eddy currents the openings are growing as the current penetrated inward. At time zero these cuts (N=8) introduce a Δ of 7to 11 cm and this doubles in 30 msec as opposed to 100msec doubling time for the shell penetration.

Let us consider a system in which $\Delta = \Delta_o$ meaning $B_\perp^\infty = B_\perp^{ext}$. In this case you notice that the eddy current are zero. This operating regime has a special advantage and is considered ideal. The reason becomes evident after an examination of the forces on the shell due to the interaction of the eddy currents along the cuts with the B_ϕ. The resulting force puts each section in a swisting action with forces attaining 42 tons for $B_\perp^{ext} = 0$. (figure 8)

From this brief introduction to the equilibrium considerations in a Tokamak, it is evident that the problem is quite complicated and that an openings such pumping ducts or fueling ports etc can have a major effect on these conditions, and therefroe demand careful consideration when designing a reactor.

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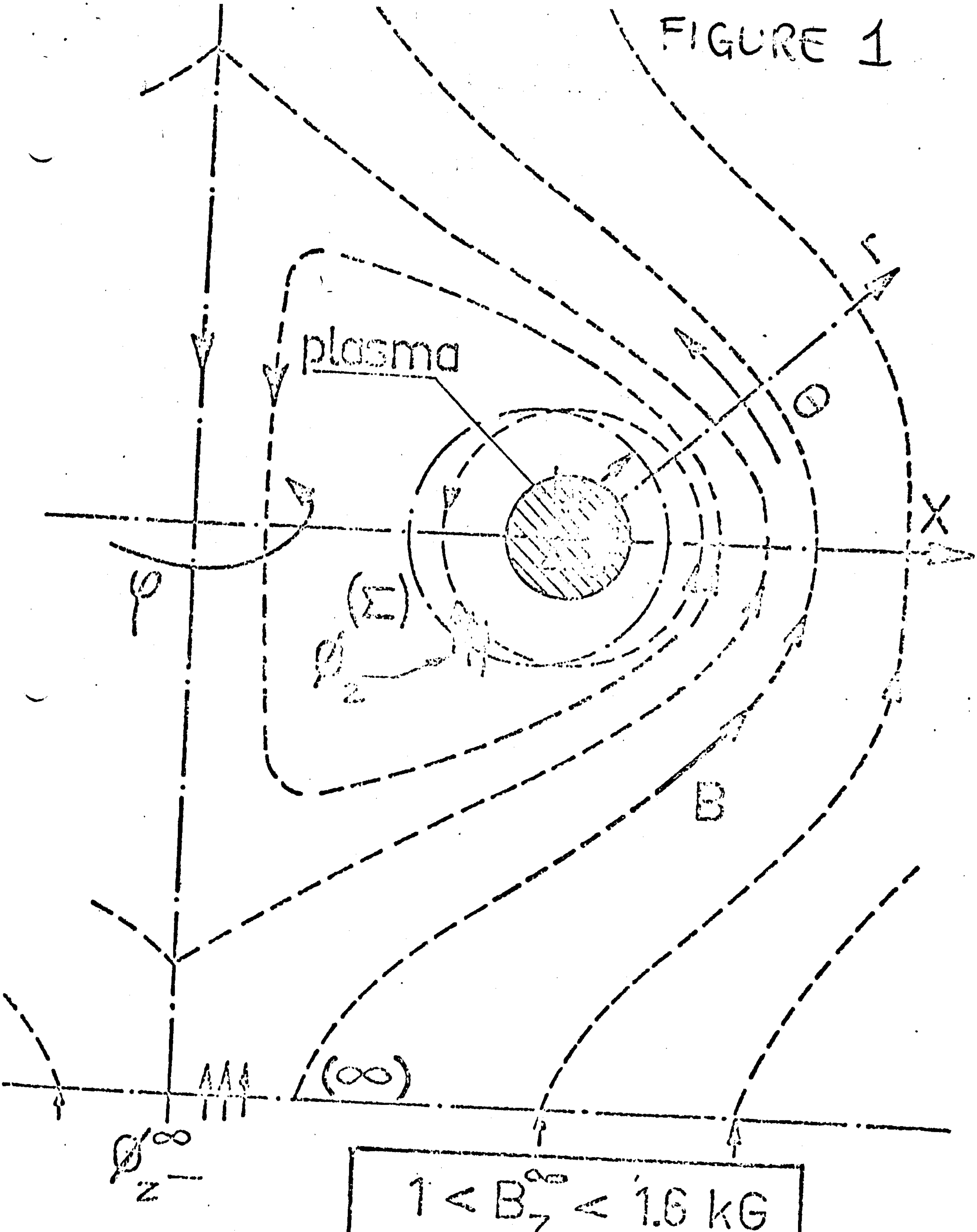
FIGURE 1



PLASMA I=400 kA
R=[illegible] a=[illegible]

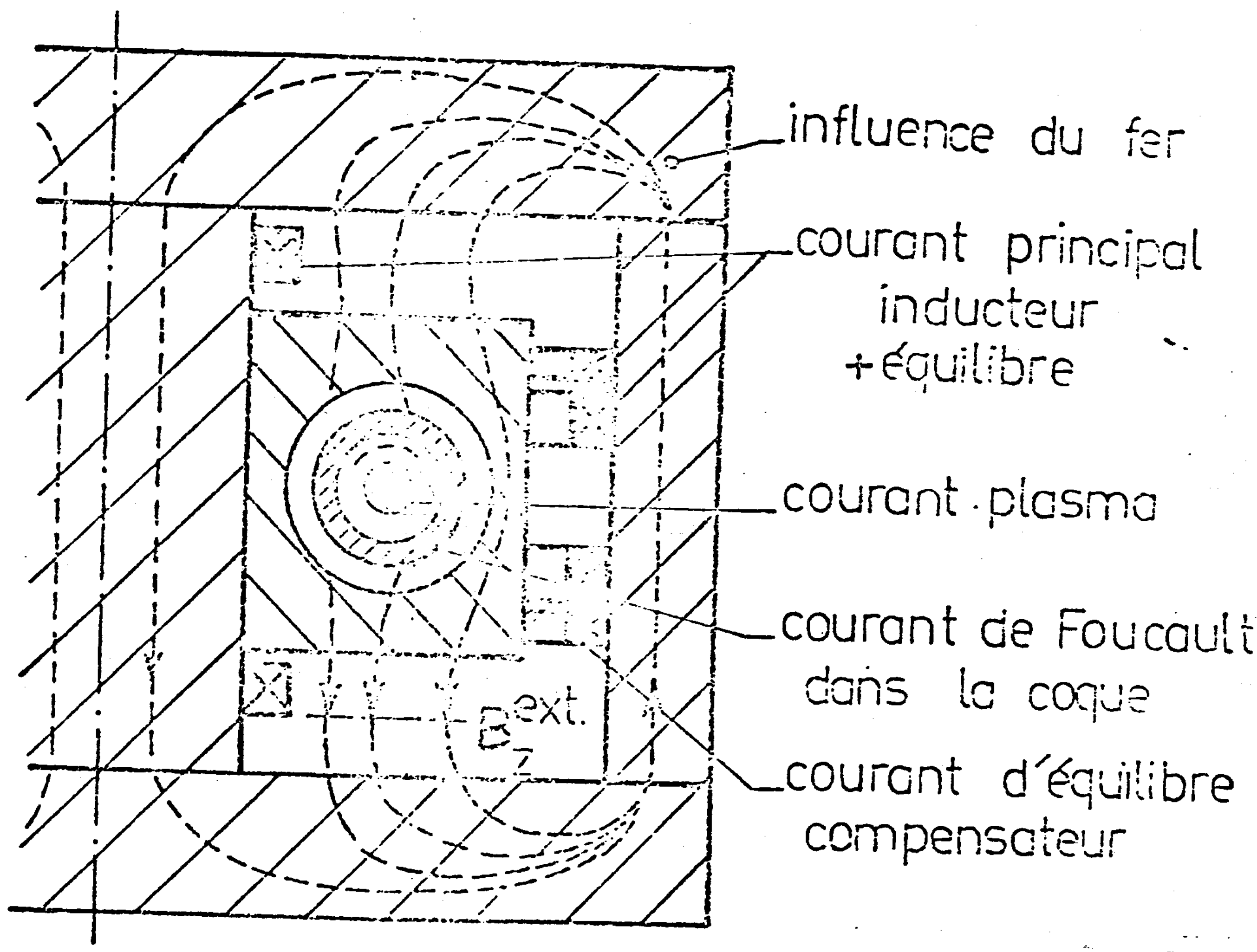
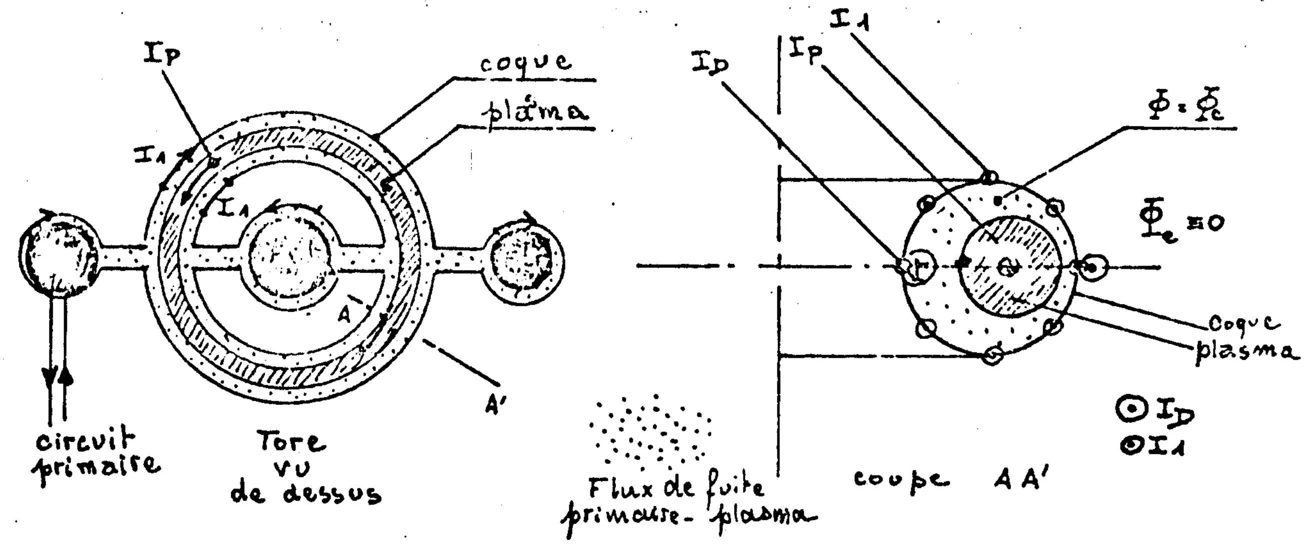

influence du fer
courant principal
inducteur
+équilibre
courant plasma
courant de Foucault
dans la coque
courant d'équilibre
compensateur
$B_z^{ext.}$

FIGURE 2

a) $\Phi_e = 0$



b) $\Phi_e = \Phi_p^{\infty}$

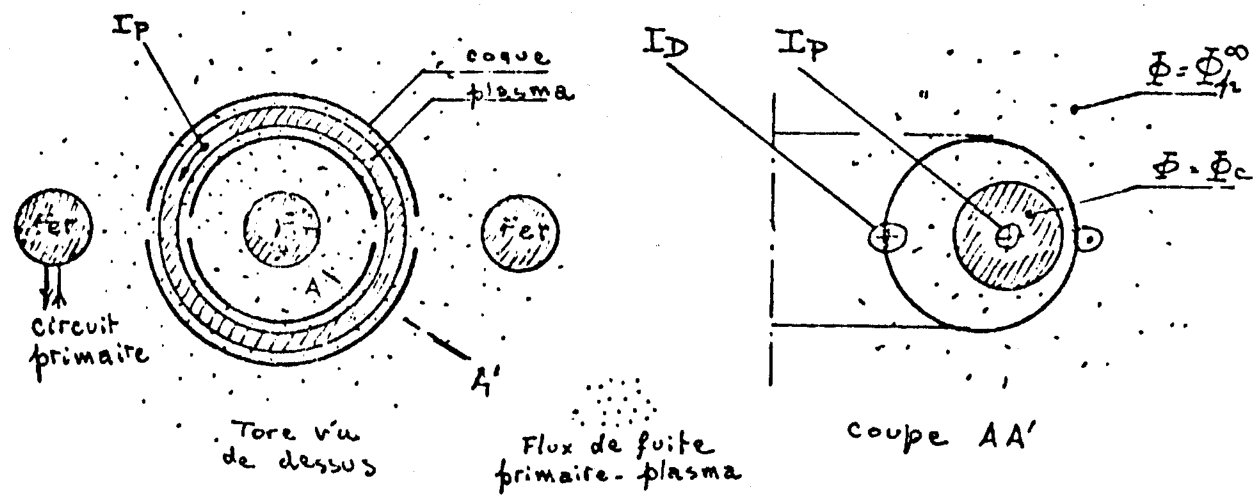
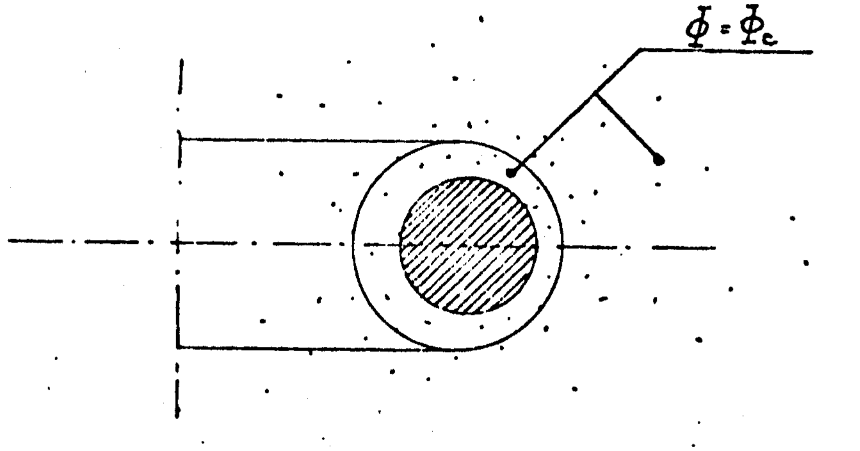


FIGURE 3

c) $\Phi_c = \Phi_c = \Phi_p^{\infty} + \Phi_1^{\infty}$



d) $\Phi_c = \Phi_{\perp}^{\infty}$

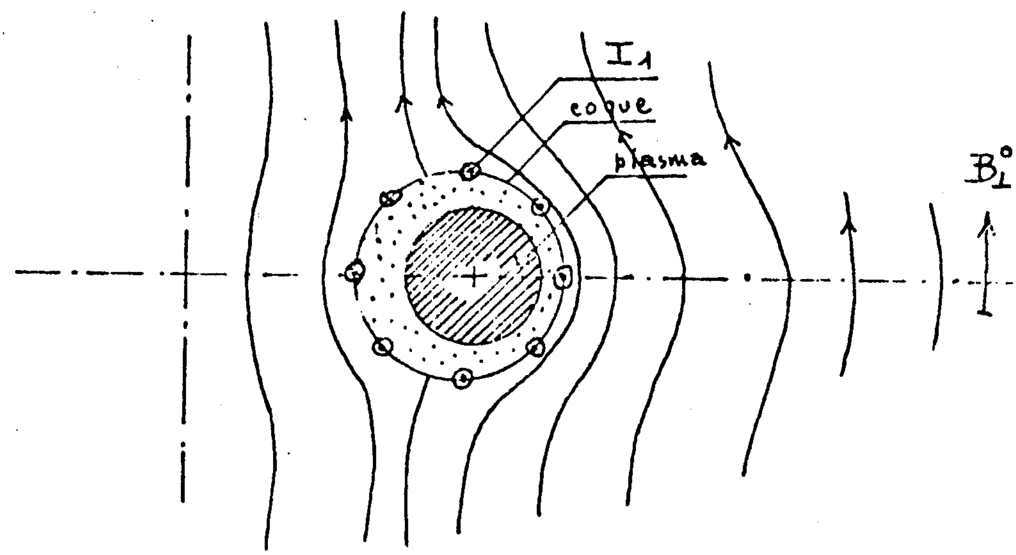


FIGURE 4

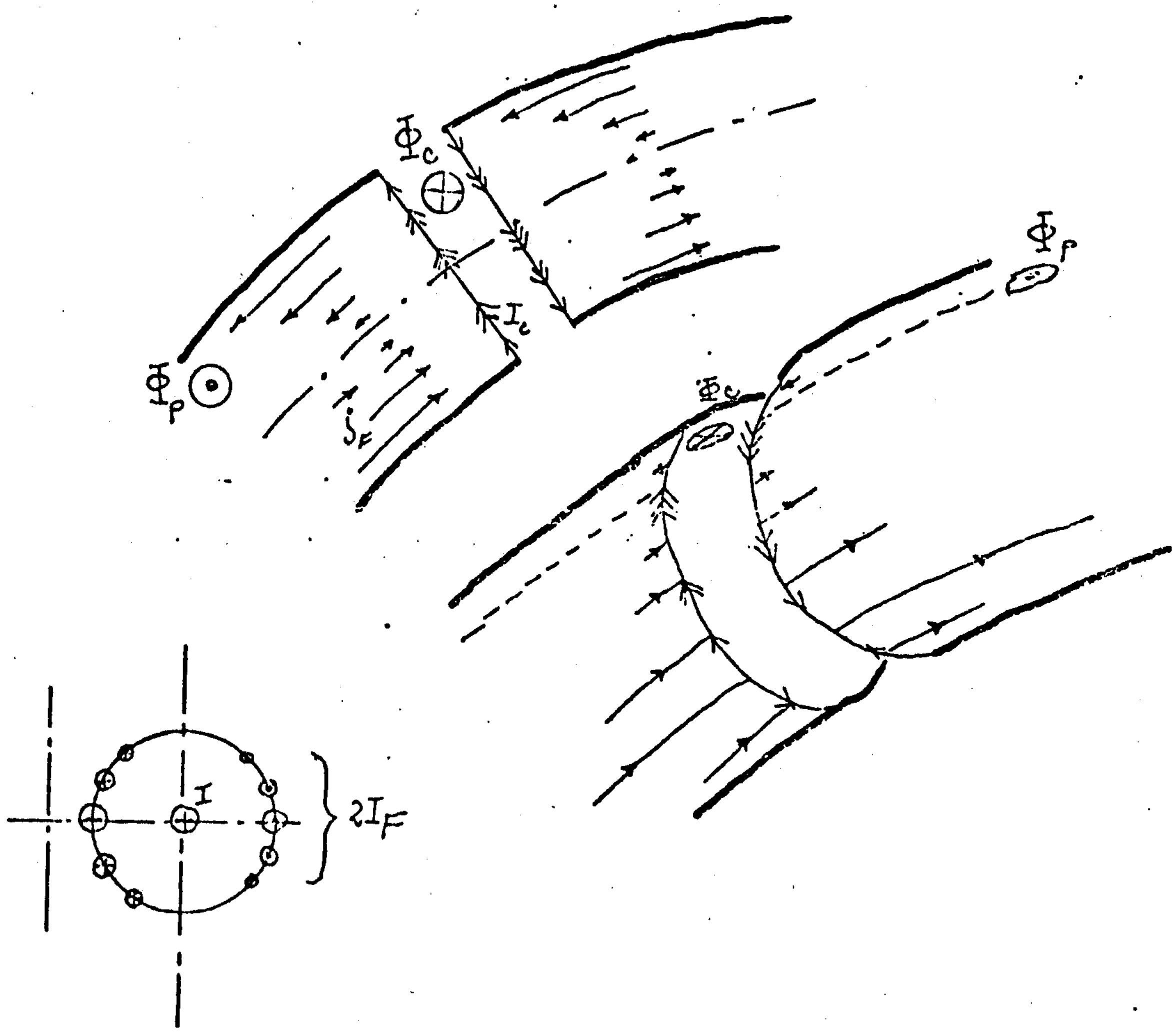


FIG. 5

COURANT DE FOUCAULT CIRCULANT
DANS LA COQUE ET DANS LA COUPURE

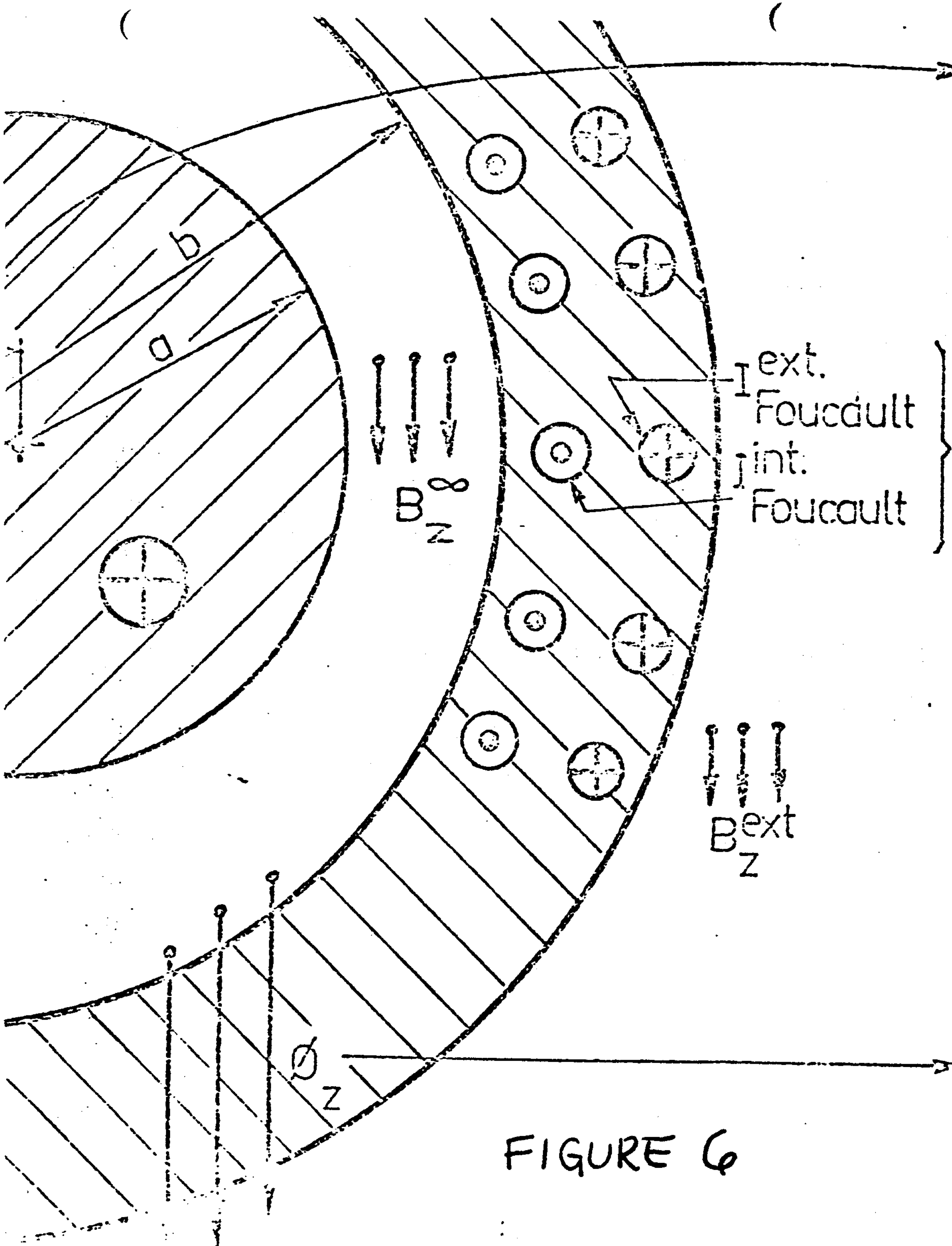


FIGURE 6

$$\frac{\Delta - \Delta_o}{b} = \frac{10^9}{8\pi R I} \times \emptyset_z (\Sigma$$

$$I_{\substack{Foucault\\(Eddy)}} = \frac{10b}{2\pi}(B_\perp^{\infty} - B_\perp^{ext})$$

$$B_\perp^{\infty} = \frac{I}{10R}\left(\mathrm{Log}\, 8\frac{R}{a} + \beta^{*} + \frac{l}{2}\right.$$

$$\frac{d\emptyset_z(\Sigma)}{dt} = R_{coque} \times I_{\substack{Foucau\\(Eddy}}$$

$$\emptyset_{z(\Sigma)}(t=0) = L_{\substack{coupure\\(cuts)}}(t=0) \times I_{\substack{Foucault\\(Eddy)}}(t=0) +$$

FIGURE 7

$\Delta(t_{sec})_{cm} =$	total displacement of plasma
$1.65 < \Delta_0 < 6.5$	variation of plasma internal energy
$+ \quad 115 \times t \times \left(1 - \frac{B_z^{ext}}{B_z^{\infty}}\right) < \Delta_{shell} < 190 \times t \times \left(1 - \frac{B_z^{ext}}{B_z^{\infty}}\right)$	resistive penetration in shell $\tau \sim 100$ ms
$+ \; 0.9 N \times (1 + 6\sqrt{t})\left(1 - \frac{B_z^{ext}}{B_z^{\infty}}\right) < \Delta_{cuts} < 1.43 N \times (1 + 6\sqrt{t})\left(1 - \frac{B_z^{ext}}{B_z^{\infty}}\right)$	resistive penetration in cuts $\tau \sim 30$ ms

position Δ is mastered by programming B_z^{ext}

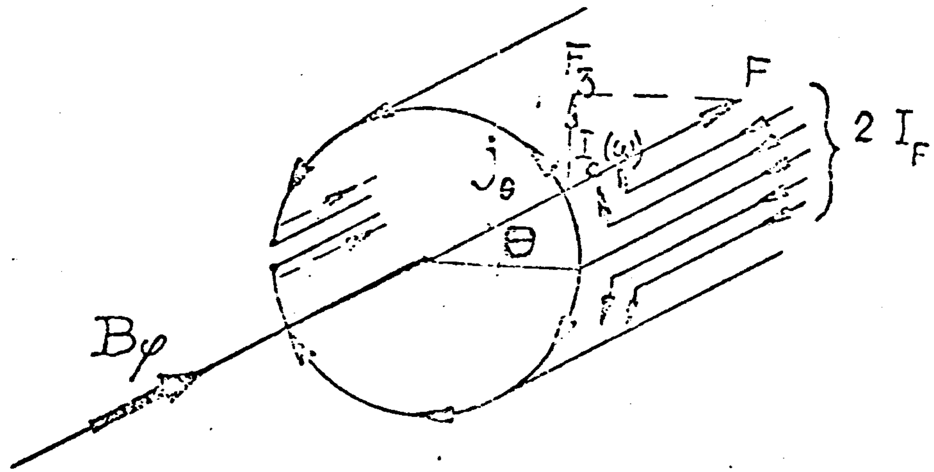


Fig.3 Courants de Foucault (Eddy currents)

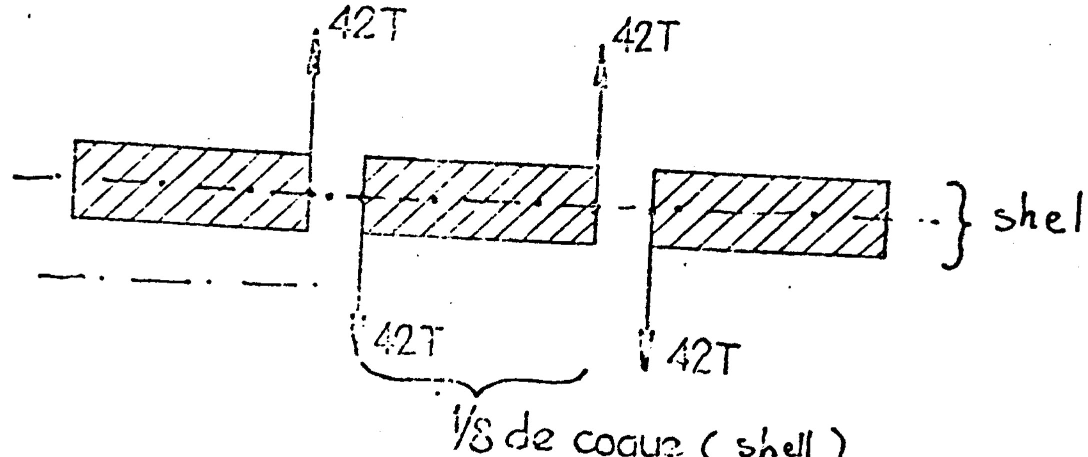


FIGURE 8