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THE INTERACTION OF A HIGH FREQUENCY MAGNETIC FIELD WITH THE DCLC MIRROR INSTABILITY

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The effect of a small amplitude, high frequency magnetic field on drift-cyclotron loss-cone instability is investigated. With $B_1/B_o = 5\%$ and $\omega_{ce}/\omega_o = 80$, the critical characteristic length of plasma density gradient reduces dramatically compared with the results of Post and Rosenbluth and is comparable to the finite β stabilization effect.

I. Introduction

The stabilization of plasma instabilities by a high frequency oscillating magnetic field has been studied by Ivanov and his co-workers.[1,2] They did not consider the drift-cyclotron loss-cone (DCLC) instability[3,4] however which is driven by the mirror loss-cone and plasma density gradient. This mode is observed in present day experiments [5] and puts a constraint on the characteristic length of the plasma density gradient. For the case of a mirror fusion reactor having a flat central density profile[3,4,6] the DCLC stability will determine the size of the plasma boundary. For tandem mirror reactors, a thin and stable plasma boundary in the plugs is necessary to maintain high Q.[6] In this paper, we study the stabilization effect of an oscillating magnetic field on the DCLC, and find that an oscillating magnetic field at frequency $\omega_{ci} \ll \omega_o \ll \omega_{ce}$ can reduce the critical characteristic length of the plasma density gradient by a factor of 2.

II. Stability Analysis

We consider a plasma which is inhomogeneous in the x direction and immersed in a fixed magnetic field $\vec{B}_o$ in the z direction. Assume that there is an externally applied high frequency magnetic field $\vec{B}_1(t) = \vec{B}_1 \cos\omega_o t$ in the y direction. We also assume the amplitude of the oscillating magnetic field is smaller than that of the fixed magnetic field ($B_1 \ll B_o$). The spatial dependence of $\vec{B}_1$ and the electric field associated with the oscillating magnetic field are neglected.

Following the analyses of Ivanov,[1,2] we can obtain the dispersion relation of the DCLC in the presence of a high frequency oscillating magnetic field ($B = B_1 \cos\omega_o t$) to be

$$1 + \frac{\omega_{pe}^2}{\omega_{ce}^2} + 2\frac{\omega_{pe}^2}{k^2 v_{Te}^2}(1-A) = A\frac{\omega_{pe}^2}{\omega_{ce}}\frac{\varepsilon}{\omega k} - \frac{8\pi^2 n_o e^2 \omega}{k^2 M} \cdot \int dv_\perp^2 \, dv_{\shortparallel} \frac{\partial f_o}{\partial v_\perp^2} \sum_{n=-\infty}^{\infty} \frac{J_n^2(k\, v_\perp/\omega_{ci})}{\omega + n\omega_{ci}} . \tag{1}$$

where $A = \int_{-\infty}^{\infty} J_o^2(\mu v_{\shortparallel}) f_{oe} dv_{\shortparallel}$, $\mu = kB_1/\omega_o B_o$, $\varepsilon = \frac{d\ell n\, n_o}{dx}$, ω_{pe} = electron plasma frequency, $\omega_{ce(i)}$ = electron (ion) cyclotron frequency, ω and k are the frequency and wave vector for the excited wave, V_{Te} is the electron thermal speed and $V_{\shortparallel(\perp)}$ is the ion velocity parallel (perpendicular) to the field direction. To obtain Eq. (1), we have assumed

(1) $k_{\shortparallel} = 0$; (2) $\frac{\omega}{\omega_{ce}}$, $ka_e \ll 1$; (3) $k\varepsilon v_\perp^2/\omega_{ci}\omega \ll 1$; (4) $\omega_{ci} \ll \omega_o \ll \omega_{ce}$, $\mu v_{Te} \ll 1$. The electron distribution function is assumed to be Maxwellian and ion distribution function is assumed to have the form[4]

$$f_{oi}(v_\perp^2) = (1/\pi v_{T_i}^2)[R/(R-1)][\exp(-v_\perp^2/v_{T_i}^2) - \exp(-Rv_\perp^2/v_{T_i}^2)], \tag{2}$$

where $R \equiv$ mirror ratio, and $v_{T_i}^2 \equiv 2T_i/M$. With the assumption that $(k\, v_\perp/\omega_{ci}) \gg 1$, Eq. (1) can be expressed as

$$\left(\frac{\omega_{ci}^2}{\omega_{pi}^2} + \frac{m}{M}\right) k^3 a_i^3 + 2(ka_i)\left(\frac{T_i}{T_e}\right)\left(\frac{R+1}{R}\right)(1-A) = (\varepsilon a_i)\frac{\pi A}{\Omega}(ka_i)^2 - D\Omega\cot\Omega, \tag{3}$$

where $a_i = (v_{T_i}/\omega_{ci})[(R+1)/R]^{1/2}$, $\Omega = \pi\omega/\omega_{ci}$, and $D = (2/\sqrt{\pi})(R+1)^{3/2}(R+\sqrt{R})$.
With straightforward algebra, Eq. (3) can be written in the dimensionless form

$$\varepsilon a_i = \Omega[x + C_1(1-A)\, x^{1/3} + \Omega\cot\Omega]/[AC_2\, x^{2/3}] \equiv G(x,\Omega), \tag{4}$$

where $C_1 = 2(T_i/T_e)[(R+1)/R]E^{-1/3}D^{-2/3}$, $C_2 = \pi E^{-2/3}D^{-1/3}$, $E \equiv (\omega_{ci}^2/\omega_{pi}^2 + m/M)$, and $x = (ka_i)^3 E/D$. The critical density gradient can then be obtained as

$$(\varepsilon a_i)_{crit} = \underset{x}{\mathrm{Min}}\ \underset{\Omega}{\mathrm{Max}}[G(x,\Omega)], \tag{5}$$

and the stability criterion for the DCLC mode becomes

$$(\varepsilon a_i) > (\varepsilon a_i)_{crit} = \Omega[x + C_1(1-A)x^{1/3} + \Omega\cot\Omega]/[AC_2 x^{2/3}] , \tag{6}$$

with

$$x = \frac{1}{2}\,\Omega^2\csc^2\Omega + \frac{3}{2}\frac{A'}{A}[x^2 + C_1x^{4/3} + x\Omega\cot\Omega] \qquad (6a)$$

and

$$\Omega = \frac{\Omega^2}{\sin 2\Omega} - \frac{1}{2}\tan\Omega[x + C_1(1-A)x^{1/3}] \;, \qquad (6b)$$

where $A' = \frac{dA}{dx}$. To solve Eqs. (6a) and (6b) numerically, we note that $A = {}_3F_3[\frac{1}{2}, 1, \frac{1}{2}; 1, 1, 1; -x^{2/3}H]$ and $A' = \frac{1}{4}\,{}_3F_3[\frac{3}{2}, 2, \frac{3}{2}; 2, 2, 2; -x^{2/3}H]\cdot(-H)(2/3)x^{-1/3}$, where ${}_pF_q$ is the generalized hypergeometric function and $H = (D/E)^{2/3}(T_e/T_i)(m/M)[R/(R+1)](\omega_{ce}/\omega_o)^2(B_1/B_o)^2$.[7,8]

In Eqs. (6a) and (6b), the terms that contain C_1 are dominant. Since $x^{2/3}H = (\mu v_{Te})^2 << 1$, we can approximate A as

$$A \cong 1 - \frac{1}{4}\,x^{2/3}H, \qquad (7)$$

and

$$A' \cong -\frac{1}{6}\,x^{2/3}H. \qquad (8)$$

Then from Eq. (6a) we have

$$x \cong \frac{2\Omega^2\csc^2\Omega}{HC_1} \qquad (9)$$

Substituting Eq. (9) into Eq. (6b), we find

$$\frac{1}{4}\Omega \cong \sin\Omega\,\cos\Omega. \qquad (10)$$

This is the same as the result obtained in Ref. (3). From Eq. (10), one then finds $\Omega \approx 1.237$ and $x \approx 3.43\,H^{-1}(C_1)^{-1}$. We can also write an approximate formula for $ka_i \approx 1.51\,H^{-1/3}(C_1)^{-1/3}\,D^{1/3}E^{-1/3}$ which is greater than 1 except at low density. In this case

$$(\varepsilon a_i)_{crit} \sim H^{2/3}(C_1)^{2/3}(C_2)^{-1}, \qquad (11)$$

i.e. $(\varepsilon a_i)_{crit}$ depends only weakly on the temperature ratio.

The critical plasma radii, $R_{crit} \equiv 1/\varepsilon_{crit}$, in various cases are given in Fig. 1-3. Fig. 1 compares our results with those of Post and Rosenbluth.(3) The saturation of the critical scale length can be understood from Eq.(1), since at high density $(\varepsilon a_i)_{crit}$ is independent of plasma density. Fig. 2 shows the effect of the amplitude of the oscillating magnetic field. The larger the amplitude, the smaller is the critical radius. This effect combined with the frequency effect shown in Fig. 3, can be understood by noting that the transverse distanceelectrons moved in one period is $v_{\shortparallel}(B_1/B_0)\omega_o^{-1}$. Thus for larger values of B_1/B_0 or ω_0^{-1}, the electron can move farther across the magnetic field and cancel the wave fields.

III. Conclusions

We have found tha an oscillating magnetic field can stabilize the DCLC mode for sufficient oscillating field amplitude. At B_1/B_0=5% and ω_{ce}/ω_o=80, the critical radius reduces dramatically compared with the Post-Rosenbluth result(3) and is comparable to the finite β stabilization effect.(4)

Acknowledgements

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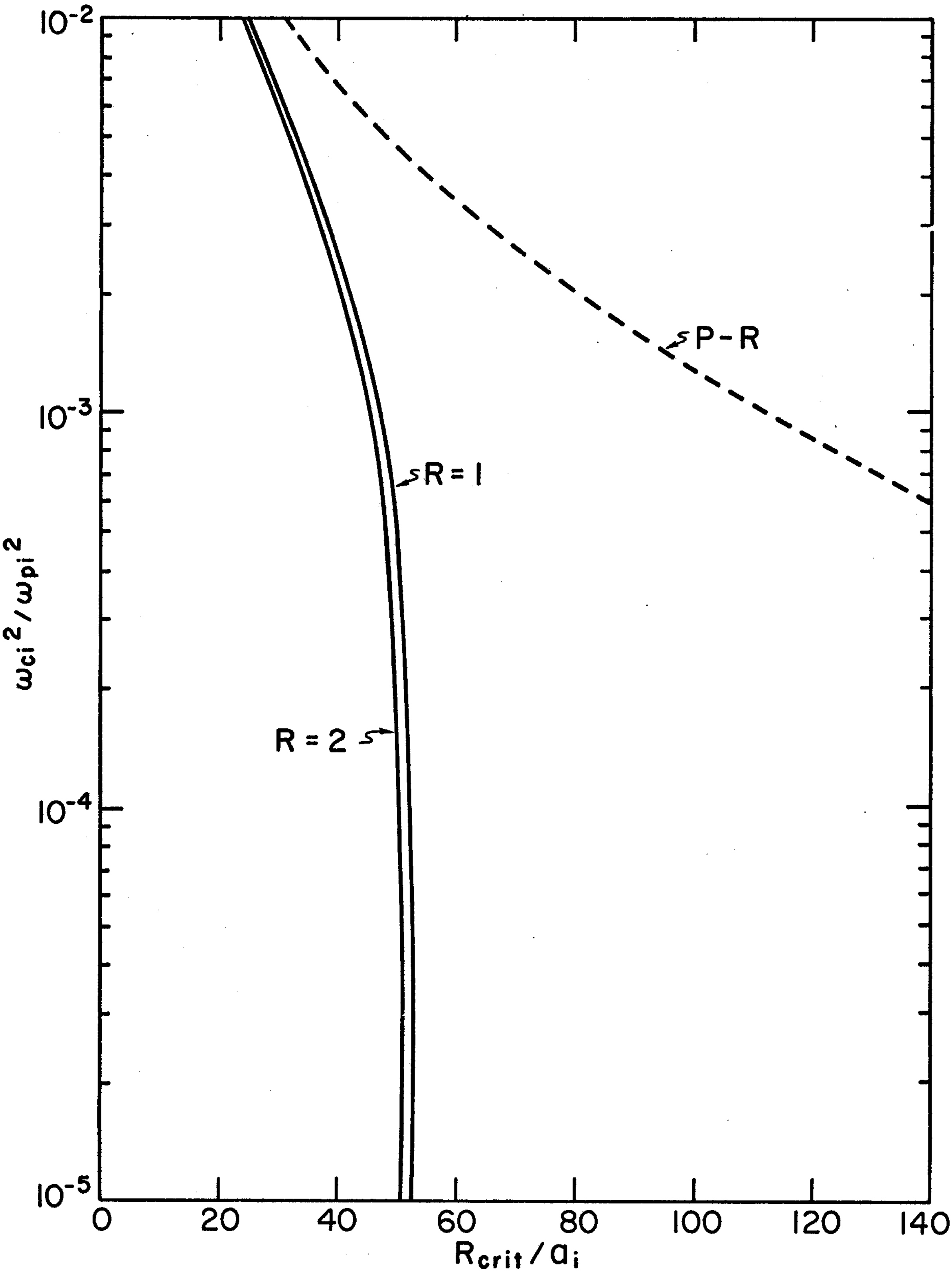
Figure Captions

Fig. 1 - Critical characteristic length (R_{crit}/a_i) as a function of density ($\omega_{ci}^2/\omega_{pi}^2$) for ω_{ce}/ω_o = 80, T_i/T_e = 20, and B_1/B_o = 5% at R=1 and R=2. Curve P-R is the result of Post and Rosenbluth at R=1.

Fig. 2 - Critical characteristic length (R_{crit}/a_i) as a function of density ($\omega_{ci}^2/\omega_{pi}^2$) at R=2, ω_{ce}/ω_o = 80, and T_i/T_e = 20 for B_1/B_o = 2%, 5% and 10% respectively.

Fig. 3 - Critical characteristic length (R_{crit}/a_i) as a function of density ($\omega_{ci}^2/\omega_{pi}^2$) at R=2, T_i/T_e = 20, and B_1/B_o = 5% for the applied frequency (ω_o) set at .025 ω_{ce} and .0125 ω_{ce} respectively.



10^{-2}
10^{-3}
10^{-4}
10^{-5}
$\omega_{ci}^2/\omega_{pi}^2$
0
20
40
60
80
100
120
140
R_{crit}/a_i
P-R
R = 1
R = 2

FIGURE 1

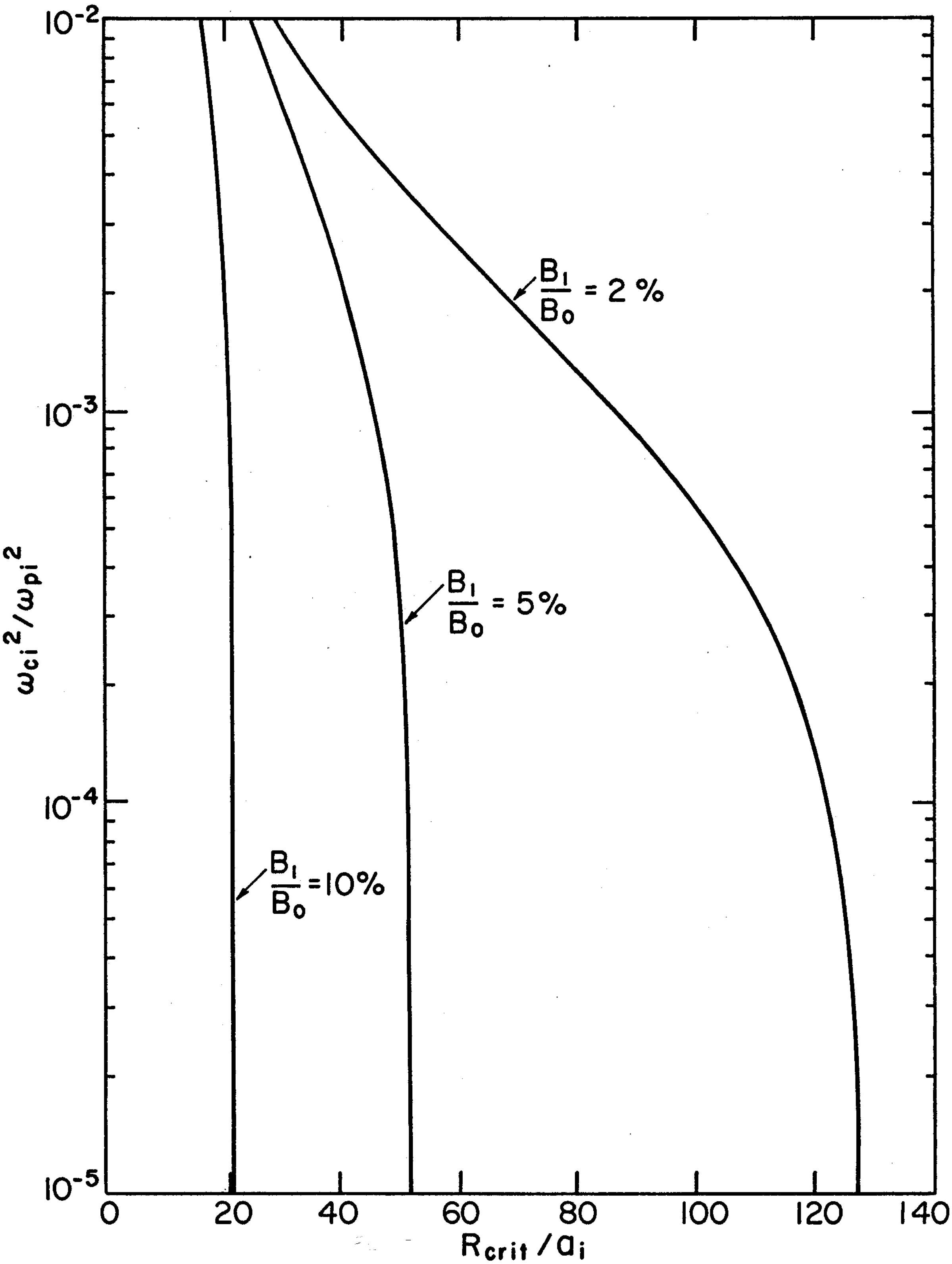

10^{-2}
10^{-3}
10^{-4}
10^{-5}
$\omega_{ci}^2/\omega_{pi}^2$
0
20
40
60
80
100
120
140
R_{crit}/a_i
$\frac{B_1}{B_0} = 2\%$
$\frac{B_1}{B_0} = 5\%$
$\frac{B_1}{B_0} = 10\%$

FIGURE 2

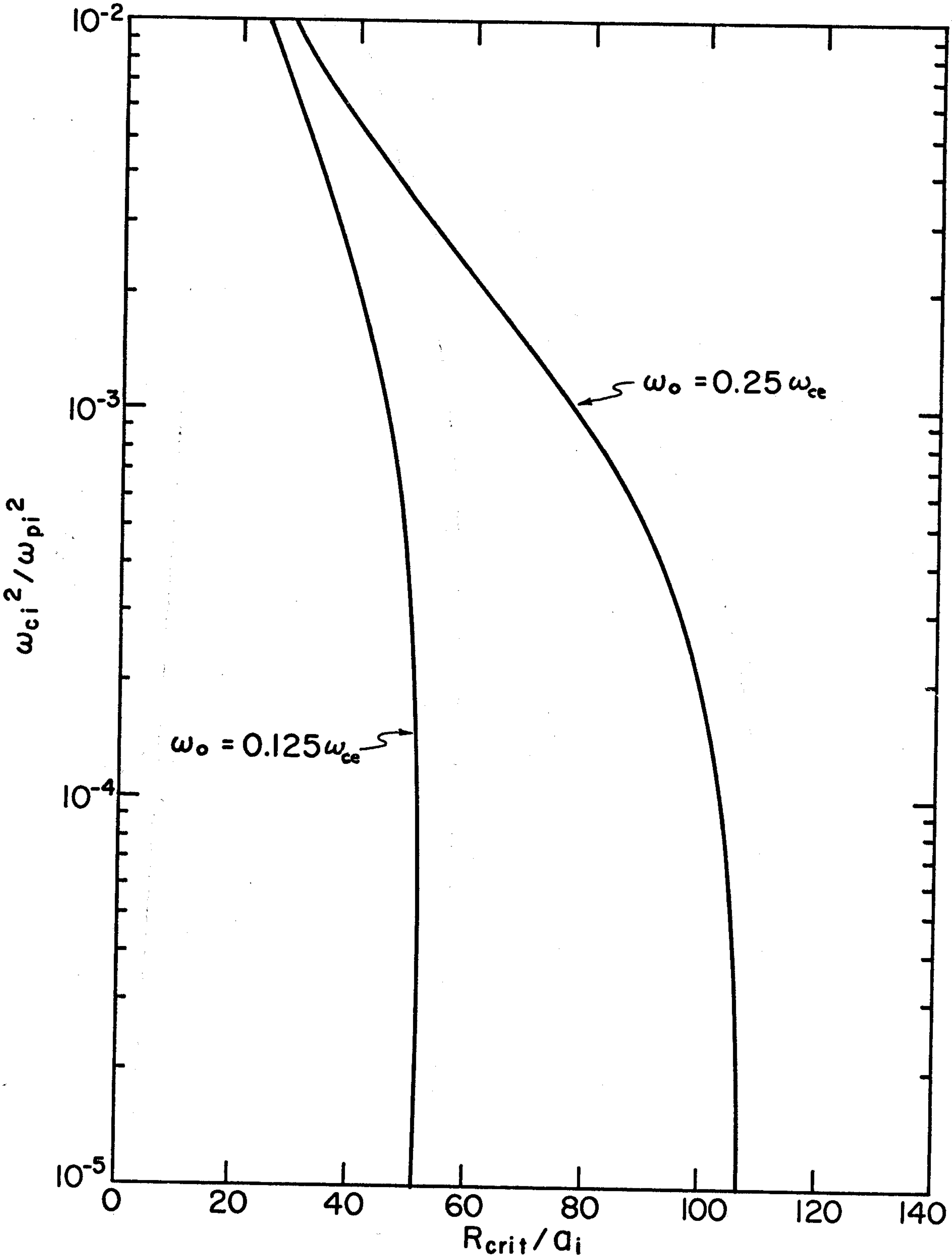

10^{-2}
10^{-3}
10^{-4}
10^{-5}
$\omega_{ci}^2/\omega_{pi}^2$
$\omega_o = 0.25\,\omega_{ce}$
$\omega_o = 0.125\,\omega_{ce}$
0
20
40
60
80
100
120
140
R_{crit}/a_i

FIGURE 3