Estimate of Minimum Heat Deposition Time to Prevent Wall Meltdown for a Fusion Reactor

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September 1972

UWFDM-25

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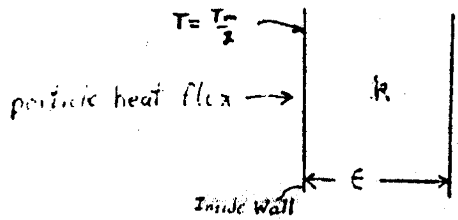
Consider the following situation

1. a wall of unit area, and thickness , thermal conductivity k, heat capacity C_p, and operating at 1/2 melting temperature on inside wall.



2. The energy content of a unit volume of this material is given by.

$$u = \rho C_p T = \left[\frac{gm}{cm^3}\right]\left[\frac{joules}{gm\text{-}{}^\circ K}\right]\left[{}^\circ K\right] = \left[\frac{joules}{cm^3}\right]$$

3. This energy <u>conducts</u> from this volume in opposite direction to temperature gradient.

$$\vec{q} = -k\nabla T = \left[\frac{joules}{sec\text{-}cm\text{-}({}^\circ K/cm)}\right]\left[{}^\circ K/cm\right] = \left[\frac{joules}{cm^2\text{-}sec}\right]$$

at a rate

$$-\nabla\cdot\vec{q} = k\nabla^2 T = \left[\frac{joules}{cm^3\text{-}sec}\right] = \frac{\partial u}{\partial t}$$

Therefore for ρ, C_p, k assumed constant, one has

$$\frac{\partial T}{\partial t} = \underbrace{\left(\frac{k}{\rho C_p}\right)} \nabla^2 T$$

thermal diffusivity [cm^2/sec] or thermal diffusion coefficient.

In analogy with most skin effect diffusion problems one assumes some type of sinusoidal imput of heat and gets a characteristic skin depth. In this case it would be the depth into the material at which the temperature reaches 37% of T_{max} (at x=0) in a time τ.

Although the analysis is strictly valid only for a single frequency sinusoidal input, one can often get an order of magnitude estimate of how a thermal pulse or "spike" might diffuse. Therefore let us suppose that a certain heat input [joules/cm^2] is deposited on the surface of this wall in the time τ. Therefore in this time 63% of the energy content of this pulse remains in 1 thermal skin depth of the material. can find this thermal skin depth from our diffusion eqn.

$$\frac{T}{\tau} = \frac{k}{c_p \rho} \frac{T}{\delta^2}$$

$$\delta = \sqrt{\frac{k\tau}{\rho c_p}} \quad [cm]$$

= thermal skin depth for a heat pulse of duration τ.

Now suppose one has an incident flux of hot plasma ions (and α particles). The energy flux to the wall would be

$$\sum_i n_i v_i E_i = \left[\frac{\text{joules}}{\text{cm}^2\text{-sec}}\right] \equiv \phi_E$$

If this is deposited in a time τ, then the energy input to ou unit area of wall is

$$\phi_E \tau \,(\text{Unit Area}) = [\text{joules}]$$

This energy then goes into heating up a volume element of uni area times thickness δ. (approximately). This energy increas the temperature of our volume in time τ. The temperature in- crease is found from an energy balance

$$\rho c_p \Delta T \,(\text{Unit Area})\, \delta = [\text{joules}]$$

or

$$\rho c_p \Delta T \,(\text{Unit Area})\, \delta = \phi_E \tau \,(\text{Unit Area})$$

substituting for δ one has

$$\rho c_p \Delta T \left(\frac{k\tau}{\rho c_p}\right)^{1/2} = \phi_E \tau$$

If the wall is supposed to be operating at 1/2 of its melting point = $T_m/2$, and one wishes to know what power input will ra the temperature to melting T_m, then

$$\frac{T_m}{2}\sqrt{\rho c_p k} = \phi_E \tau^{1/2}$$

where this formulation is valid for τ which correspond to δ's which are much less than the thickness of the wall (ℓ).

Let us ask ourselves the following question, if I took all of the hot plasma (($\sum_i N_i$) x Volume of plasma) and let it go to the wall of the reactor (wall area = A_w) in a time τ, what is the minimum (fastest) time I may allow this to happen without the worry of melting the wall?

Take for example

$$n = 10^{14} \text{ particles/cm}^3 , \quad \text{Vol. of plasma} = (2\pi R)(\pi r_p^2) \equiv V_p$$

$$A_w = (2\pi R)(2\pi r_w)$$

then ϕ_E becomes

$$\frac{n V_p}{A_w \tau}\left(\frac{3}{2} k_B T\right) = \left[\frac{\text{joules}}{\text{cm}^2\text{-sec}}\right]$$

or using (1 one has

$$\frac{T_m}{2}\sqrt{\rho c_p k} = \frac{n V_p}{A_w \tau}\left(\frac{3}{2} k_B T\right)\tau^{1/2}.$$

$$\tau^{1/2} = \frac{\frac{3}{2}\, n k_B T}{\frac{T_m}{2}\sqrt{\rho c_p k}}\,\frac{V_p}{A_w}$$

$$\tau^{1/2} = \frac{3}{2}\,\frac{P}{T_m\sqrt{\rho c_p k}}\, r_p Y$$

Giving the result

$$\tau_c = \frac{9}{4} \frac{P^2}{T_m^2 \rho C_p k} r_p^2 y^2$$

This savs that for deposition times τ shorter than τc, the wall melts.

$$\tau_{deposition} \geq \tau_c = \frac{9}{4} \frac{P^2}{T_m^2 \rho C_p k} r_p^2 y^2$$

Given some of these physical parameters for materials of fusion reactor interest.

	Nb	Mo	Va	304-SS
T_m [°K]	2688	2883	2192	1400
ρ $[\frac{gm}{cm^3}]$	8.57	9.01	5.87	7.9
C_p $[\frac{joule}{gm-°K}]$	0.33 @ 1500°K	0.33 @ 1500°K	0.66 @ 1200°K	0.50 @ 500°K
k $[\frac{watt}{cm-°K}]$	0.721 @ 1500°K	0.97 @ 1500°K	0.40 @ 1200°K	0.19 @ 500°K
$\rho C_p k$	2.039	2.884	1.550	0.7505

Look at range of energy fluxes.

$$\left.\begin{array}{l} n \sim 10^{14}\ \#/cm^3 \\ k_B T \sim 10\ keV \rightarrow 50\ keV \end{array}\right\} \Rightarrow p \sim .16 \rightarrow .8 \quad dynes/cm^2$$

$$r_p \sim 100\ cm \rightarrow 250\ cm \ \} \text{ pick } \ r_p \sim 200\ cm$$

$$y \sim .8 \rightarrow .9 \ \} \text{ pick } \ y \sim .85$$

	τ_c for T=10keV	τ_c for T=50keV
Nb	112.7 μsec.	2.81 msec.
Mo	68.4 μsec.	1.73 msec.
V	223.0 μsec	5.58 msec.
S.S. 304	1.13 msec.	28.3 msec.

$$\tau_c = \frac{9}{4}\frac{p^2}{T_m^2\,(\rho c_p k)}\, r_p^2 y^2$$

1. Which says that s.s. can not stand a 10keV particle flux of greater than 7.5×10^{18} particles/cm^2-sec., or a 50keV particle flux greater than 3.0×10^{17} particles/cm^2-sec.
2. If our α-particle density were to build up to say 10^{12}#/cm^3, and have an average energy of ~250keV then for α's $\tau_c \sim 9.2$ μsec., which corresponds to a flux of 9.0×10^{18} α's/cm^2-sec. which is higher I believe than anticipated.

Therefore unless one has some catastrophically unimaginable accident which would drive the plasma to the wall at its thermal speed (time scale of a few micro seconds) one should be safe from wall meltdown. This calculational technique however is very useful for divertor work.

This work is a slight extension of a calculation performed by H. K. Forsen in his Fusion Feasibility lecture on Divertors in the Fall of 1971.