IRF Fueling of Tandem Mirror End Plugs

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FUSION TECHNOLOGY INSTITUTE

UNIVERSITY OF WISCONSIN

MADISON WISCONSIN

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J. Kesner

Fusion Technology Institute
University of Wisconsin
1500 Engineering Drive
Madison, WI 53706

http://fti.neep.wisc.edu

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Jay Kesner

Abstract

Ion heating by IRF may be useful for fueling as well as heating the high energy and density mirror end plugs required by a tandem mirror. Fueling may be accomplished by trapping the ions that are ejected from the central cell electrostatic well as they flow out through the end plugs. It is shown that for a reasonable level of RF power this trapping can be quite efficient. Furthermore for a fixed RF power level the trapping efficiency is seen to be adjustable by tuning the wave frequency.

I. Introduction

Recently there has been interest in an open-ended magnetic confinement device known as a tandem mirror[1,2], a configuration in which high energy mirror "plugs" are placed at each end of a long uniform field region called the central cell. The plug density and temperatures are adjusted relative to the central cell parameters such that an ambipolar potential is created to stopper the loss cones of the central cell. Central cell confinement can thereby be significantly improved over classical mirror confinement. For a conceptual tandem mirror reactor in which the central cell volume is much larger than that of the plugs, the power amplification factor (Q) can be increased by a factor of 3 to 10 compared with a simple mirror.

One means for creating the temperatures and densities required by the plugs is use of high energy neutral beams[1-3]. Studies so far indicate that in the absence of auxiliary electron heating, a fusion reactor based on neutral beam heating alone will require the beams to have an energy of about 1 MeV. An alternative method to heat the plug ions is the use of RF at the cyclotron frequency and its harmonics (IRF). The Phaedrus experiment[4] is being designed to study this process. IRF plug heating would require an additional plug fueling mechanism. One such possibility is to combine low energy beams for fueling with RF heating and recent studies[5] have shown that this may in fact offer a practical alternative approach.

A second approach which will be discussed in this paper is the use of IRF to trap the stream of plasma escaping from the central cell as it flows through the plugs. This scheme is based on the understanding that IRF tends to add energy to the ions in the direction perpendicular to the field. Therefore a central cell ion which reaches the plug can become trapped there. Since a fraction of central cell ions are electrostatically trapped and will penetrate the plugs up to their midplane, this scheme requires that the region of ion-wave interaction (the resonance zone) be located on the central cell side of the plug midplane.

We will demonstrate a method for calculating the fraction of central cell ions that can be trapped in the plug. It is found that a significant fraction of the stream flowing through the plugs may be trapped without seriously degrading central cell confinement. Trapping requires a sufficiently peaked ambipolar potential and the trapping fraction depends sensitively on the RF power level. Varying the wave frequency tends to move the resonance zone and, as this zone moves away from the plug midplane, the trapping fraction increases. However, as the resonance zone moves towards the central cell, confinement in the central cell begins to decrease. Thus the position of the resonance zone must be chosen to optimize plug fueling as well as central cell confinement.

IIa. Characteristics of Tandem Mirror Confinement

Fig. 1 indicates the magnetic field, ambipolar potential and density distributions typical of a tandem mirror. We can view the tandem mirror velocity space by plotting the ε-μ space shown on Fig. 2 where ε is total ion energy ($\varepsilon = \frac{m}{2}V_{\parallel}^2 + \frac{m}{2}V_{\perp}^2 + e\phi$), μ is the ion adiabatic invariant ($\mu = \frac{mV_{\perp}^2}{2B}$) and ϕ the local ambipolar potential. We will define B_c to be the central cell magnetic field, B_o the plug midplane field, B_m the plug peak field, and ϕ_c and $-\phi_e$ the ambipolar potentials at the plug midplane and outer mirror respectively relative to the central cell. Typically $|\phi_e| \sim 3 - 6\ \phi_c$. Central cell ions with $\varepsilon > \mu B_m$ or plug ions with $\varepsilon > \mu B_m - e\phi_e$ cannot be mirror trapped and so the lines $\varepsilon = \mu B_m$ and $\varepsilon = \mu B_m - e\phi_e$ define the respective loss cone boundaries. The parallel energy of an ion is defined by $W_{\parallel} = \varepsilon - \mu B - e\phi$ and ions that cross the plug midplane, located at Z_p, require that $W_{\parallel}(Z_p) > 0$ or $\varepsilon > \mu B_o + e\phi_c$. Therefore the confined plug ions will find themselves in the region between the lines $\varepsilon = \mu B_m - e\phi_e$ and $\varepsilon = \mu B_o + e\phi_c$ labeled M_p.

The crossing point of these curves is the lowest energy of mirror trapped plug ions and is defined by $\varepsilon_o = e(R_p\phi_c+\phi_e)/(R_p-1)$ and $\mu_o = e(\phi_c+\phi_e)/(B_m-B_o)$ with R_p the plug mirror ratio $(R_p \equiv B_m/B_o)$.

The region labeled "E" contains the central cell electrostatically trapped ions (to be referred as e-trapped) which are of primary interest here. The region labeled "M_c, M_p^*" is accessible to central cell ions as well as to a class of plug ions which may be trapped between the inner mirror and the electrostatic barrier, thereby never crossing the plug midplane. This latter class of ions, first discussed by Yushamanov[6] will exist in part of the region where $\mu B_o<\varepsilon<\mu B_o+\phi_c$ for sufficiently peaked ambipolar potential barriers. The central cell ions can be contained in all the regions of ε-μ space labeled M_c or E.

We consider the case where the resonance zone of ion wave interaction is located on the central cell side of the plug at a magnetic field B* $(B_o < B^* < B_m)$ and an ambipolar potential ϕ^* $(\phi_c > \phi^* > 0)$. If we map Fig. 2 onto this plane, it produces a velocity space plot given in Fig. 3a. The ellipse defined by $v_\parallel^2 + v_\perp^2\,(1 - B_o/B^*) = 2e(\phi_c-\phi^*)$ contains ions which reflect from the ambipolar barrier, the hyperbola defined by $v_\perp^2(B_m/B-1) - v_\parallel^2 = 2e(\phi+\phi_e)$ contains the plug mirror trapped ions and the hyperbola defined by $v_\perp^2(B_m/B-1) - v_\parallel^2 = 2e\phi$ contains those ions which reflect from the inner mirror. The region between the ellipse and the latter hyperbola contains those ions that are trapped between the potential barrier and the inner mirror.

IIb. Fractions of Ions Trapped by IRF

To make an estimate of the fraction of the ions leaving the central cell that can be trapped in the plugs, we assume that ion cyclotron heating adds only perpendicular energy $(W_\perp)$ to the ions on each pass through the resonance zone. If the resonance zone is located at a field B* and potential ϕ^* and if an ion gains $\delta W_\perp$ on passing through this resonance zone, then $\delta\mu = \frac{\delta W_\perp}{B^*}$ and

$\delta\varepsilon = \delta W_\perp$. Therefore, $d\varepsilon/d\mu = B^*$ and IRF tends to move an ion along a trajectory in ε-μ space having a slope B^*. This assumes that the Doppler shift in the resonance frequency is small, i.e., $V_\parallel k_\parallel << (\omega - \omega_{ci})$ so that all ions are heated at the same location in real space.

In $V_\perp$-$V_\parallel$ space of Fig. 3a IRF tends to move an ion vertically and if there is overlap between the ellipse and upper hyperbola ions can become trapped in the plugs. In the limiting case where the curves are tangent, one finds $(\phi_c - \phi^*)/(\phi + \phi_e) = (B^* - B_o)/(B_m - B^*)$. If we assume a sinusoidal magnetic field dependence

$$R(z) = B/B_o = 0.5\ (R_m + 1) - 0.5\ (R_m - 1)\cos\ (2\pi Z'/L_p) \qquad (1a)$$

with 2 L_p the distance between mirrors in the plug we can set $B = B^*$ and solve for ϕ^* to get

$$\phi^* = (\phi_c + \phi_e)\cos^2\ (\pi Z'/L_p) - \phi_e$$

$$\text{with } Z' = |Z - Z_p|$$

This indicates that if the ambipolar potential is broader than this, the e-trapped and mirror trapped regions no longer overlap and no trapping is possible in a sinusoidal magnetic well. In the calculations that follow, we will chose the normalized potential on the inner side of the plug midplane χ_n to be

$$\chi_n = \phi(Z)/\phi_c = \cos^n\ (\pi Z'/L_p)$$

$$= ((R_m - R)/(R_m - 1))^{n/2} \qquad (1b)$$

For sufficiently peaked ambipolar potential distributions trapping can occur and we can estimate the trapping fraction by modeling Fig. 3a by an infinite strip as shown in Fig. 3b. The dashed boundary, located at $V_\perp^2 = 2e\phi^*/(B_m/B^* - 1) \equiv V_T^2$ is the approximate location in velocity space where the e-trapped ions would become Yushmanov trapped within the mirror plug. (This model does not take into account the shrinking of the overlap region for broad ambipolar potentials). By setting a unit point source at the origin, we can estimate the fraction of flux that crosses the dashed boundary and is trapped. The remaining flux diffuses across the boundary of the strip and is lost. We are assuming here that once an ion crosses the dashed boundary and becomes Yushmanov trapped its bounce frequency and therefore the RF diffusion coefficient increases and it will quickly become mirror trapped. We will model this process by keeping only the diffusion terms of a collision operator

$$\frac{\partial f}{\partial t} = D_{\parallel} \frac{\partial^2 f}{\partial V_{\parallel}{}^2} + D_{RF} \frac{\partial^2 f}{\partial V_{\perp}{}^2} + S \qquad (2)$$

setting the distribution function f to zero at the boundary of the infinite strip defined by

$$V_{\parallel} = V_L \equiv \sqrt{2e\ \phi_c(1 - \chi_n)}$$

D_{RF} represents the perpendicular diffusion coefficient due to RF and $D_{\parallel}$ is the parallel diffusion coefficient due to Coulomb collisions of the e-trapped ions between passes through the resonance zone. We will assume $D_{RF} > D_{\parallel}$ and ignore perpendicular diffusion due to collisions. We have also ignored electron drag on the ions. This model, though greatly simplified, is seen to yield interesting results.

Following Ref. 7 the Green's function response to a point unit source at time τ is

$$u = \frac{0.5}{V_L \sqrt{\pi D_{RF}(t-\tau)}} \exp\left[-V_{\perp}^2/(4\ D_{RF}(t-\tau)\right] \sum_{m=1}^{\infty} (-1)^{m+1} \sin\left[(m-.5)\ \pi\ (1+V_{\parallel}/V_L)\right]$$

$$\times \exp\left(-\ D_{\parallel}\ \pi^2\ (m-.5)^2\ (t-\tau)/V_L^2\right) \qquad (3)$$

The steady state solution for a constant source of unit strength may be gotten by integrating from $\tau = -\infty$ to $\tau = 0$. This yields

$$\bar{u}\ (V_{\parallel},V_{\perp}) = \frac{1}{\pi\sqrt{D_{RF}D_{\parallel}}} \sum_{m=1}^{\infty} \frac{(-1)^{m+1}}{2m-1} \sin\left[(m-.5)\pi(1+V_{\parallel}/V_L)\right]$$

$$\times \exp\left[-\pi(m-.5)\ (V_{\perp}/V_L)\ \sqrt{D_{\parallel}/D_{RF}}\right] \qquad (4)$$

The estimate of the flux that will become trapped is

$$F_T = 2\, D_{RF} \int_{-V_L}^{+V_L} dV_{\parallel} \left. \frac{\partial \bar{u}}{\partial V_{\perp}} \right|_{V_{\perp}=V_T}$$

$$= \frac{4}{\pi} \sum \frac{(-1)^{m+1}}{(2m-1)} \exp\left[- \pi\, (V_T/V_L)\, (m-.5)\, \sqrt{D_{\parallel}/D_{RF}} \right] \qquad (5)$$

$$\text{with } V_T/V_L = \left[\chi_n/(1 - \chi_n)(B_m/B - 1) \right]^{1/2}$$

It is clear that the trapped fraction (F_T) is a function of the ratio of RF to collisional diffusion coefficients ($D_{RF}/D_{\parallel}$) and the location of the resonance plane (which enters through χ_n and the normalized ambipolar potential at the resonance zone location). The RF diffusion coefficient is proportional to the IRF power that is depositing into the plasma, or more specifically to the square of the wave field, as will be discussed below.

Fig. 4 indicates the variation of the fraction trapped with IRF power at several locations of the resonance zone. As we will discuss it is desirable to have $\phi/\phi_c > 0.9$. We see that it is possible to trap a large part of the central cell stream which passes through each plug. The sensitivity of the result to the resonance zone location (or ϕ/ϕ_c) suggests that the trapped fraction could be modulated by tuning the wave frequency, which moves the resonance zone. When the normalized potential approaches 1 the resonance zone approaches the plug midplane and trapping is no longer possible. The trapping fraction is seen to rise rapidly as the resonance location moves away from the plug midplane and the potential decreases by up to about 15% from its midplane value. This result depends weakly on the shape of the potential as long as the potential is sufficiently peaked to allow trapping.

By taking all of the e-trapped ions above a certain energy and ejecting them into either the plug or loss cone we necessarily degrade central cell confinement. This occurs because the central cell confinement may be approximated by $\tau_c = \tau_{ii} G(R_c)\, e^{-e\phi/kT_c}$ with τ_{ii} the Coulomb collision pitch angle scattering time and R_c the central cell mirror ratio ($R_c = B_m/B_c$). If the confinement time in the presence of IRF is τ_c^{RF}, then

$$\tau_c^{RF}/\tau_c = \exp[-\frac{e\phi_c}{kT_c}(1-\chi_n)]$$

For $e\phi_c/kT_c = 2.8$ and $\chi_n = 0.9$ we get $\tau_c^{RF}/\tau_c = 0.76$.

Therefore we would like to have $\chi_n \gtrsim 0.9$, i.e., we would want the resonant heating plane close to the plug midplane.

IIc. IRF Quasi-Linear Diffusion Coefficient

In the calculation just discussed we have seen that a significant fraction of the central cell stream can be trapped when IRF dominates Coulomb collisions. The critical parameter which determines the effectiveness of this process is the ratio of RF to collisional diffusion coefficients. An estimate of the RF diffusion coefficient can be obtained from quasi-linear theory. Following ref. 8,

$$D_{RF} = \frac{\pi^2 Z^2 e^2}{8m^2 \nu} |E_x + iE_y|^2 |J_{\nu-1}(\frac{k_\perp V_\perp}{\omega_{ci}})|^2 \delta(\omega_{ci} - (\omega + k_\parallel V_\parallel)/\nu) \qquad (5)$$

where $E_x + iE_y$ is the left-hand polarized electric field, ω is the wave frequency, $J_{\nu-1}$ is the Bessel function of order $\nu-1$ (corresponding to harmonic number ν), $k_\perp$, $k_\parallel$ are the perpendicular and parallel wave numbers respectively and ω_{ci} is the ion cyclotron frequency. We define the bounce average as

$$\langle D_{RF} \rangle = 1/\tau_b \oint dz\, D_{RF}/V_\parallel \qquad (6)$$

with the integral taken over the bounce period for the e-trapped ions and the bounce time $\tau_b = \oint dz/V_\parallel$. If we substitute the field equations (1) and approximate the central cell bounce time by $\tau_b = L_c/V_{\parallel c}$, eqn. (6) becomes

$$\langle D_{RF}\rangle = \frac{\pi Z^2 e^2 \, |E_x + iE_y|^2 \, |J_{\nu-1}(k_\perp V_\perp/\omega'_{ci})|^2 \, (L_p/L_c)(V_{\parallel c}/V'_\parallel)}{16 \, \nu \, m^2 \, (R_m - 1) \, \chi^{1/n} \sqrt{1-\chi^{2/n}} \, \omega_{cio}} \qquad (7)$$

where the primes mean the quantities are evaluated at the resonance surface ($\omega_{ci} \simeq \omega/\nu$). Notice that for a given amplitude of the left hand polarized wave field, the diffusion coefficient decreases with increasing relative central cell length and increases as the resonance zone approaches the plug midplane ($\chi \to 1$).

To evaluate $V'_\parallel$ we note that the ions which surmount the potential barrier appear at the resonance layer with near to zero velocity. We must then determine the velocity at the edge of the region of width δ in which the particle-wave interaction occurs. δ is defined by the condition $(\omega_{ci} - \omega_{cio})^{-1} \simeq \delta/V_\parallel$. For an ion that stops at the resonance layer, its velocity at a distance δ is $V'_\parallel \sim (\frac{2e}{m} \frac{d\phi}{dZ} \delta)^{1/2}$. We can combine these expressions to find

$$V'_\parallel \sim \left(\frac{2e}{m} \frac{d\phi}{dZ}\right)^{2/3} \, (d\omega_{ci}/dZ)^{-1/3}$$

Evaluating ϕ and ω_{ci} from (1) for $n=6$, $R_m=2$, $\chi=0.9$, and using Phaedrus parameters $\phi_c=36$ eV, $L_p=50$ cm, we find $V'_\parallel \simeq 10^5$ cm/sec. For fundamental heating, a peaked ambipolar potential
($n=6$), and other parameters chosen from the Phaedrus experiment of Ref. 4 ($R_m=2$, $\omega_{cio}=1.9 \times 10^7$ sec^{-1}, $L_p/L_c=.25$, $k_\perp \simeq \pi/L_p=.06$ cm^{-1}, $T_c=13$ eV, $\phi_c=36$ eV, $n_c=2 \times 10^{12}/\text{cm}^3$) (7) becomes

$$\langle D_{RF}\rangle \simeq 4.2 \times 10^{22} \, |E_x + iE_y|^2 \qquad (7a)$$

with the electric field is in statvolts/cm.

Using the central cell collision time (τ_c) from Pastukhov[7] and estimating $D_{\parallel} \sim e\phi_c/m\tau_c$ yields the result

$$D_{\parallel} \approx kT_c \exp(-T_c/\phi_c)/m\ \tau_{ii}\ G(R_c)$$

$$\text{with } G(R_c) = \sqrt{\pi}\ (2\ R_c + 1)\ \ln\ (4\ R_c + 2)/4\ R_c.$$

τ_{ii} is the ion-ion pitch angle scattering time and R_c the central cell mirror ratio. For $R_c = 15$ we find that $D_{RF} = D_{\parallel}$ for $|E_x + iE_y| =$ 2.0 V/cm which is quite a modest field. The ratio $D_{RF}/D_{\parallel}$ will then increase as the square of the wave field.

V. Consequences for a Tandem Mirror Reactor

In a tandem mirror, the central cell volume (V_c) would be much larger than the plug volume (V_p). If the central cell source is S_c, the corresponding plug source would be $S_p = 0.5\ F_T\ S_c\ (V_c/V_p)$. In view of the volume ratio, this source could be quite large. It should be noted however that the higher density and lower confinement of the plugs requires a higher source density in the plugs than in the central cell.

In a D-T tandem mirror reactor the deuterium and tritium streaming through the plug will interact with the wave at different positions. If, for example, the deuterium resonance zone is located near the plug midplane, tritium will have a resonance at a local mirror ratio of 1.5. These resonance zones located well off the plug midplane would adversely affect central cell confinement of the respective species. Eliminating them would require moving the desired resonance layer further from the midplane.

In a high beta plug there is a significant diamagnetic effect which would serve to steepen the magnetic field gradient within the well. Since trapping requires that the ambipolar potential be more sharply peaked than the magnetic field, this effect must be examined self-consistently.

Conclusions

We have seen that IRF has the potential for fueling the plugs of a tandem mirror by trapping the stream of ions escaping from the central cell. Ideally the IRF power level would be determined so as to produce a desired mean plug energy. It is then seen that the plug density level could be adjusted by varying the IRF wave frequency, thereby moving the resonance zone location.

This result would make possible the consideration of a tandem mirror which does not rely on neutral beams to sustain the end plug plasma.

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Figure Captions

1. Magnetic field, ambipolar potential and density axial distributions typical of a tandem mirror.

2. ε-u space for a tandem mirror. The various regions that are labelled correspond to a) ions electrostatically trapped in the central cell (E), ions magnetically trapped in the plugs (M_p), ions magnetically trapped in the central cell (M_c), and ions trapped between the electrostatic and magnetic barrier of the plug (M_p*).

3. a) Velocity space at a region located close to the plug midplane and on the side towards the central cell.

   b) Velocity space model which substitutes an infinite strip for the elliptic electrostatic trapping boundary.

4. Fraction of ions flowing through the plug that would be trapped vs. the RF to collisional diffusion coefficient ratio. Curves are plotted for various resonance zone locations specified by the ambipolar potential normalized to its plug midplane value for $\phi/\phi_c = \cos^6 (\pi Z'/L_p)$.

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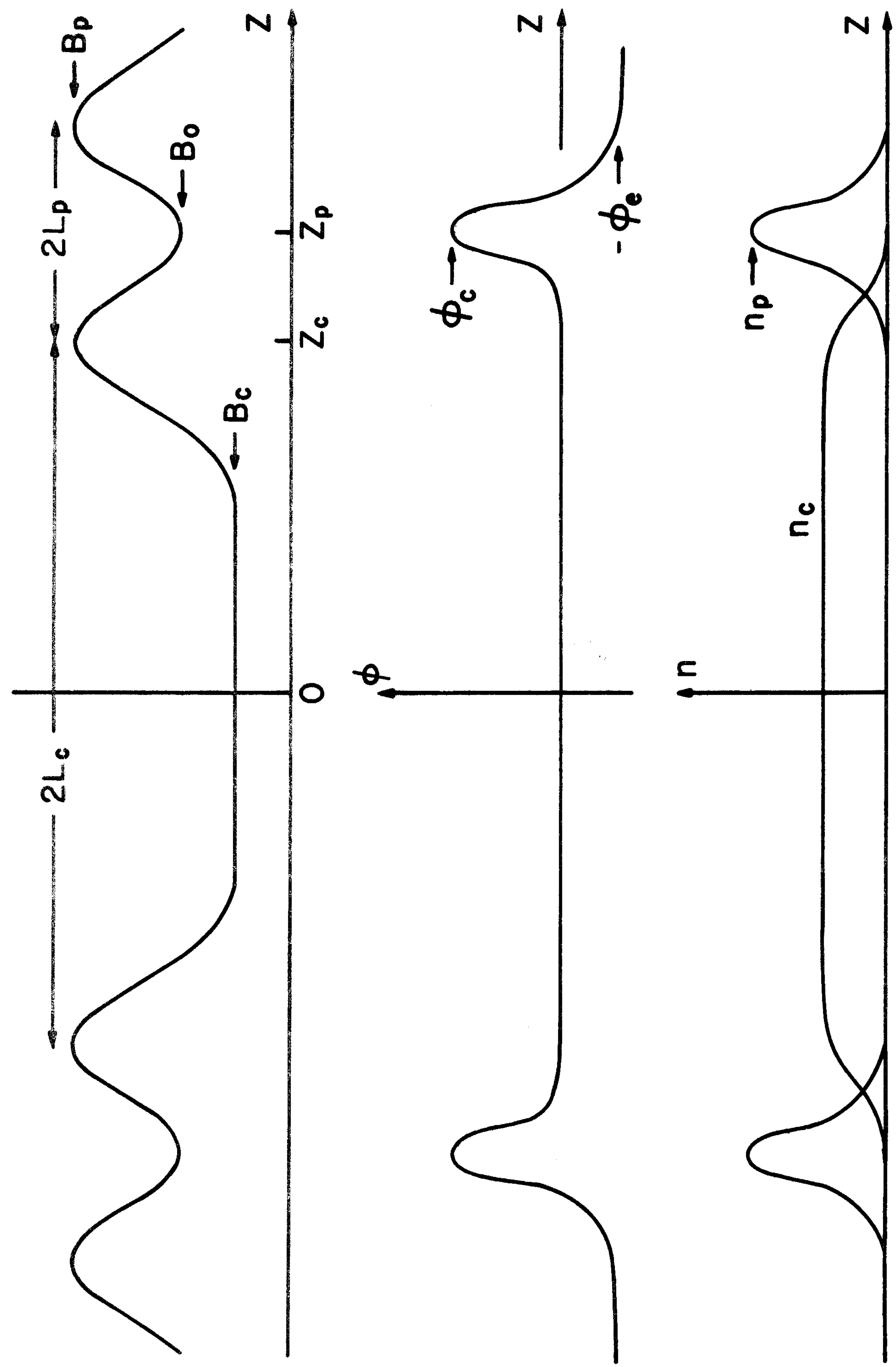
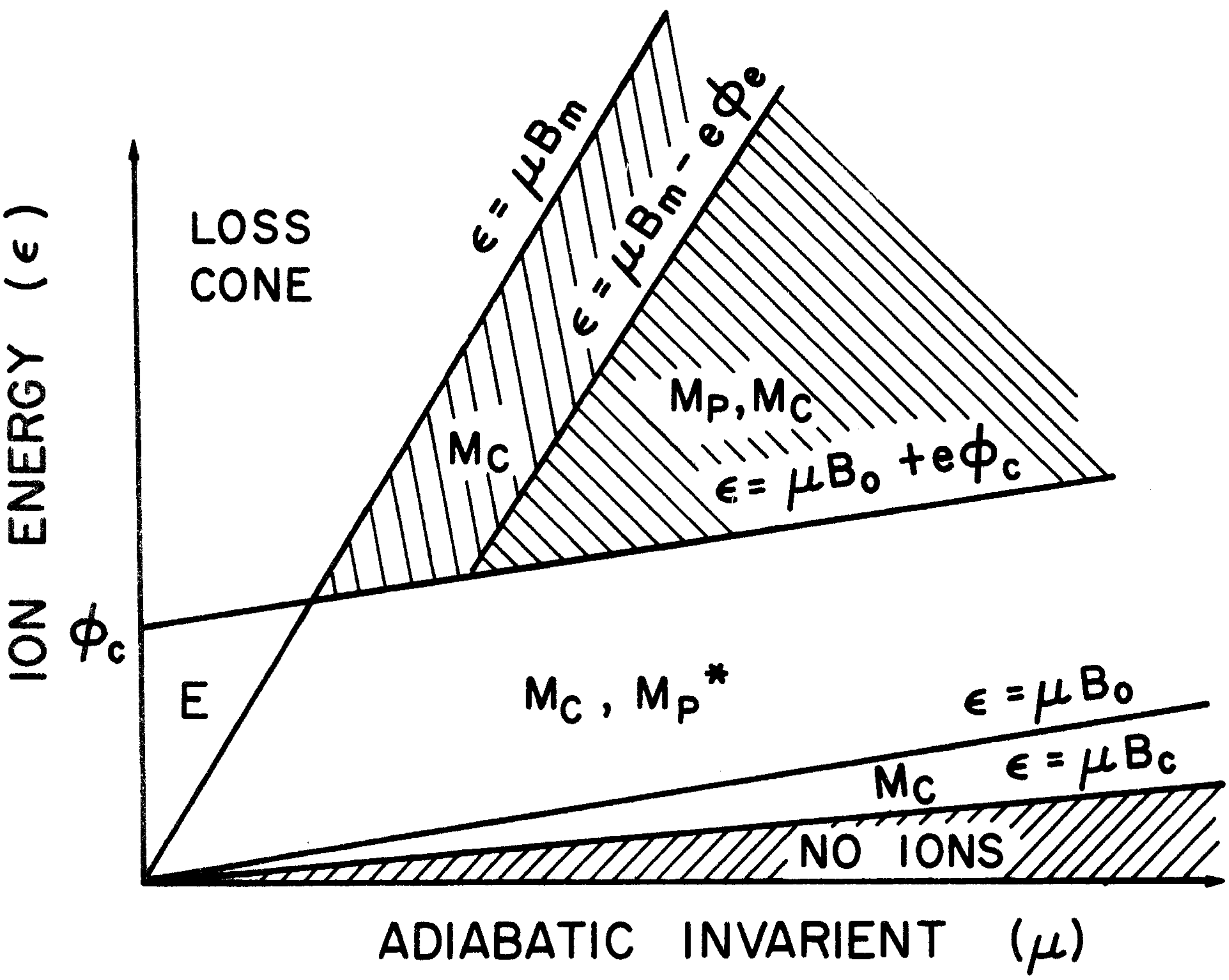


FIGURE 1



ION ENERGY (ε)
LOSS CONE
ε = μBm
ε = μBm − eφe
MP, MC
MC
ε = μBo + eφc
φc
E
MC , MP*
ε = μBo
MC
ε = μBc
NO IONS
ADIABATIC INVARIENT (μ)

FIGURE 2

FIGURE 3A

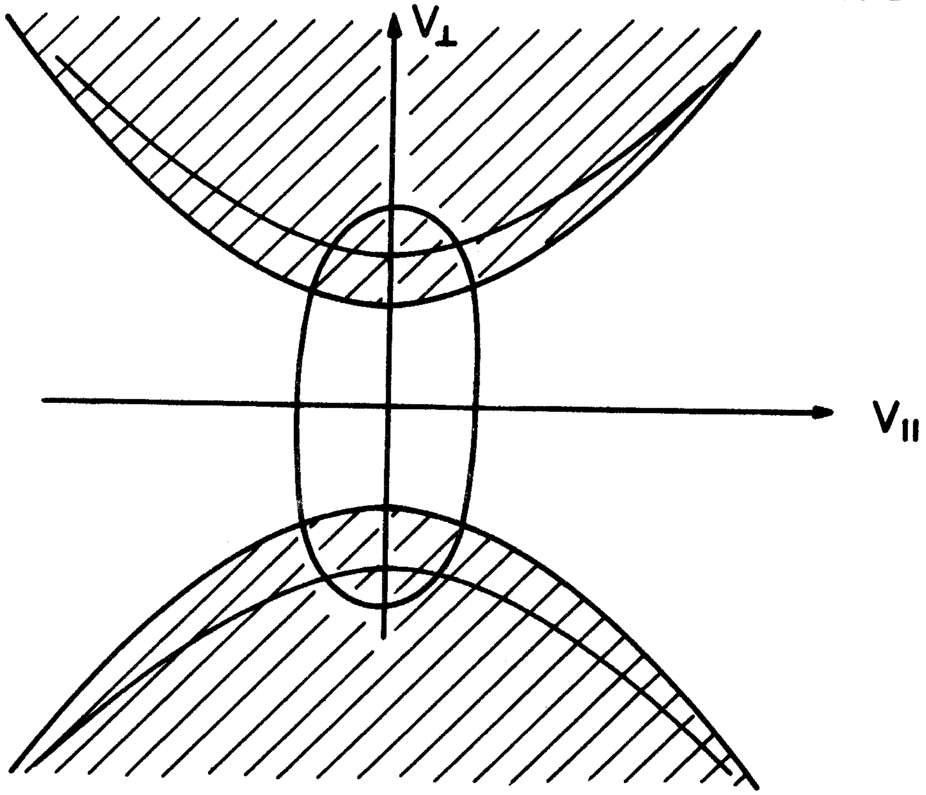
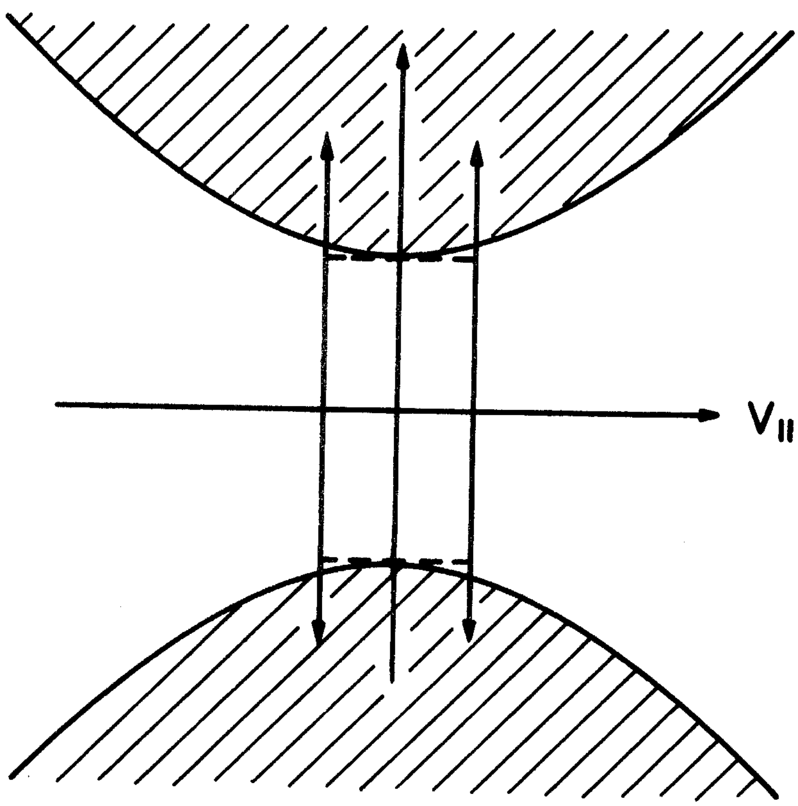


FIGURE 3B



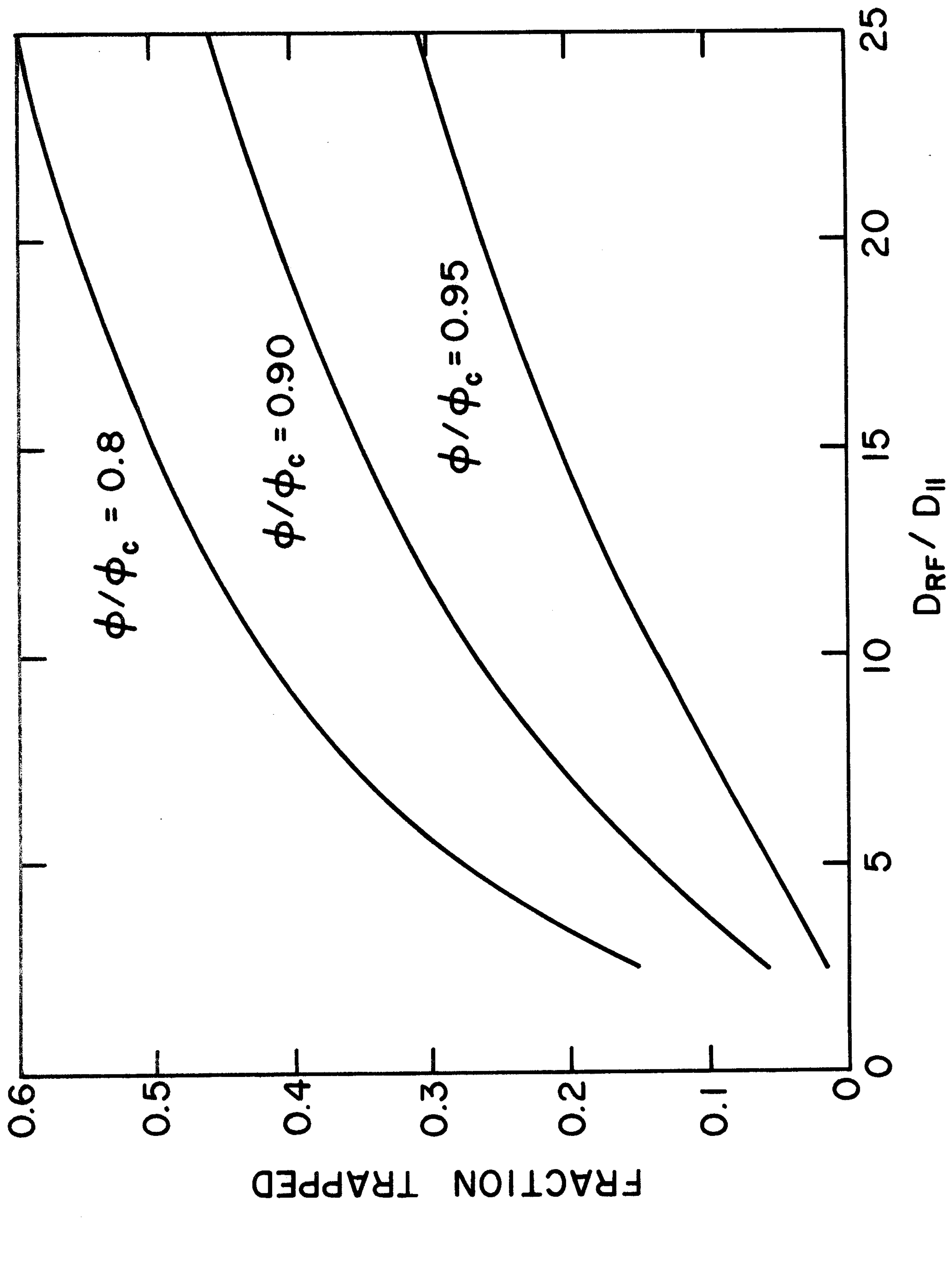

FRACTION TRAPPED
0.6
0.5
0.4
0.3
0.2
0.1
0
0
5
10
15
20
25
$D_{RF} / D_{\parallel}$
$\phi/\phi_c = 0.8$
$\phi/\phi_c = 0.90$
$\phi/\phi_c = 0.95$

FIGURE 4