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January 1978

UWFDM-230

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Abstract

Mirror surface damage thresholds are calculated analytically for two temporal pulse forms relevant to laser fusion experiments; a truncated Gaussian, and the "ideal" isentropic pulse. The results show that 20% more laser energy/cm^2 may be handled using a two e-folding Gaussian of the same FWHM as a square pulse of equal peak power. The surface temperature rise is given in completely closed form for the isentropic pulse and applied to a conceptual laser fusion design.

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I. INTRODUCTION

The maximum allowable energy density (J/cm^2) incident on bare metal mirrors is usually thought to be limited by surface melting under intense laser radiation. The damage threshold has been calculated analytically for (temporally) square pulses[1] using classical one-dimensional thermodynamics.[2] Numerical computations have also been performed[3] for an asymmetric model pulse typical of saturated operation of a high-power CO_2 laser.

In this note we calculate damage thresholds for two pulse shapes relevant to laser fusion; the truncated Gaussian, and the semi-empirical "ideal" pulse designed to produce isentropic compression of spherical targets.[4] A Gaussian pulse is characteristic of Nd: glass lasers operated below saturation, whereas the steeply rising ideal pulse would have to be produced by a device such as a pulse-stacker. The result for Gaussian pulses is reduced to a simple integral equation, while the ideal pulse threshold is given in completely closed form.

Although the emphasis here is on illumination of metallic surfaces, the results apply to any surface for which the depth of the heated zone is small compared to the beam radius, and the pulse width is long enough that a local temperature may be defined. It may be shown that radiation losses are almost always negligible in cases of practical interest.[1] We shall also ignore the small variation in thermal parameters with temperature.

II. SQUARE PULSE

Assuming zero penetration depth, the surface temperature rise for arbitrary flux $I(t)$ W/cm^2 incident on a semi-infinite medium is[1]

$$\Delta T = \frac{(1-R)}{K} \left(\frac{k}{\pi}\right)^{1/2} F(t) \quad , \qquad (1)$$

where K is the thermal conductivity, $k = K/\rho c_p$ is the thermal diffusivity, R is the reflectivity and

$$F(t) \equiv \int_0^t I\ (t - t') \frac{dt'}{(t')^{1/2}} \quad . \qquad (2)$$

We are interested here in the maximum surface temperature rise:

$$\Delta T_{max} = AF_{max} \quad , \qquad (3)$$

where

$$A \equiv \frac{(1-R)}{K} \left(\frac{k}{\pi}\right)^{1/2} \quad . \qquad (4)$$

For a square pulse of magnitude I_o and width τ the solution of Eq. (1) is well known:

$$\Delta T = 2A\ I_o\ t^{1/2}, t \leq \tau \quad . \qquad (5)$$

The damage threshold is customarily expressed in terms of the energy density,

$$\Phi = \int_0^t I(t')dt' = I_0\, t \,, \tag{6}$$

Combining Eqs. (5) and (6) we arrive at the critical energy density for surface melting,

$$\Phi^* = \frac{(T_m - T_0)}{2A}\, \tau^{1/2} \,, \tag{7}$$

where T_0 and T_m are the initial and melting temperatures.

III. GAUSSIAN PULSE

Consider a Gaussian pulse of maximum intensity I_0 and total width 2τ, with origin at the leading edge, as indicated in Fig. 1.

$$I(t) = I_0 \exp\,[-a^2(t - \tau)^2] \,. \tag{8}$$

Then

$$F(t) = I_0 \int_0^t \exp\,[-a^2\,(t - t' - \tau)^2]\,(t')^{-1/2}\, dt' \tag{9}$$

or

$$F(t) = I_0\, (\tau/Y)^{1/2} \int_0^y \frac{\exp[-(Y-u)^2]du}{(y-u)^{1/2}} \,, \tag{10}$$

where $y = at$, $Y = a\tau$ and $u = a(t-t')$. Integrating by parts, we obtain

$$F(t) = 2\, I_0\, \tau^{1/2}\, \frac{G(y,Y)}{Y^{1/2}} \,, \tag{11}$$

with

$$G(y,Y) = y^{1/2} e^{-Y^2} + 2 \int_0^y (Y-u)\ (y-u)^{1/2} \exp\ [-(Y-u)^2] du \ . \qquad (12)$$

Figure 2 depicts the function $GY^{-1/2}$ for $Y = 1,\ \sqrt{2},\ \sqrt{3}$ and 2, Y^2 corresponding to the number of e-foldings at the pulse edge. The decrease in T_{max} with increasing Y is primarily due to the decrease in area (energy). Figure 3 depicts the same function for Y = 1, 2, 4 and 8, illustrating the rather slow decay in temperature for large Y.

The scaled time at which the maximum temperature occurs is given by solving $\partial G/\partial y = 0$ for y. This leads to the equation

$$\int_{-Y}^{\tilde{y}} \frac{v e^{-v^2} dv}{(\tilde{y}-v)^{1/2}} = \frac{e^{-Y^2}}{2(\tilde{y}+Y)^{1/2}} \ , \qquad (13)$$

to be solved for $\tilde{y} \equiv y - Y$. As $Y \to \infty$, this reduces to

$$\int_{-\infty}^{\tilde{y}} \frac{v e^{-v^2} dv}{(\tilde{y}-v)^{1/2}} = 0 \ . \qquad (14)$$

We have not found analytic solutions to Eqs. (13) or (14). Numerically obtained values of $\tilde{y}$ and G_{max} are shown in Fig. 4 as functions of Y. Convergence is seen to be quite rapid, the limiting values being

$$\lim_{Y\to\infty} \tilde{y} = 0.5409$$

$$\lim_{Y\to\infty} G_{max} = 1.0760 \ . \qquad (15)$$

It is of interest to compare the damage threshold for a truncated Gaussian pulse with that for a square pulse of equal peak power and total width equal to the FWHM (Fig. 1). The total energy is then equal to within 2% for a two e-folding Gaussian ($Y = \sqrt{2}$). In general,

$$\Phi = 2\, I_o \int_0^{\tau} \exp\,(-a^2t^2)dt = \frac{\pi^{1/2} I_o \tau}{Y}\, \mathrm{erf}(Y)\ . \tag{16}$$

Combining Eqs. (3), (11) and (16) gives

$$\Phi = \left(\frac{\Delta T_m}{2A}\tau^{1/2}\right) \frac{\pi^{1/2}\mathrm{erf}(Y)}{Y^{1/2} G_{max}}\ . \tag{17}$$

Substituting for the FWHM,

$$\tau_{1/2} = \frac{(\ell n 16)^{1/2}}{Y}\,\tau \tag{18}$$

yields

$$\Phi^* = \frac{\pi^{1/2}\mathrm{erf}(Y)}{G_{max}(\ell n 16)^{1/4}}\,\Phi^*_{sp}\ , \tag{19}$$

where Φ^*_{sp} is the square pulse threshold, Eq. (7), with $\tau = \tau_{1/2}$. Taking $Y = \sqrt{2}$ in Eq. (19), we find

$$\Phi^* = 1.22\ \Phi^*_{sp}\ . \tag{20}$$

This 20% energy dividend is primarily due to the fact that the surface temperature reaches its maximum somewhat before the end of the pulse; any energy transmitted after this time cannot contribute to surface damage. Damage threshold experiments using Gaussian pulses should, therefore, be compared with a theoretical value about 20% higher than the square pulse value.

IV. "IDEAL" ISENTROPIC PULSE

A great deal of numerical simulation has been done using the steeply rising pulse form[4]

$$I(t) = I_o(1 - t/t_1)^{-2}, \quad 0 \leq t \leq \tau < t_1 \tag{21}$$

to obtain isentropic compression of spherical laser fusion targets. It should be noted that this semi-empirical pulse shape differs from Kidder's theoretical form,[5] in which the quantity t/t_1 is squared. As Fig. 5 illustrates, I_o now denotes the initial irradiance, the peak value being

$$I_{max} = I_o \, (1 - \eta)^{-2} , \tag{22}$$

where $\eta = \tau/t_1$. The energy density and peak intensity are related by

$$\Phi = I_{max} \, \tau \, (1 - \eta) . \tag{23}$$

Thus,

$$\eta = 1 - \frac{\Phi}{I_{max} \, \tau} . \tag{24}$$

The integral in Eq. (2) may be carried out, with the result

$$F(t) = \frac{I_o t_1^{1/2}}{(1-y)^{3/2}} [\sin^{-1} y^{1/2} + y^{1/2} \, (1 - y)^{1/2}], \tag{25}$$

where $y = t/t_1$. Thus, the temperature rise is monotone increasing, reaching the maximum

$$F_{max} = I_{max} \, \tau^{1/2} \, [\varepsilon + (\frac{\varepsilon}{1-\varepsilon})^{1/2} \cos^{-1} \varepsilon^{1/2}] , \tag{26}$$

where

$$\varepsilon = 1 - \eta = \frac{\Phi}{I_{max}\ \tau} << 1 \ , \qquad (27)$$

at the end of the pulse. Typical reactor conditions are a total energy $E = 10^6$ J, peak power $P_{max} = 10^{15}$ W and $\tau = 40$ nsec[6], giving $\varepsilon = 0.025$. (Note that $\Phi/I_{max} = E/P_{max}$, independent of the shape of the spatial profile and the aperture area). Taylor expanding Eq. (26) in powers of ε we find

$$F_{max} \simeq \frac{\pi}{2}\, I_{max}\ (\varepsilon\tau)^{1/2} \ . \qquad (28)$$

Combining Eqs. (3), (23) and (28), we obtain the damage threshold

$$\Phi^* = \frac{2\Delta T_m}{\pi A}\,(\varepsilon\tau)^{1/2} \ . \qquad (29)$$

This result may be compared with the square pulse threshold by defining an effective pulse width such that $I_{max}\ \tau_{eff} = \Phi$. Then $\tau_{eff} = \varepsilon\tau$ and Eq. (29) may be written

$$\Phi^* = \frac{4}{\pi}\,\Phi_{sp} \ . \qquad (30)$$

Measured in this way, about 27% more energy may be safely handled using the ideal pulse. Due to the complicated pulse shape the physical reason for this increase is not obvious. One can only say that the slowly rising initial portion of the pulse dominates the rapidly rising final portion.

In conclusion, we have found rather modest enhancements of damage thresholds over the square pulse value for two pulse shapes commonly encountered in laser fusion.

Acknowledgement

This work was supported by the Electric Power Research Institute. Support for T.O.H. was provided by U.S.D.O.E.

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FIGURE CAPTIONS

Fig. 1. A Gaussian pulse of full width 2τ truncated after two e-foldings ($Y = \sqrt{2}$) superimposed on a square pulse of width equal to the FWHM, $\tau_{1/2}$.

Fig. 2. Scaled temperature time histories for moderate Y-values. The maxima occur at $\tilde{y} = 0.5409$ for $Y > 2$.

Fig. 3. Scaled temperature time histories for larger Y-values.

Fig. 4. Location and magnitude of the maximum of $G(y,Y)$.

Fig. 5. Ideal isentropic compression pulse. An atypical value of $\varepsilon = 0.25$ has been chosen for clarity.

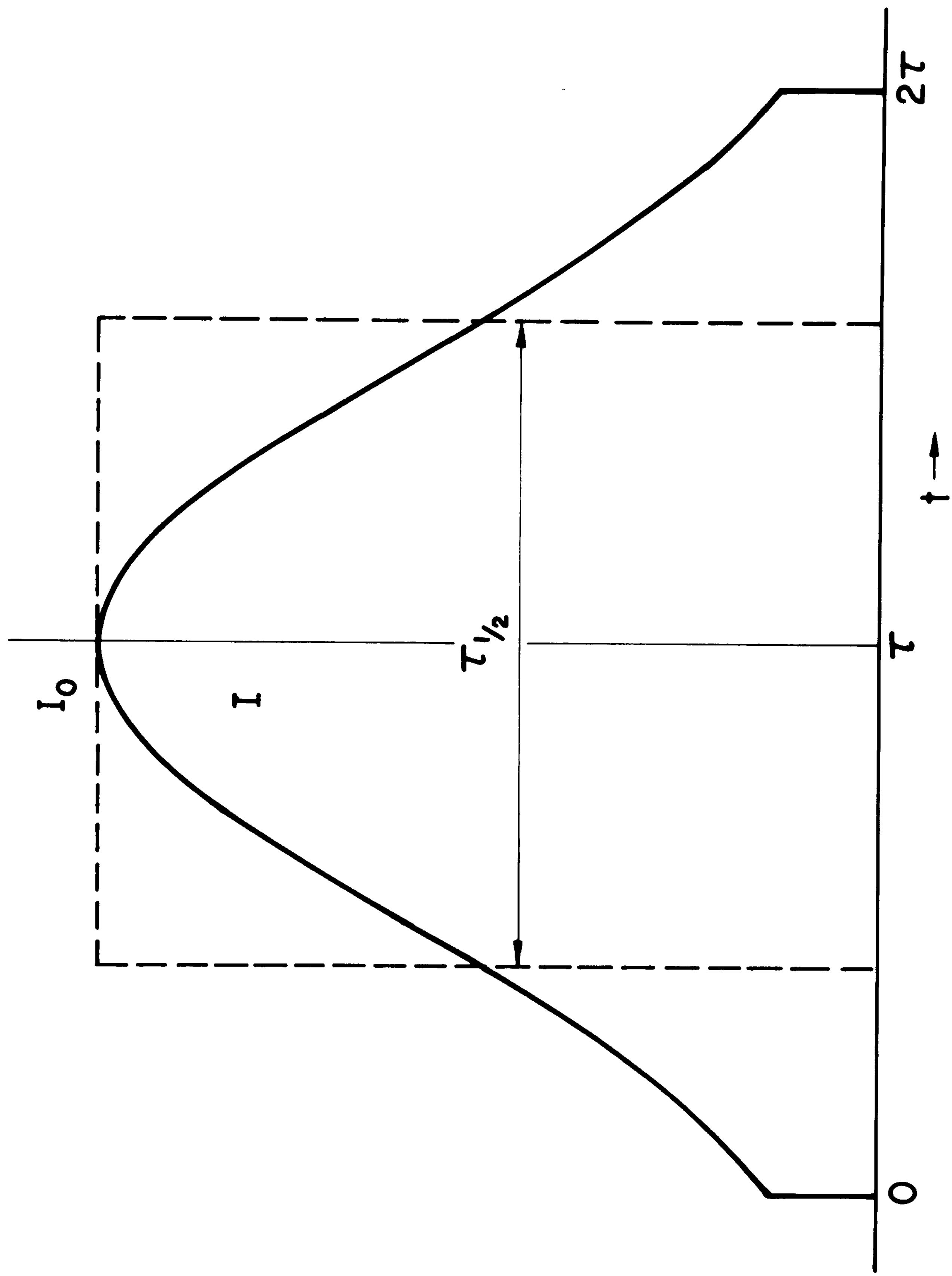

Fig. 1

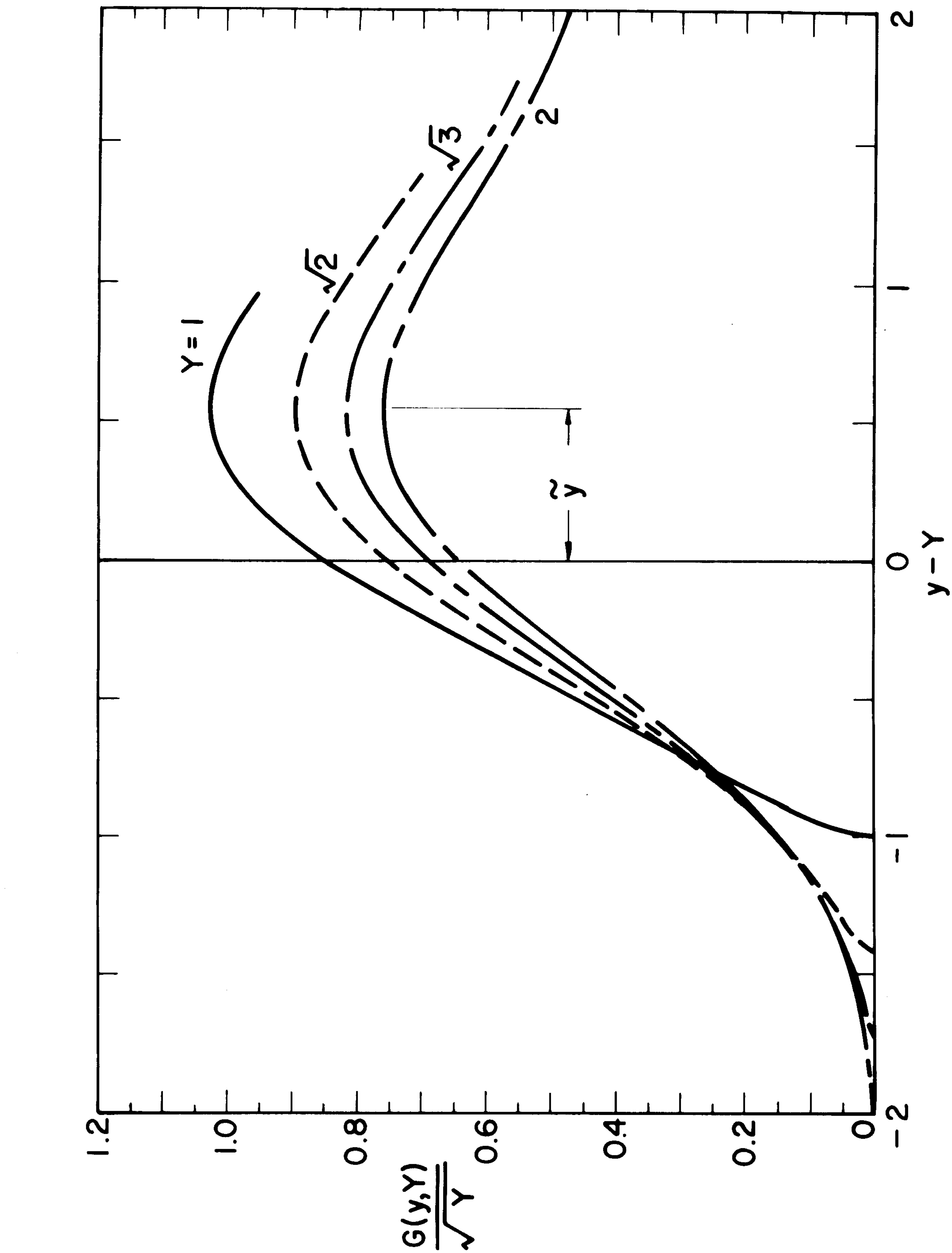

Fig. 2

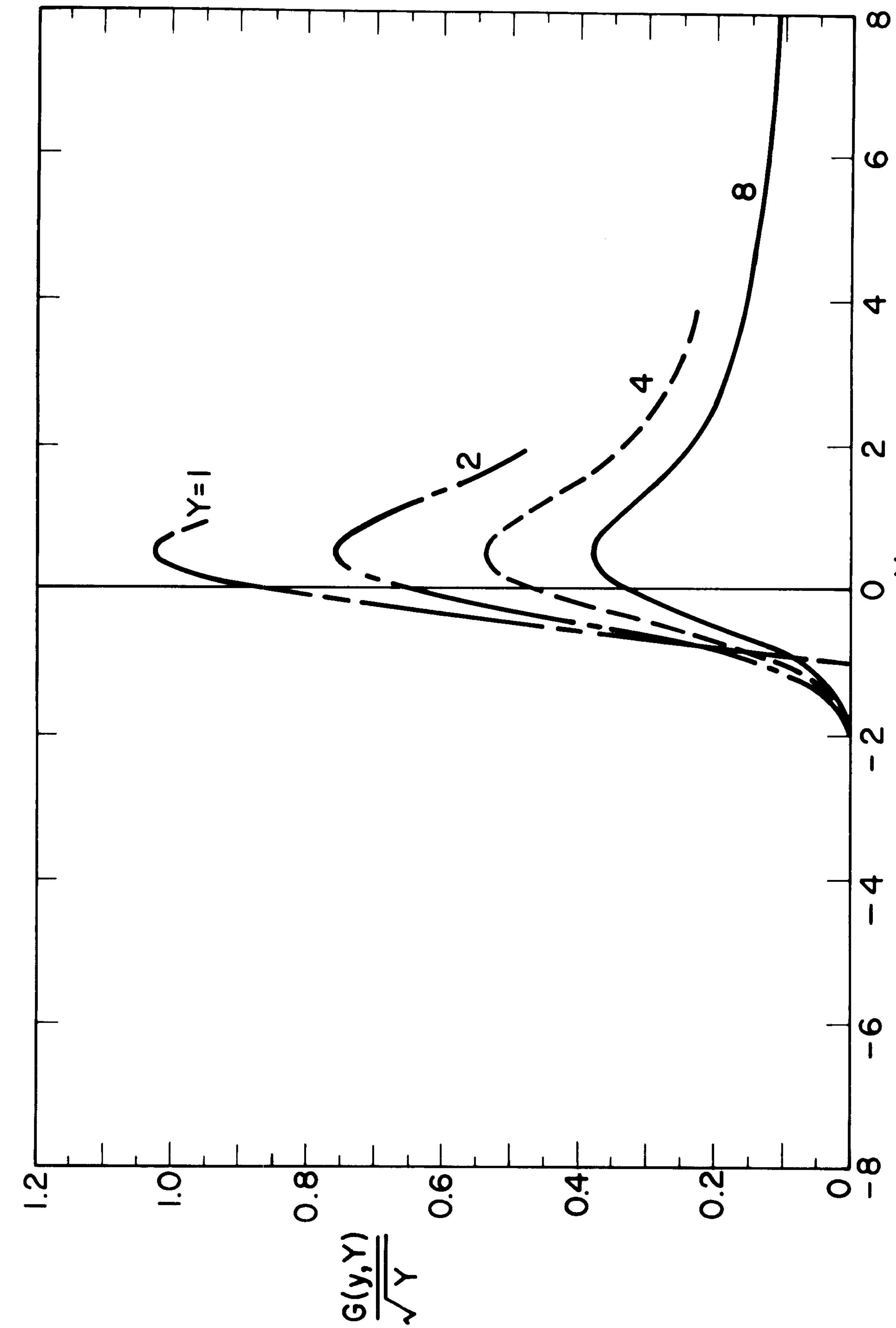

Fig. 3

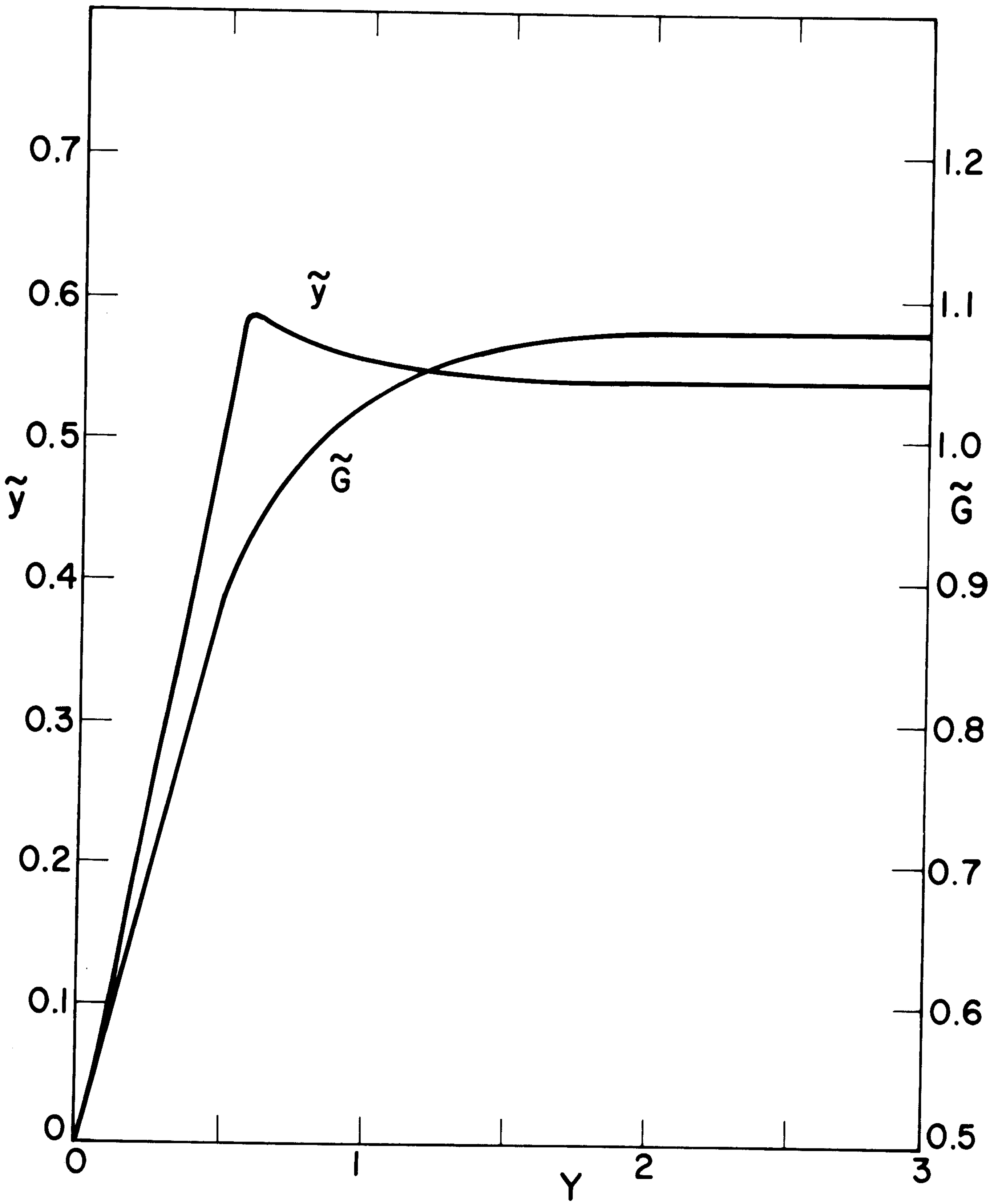
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
$\tilde{y}$
$\tilde{y}$
$\tilde{G}$
$\tilde{G}$
1.2
1.1
1.0
0.9
0.8
0.7
0.6
0.5
0
1
2
3
Y

Fig. 4

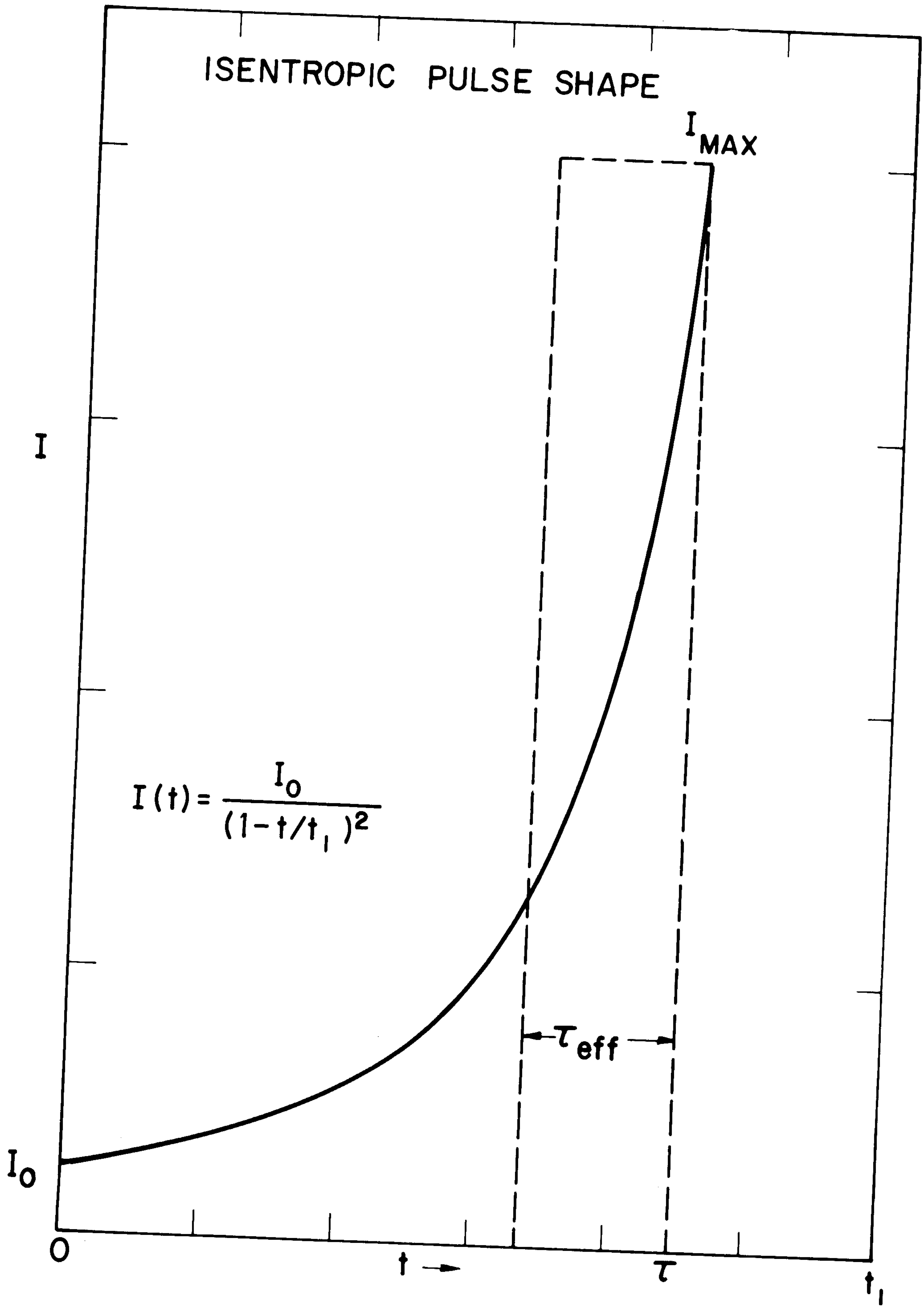
ISENTROPIC PULSE SHAPE
I_{MAX}
I
$I(t) = \frac{I_0}{(1-t/t_1)^2}$
τ_{eff}
I_0
0
t →
τ
t_1

Fig. 5