Quasi-Linear Model for Ion Cyclotron Heating

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Abstract

A quasi-linear numerical model for RF heating is developed in a form so as to be applicable both to toroidal and to magnetic mirror plasma containment devices. It is then combined with a two-dimensional multi-species Fokker-Planck code. It is found that as much as 40% of beam power injected into a two-component tokamak may be replaced by RF power without significantly degrading the fusion power produced. Furthermore, results indicate that in a magnetic mirror, RF at the fundamental or second harmonic may be useful for heating as well as for trapping of a low energy ion source.

I. Introduction

Ion heating at the cyclotron frequency and its harmonics may play a significant role in allowing plasma devices such as tokamaks or mirrors to approach the ion temperatures required for fusion. To reach ignition, tokamaks will probably need supplementary heating in addition to ohmic heating. Driven power producing devices such as a two-component tokamak,[1] sub-ignited[2] tokamaks or magnetic mirror devices require significant ion heating and the heating efficiency figures importantly in the economics of these devices.

A great deal of effort has recently been directed towards RF heating of tokamaks by means of the compressional hydromagnetic wave or "fast wave" (see (3) for references). Stix[3] has studied fast wave cyclotron heating of tokamaks analytically by including the quasi-linear diffusion coefficients[4] among the Fokker-Planck terms in the Boltzmann equation. We have followed a similar approach in a numerical treatment of the problem, combining the full quasi-linear diffusion coefficients with the Livermore two-dimensional multi-species Fokker-Planck code.[5]

RF heating is also potentially of great interest for heating magnetic mirror devices. In a magnetic mirror, plasma containment depends sensitively on the ion distribution function. The solution of the Fokker-Planck quasi-linear equation will therefore serve to indicate the effect of RF heating on mirror confinement.

In section II, we will present the quasi-linear model for studying RF heating. Sections IIIa and IIIb will then present results of calculations for tokamaks and mirrors respectively. Section IV contains a summary of results.

II. Quasi-Linear Mathematical Model

Quasi-linear theory described the evolution of the distribution function under the action of a spectrum of uncorrelated waves and is therefore a stocastic heating process. Stix[3] has argued that this formalism is applicable for studying RF plasma heating when the plasma is sufficiently collisional so that collisions destroy phase information between successive passes through the resonances zone of heating. Typically, the time between successive transits is $\tau_p \approx 2L/V_{\shortparallel}$, where L is the distance a particle travels between successive passes through the resonance zone, so that the variation in τ_p is $\Delta\tau_p = 2L\Delta V_{\shortparallel}/V_{\shortparallel}^2$. The dispersion by coulomb collisions would then lead to a rms $\Delta V_{\shortparallel}$ value of $(<(\Delta V_{\shortparallel})^2> \tau_p)^{1/2}$ where $<(\Delta V_{\shortparallel})^2>$ is the diffusion coefficient (see for example Spitzer). Requiring that the ions "forget" the cyclotron phase between transits says $\omega_{ci}\Delta\tau_p \gg 1$ or $\dfrac{<(\Delta V_{\shortparallel})^2>}{V_{\shortparallel}^2} \gg \dfrac{1}{(\omega_{ci}\tau_p)^2}\ \dfrac{1}{\tau_p}$. This is easily satisfied for typical mirror or tokamak fusion plasmas.

The fluctuation induced rate of change in f_0 by the quasi-linear approximation is then

$$\frac{\partial f_0}{\partial t} = \frac{q}{m} \ <\nabla\phi_1 \ \cdot \ \frac{\partial f_1}{\partial \tilde{v}}> \ .$$

where f_1 is the linear response to ϕ_1, the wave potential. The brackets represent a sum over uncorrelated modes. For a given wave frequency and $k_{\shortparallel}$ (the wave number parallel to the field direction) the linearized Vlasov-Maxwell equations will determine the dispersion relation and therefore $k_\perp$ (the wave number perpendicular to the field direction) as well as the relative magnitudes of the electric field. Both $k_{\shortparallel}$ and wave frequency are expected to be experimentally controllable parameters.

When the wave amplitude becomes sufficiently large, non-linear effects such as wave particle trapping or resonance broadening can come into play and should be considered in a more complete treatment of the problem. Furthermore, it has been shown that in tokamaks under some circumstances several modes are excited[3,6] or a coupling between wave modes[7] takes place and these effects must also be looked into.

From the point of view of the cyclotron damping mechanism, fundamental and higher harmonic heating ($2\omega_{ci}$, $3\omega_{ci}$) behave quite differently. For an electric field at the fundamental frequency rotating in the ion direction, each ion passing through the resonance zone (where $\omega = \omega_{ci}$) will get a "kick" in $v_{\perp}$ and this results in a velocity space diffusion. Heating at the second harmonic, however, invokes a finite Larmor radius effect, and the velocity space diffusion coefficient is proportional to $v_{\perp}^2$. Thus, second harmonic heating will preferentially heat ions with high perpendicular energy and tend to create tails, whereas fundamental heating tends to heat the bulk plasma.

The Hybrid-2 Fokker-Planck code[5] solves a set of coupled equations (one for each species) in spherical coordinates (v, θ) in velocity space (where θ is the pitch angle) of the form

$$\frac{\partial f}{\partial t} = n_0 \Gamma \left\{ \frac{1}{v^2} \frac{\partial}{\partial v} \left(\bar{A}f + \bar{B}\tilde{v} \frac{\partial f}{\partial v} + \bar{C} \frac{\partial f}{\partial \theta} \right) + \frac{1}{\tilde{v}v^2 \sin^2\theta} \frac{\partial}{\partial \theta} \left(\bar{D}f + \bar{E}\tilde{v}\frac{\partial f}{\partial v} + \bar{F} \frac{\partial f}{\partial \theta} \right) \right\} + S - \frac{f}{\tau_p} \quad (1)$$

where Γ is a constant defined by $\Gamma = 4\pi Z^4 e^4 / m^2$, n_0 is a normalizing density, $\tilde{v}$ is the normalizing velocity and the terms, $\bar{A}$, $\bar{B}$ . . . $\bar{F}$ contain integrals of the distribution function (f) as well as the independent variables and so this equation is non-linear. S contains the source term, charge exchange loss,

and alpha heating terms and τ_p is the particle containment time. For a mirror $\tau_p = L/V_{\shortparallel}$, for ions in the loss cone (L is the mirror length) and $\tau_p = \infty$ otherwise. For a tokamak, there is an additional loss term added

$$\frac{\partial f}{\partial t}\bigg|_{tok} = + \frac{1}{2v^2} \frac{\partial}{\partial v} \frac{v^3 f}{\tau_E} \tag{1a}$$

with τ_E the energy containment time.

Quasi-linear theory solves the Vlasov equation by separating the distribution function and fields into spacially independent components and small, rapidly oscillating wave fields. We then can write the space-averaged Vlasov equation, which gives the rate of change of the distribution function due to a spectrum of wave.

The quasi-linear diffusion equation of Kennel and Engelmann[4] can be put into a form identical to (1) as shown in Appendix I. In this case, the coefficients $\bar{A}$, etc. are replaced by coefficients A', B', etc. which are functions only of the independent variables (v, θ) and the externally imposed wave parameters, and therefore the operator is linear.

These coefficients become:

$$A' = 0 \tag{2a}$$

$$B' = Cv^2 \sin^2\theta/\tilde{v} \tag{2b}$$

$$C' = -Cv(k_{\shortparallel} v/\omega - \cos\theta)\sin\theta \tag{2c}$$

$$D' = 0 \tag{2d}$$

$$E' = -Cv \sin^2\theta(k_{\shortparallel} v/\omega - \cos\theta) \tag{2e}$$

$$F' = C\tilde{v}(k_{\shortparallel} v/\omega - \cos\theta)^2 \sin\theta \tag{2f}$$

with $C = \dfrac{|E_+|^2}{32Z^2 e^2 n_o}\ \delta(n\omega_{ci} - \omega + k_{\shortparallel} v_{\shortparallel})J_{n-1}^2\left(\dfrac{k_\perp V_\perp}{\omega_{ci}}\right)$ \hfill (2g)

Here $\tilde{v}$ = normalized velocity, ω is the wave frequency, $k_{||}$, $k_{\perp}$ are parallel and perpendicular wave numbers, J_{ν} is the Bessel function or order ν, n is the harmonic at which we are heating, and E_{+} is the electric field rotating with the ions.

Since ω_{ci} is a function of $B(Z)$, (2g) can be integrated along a field line to eliminate the delta function. If we assume a parabolic magnetic mirror of mirror ratio $R(= B_{max}/B_{min})$, after bounce averaging, the expression for C becomes

$$C \to |E_{+}|^2 \, J_n^2 \, (\frac{v_{\perp}k_{\perp}}{\omega_{ci}})/(32\pi Z^2 e^2 n_0 \, n \, \omega_{ci} \, (z=0)\xi) \qquad \text{2h}$$

with $\xi = ((B'/B_0 - 1)(R - B'/B_0))^{1/2}$ and $B' = (\omega - k_{||}v_{||}) \frac{mc}{ne}$. This is valid for $B_{min} < B' < B_{max}$. Otherwise, we set $C = 0$, i.e. the particle does not pass through a resonance layer. Similarly, for a tokamak we can integrate 2g around the minor radius to find a similar expression for C

$$C \to |E_{+}| \, J_n^2 \, (\frac{v_{\perp}k_{\perp}}{\omega_{ci}}) \, \frac{\rho_0}{a} \, / \, (32\pi \, Z^2 \, e^2 \, n_0 \, n \, \omega_{cio}\xi) \qquad \text{2i}$$

with $\xi = (1 - (\frac{\rho_0}{a})^2 \, (\frac{n\omega_{cio}}{\omega - k_{||}v_{||}} - 1)^2)^{1/2}$ with ω_{cio} the cylcotron frequency at the major axis (ρ_0), and (a) the minor radius at which we heat.

Similar equations may be derived for the $E_{\perp}(J_{n-1}$ is replaced by $J_{n+1})$ and the $E_{||}$ $(J_{n+1}$ is replaced by $J_n)$ terms. The heating that results from these terms is small, but they are included in the code.

The effect of varying the wave vector $(k_{\perp}, k_{||})$ is apparent from eqn. 2g. The argument of the Bessel function $(k_{\perp}\rho_i$ with ρ_i the Larmor radius) is generally small. For fundamental heating, $J_0^2(k_{\perp}\rho_i) \approx 1$ and heating is independent of ρ_i (unless $\rho_i \sim k_{\perp}^{-1}$ which corresponds to very high energies). For heating at

$2\omega_{ci}, J_1^2(k_\perp \rho_i) \approx (k_\perp \rho_i)^2$ which is the finite Larmor radius effect mentioned previously. The parallel wave number comes into 2g in the doppler shifted resonance condition. For $\omega/k_{\shortparallel} \sim v_A \gg v_{\shortparallel}$, the doppler shift is small and most ions interact with the wave close to the resonance surface, defined by $n\omega_{ci}(Z) = \omega$. If, however, $k_{\shortparallel}$ becomes large, only ions with $v_{\shortparallel} < (n\omega_{ci}^* - \omega)/k_{\shortparallel}$ will interact with the wave, where ω_{ci}^* is the maximum cyclotron frequency corresponding to the maximum magnetic field felt by the ion along its orbit.

We can calculate the total RF heating rate by integrating the equation(1) in velocity space using the RF coefficients of equation (2). It is then seen that only the B' and C' terms will give contributions to heating, while the terms resulting from E' and F' coefficients correspond to pitch angle scattering.

III. Results of Sample Calculations

III-a. Second Harmonic Heating of Two Component Tokamaks

There has been much interest in two component tokamaks as a driven reactor or a high flux neutron source[2,8]. The two component tokamak concept generally involves injecting deuteron beams of energy 100 to 200 keV into a tritium target plasma. For high deuteron currents this approach will necessarily degrade the purity of the tritium target and thereby the amplification factor Q. Here we defined Q as the ratio of the instantaneous fusion power produced (at 17.6 MeV/fusion) to the power injected into the plasma either by beams or RF. RF heating of the minority deuteron species would then offer the possibility of heating deuteron tails without degrading the purity of the background plasma.

The parameters chosen for these calculations are shown in Table I and were chosen to be typical of next generation large tokamaks. The tritium density is

set at 9×10^{12} and tritium and electrons are given an energy containment time (τ_E) of 100 ms. The deuterium density is determined by the beam current and a particle containment time (τ_{pD}) of 100 ms.

There is reason to believe that turbulence induced particle loss is only significant for deuterons that have thermalized.[9] The effect of this assumption has been tested and Fig. 1 displays the results. Here we consider injection of a 150 keV deuteron beam into a tritium target of density $9 \times 10^{12} cm^{-3}$. With no deuteron loss ($\tau_{pD} \to \infty$) the deuteron density builds up with time so Q decreases monotonically after reaching a peak value of 0.81. Setting τ_{pD} to 100 ms causes Q is degraded and it reaches a plateau level of only .67. However for $\tau_{pD} = 100$ ms for only thermal deuterons below 30 keV is the equilibruim Q higher, attaining a value of 1.04. In the calculations shown below, we will assume the latter case, i.e. only deuterons that have nearly thermalized (E < ~30 keV) are removed from the device.

We have been interested in considering the usefulness of combining beams with RF heating at $2 \omega_{ci}$. The wave vector $\vec{k}$ was chosen to satisfy $k_{\shortparallel} \approx \frac{n}{2R}$ and $k_{\perp} \approx \frac{\omega_{ci}}{v_A}$ with v_A the Alfven speed. Fig. 2 compares the time of evolution of Q for different mixtures of beam and RF. At times early compared with the slowing down time (τ_{SD}) or the containment time (τ_p) (note $\tau_{SD} \sim \tau_p$ ~100 ms),beams without RF give a higher Q value. At later times, however, an equilibrium is approached and we see that as much as 40% of the input power could be supplied by RF without a significant degrading of Q. This is also indicated in Fig. 3 where the asymptotic Q values are shown as a function of percentage of power supplied by the beams.

Fig. 4 indicates a typical distribution function in perpendicular ana parallel velocities. The tendency of RF to drive tails in velocity space is clearly displayed.

III-b. RF Heating of Magnetic Mirrors

RF heating may also be useful as an alternative to beams in mirror machines. Heating has been demonstrated in several mirror-like configurations.[10,11]

At first glance, we may think that RF heating, being a stocastic process, will tend to diffuse ions into the loss cone as well as up in velocity space. We have found, however, that the RF will tend to modify the ion distribution function so that most of the power goes into heating and confinement is not significantly affected.

We have considered heating both at the fundamental $(\omega = \omega_{cio})$ and second harmonic $(\omega = 2\omega_{cio})$ where ω_{cio} is the cyclotron frequency at the midplane of the mirror. As expected, fundamental heating acts on the bulk plasma whereas heating at $2\omega_{cio}$ tends to drive tails in $v_{\perp}$, but these effects must be considered in combination with the ambipolar potential of the mirror and the velocity dependence of the ion source (if one is present).

Livermore results have shown that plasma stability against the drift loss cone and convective loss cone can be obtained by passing a cold streaming plasma through the mirror machine. We have, therefore, looked at the cases with and without a cold streaming plasma.

Table 2 indicates plasma and machine parameters used in this series of calculations. We are considering a plasma with midplane radius of 6.5 cm confined in a mirror well with a vacuum mirror ratio of 2. The plasma occupies about 7,600 cm^3 at low β, and   as it heats and β increases, the

occupied volume can decrease by up to 20%. Furthermore, since the Fokker-Planck code calculates the midplane density, as the $P_\perp/P_\parallel$ ratio increases the plasma becomes more localized near the midplane resulting in a local density increase. Fig. 5 indicates the density loss from the well with injection of about 9.4 kW of RF power (absorbed in the plasma) at the fundamental and second harmonic. No stream is present. The decay appears to be similar for the two cases. Examination of the distribution functions for these cases (Fig. 6) leads to the conclusion that whereas fundamental heating tends to enhance diffusion into the loss cone, second harmonic heating heats the high energy ions preferentially. They will in turn heat the electrons and drive up the ambipolar potential which will eject the lower energy ions. In either case, there is a runaway of ion energy due to the preferential loss of low energy ions.

Fig. 7 indicates the density evolution in the mirror when a source of "streaming", 10 eV plasma is added. These calculations also include the charge exchange losses with a 1 ms characteristic time (which corresponds to a base pressure of ~5 x 10^{-7} torr). The density seeks an equilibrium level because the high energy tail of the cold streaming plasma behaves as a plasma source. If the RF power level is increased, we see that the equilibrium density level also increases along with the ion and electron temperature. If the power is deposited by means of second harmonic heating, the equilibrium density is lower since the second harmonic heating is not as effective in trapping the streaming plasma which is the density source. These results serve to indicate that in addition to a heating mechanism, RF can also serve to fuel a mirror plasma when properly combined with a plasma stream. Figure 8 indicates the

equilibrium to which the distribution function evolves. Notice f(E) has a dip near the ambipolar potential (1.2 keV) loss cone boundary and exhibits a hot monotonic tail going up to ~10 keV.

Fig. 9a indicates the electron temperature evolution for two RF power and streaming plasma levels. If a stream of 10^{17} particles/cm^3-s is necessary for stabilization, RF power levels of ~ 47 kW (absorbed) would be required to heat the electrons to ~150 eV (this corresponds to an ambipolar potential of ~550 eV). Figure 9b indicates the resultant ion temperatures for these cases.

These results lead to speculation as to the possible merits of RF heating with comparison to beams in a future mirror reactor. Fig. 10 indicates schematically the energy distribution that would be expected to accompany beam injection. The distribution function peaks at the energy of injection of the beams and goes to zero close to the ambipolar potential low energy cutoff (ϕ_a/R-1). Between these two energies $\frac{\partial f}{\partial E} > 0$ and diffusion will tend to drive ions into the loss cone (E < ϕ_a/R-1). Furthermore, cyclotron frequency turbulence can greatly enhance velocity space diffusion and is, therefore, deleterious to containment.

The distribution function resulting from RF heating combined with fueling at the ambipolar potential would tend to produce a distribution function also sketched in Fig. 10. Since $\frac{\partial f}{\partial E} < 0$ throughout most of velocity space, the RF generated velocity space diffusion is mostly driving ions up to higher energy and so the imposed cyclotron frequency "turbulence" will heat without causing a catastrophic loss of confinement.

IV. Summary of Results

A quasi-linear model for RF heating can lead to several interesting predictions. For a two component tokamak, it appears that in order to avoid a serious degradation of the fusion rate it is necessary that the beams are well confined until they have thermalized. It is seen that up to 40% of the injected power could be replaced by RF at $2\omega_{ci}$ without seriously decreasing the fusion rate. This may be desirable from the point of view of source efficiency and neutral buildup.

Magnetic mirrors may similarly be heated using RF at the fundamental and/or second harmonic. Unlike beams, RF would require a plasma source. When no source is present, decay from the mirror is comparable for heating at ω_{ci} and $2\omega_{ci}$ although it is seen that the fundamental tends to do more bulk heating while higher harmonic heating tends to drive tails. If a particle source is present near the ambipolar potential cutoff, the interaction of the source with RF will fuel the system. This appears to indicate the possibility of fueling at the minimum confinable energy of the mirror.

Appendix I

The quasi-linear diffusion equation of Kennel and Engelmann[4] may be put in the form

$$\frac{\partial f}{\partial t} = \lim_{V \to 0} \sum_n \frac{\pi Z^2 e^2}{m^2} \times \int \frac{d^3 k}{(2\pi)^3 V} \, L \, V_\perp \delta(\omega - k_{||} v_{||} - n\omega_{ci}) |\theta_{n,k}|^2 V_\perp L f \qquad (A-1)$$

where

$$L = (1 - \frac{k_{||} v_{||}}{\omega}) \frac{1}{V_\perp} \frac{\partial}{\partial V_\perp} + \frac{k_{||}}{\omega} \frac{\partial}{\partial V_{||}} \qquad (A-1a)$$

and

$$\theta_{n,k} = \frac{1}{2} e^{i\psi}(E_x - iE_y)_k \, J_{n+1}(\frac{k_\perp V_\perp}{\omega_{ci}}) \qquad (A-1b)$$

$$+ \frac{1}{2} e^{-i\psi}(E_x + iE_y) {}_k J_{n-1}(\frac{k_\perp V_\perp}{\omega_{ci}})$$

$$+ \frac{v_{||}}{v_\perp} E_{zk} J_n(\frac{k_\perp V_\perp}{\omega_{ci}})$$

where V is plasma volume and $\vec{E}_k$ the Fourier amplitudes of electric field and the phase angle. Assuming $\psi = 0$, and a monochromatic spectrum of waves, we can integrate A-1 so that $E_k \to Re(\vec{E}(\vec{x},t))$.

By transforming the operator L to spherical velocity coordinates $(v, \mu = \cos\theta)$ it becomes

$$L_s = \frac{1}{v} \frac{\partial}{\partial v} + (\frac{k_{||}}{\omega v} - \frac{\mu}{v^2}) \frac{\partial}{\partial \mu} = \frac{1}{v^2}(\frac{\partial}{\partial v} v - \frac{\partial}{\partial \mu} \mu + \frac{\partial}{\partial \mu}(\frac{k_{||} v}{\omega})) \qquad (A-1c)$$

This then gives for the terms containing the electric field rotating with $(E_x - iE_y)$ and counter $(E_x + iE_y)$ to the ion gyration.

$$\frac{\partial f}{\partial t} = \frac{\pi Z^2 e^2}{8m^2} \left| E_x \pm i E_y \right|^2 \{\frac{1}{v^2} \frac{\partial}{\partial v}[J_{n\mp1}^{\ 2}(\frac{k_\perp V_\perp}{\omega_{ci}}) \ \delta(n\omega_{ci} - \omega + k_{,,}V_{,,})(v^2 sin^2\theta$$

$$\frac{\partial f}{\partial v} - v \ sin\theta(\frac{k_{,,}v}{\omega} - cos\theta)\frac{\partial f}{\partial \theta})]$$

$$+ \frac{1}{v^2 sin\theta} \frac{\partial}{\partial \theta}[J_{n\mp1} \ (\frac{k_\perp V_\perp}{\omega_{ci}}) \ \delta(n\omega_{ci} - \omega + k_{,,}V_{,,})$$

$$(-v \ sin^2\theta(k_{,,} \ v/\omega - cos\theta)\frac{\partial f}{\partial v} + sin\theta(\frac{k_{,,}V}{\omega} - cos\theta)^2 \ \frac{\partial f}{\partial \theta})]\} \qquad (A-2)$$

The $E_{,,}$ term is similar with $J_{n\mp1} \rightarrow J_n ctn\theta$. This then leads to the diffusion coefficients given in equation (2).

The diffusion coefficients all contain a term (sec equation 2g)

$$C' = \delta(n\omega_{ci} - \omega + k_{,,}V_{,,}) \ J_\nu^{\ 2}(\frac{k_\perp V_\perp}{\omega_{ci}}) \qquad \text{A-3}$$

For a mirror machine, we can bounce average (A-3) assuming a parabolic B field to give

$$C' = J_\nu^{\ 2}(\frac{k_\perp V_\perp}{\omega_{\rho i}})/(n\omega_{ci}' \ (B'/B_0 - 1)^{1/2}(B^*/B_0 - B'/B_0)^{1/2})$$

where B_0 is the midplane mirror field, B' is the field where the doppler shifted resonance occurs ($B' = \frac{mc}{en}(\omega - k_{,,}v_{,,})$), and B^* is the field where the particle bounces $B^* = \epsilon/\mu$. Note that $B^* \leq B_m$ (B_m is the field at the mirror throat) for mirror trapped ions. This expression will blow up either for ions for which the bounce position coincides with the resonance position ($B^* = B'$) or when the resonance occurs at the midplane ($B' = B^* = B_0$ and $v_{,,} = 0$). In this case, a maximum level is imposed on C' which reflects

the fact that the δ function approximation become inaccurate due to the need to include an intrinsic particle correlation rate due to the transit rate of particles in an inhomogeneous magnetic field, or a turbulent correlation due to orbit diffusion.

For a tokamak, we can integrate along an orbit as done in ref. 3 to obtain the result 2i.

Acknowledgement

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Table I

Tokamak Parameters

Toroidal Field	40 kG
Major Radius	3.25 m

Plasma Parameters

Tritium Density	9×10^{13} cm^{-3}
Initial Deuterium Density	1×10^{12} cm^{-3}
Tritium Energy Containment Time	100 ms
Electron Energy Containment Time	100 ms
Neutral Beam Energy	150 keV

RF Parameters

Parallel Wave Number ($k_{\shortparallel}$)	0.063 cm^{-1}
Perpendicular Wave Number ($k_{\perp}$)	0.31 cm^{-1}
Minor Radius of Heating Calculation (a)*	25 cm

*This is the minor radius at which we do the heating calculation. Results are scaled to other radii by equation 2i.

Table II

<u>Mirror Parameters</u>

Vacuum Mirror Ratio (R)	2
Mirror Length (1)	100 cm

<u>Initital Plasma Parameters</u>

Trapped Plasma Density (N_H)	$5 \times 10^{13}/cm^3$
Electron Energy (E_e)	100 eV

<u>RF Heating Parameters</u>

Perpendicular Wave Number ($k_\perp$)	$.157\ cm^{-1}$
ω_{ci} (Z = 0)	4.6×10^6 hz
Parallel Wave Number for Fundamental Heating	$0.6\ cm^{-1}$
Parallel Wave Number for Second Harmonic Heating	$0.063\ cm^{-1}$

Figure Captions

Fig. 1 - Time evolution of fusion amplification factor (Q) for a two component tokamak with 150 keV deuteron beams (3.2 watts/cm^3) injected into a tritium plasma of density 9×10^{13}. Curves shown for deuterium particle containment times (τ_{pD}) of (a) $\tau_{pD} = \infty$, (b) $\tau_{pD} = 100$ ms, (c) $\tau_{pD} = 100$ ms for $E_D < 30$ keV, $\tau_{pD} = \infty$ otherwise.

Fig. 2 - Time evolution of Q for different mixtures of 150 keV beams and RF at $2\omega_{ci}$. Power density in plasma fixed at 3.2 w/cm^3.

Fig. 3 - Maximum Q vs. percentage of total power input in 150 keV deuteron beams (remainder of power is in RF at $2\omega_{ci}$).

Fig. 4 - Deuteron distribution function for injection of 3.2 w/cm^3 power, half in 150 keV beams and half in RF at $2\omega_{ci}$.

Fig. 5 - Density decay from magnetic mirror. Cases shown are (1) without RF, (2) RF at the cyclotron frequency of the plasma midplane (ω_{ci}), (3) RF at $2\omega_{ci}$. No charge exchange losses are considered and the ion species is hydrogen.

Fig. 6 - Ion distribution function at $t = 1.24$ ms during plasma decay in a magnetic mirror.

Fig. 7 - Density evolution in a mirror when a streaming 10 eV plasma is present. Charge exchange losses are considered with $\tau_{cx} = 100$ ms. Curves indicate effect of varying stream intensity, RF power level and harmonic heating.

Fig. 8 - Equilibrium distribution function for mirror confined plasma with RF fundamental heating and a 10 eV stream of 10^{17}/cm^3-sec.

Fig. 9a - Electron temperature evolution for a variation of streaming plasma source strengths and of RF power levels.

9b - Ion temperature evolution for varying stream and RF power levels.

Fig. 10 - Sketch indicating the different distribution functions that would be expected for a mirror confined plasma heated and fueled by (a) high energy neutral beams, (b) low energy beams and RF.
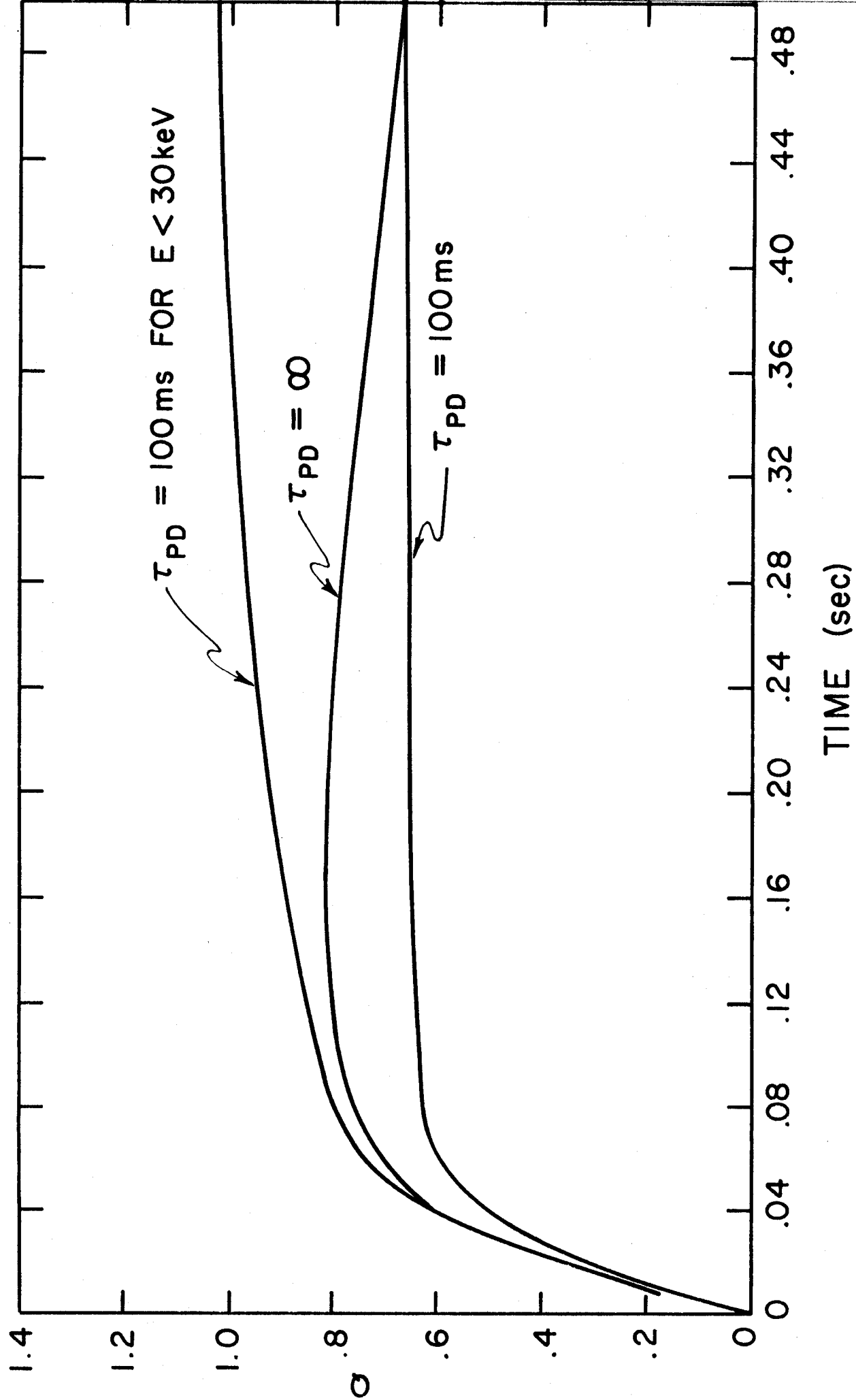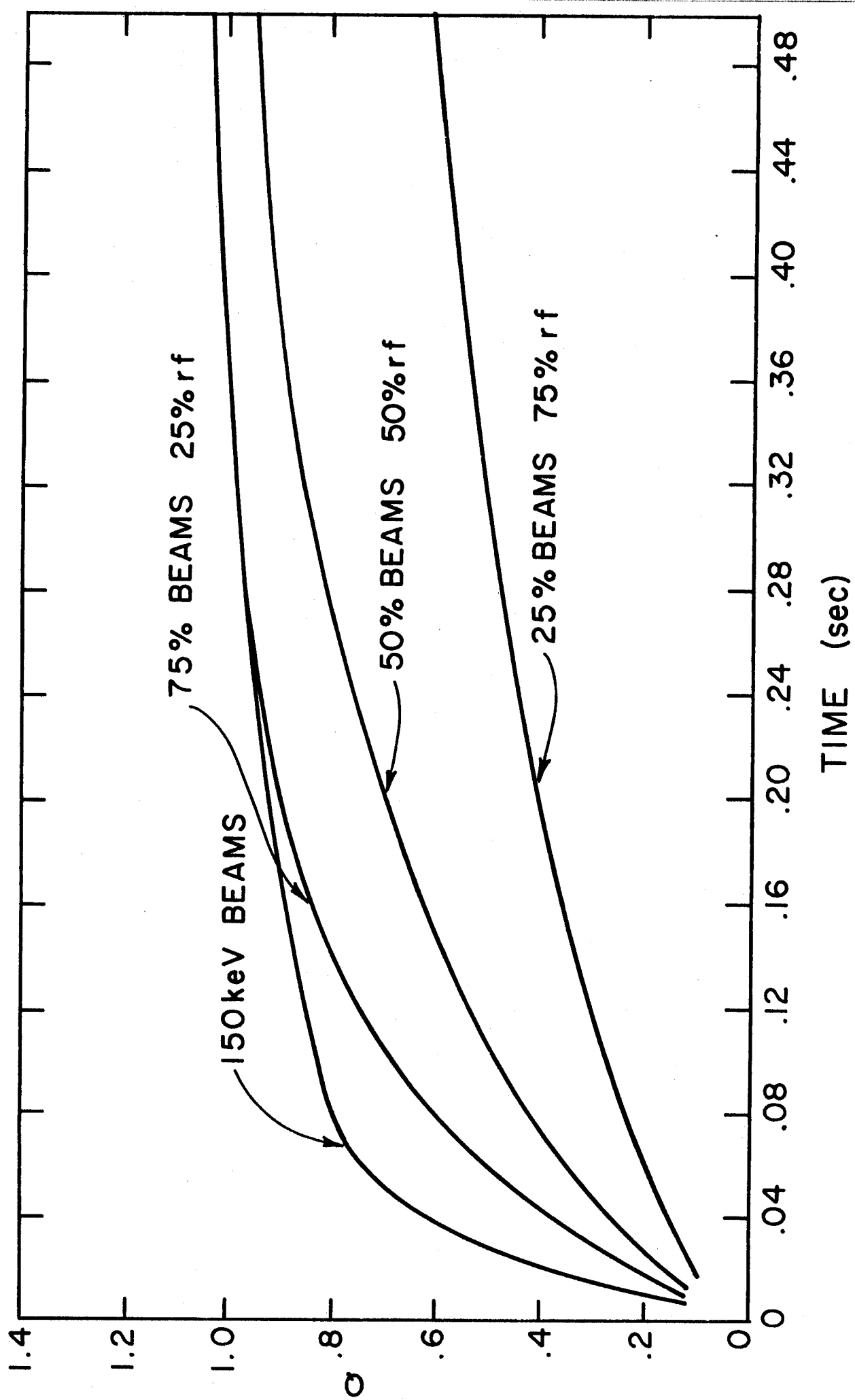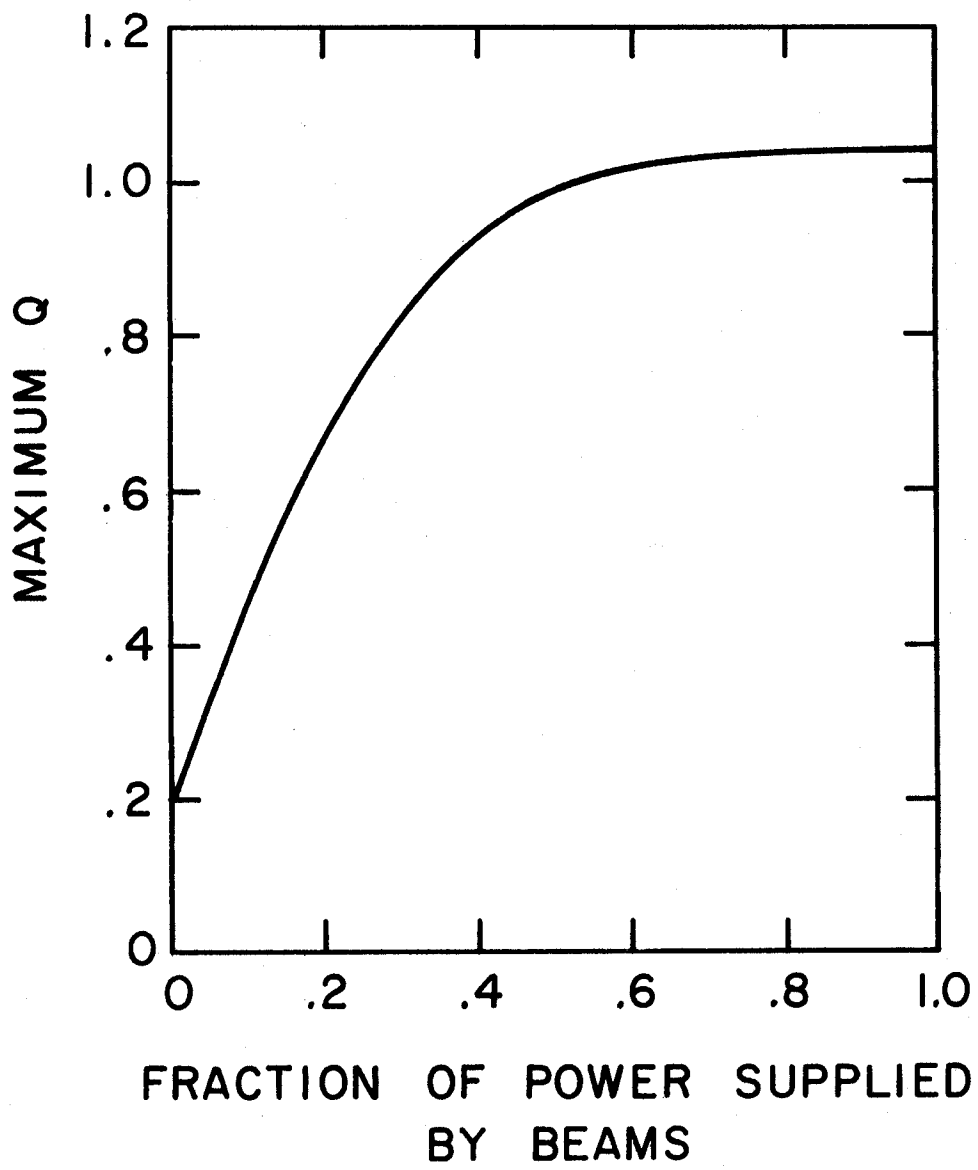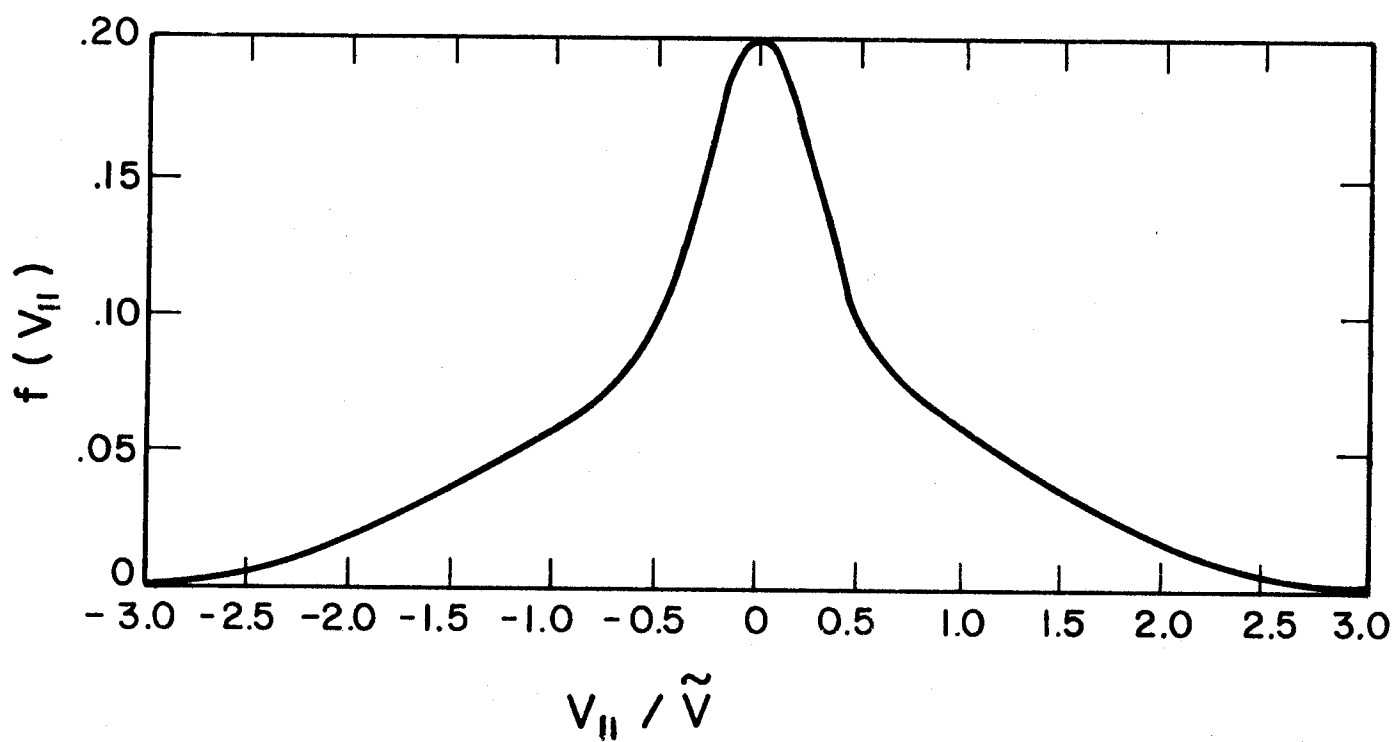
FIG. 1

19

FIG. 2

TIME (sec)

Q

75% BEAMS 25% rf

50% BEAMS 50% rf

25% BEAMS 75% rf

150keV BEAMS

20

FIG. 3

FIG. 4

FIG. 5

SECOND HARMONIC

FUNDAMENTAL

$P_{rf} = 9.4\,kW$

NO STREAM
PRESENT

$f(V_\perp)$

$V_\perp / \tilde{V}$

SECOND HARMONIC

FUNDAMENTAL

$f(V_{\parallel})$

$V_{\parallel} / \tilde{V}$
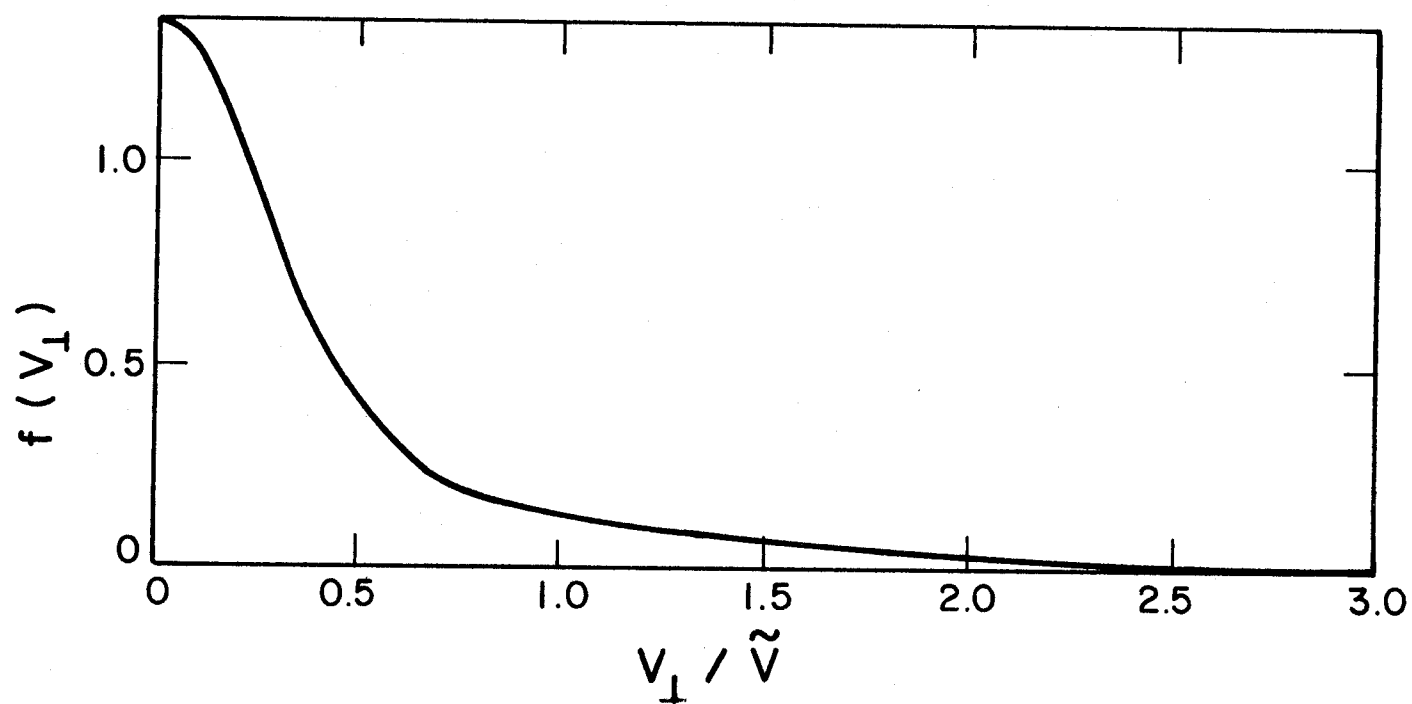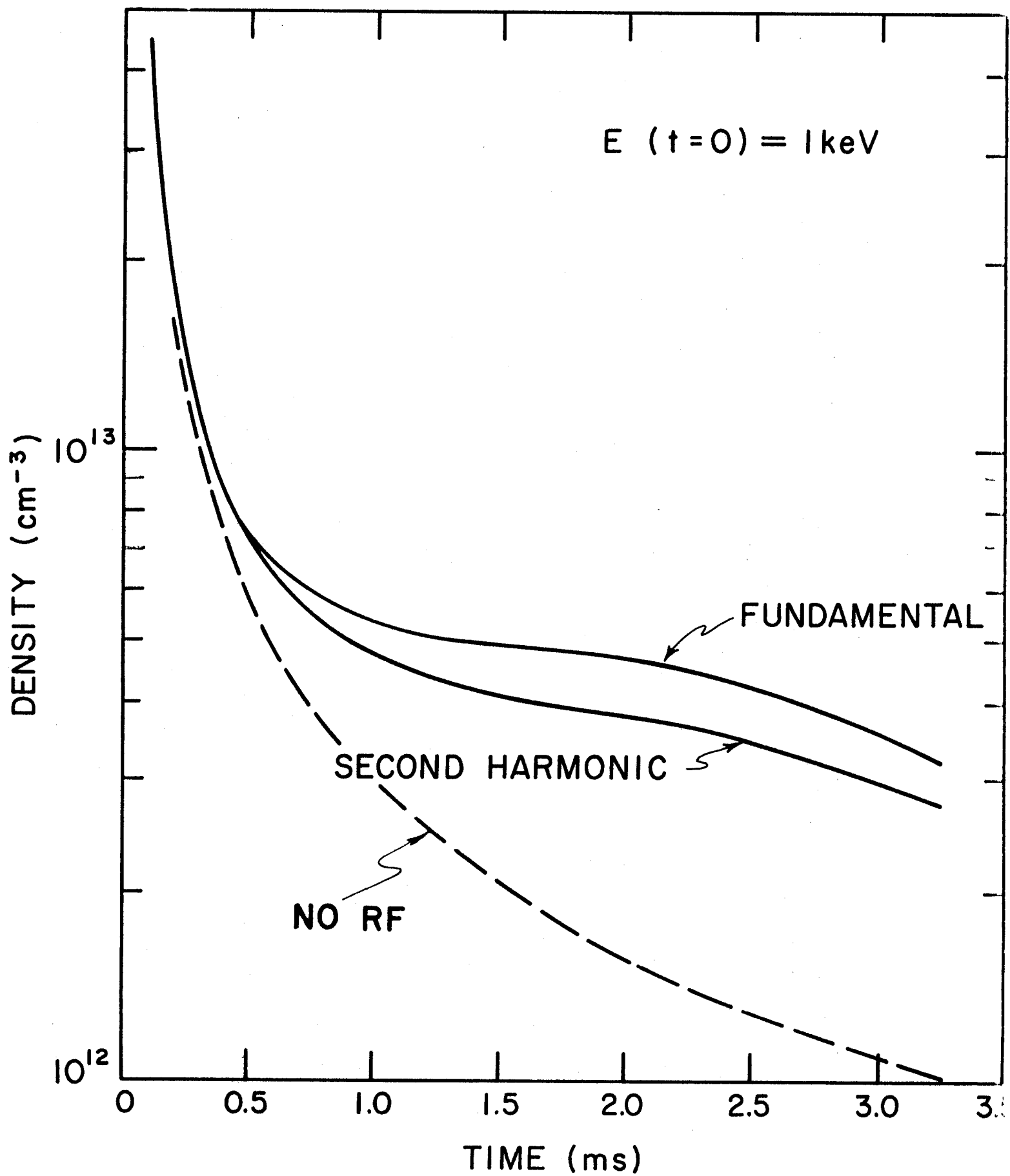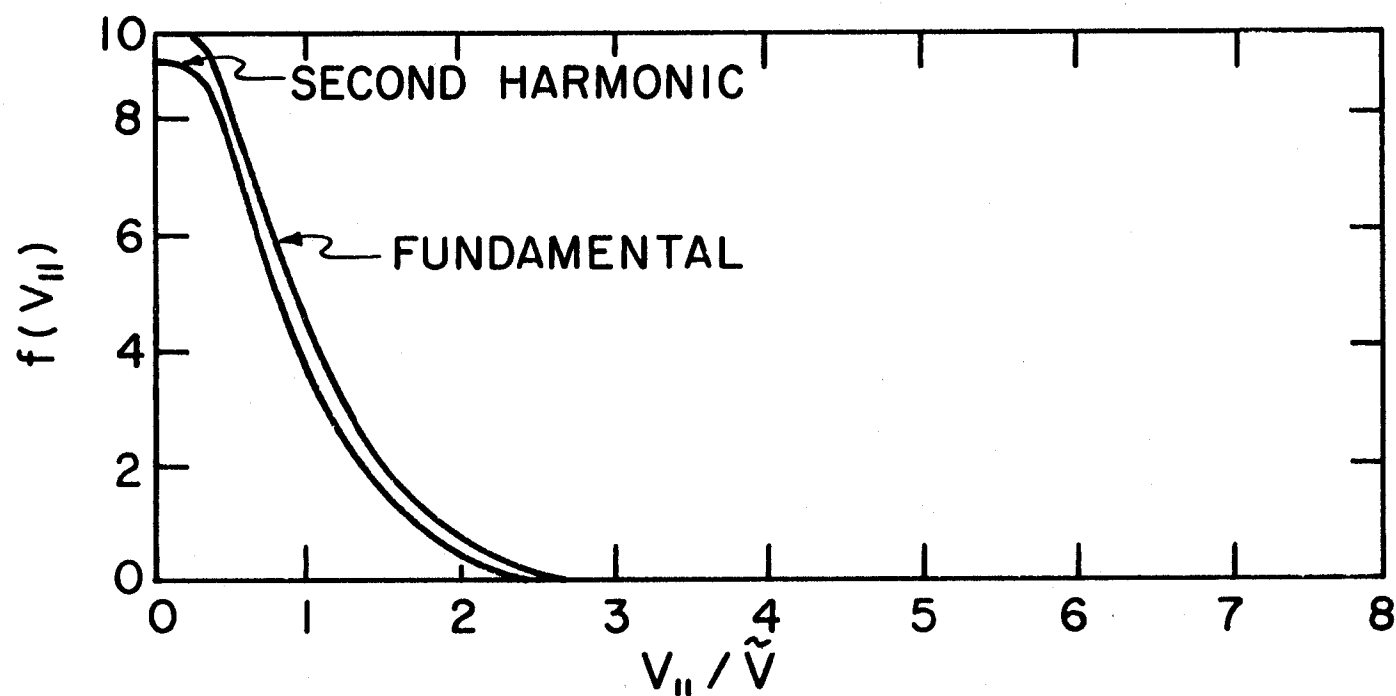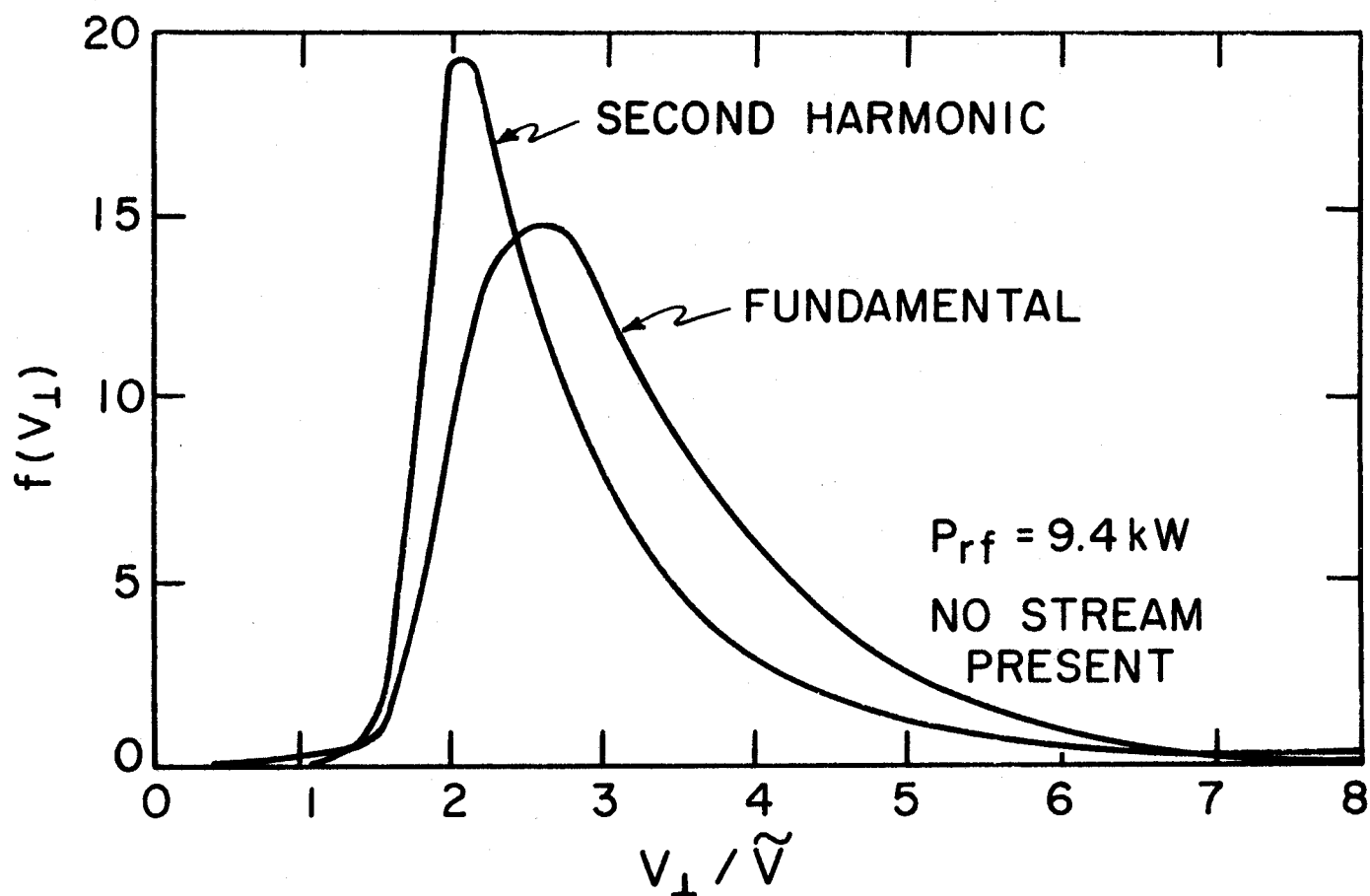
TIME = 1.24 MS $\tilde{V} = 3 \times 10^{7} cm/s$ $\frac{1}{2} m \tilde{V}^{2} = 470 eV$

FIG. 6

P$_1$ = RF POWER AT ω_{ci} (kW)
P$_2$ = RF POWER AT 2ω_{ci} (kW)
S = COLD STREAM (AMPS)

CHARGE EXCHANGE TIME = 1 ms
PLASMA VOLUME = 7600 cm^3

S = 120 A P$_1$ = 47 kW

S = 12 A   P$_1$ = 47 kW

S = 120 A P$_2$ = 47 kW

S = 12 A   P$_1$ = 9.4 kW

S = 12 A   P$_2$ = 47 kW

S = 0   P$_1$ = 9.4 kW

DENSITY (cm^{-3})

TIME (MS)

FIG. 7

FUNDAMENTAL HEATING
SOURCE = 10^{17}/cm^3-s

CHARGE EXCHANGE TIME = 1 MS

COLD STREAMING PLASMA

$E^{1/2}f(E)$ ARBITRARY UNITS

ENERGY (keV)

FIG. 8

26

FIG. 9a

P_1 = RF POWER AT FUNDAMENTAL

CHARGE EXCHANGE TIME = 1 MS

$S = 10^{17}$ $P_1 = 47$ kw

$S = 10^{16}$ $P_1 = 9.4$ kw

$S = 10^{17}$ $P_1 = 9.4$kw $\tau_x = 1$

ELECTRON ENERGY (eV)

TIME (ms)

27

FIG. 9b

28

FIG. 10