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ABSTRACT

The equations required to study the dependence of the steady state operating conditions in a tokamak-like CTR plasma on various system parameters are solved. Particle diffusion is explicitly represented by neo-classical theory and β_p, the ratio of plasma pressure to poloidal magnetic field pressure, is limited to a maximum value of $\sqrt{A}$, where A is the aspect ratio. The particle and energy balance equations are solved self-consistently with the confinement time equation and the constraint equation on plasma pressure. The dependence of the plasma operating parameters on enhanced bremsstrahlung radiation losses and enhanced diffusional losses is studied. A viable tokamak-like CTR plasma is suggested consistent with the parametric study.

I. Introduction

An important part of any detailed examination of the feasibility of controlled fusion is to establish, as best possible, the plasma operating conditions. A number of recent studies[1,2,3,4] have attempted to calculate possible steady-state plasma operating conditions in CTR systems. The most complete study on closed systems is that of Rose,[2] who examined in detail the dependence of the steady-state conditions on the levels of bremsstrahlung and synchrotron radiation, alpha particle heating, convective particle and energy losses, and details such as electron-ion rethermalization and energetic particle injection. However, it is difficult to apply these results to a specific closed system, such as a Tokamak[5] or Stellerator,[6] because of the schematic used to obtain the solutions. The steady-state conditions (such as the temperature of various plasma components) are derived from particle number and energy conservation equations, including an equation for the particle confinement time; τ_c. In general, τ_c depends on density, temperature, and system parameters. In the work of Rose,[2] however, the solution schematic is to choose a value of the electron temperature, solve for the ion temperature using one of two energy balance equations, and determine a confinement time, τ_c, such that a second energy balance equation is satisfied. It is therefore difficult to uncover the dependence of the steady-state solutions on the functional variation of τ_c with temperature and density.

In this paper, we use a set of point reactor equations derived previously[7] to study steady-state solutions for Tokamak-like CTR systems and to examine the dependence of these solutions on system parameters. The parameters of interest include β_p, the ratio of plasma pressure to poloidal magnetic field pressure, the stability factor, q, the aspect ratio, A, and the toroidal magnetic field strength, B_T. The steady-state solutions are determined self-consistently from a set of particle and energy conservation equations together with confinement time formulas that have functional forms of particular applicability to Tokamak and/or Stellerator systems,[8,9,10,11]. The solutions include restrictions on q and β_p.[10,12] In addition, within the limitations imposed on the solutions either by the method of analysis or by the plasma physics itself, a solution representing a viable CTR system, from the plasma point of view, is sought.

II. Basic Equations

The basic equations are solved after imposing the following set of assumptions:

a. In the parameter range of interest, synchrotron radiation losses are negligible compared to bremsstrauhlung, especially if enhanced,

b. Zero energy particles are injected to make up particle losses due to fusion and diffusion,

c. The CTR plasma system operates with a divertor so that conductive energy losses can be considered negligible,

d. Diffusion is neoclassical and in the banana regime[8,9] i.e., the equations for the electron and ion confinement times predicted by neoclassical theory are used as additional equations and solved self-consistently with the other governing equations,

e. β_p is limited[8] to a maximum value of $\sqrt{A}$, although of course other values, such as $\beta_p = A$, can be used if desired,

f. Ohmic heating power is negligible during steady state operation.

Under these conditions, the particle and energy conservation equations become:

a. Particle conservation:

$$n_e = n_i + 2n_\alpha + \sum_{im} z_{im} n_{im} \quad (1)$$

$$n_\alpha = \left(\frac{\tau_c^\alpha}{\tau_c^i}\right) \frac{n_i X_i \langle\sigma v\rangle}{4} , \quad (2)$$

$$X_i = n_i \tau_c^i$$

b. Ion energy conservation:

$$\frac{n_i^2 \langle\sigma v\rangle}{4} E_\alpha U_{\alpha i} - \frac{7T_i}{2}\left(\frac{n_i}{\tau_c^i}\right) - \frac{3}{2} T_i n_i^2 \frac{\langle\sigma v\rangle}{2} + Q_{ei} = 0 \quad (3)$$

c. Electron energy conservation:

$$n_i^2 \frac{\langle\sigma v\rangle}{4} E_\alpha U_{\alpha e} - \left(\frac{5}{2} - \frac{1.53}{1.12} - \frac{T_i}{T_e}\right) \frac{n_e T_e}{\tau_c^e} - Q_{ei}$$

$$- Q_{e\alpha} - A_x\left(n_i + 4n_\alpha + \sum_{im} z_{im}^2 n_{im}\right) n_e T_e^{1/2} = 0 \quad (4)$$

It should be noted that the electron-alpha rethermalization term $Q_{e\alpha}$, is non-zero whereas the ion-alpha term, $Q_{i\alpha}$, is zero. This approximation is justified under CTR conditions with particle confinement times on the order of several seconds because the alpha slowing down time is short compared to the confinement time and the alpha particles fully thermalize.[13] Under these circumstances, it can be shown[14] that $T_\alpha \cong T_i$.

d. Confinement time:

$$\tau_c^i = \frac{C_o\, B_T^2\, r_p^2}{\Lambda^{3/2}\, q^2} \frac{T_e^{1/2}}{n_e} \tag{5}$$

where C_o, consistent with MKS units, is given by,

$$C_o = 0.24 \times 10^{23}$$

$$q = \frac{1}{\Lambda} \frac{B_T}{B_p} \tag{6}$$

For a more complete discussion of τ_c^i, see Appendix A.

III. Approximations, Self-Consistent Plasma Limitations and Equations

It has been pointed out in recent work[8] that to operate at steady state, β_p should be limited to $\sim\sqrt{\Lambda}$. Although it is theoretically possible to operate with $\beta_p > \sqrt{\Lambda}$ in a time dependent manner, steady state operation is not possible since reversed electric fields are required in the plasma, i.e., the electric field is not curl free. A system which operates with $\beta_p \approx \sqrt{\Lambda}$ is attractive since this limitation implies that the poloidal magnetic field can be maintained by currents in the plasma resulting directly from particle diffusion.[8,10,15] As such, externally induced electric fields are not required during steady state operation. Thus, the additional equation,

$$\beta_p = \sqrt{\Lambda} \tag{7}$$

is solved self-consistently with the remainder of the equation set.

The following restrictions and approximations are imposed to simplify the solution of the equations:

1. $\langle \sigma v \rangle_{DT}$, the velocity averaged cross section for fusion in a D-T system, is reasonably linear in the range 7 KeV $\leq T_i \leq$ 30 KeV. This is shown in figure 1 where data from Rose[2] is plotted. Therefore, we have represented the cross section by

$$\langle \sigma v \rangle = \gamma_1 + \gamma_2 T_i \ m^3\text{-sec}^{-1}, \ T_i \text{ in KeV} \tag{8}$$

$$\gamma_1 = -0.206 \times 10^{-21}$$

$$\gamma_2 = +0.314 \times 10^{-22}$$

and restricted solutions for T_i to be less than 30 KeV. In general, this has been the range of interest in previous studies.[1,2,3,4]

2. Alpha particles are produced with an energy of 3.5 MeV in the D-T fusion reaction. The fraction of the initial α-energy deposited in the electrons (or ions) as the α-particles slow down is dependent on T_e only.[16] If one assumes $n_e = n_i$, then the fraction of the α-energy deposited in the ions, $U_{\alpha i}$, can be represented by

$$U_{\alpha i} = 0.97 \times 10^{-1} + 0.13 \times 10^{-2}\, Te + 0.11 \times 10^{-2}\, Te^2 - 0.30 \times 10^{-4}\, Te^3, \ Te \text{ in KeV} . \tag{9}$$

This function results from a least squares fit to data calculated by Conn.[14]

3. The steady state α-particle balance equation,

$$\frac{dn_\alpha}{dt} = 0 = n_i^2 \frac{<\sigma V>}{4} - \frac{n_\alpha}{\tau_c^\alpha} \tag{10}$$

can be solved for n_α to yield

$$n_\alpha = (\tau_c^\alpha n_i)(n_i \frac{<\sigma V>}{4}) . \tag{11}$$

This can be written more conveniently as,

$$\frac{n_\alpha}{n_i} = \frac{\tau_c^\alpha}{\tau_c^i} (n_i \tau_c^i) \frac{<\sigma V>}{4} , \tag{12}$$

The ratio, n_α/n_i, can be written in terms of the fractional burnup. Recalling that the fractional burnup is given by

$$f_b = \frac{1}{1 + \frac{2}{n_i \tau_c^i <\sigma V>}} , \tag{13}$$

one finds for small f_b that

$$f_b \simeq n_i \tau_c^i \frac{<\sigma V>}{2} . \tag{14}$$

Also, the ratio of α-confinement time to ion confinement time is assumed to be given by

$$\frac{\tau_c^\alpha}{\tau_c^i} = \frac{1}{2} . \tag{15}$$

With these last two equations, the expression for n_α/n_i reduces to

$$\frac{n_\alpha}{n_i} = \frac{f_b}{4} \quad . \tag{16}$$

4. We will assume that at CTR conditions, the diffusion laws for a tokamak plasma will be governed by the banana regime of neoclassical theory.[8,9] In this case, the expression for the confinement time is[10]

$$\tau_c^i = \frac{C_0 T_e^{1/2}}{n_e} \; \frac{B_T^2 r_p^2}{q^2 A^{3/2}} \tag{17}$$

$C_0 = 0.24 \times 10^{23}$ (consistent with MKS units)

In the equation for τ_c^i, the tokamak machine parameters, B_T, the toroidal magnetic field strength, r_p, the plasma radius, q, the stability margin, and A, the aspect ratio, occur in the grouping

$$Mc = \frac{r_p^2 \; B_T^2}{q^2 \; A^{3/2}} \tag{18}$$

We have defined this parameter as the McAlees Number, Mc. It is possible to study the variation of solutions for T_e, T_i, $n\tau$, etc., as functions of Mc. It is not necessary to define each machine parameter separately.

5. To preserve macroscopic neutrality in the plasma, we require

$$\tau_c^e = \frac{\tau_c^i}{\sum_j z_j^2 \frac{n_j}{n_e}} = \frac{n_e \; \tau_c^i}{n_e + 2n_\alpha} \tag{19}$$

In addition, note that

$$\frac{n_e}{n_e + 2n_\alpha} = \frac{n_i + 2n_\alpha}{n_i + 4n_\alpha} = \frac{1 + f_b/2}{1 + f_b} \tag{20}$$

We have restricted $f_b << 1$, (in practice $f_b \leq 0.2$), which permits expansions such as

$$\frac{1 + f_b/2}{1 + f_b} \cong 1 - \frac{f_b}{2} + 0(f_b^2) \tag{21}$$

Under this condition, one finds that

$$\tau_c^e \cong \tau_c^i \left(1 - \frac{\tau_c^\alpha}{\tau_c^i} n_i \ \tau_c^i \ \frac{<\sigma V>}{2}\right) + 0(f_b^2) \ . \tag{22}$$

This approximation greatly simplifies the equations to be solved. Using the same criteria, one can express $n\tau$ as

$$n_i \ \tau_c^i \cong Mc \ C_0 \ T_e^{1/2}(1 - f_b/2) + 0(f_b^2) \ . \tag{23}$$

In solving the general equations, we have taken $f_b = 0.1$. Thus, solutions that result in $f_b = 10\%$ are exact relative to this approximation and, in other cases, the errors are small since we have restricted f_b to the interval $0 \leq f_b \leq 0.2$.

6. To permit solutions corresponding to an ion confinement time with functional dependence governed by neoclassical theory, (specifically, banana regime), but with a variable numerical coefficient, we write

$$\tau_c^i = \frac{C_0 \ T_e^{1/2} \ Mc}{n_e \ S} \tag{24}$$

where S is a "spoiling factor." S = 1 corresponds to the theoretical neoclassical value of τ_{banana}. Using S = 10, for example, would represent a confinement time that is a factor of 10 smaller than the theoretical value.

7. The bremsstrahlung radiation released by a plasma is strongly dependent on impurity ion density and the general formula for bremsstrahlung radiation losses is[17]

$$\omega_X = -\Lambda_X(n_i + 4n_\alpha + \sum_{im} z_{im}^2 n_{im})n_e T_e^{1/2} . \qquad (25)$$

With no impurities,

$$\omega_X = -\Lambda_X(n_i + 4n_\alpha)n_e T_e^{1/2} \qquad (26)$$

which is defined as the normal bremsstrahlung loss. The effect of impurities on the enhancement of bremsstrahlung can be accounted for most simply by writing

$$\omega_X = -\Lambda_X N_H(n_i + 4n_\alpha)n_e T_e^{1/2} \qquad (27)$$

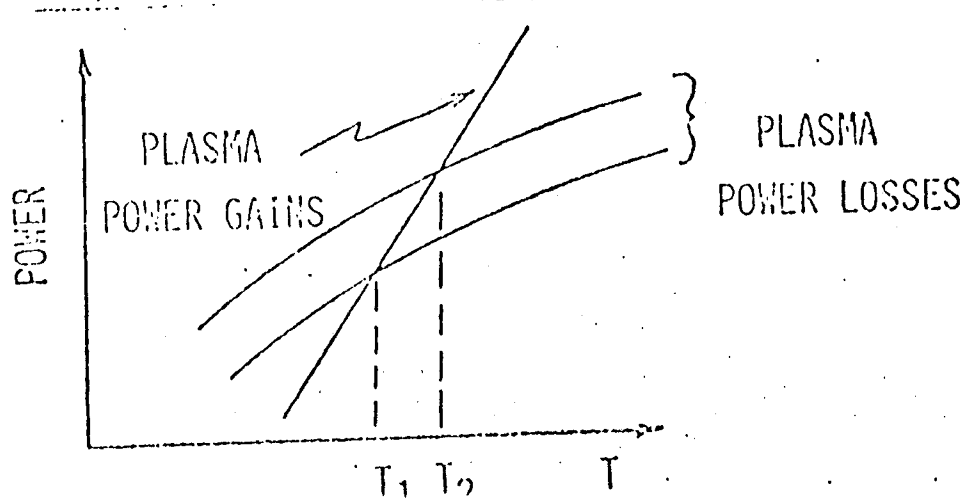
where the factor, N_H, is defined as the bremsstrahlung enhancement factor. It should be mentioned that plasma instabilities may limit the maximum impurity density permitted in the plasma.[18,19] Considering only the increased bremsstrahlung due to impurities and neglecting line and recombination radiation losses, as defined above, $N_H \leq 10$ is required.

Finally, the particle and energy balance equations, (1-4), must be solved self-consistently with equation (5) for τ_c^i and equation (7) for β_p. The importance of the functional dependence of τ_c^i on density and temperature and the impact of this dependence on the energetic stability of the system was discussed previously.[1,20,7] The numerical solution method is outlined in Appendix B.

IV. Results

To present the results in an orderly fashion, we have attempted to evolve a tokamak-like CTR plasma system of viable power, power density, size, etc., by solving the basic equations self-consistently for specified values of Mc, N_H and S. For normal bremsstrahlung ($N_H = 1$) and theoretical banana regime confinement time (S = 1), the solutions yield ion temperatures less than 7 KeV, except for Mc << 1. Yet, economically acceptable power densities demand $T_i \gtrsim 10$ KeV. Further, assuming $n_i = n_e$ and $T_e = T_i$, with a β_p limitation imposed on the system, it has been shown[2] that the optimum power density, and therefore the maximum total power for a machine of fixed size, occurs at 12 to 15 KeV. Figure 2 illustrates this point. In a self-consistent study such as this one, the plasma component number densities and temperatures are not equal but differences are small and we expect the optimum power density to occur in approximately the same temperature range as above.

To increase the steady state value of T_i, we note by inspection of the governing equations that one must increase the power loss from the plasma. If bremsstrahlung is enhanced, but radiation losses continue to vary as $\sqrt{T}$, then the power into the plasma must be increased to balance this increased loss. This can be done by operating at a higher ion temperature, as can be seen qualitatively by examining the figure below.



In a similar way, one can enhance particle diffusion by increasing the spoiling factor, S, to achieve the same purpose. Figure 3 shows the steady state ion temperature versus Mc for various N_H and S. The value of $S \equiv 27$ leads to a particle confinement time that is numerically equivalent to the energy confinement time suggested by Strelkov and Popkov.[21] Recall that solutions for T_i corresponding to the theoretical value of τ_c^i, (S = 1), resulted in unacceptably low power densities. Figure 4 is a plot of fractional burnup versus Mc. By examining figures 3 and 4 together, one is led to the conclusion that even with S = 27, the confinement time is too good in the sense that:

1. With normal bremsstrahlung, $N_H = 1$, plasma losses are balanced at $T_i < 10$ KeV, (except for extremely small Mc), and this leads to both low total power and very low power densities.
2. With bremsstrahlung enhanced by a factor of 10, fractional burnup exceeds 20% except for very small Mc.

Note further that physically S and N_H play two separate roles. This can be seen by examining figures 3, 4, and 5 simultaneously. Figures 4 and 5 illustrate that the effect of increasing S, or physically increasing particle diffusion, is to significantly decrease $n\tau$ and f_b. The same figures show that increasing N_H for a fixed S, or physically injecting impurities to increase bremsstrahlung losses, significantly increases f_b but has little effect on $n\tau$. This is reasonable since S is directly related to particle loss whereas N_H is related only to energy loss.

Thus far, it is clear that both N_H and S must be greater than one. To determine a more precise range of values for these parameters, the total power of the system must be considered. We shall fix, at this point, two conditions as constraints on the system:

$$T_B \text{ (blanket thickness)} = 2.25 \text{ m}$$ [22]

$$B_{TM} \text{ (toroidal field at the magnets)} = 85 \text{ kg}$$ [23].

Further, since power increases monotonically with increasing β_p and decreasing q, the following reasonable values have been taken for A and q:

$$A = 5$$

$$q = 1.5$$

(We shall note the effect of choosing $q \geq 2$ shortly.)

Finally, system power will also increase with increasing r_p. We have taken $r_p = 2.5$ m, $r_p/r_\omega = 0.9$ and want to determine if a viable power can be produced from this size machine. Note the important point that a value for the ion density cannot be arbitrarily selected. When the parameters B_T, A and q are fixed and the equation, $\beta_p = \sqrt{A}$, is solved self-consistently with the general equation set, the particle densities are determined as a consequence and are not independent variables.

The mean α-energy is not $3/2\ T_i$ even though the α-particles thermalize to T_i before leaving the plasma because the plasma also contains alpha particles that are in the process of slowing down. The mean α-energy, averaged over the alpha distribution function, can be estimated by computing the mean energy of one α-particle over its confinement time. Using the slowing down rate from the Fokker-Planck equation,[16] we find the approximate result that the mean α-energy is

$$\bar{E}_\alpha \approx \frac{600}{\tau_c^\alpha} \text{ (KeV)}. \tag{28}$$

Thus, the total alpha pressure is

$$p_\alpha = \frac{2 \cdot \overline{E_\alpha}}{3} + n_\alpha T_i .$$

With the parameters B_{TM} = 85 kg, A = 5, q = 1.5, r_p = 2.5m, the value of the McAlees number is 6.4. Figures 6 and 7 show average power, average power density and ion confinement time versus N_H for fixed values of S. The curves indicate that bremsstrahlung enhancement is effective in raising power for all $N_H \leq 10$ (the maximum enhancement considered). Thus, maximum bremsstrahlung enhancement, N_H = 10, should be used in the system.

For the same machine parameters and with N_H = 10, a value of S must be determined. Figures 8 and 9 show average power, average power density and ion confinement time versus S, with other parameters held constant. As opposed to the bremsstrahlung enhancement results, confinement spoiling becomes ineffective in raising the total power once S becomes greater than ~300. Since the effect of spoiling is to reduce τ, (i.e., enhance diffusion losses), this is reasonable. From the equation,

$$\tau_c^i = \frac{Mc\ Co\ Te^{1/2}}{n_e S} ,$$

it can be seen that $\tau_c^i \propto 1/S$ since Mc and Co are constant and $T_e^{1/2}$ and n_e are slowly varying compared to S. Thus, from a point of view of the total power of the system, a spoiling factor greater than 300 is not beneficial. Again, we expect this result from figure 2. By both enhancing bremsstrahlung and spoiling confinement, the system operating temperature has been raised into the optimum range. Since the maximum in figure 2 is reasonably broad, increasing T_i further has little effect. Ultimately, the power density will decrease with a continued increase in T_i. When fuel handling problems are considered, it is clear that a large τ^i is desirable. Since approximately the same system power can be provided for a range of τ_c^i values, the final choice of τ_c^i can be dictated by design considerations other than total power.

V. Conclusions

Within the limitations outlined at the beginning of this paper, the following conclusions can be drawn concerning a tokamak-like CTR plasma operating at steady state:

1. To attain viable power production, ($\langle P\rangle \geq 1000$ MW_t), both bremsstrahlung enhancement and confinement spoiling, relative to reference values, are required. See figure 10. (Calculation of $\langle P\rangle$ is discussed in Appendix B).
2. Bremsstrahlung enhancement is effective in raising power levels to the maximum acceptable level of enhancement, i.e., $N_H \simeq 10$. It should be noted at this point that, consistent with the reactor parameters given below, the required bremsstrahlung enhancement factor of 10 can be achieved by maintaining an impurity density in the plasma equal to 0.8 per cent of the ion density when the impurity used is $Z = 18$. This is based on work by Hopkins.[24]
3. Confinement spoiling is not effective in increasing power levels once τ_c^i (theoretical) is reduced by roughly 250 to 300.
4. From results of calculations analogous to those discussed herein, the steady state operating parameters listed in table 1 are suggested for a tokamak-like CTR plasma system limited by $\beta_p = \sqrt{A}$.
5. Higher powers can be attained only by increasing the size of the machine. Thus, $r_p > 2.5$m or $A > 5$ will result in more attractive power levels but at increased cost.
6. Operation with a stability margin, q, greater than 1.5 has a

Table 1

Mc	6.4	τ_c^i	12.2 sec
N_H	10.0	q	1.5
S	430.0	r_p	2.5 m
T_e	11.8 KeV	y	0.9
T_i	12.4 KeV	B_{TO}	5.1 T (on axis)
$<\sigma V>$	1.8×10^{-22} m^3-sec^{-1}	B_p	0.68 T
$n_i \tau_c^i$	1.16×10^{21} m^{-3}-sec	T_B	2.25 m
f_b	10.6%	β_p	$\sqrt{5}$
n_i	$0.95 \times 10^{20} m^{-3}$	<P>	1140 MW_T
n_e	$1.0 \times 10^{20} m^{-3}$	<P/V>	0.74 MW - m^{-3}
n_α	0.25×10^{19} m^{-3}	Neutron Wall Loading	0.53 MW/m^2
I	8.5×10^6 amps	γ- Wall Loading	0.10 MW/m^2
$\frac{n_{im}}{n_i} = .008$		Leakage Power to Divertor	36.60 MW
$Z_{im} = 18$			

severe effect on power. For example, when q is taken equal to 2, a machine with $r_p = 3.2m$ and $A = 6$ is required to attain a total power of 1000 MW_t.

For comparison with other work,[2,3,4] we have calculated <P> using $\beta_p = A$ for the same plasma parameters as in table 1. Since <P> scales like β_p^2, we expect, and find, about a factor of four increase in <P>. The effect on <P> of different limits on β_p is most clearly seen in figure 11. We plot <P> versus q for $\beta_p = A$, keeping the other plasma parameters fixed. As discussed earlier, if the bootstrap current materializes, it may not be possible to exceed $\beta_p = \sqrt{A}$.

In closing, it is worth noting several qualifications on these results which place them in a somewhat different context. First, it has been pointed out previously[1,20,7] but is worth noting again that steady state ion temperatures less than 40-50 KeV correspond to energetically unstable operating points (the plasma has a positive "temperature coefficient of reactivity") if the banana regime of neoclassical theory properly describes the diffusion processes. Therefore, operation at lower temperatures would require a system of feedback stabilization. Secondly, the enhancement of bremsstrahlung losses has no effect on the stability classification of this operating point because the functional dependence on temperatures of these losses is not affected. Thirdly, when β_p is limited by $\sqrt{A}$ (or any other limit for that matter), then as T_i increases past the 12-15 KeV range, we start losing ground in terms of power density and total power in a D-T plasma. That is, operation in the 20-40 KeV range will mean lower power densities compared to the lower temperature range. Therefore, larger machines would be required to achieve a given total power. Thus, even though synchrotron losses or altered forms of the confinement time formula (such as temperature dependencies characterizing Bohm diffusion or the plateau regime of neoclassical theory) make it possible to achieve a stable operating temperature in the 20-40 KeV range, such high temperatures will have adverse consequences for the economics of the machine.

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APPENDIX A

To derive an expression for τ_c^j, we assume the tokamak is an axisymmetric torus and follow Rutherford[6]. In the banana regime of neoclassical theory, the diffusion coefficient for electrons due to collisions with species j is given by,

$$D_{ej} = 1.6\langle\nu_{ej}\rangle \rho_e^2 q^2 A^{3/2} \qquad \text{(A-1)}$$

where, $\rho_e \equiv$ electron gyroradius

$\langle\nu_{ej}\rangle \equiv$ average electron-species j collision frequency

We also require the current of electrons due to scattering with species j to be equal to the current of species j due to scattering with electrons, i.e.,

$$Z_j \vec{J}_{je} = \vec{J}_{ej} \qquad \text{(A-2)}$$

From the continuity equation, for a given species,

$$\frac{\partial n_j}{\partial t} + \nabla \cdot \vec{J}_{je} = 0 \qquad \text{(A-3)}$$

$$\vec{J}_{je} = \frac{\vec{J}_{ej}}{Z_j} = \frac{-D_{ej}\nabla n_e}{Z_j} \qquad \text{(A-4)}$$

Therefore,

$$\frac{\partial n_j}{\partial t} = \frac{D_{ej}}{Z_j} \nabla^2 n_e \qquad \text{(A-5)}$$

Using the heuristic identifications,

$$\frac{\partial n_j}{\partial t} \rightarrow \frac{n_j}{\tau_c^j} \qquad (A\text{-}6)$$

$$\nabla^2 n_e \rightarrow \frac{n_e}{r_p^2} \qquad (A\text{-}7)$$

we find,

$$\tau_c^j = \frac{Z_j n_j r_p^2}{D_{ej}\, n_e} \qquad (A\text{-}8)$$

or,

$$\tau_c^i = \frac{n_i r_p^2}{D_{ei} n_e} . \qquad (A\text{-}9)$$

Finally, taking

$$\langle \nu_{ei} \rangle = \frac{4\pi e^4 n_i \ln\Lambda_{ei}}{m_e^{1/2} \, T_e^{3/2}} , \qquad (A\text{-}10)$$

$$\rho_e = \sqrt{2T_e/m_e}\ \frac{e}{m}/B_T \qquad (A\text{-}11)$$

$$D_{ei} = 1.6 \langle \nu_{ei} \rangle \rho_e^2 \ q^2 A^{3/2} \qquad (A\text{-}12)$$

we find,

$$\tau_c^i = \frac{T_e^{1/2}}{n_e} \frac{r_p^2 B_T^2}{q^2 A^{3/2}} C_0$$

$$\tau_c^i = C_0 \ M_c \ \frac{T_e^{1/2}}{n_e} \qquad (A\text{-}13)$$

where C_0 is a numerical constant.

It should be noted that ion-alpha collisions are neglected in this analysis.

In the same manner as above, the alpha particles can be considered a second ion component. Then,

$$\tau_c^\alpha = \frac{1}{Z_\alpha} C_0 M_c \frac{T_e^{1/2}}{n_e} = \tau_c^i / 2 \qquad \text{(A-14)}$$

For charge neutrality we require,

$$\frac{n_e}{\tau_c^e} = \sum_j \frac{Z_j n_j}{\tau_c^j} \qquad \text{(A-15)}$$

$$\tau_c^e = \frac{n_e}{\frac{n_i}{\tau_c^i} + \frac{2n_\alpha}{\tau_c^\alpha}} \qquad \text{(A-16)}$$

Combining this expression and,

$$n_e = n_i + 2n_\alpha \qquad \text{(A-17)}$$

yields the equation desired for τ_c^e,

$$\tau_c^e = \frac{n_e \tau_c^i}{2n_e - n_i} \qquad \text{(A-18)}$$

APPENDIX B

With the approximations discussed in the main text of this paper, the ion energy conservation equation can be written as,

$$n_i^2 \frac{\langle\sigma v\rangle}{4} E_\alpha U_{\alpha i} = \frac{7T_i}{2}\left(\frac{n_i}{\tau_c^i}\right) + \frac{3}{2} T_i \; n_i^2 \frac{\langle\sigma v\rangle}{2} \quad \text{(B-1)}$$

$$- \frac{3}{2} n_e \frac{(T_i - T_e) F n_i}{T_e^{3/2}} = 0 \quad \text{(B-2)}$$

where MKS units are used throughout except that E_α and T are in KeV. F is a numerical constant when $\ln \Lambda_{ei}$ is a constant. Defining,

$$n_i \tau_c^i = X_i \quad \text{(B-3)}$$

and noting that

$$n_e = n_i \left(1 + X_i \frac{\langle\sigma v\rangle}{4}\right), \quad \text{(B-4)}$$

$$\langle\sigma v\rangle = \gamma_1 + \gamma_2 T_i, \quad \text{(B-5)}$$

the ion equation can be written in theform,

$$\alpha_2 T_i^2 + \alpha_3 T_i - \alpha_4 = 0 \quad \text{(B-6)}$$

where,

$$\alpha_j = \alpha_j (T_e, M_c). \quad \text{(B-7)}$$

Thus, for a given electron temperature and McAlees number, there are two possible steady state values of T_i. However, only one of these values is physically acceptable and since every assumed T_e results in a (T_e, T_i) pair that satisfies the ion energy equation, the electron energy equation must be used to determine the (T_e, T_i) pair that self-consistently satisfies both the electron and the ion equations simultaneously. The electron equation is,

$$\frac{n_i^2 \langle\sigma v\rangle}{4} E_\alpha U_{\alpha e} - \left(\frac{5}{2} - \frac{1.53}{1.12} - \frac{T_i}{T_e}\right)\frac{n_e T_e}{\tau_c^e}$$

$$- \frac{3}{2} n_e \left(1 + 2.5 \frac{n_e}{n_i}\right)\left(\frac{T_e - T_i}{T_e^{3/2}}\right) F n_i$$

$$-\Lambda_x (n_i + Z_\alpha^2 n_\alpha) n_e T_e^{1/2} = 0 \quad . \tag{B-8}$$

We shall assume that $T_T = T_e$ to determine the numerical coefficient of $n_e T_e / \tau_c^e$. This introduces negligible errors in the analysis.

The numerical approach is : first, for a given M_c, determine the possible (T_e, T_i) solutions from the ion energy equation. Second, determine which (T_e, T_i) pair satisfies the electron energy equation. This is done iteratively by guessing T_e, solving for T_i and substituting these temperatures into the electron equation. One continues in this way until a self-consistent (T_e, T_i) pair is found. Following this, values of $\langle\sigma v\rangle$, f_b, and $n_i \tau_c^i$ are deduced directly.

To determine the particle number densities, the equations for plasma pressure and β_p,

$$\beta_p = \frac{p}{B_p^2/2\mu} = \sqrt{A} \tag{B-9}$$

$$p = n_i T_i + n_e T_e + n_\alpha T_i + \frac{2}{3} n_\alpha \bar{E}_\alpha \tag{B-10}$$

are solved simultaneously with the definition of the stability margin

$$q = \frac{1}{A}\frac{B_T}{B_p} \qquad \text{(B-11)}$$

to obtain a value for the steady state ion density, n_i. The electron and alpha densities, n_e and n_α, as well as the average power, $\langle P \rangle$, follow directly.

Finally, a reasonable assumption for the ion density profile is required. The ion flux in the radial direction in cylindrical geometry may be written as,

$$\Gamma = -D\,\frac{dn}{dr} \qquad \text{(B-12)}$$

Further, at steady state, ions must be supplied to the plasma by a source. Assuming the source can be represented by a constant,

$$\frac{d\Gamma}{dr} = S_o \qquad \text{(B-13)}$$

$$\Gamma = S_o r \qquad \text{(B-14)}$$

For neoclassical diffusion

$$D \propto n \qquad \text{(B-15)}$$

Then,

$$S_o r = -Kn\,\frac{dn}{dr} \qquad \text{(B-16)}$$

$$n^2 = n_0^2\,(1 - r^2/r_p^2) \qquad \text{(B-17)}$$

where the boundary condition used to evaluate the constant of integration is $n = 0$ at $r = r_p$ and n_0 is the peak density.

As a result of this density profile, the average power is found to be one-half of the peak power computed using n_0.

FIGURE CAPTIONS

1. $<\sigma v>$ versus T_i for the D-T reaction. The curve is approximately linear from 7 to 30 KeV.
2. The D-T reaction parameter as a function of T assuming $T_i = T_e$, $n_i = n_e$.
3. Equilibrium ion temperature versus machine number for various choices of bremsstrahlung enhancement and confinement time spoiling. For large M_c, diffusion losses become negligible and curves with the same N_H tend to the same asymptotic value of T_i.
4. Fractional burnup versus machine number. f_b increases with N_H but decreases with increasing S.
5. $n\tau$ versus machine number for various values of bremsstrahlung enhancement and diffusion spoiling.
6. Average total power versus enhancement of bremsstrahlung for fixed values of M_c, A, q, T_B, B_{Tm}.
7. Average power density and ion confinement time versus bremsstrahlung enhancement.
8. Average power versus confinement time spoiling for bremsstrahlung enhanced by a factor of 10.
9. Average power and ion confinement time versus confinement time spoiling factor for $N_H = 10$. Spoiling beyond factor of 400 over the theoretical value of τ_i has little effect on the power density.
10. Average power versus machine number for differnet values of N_H and S. This figure indicates the need for bremsstrahlung enhancement and confinement time spoiling to achieve reasonable power densities with the indicated values of q, A, and magnetic field strength.

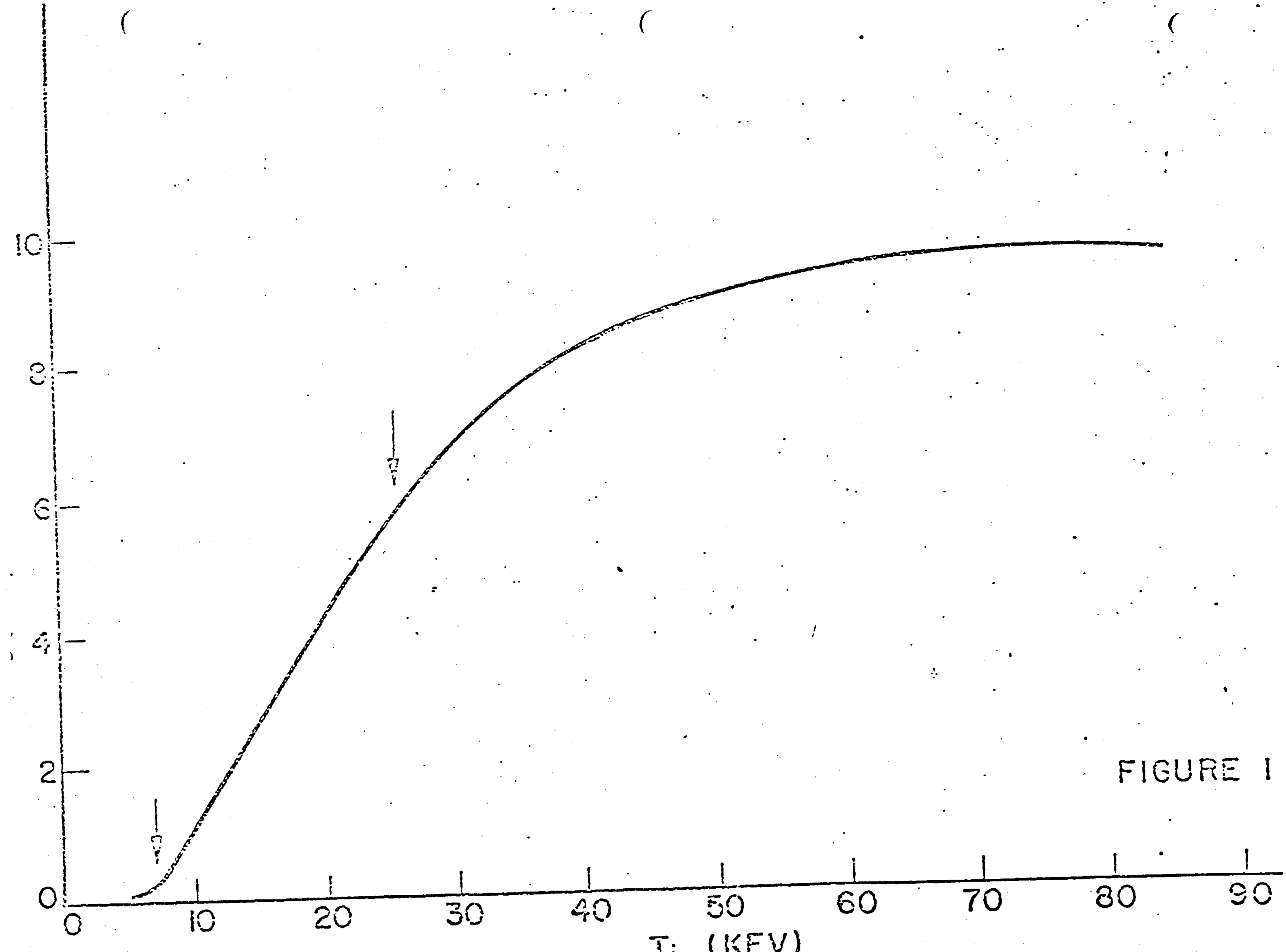
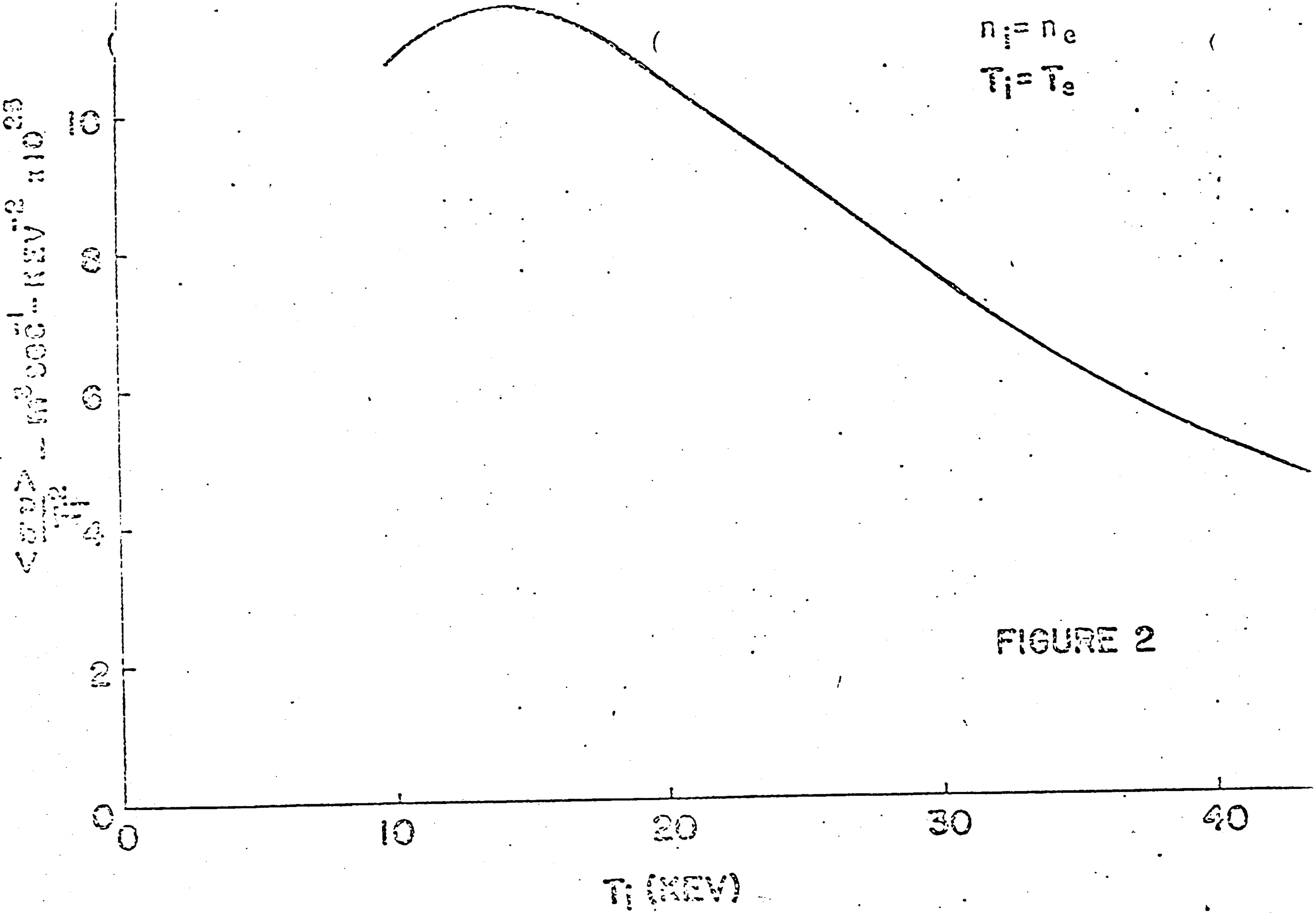


FIGURE I



$n_i = n_e$
$T_i = T_e$
$\langle \sigma v \rangle / T_i^2$ – m³ sec⁻¹ – KEV⁻² × 10²³
0
2
4
6
8
10
0
10
20
30
40
T_i (KEV)

FIGURE 2

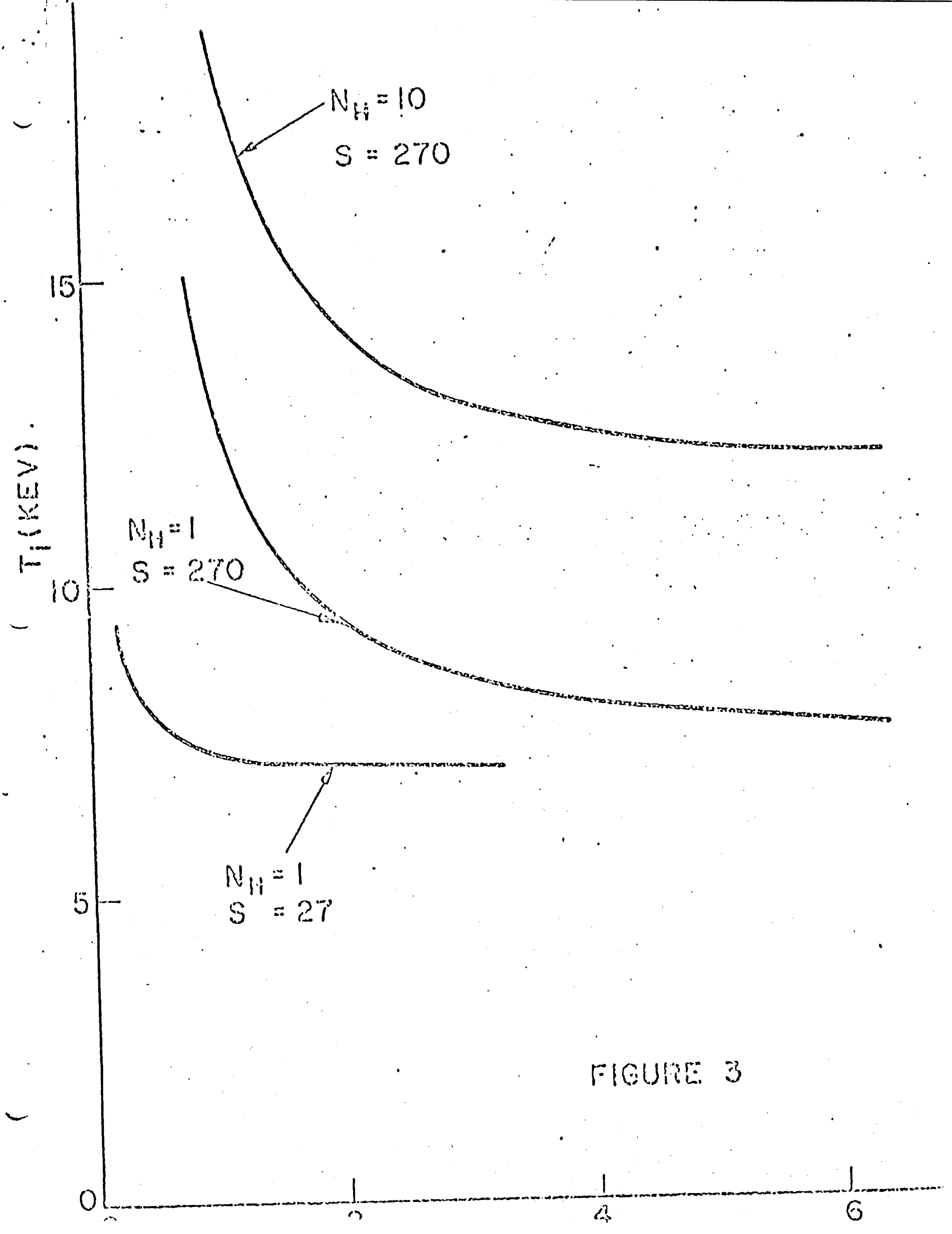


FIGURE 3

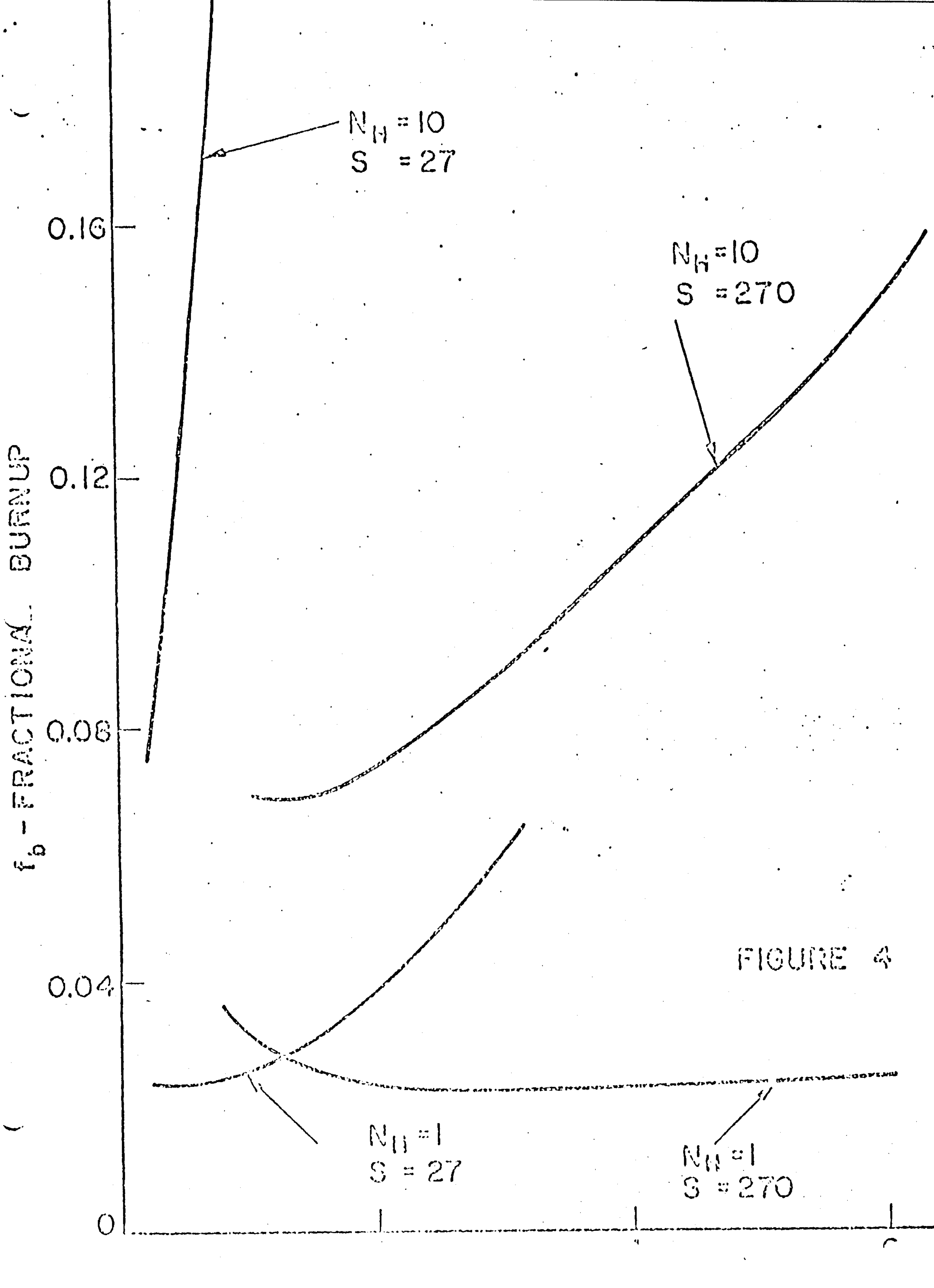
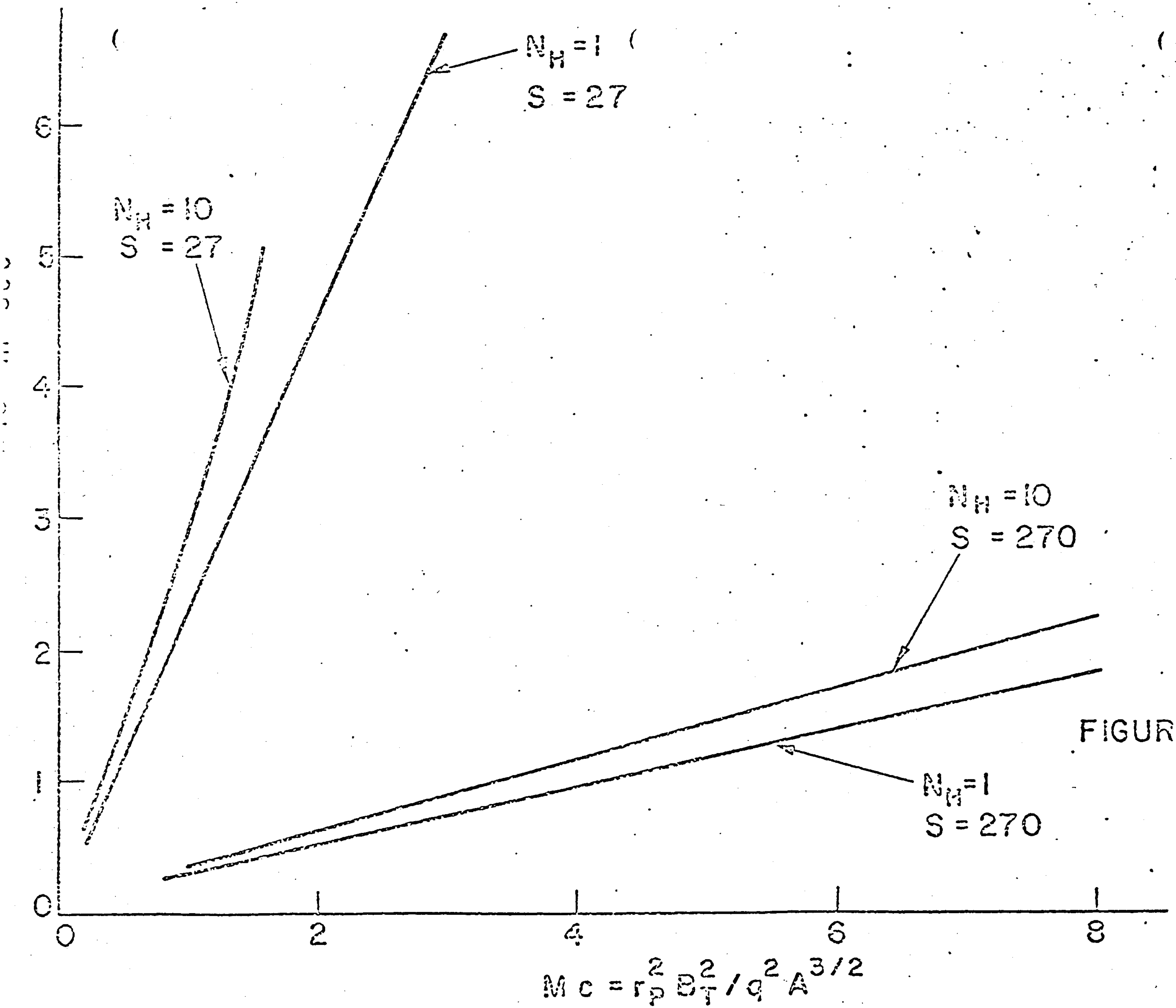
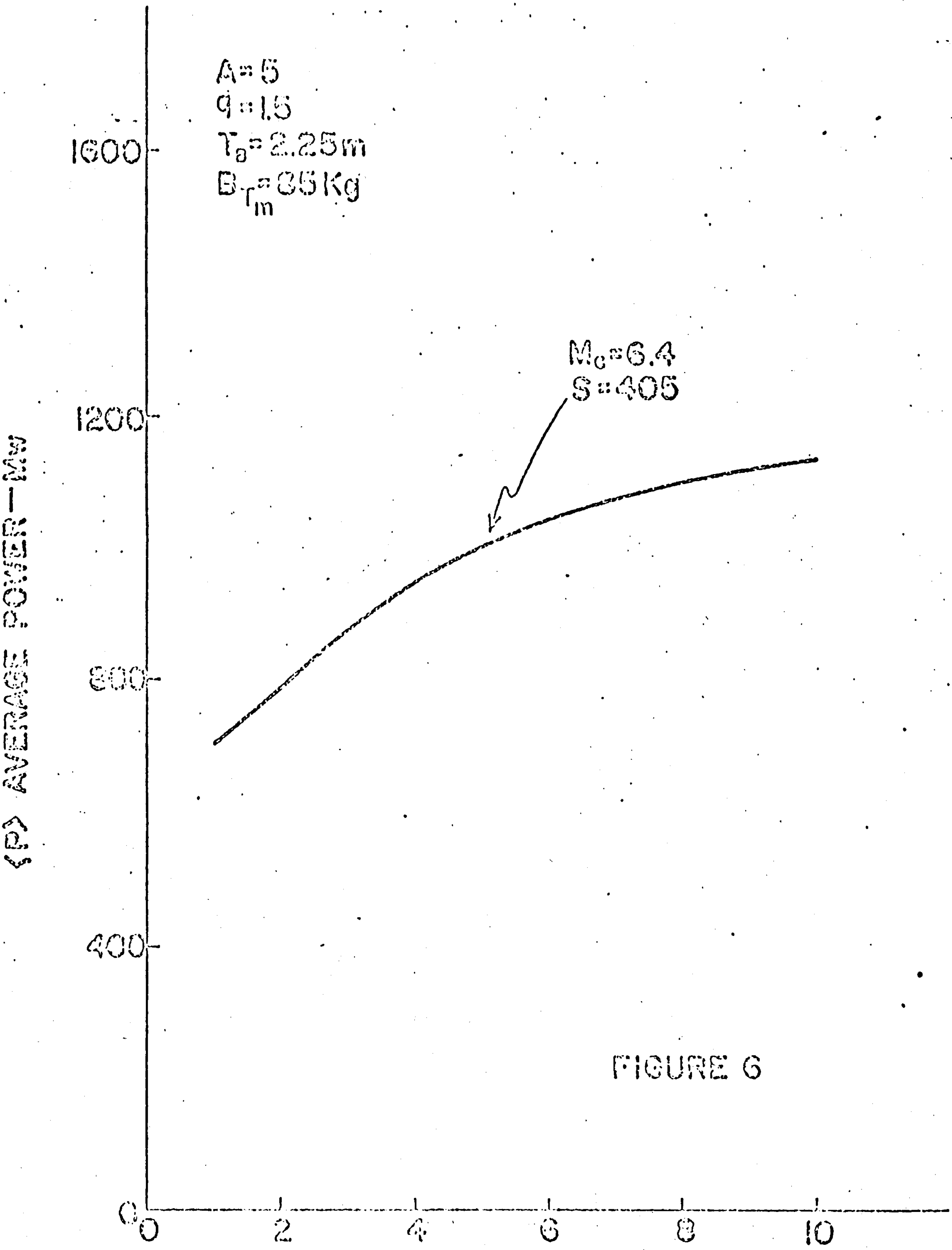

f_b - FRACTIONAL BURNUP
0.16
0.12
0.08
0.04
0
N_H = 10
S = 27
N_H = 10
S = 270
N_H = 1
S = 27
N_H = 1
S = 270

FIGURE 4



$N_H = 1$
$S = 27$
$N_H = 10$
$S = 27$
$N_H = 10$
$S = 270$
$N_H = 1$
$S = 270$
0
1
2
3
4
5
6
0
2
4
6
8
$Mc = r_p^2 B_T^2 / q^2 A^{3/2}$

FIGURE 5



A= 5
q = 1.5
T_a = 2.25 m
B_{T_m} = 35 Kg
M_c = 6.4
S = 405
⟨P⟩ AVERAGE POWER — Mw
1600
1200
800
400
0
0
2
4
6
8
10

FIGURE 6

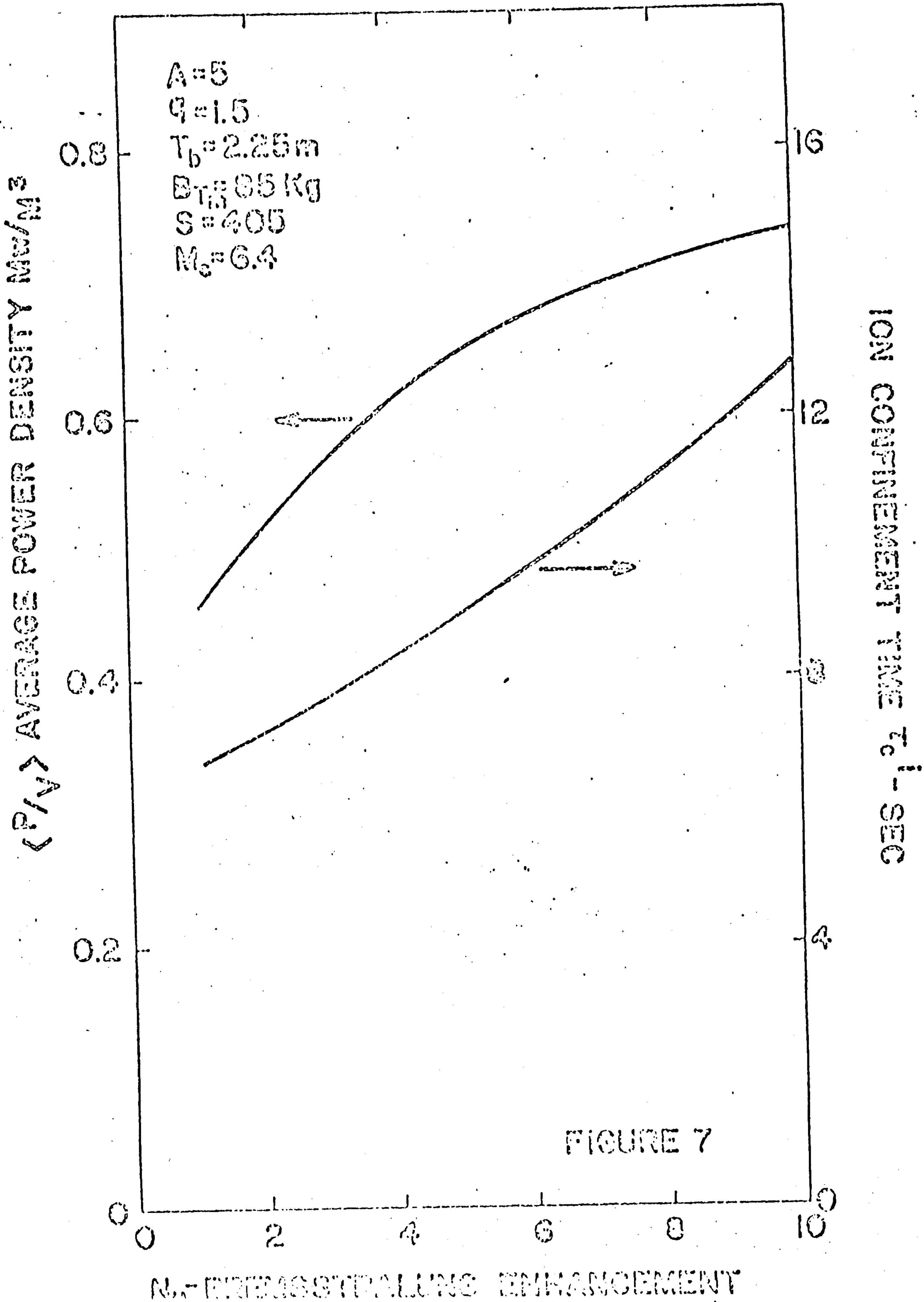
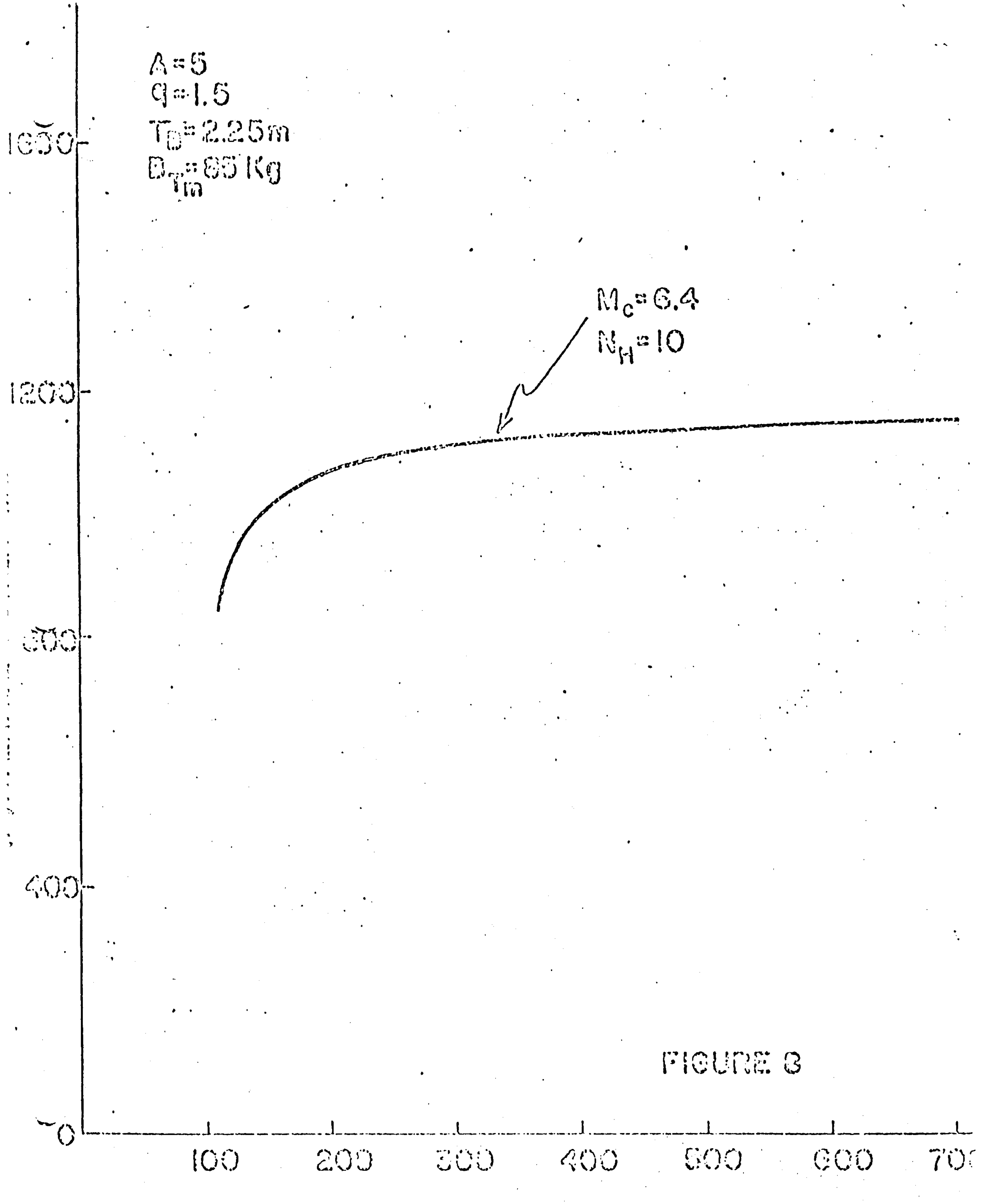
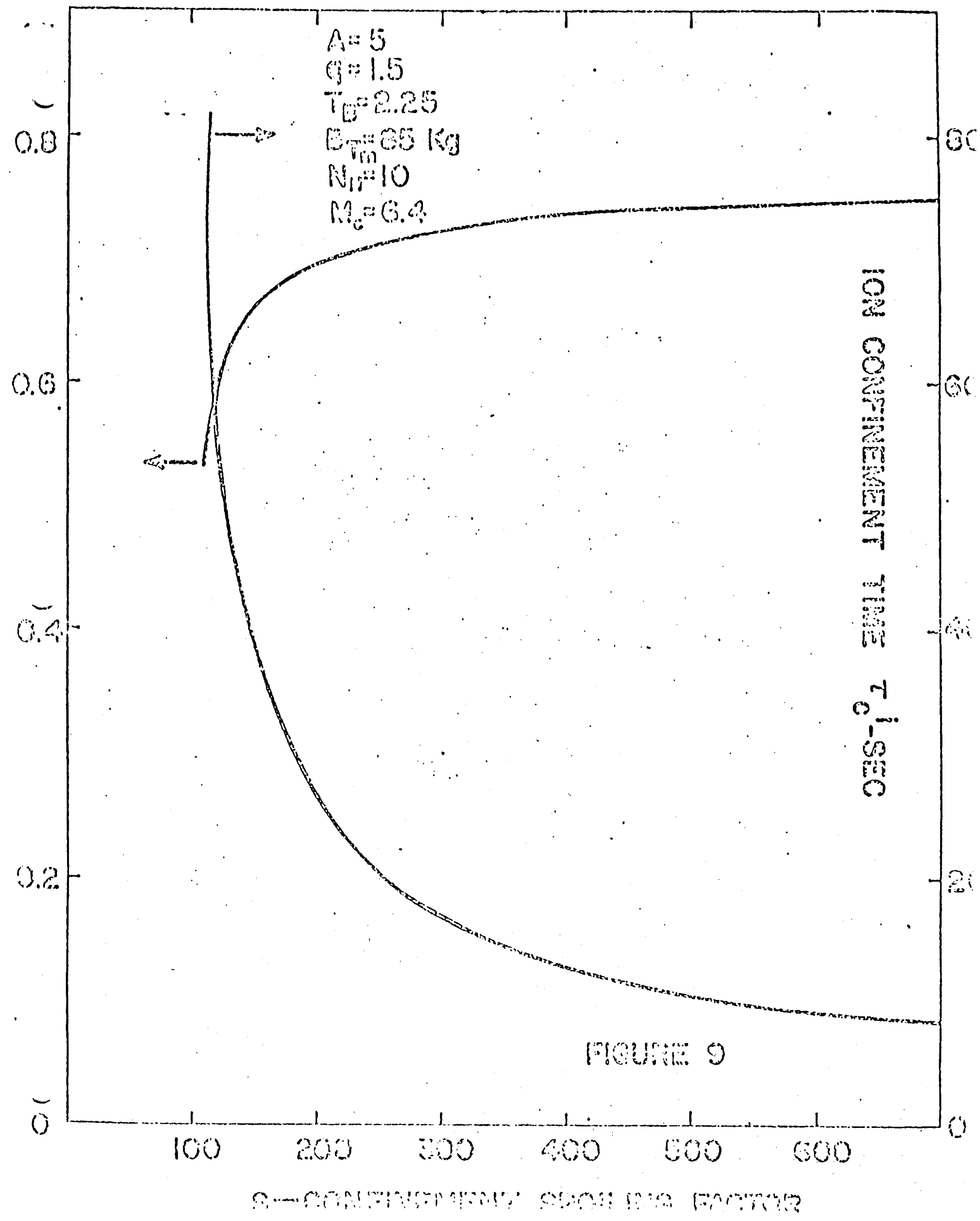

A=5
q=1.5
T_b=2.25m
B_{TR}=85 Kg
S=405
M_s=6.4
⟨P/V⟩ AVERAGE POWER DENSITY Mw/M³
ION CONFINEMENT TIME τ_c^i SEC
0.8
0.6
0.4
0.2
0
16
12
8
4
0
2
4
6
8
10

FIGURE 7



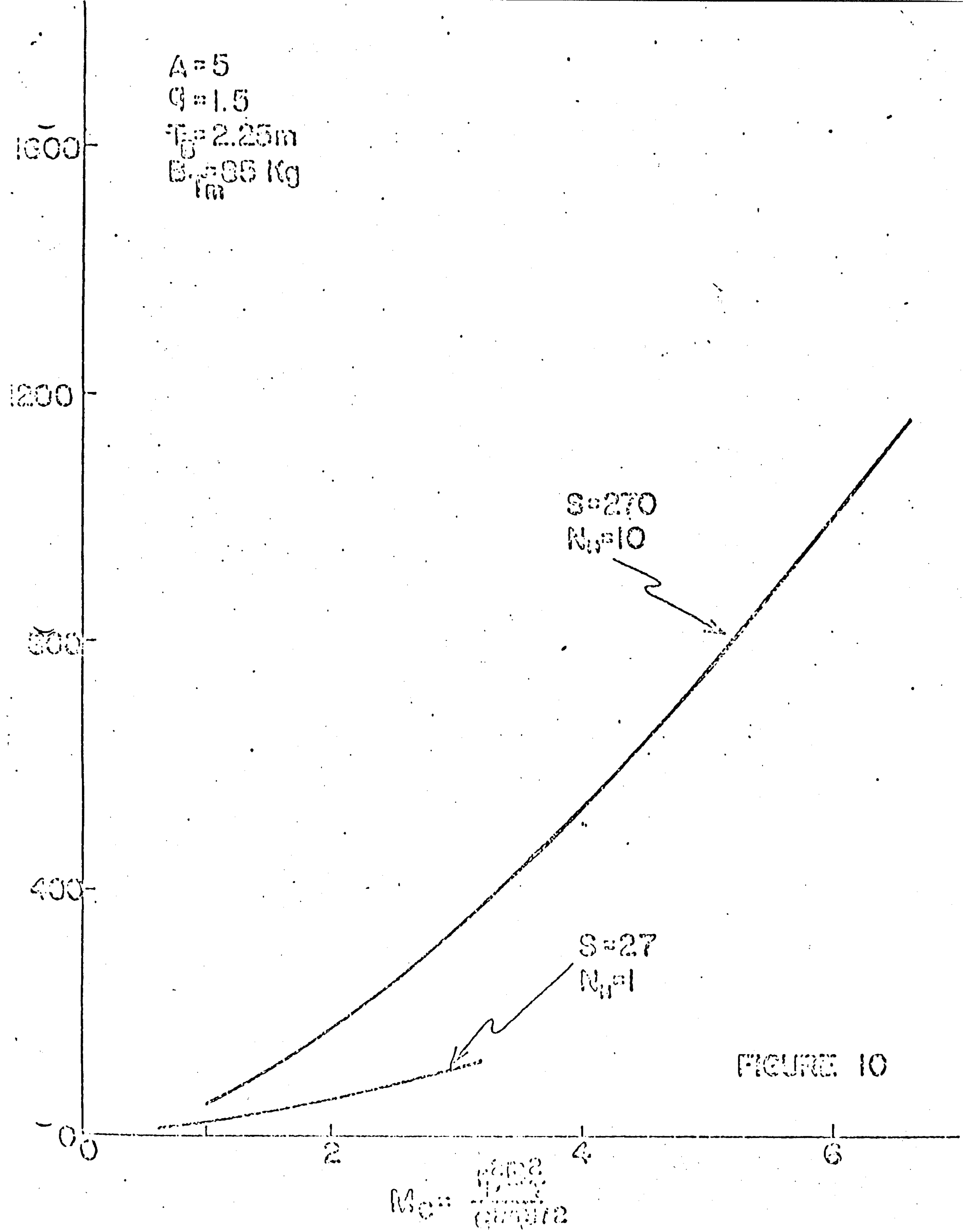
A=5
q=1.5
T_D=2.25m
B_Tm=85 Kg
M_c=6.4
N_H=10
1600
1200
800
400
0
100
200
300
400
500
600
700

FIGURE 8



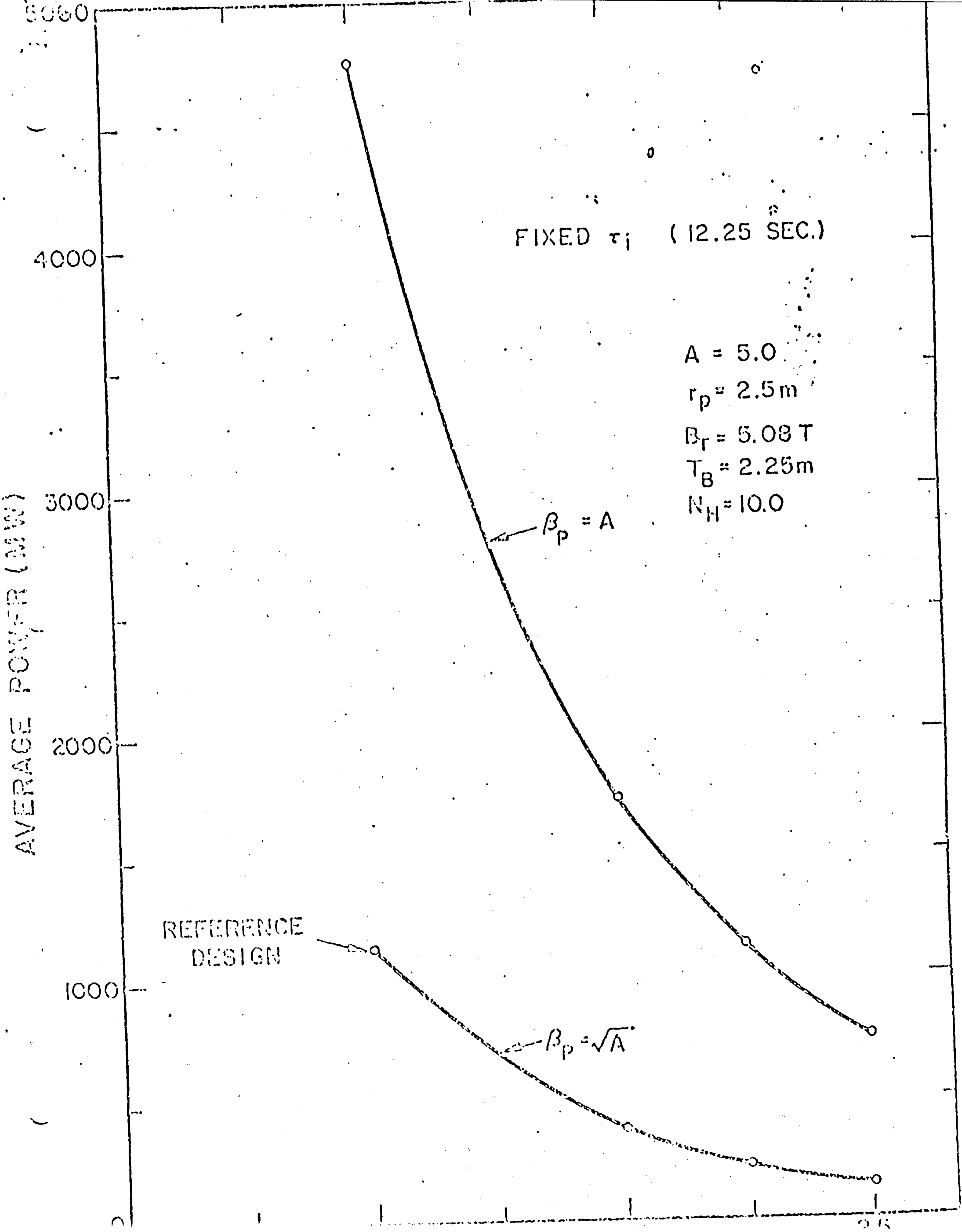
A= 5
q= 1.5
T_D= 2.25
B_{Tm}= 85 Kg
$N_{//}$= 10
M_c= 6.4
ION CONFINEMENT TIME τ_i SEC
0.8
0.6
0.4
0.2
0
100
200
300
400
500
600

FIGURE 9



A = 5
q = 1.5
r_0 = 2.25m
B_{fm} = 35 Kg
1600
1200
800
400
0
0
2
4
6
S = 270
N_H = 10
S = 27
N_H = 1

FIGURE 10



FIXED τi (12.25 SEC.)
A = 5.0
rp = 2.5 m
Br = 5.08 T
TB = 2.25 m
NH = 10.0
βp = A
REFERENCE DESIGN
βp = √A
AVERAGE POWER (MW)
4000
3000
2000
1000
0