



**On the Stability of the Flow of Thin Liquid
Lithium Films**

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Abstract

The stability of a thin layer of liquid lithium down an inclined surface is investigated. Stability limits and growth rates are calculated and compared with values for water. Applications to the design of a laser fusion reactor are discussed.

I. INTRODUCTION

A problem of considerable interest to the laser fusion program is the maintenance of a steadily flowing layer of liquid lithium on a metallic substrate. For example, it has been proposed as a means of dissipating blast energy in a laser fusion reactor (LFR), thus extending the first wall lifetime. In the "wetted wall" design⁽¹⁻²⁾, a 1 mm thick film of lithium would coat the wall of a spherical chamber; in the "suppressed ablation" design⁽³⁾ about 300 μm would coat the walls of a larger chamber. As an approximation to flow down a spherical surface where the velocity and thickness change with the local slope, we consider steady flow down a plane surface of constant slope. The stability characteristics of these flows are then expected to be similar provided that a periodic travelling wave disturbance does not experience a large change in slope in one period. The correspondence is closer in the suppressed ablation design where the inner wall is prismatic (to increase its area) and locally planar.

The theory of planar laminar flows is now well developed and in fair agreement with experiment. The classic "falling film" stability problem was first correctly solved by Benjamin⁽⁴⁾ for vertical flows and extended by Binnie⁽⁵⁾ to inclined flows, with important contributions by Yih.⁽⁶⁾ The falling film problem has been reviewed by Fulford⁽⁷⁾ and more recently by Hoffman and Munir⁽⁸⁾ in a study of the wetted wall concept. The present calculation was largely anticipated by Binnie in slightly different form for water, as was discovered after the present work was completed. Nevertheless, the present treatment is somewhat simpler and offers certain advantages of interpretation. In addition, Binnie's numerical work was limited to water, at very small angles of inclination. It turns out that lithium at 260°C and water at 20°C have almost identical kinematic viscosities, so that the stability limit for the growth

of surface waves is the same in both cases. Such detailed properties as growth or damping rates, however, will be seen to depend on the surface tension as well.

II. THE PRIMARY FLOW

The basic geometry is illustrated in Fig. 1. Steady Newtonian flow is maintained down a smooth plane surface. (It has to be verified that the liquid will wet the surface in question.) Incompressibility of the flow guarantees that the thickness h will remain constant along the surface to perhaps one part in 10^7 .

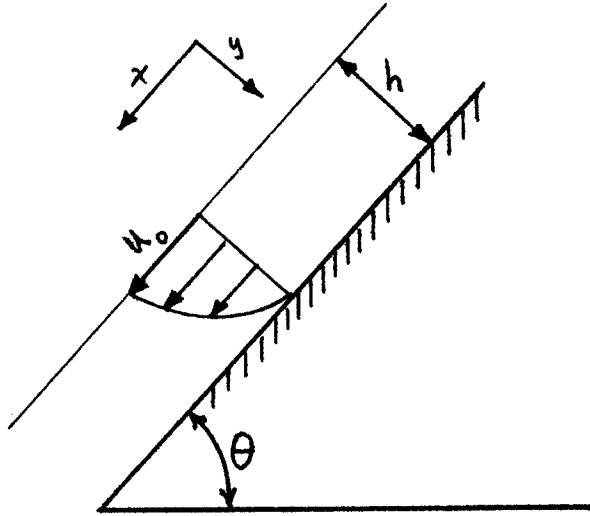


FIGURE 1

The film is subject to gravitational and viscous forces, pressure and tension. With the coordinate axes chosen as indicated in Fig. 1 and the origin located on the free surface, the Navier-Stokes equations reduce to

$$0 = -\frac{\partial P}{\partial y} + \rho g \cos\theta \quad (1)$$

$$0 = -\frac{\partial P}{\partial x} + \rho g \sin\theta + \mu \frac{d^2 u}{dy^2} \quad (2)$$

or

$$\frac{d^2 u}{dy^2} = -\frac{g}{\nu} \sin\theta, \quad (3)$$

where P is the pressure and $\nu = \mu/\rho$ is the kinematic viscosity.

The boundary conditions are the no-slip condition at the solid surface,

$$u = 0 \quad \text{at } y = h,$$

and that the shearing force vanish on the free surface;

$$u' = 0 \quad \text{at } y = 0.$$

Equation (3) then integrates to

$$u = \frac{g \sin \theta}{2\nu} (h^2 - y^2), \quad (4)$$

from which we read

$$\text{velocity at free surface} \quad u_0 = \frac{gh^2}{2\nu} \sin \theta \quad (5)$$

$$\text{average velocity} \quad \bar{u} = \frac{2}{3} u_0. \quad (6)$$

The Reynolds number is customarily defined in terms of the average velocity;

$$R = \frac{\bar{u} h}{\nu} = \frac{2u_0 h}{3\nu} = \frac{gh^3 \sin \theta}{3\nu^2}. \quad (7)$$

This defines the primary flow in terms of θ, ν and h or R . Thus,

$$h = \left(\frac{3\nu^2 R}{g \sin \theta} \right)^{1/3} \quad (8)$$

$$u_0 = \frac{1}{2} (9g\nu R^2 \sin \theta)^{1/3}. \quad (9)$$

As a specific example, consider lithium at 500°F (260°C) flowing down a 30° incline. At this temperature, $\mu = 0.00475$ poise, $\rho = 0.50 \text{ gm/cm}^3$, so that $\nu = 0.0095$. Eq. (5) then gives $\bar{u} = 1.77 \times 10^4 h^2 \text{ cm/sec}$.

$h(\mu\text{m})$	$\bar{u}(\text{cm/sec})$	R
100	1.72	1.8
300	15.5	49
1000	172	1800 (turbulent)

Table 1 Flow Parameters for Different Film Thicknesses

III. STABILITY OF THE FLOW

In general, the character of the flow passes through several regimes as the Reynolds number is increased; smooth laminar, wavy laminar, a large, constant-amplitude phase, and finally a fully turbulent randomized regime. Here we shall be concerned only with the transition from the smooth laminar flow to the small-amplitude wavy laminar regime. This problem has been thoroughly treated by Benjamin⁽⁴⁾ and Yih.⁽⁶⁾ Both of these authors obtain stability limits and growth rates via linear perturbation theory. Nonlinear saturation effects have been investigated analytically⁽⁹⁾ and by numerical solution of the Navier-Stokes equation.⁽¹⁰⁾ A temperature gradient normal to the surface would effectively make viscosity a function of film thickness, resulting in a non-parabolic velocity profile, which could conceivably affect the stability of the flow. Another important effect in the 100-1000 μm thickness range is the formation and spreading of dry areas.⁽¹¹⁾ In this respect, it should be noted that the experiments of Binnie⁽¹²⁾ were successfully carried out for a 100 μm film of water on glass.

Following Benjamin, we assume a periodic traveling-wave perturbation of the free surface of the form

$$\Delta y = A \exp[i\alpha(x-ct)] \quad (10)$$

as depicted in Fig. 2. Here x and Δy are measured in units of h and time is measured in units of h/u_0 , so that α and c are dimensionless.

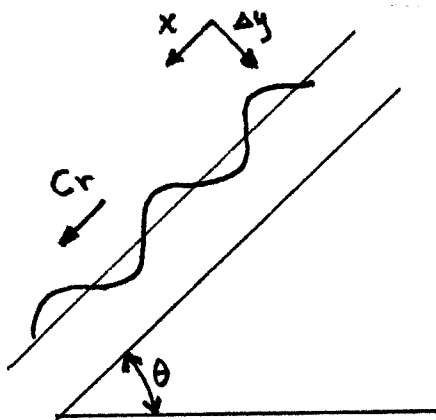


FIGURE 2

Using eq. (10) in the full set of Navier-Stokes equations and linearizing, with appropriate boundary conditions at the free surface, Benjamin obtains the following growth factor for wave lengths much greater than h ;

$$\Delta y \sim e^{\alpha c_i t} \quad (11)$$

Writing $c = c_r + ic_i$, he finds $c_r = 2$ and

$$c_i = \frac{1}{2} R \left(\frac{8}{5} \alpha - \frac{\alpha^3 \Gamma}{hu_0^2} - \frac{\alpha hg}{u_0^2} \cos\theta \right), \quad (12)$$

where Γ is the kinematic surface tension, in cm^3/sec^2 . The growth rate is $\gamma = \alpha c_i$, or

$$\gamma = \frac{2\alpha^2}{3\sin\theta} \left[\left(\frac{6}{5} R \sin\theta - \cos\theta \right) - \frac{\Gamma}{gh^2} \alpha^2 \right]. \quad (13)$$

It follows that the surface is unstable when

$$\alpha^2 < \alpha^{*2} = \frac{gh^2}{\Gamma} \left(\frac{6}{5} R \sin\theta - \cos\theta \right). \quad (14)$$

That is, the surface is stable for shorter wavelengths;

$$\lambda < \lambda^* \Rightarrow \text{stable},$$

where $\lambda^* = 2\pi h/\alpha^*$.

The surface is stable to disturbances of all wavelengths $\gg h$ when

$$\boxed{\frac{6}{5} R \tan\theta < 1.} \quad (15)$$

Note that the stability limit (15) is independent of the surface tension Γ .

Equation (15) may be written

$$h^3 < h^{*3} = \frac{5v^2}{2g \sin\theta \tan\theta}, \quad (16)$$

which has the solution

$$\cos\theta = \sqrt{1 + \frac{B^2}{h^6}} - \frac{B}{h^3}. \quad (17)$$

where

$$B = \frac{5v^2}{4g} = 1.27 \times 10^{-7}$$

for lithium at 260°C or H₂O at 20°C. The neutral stability curve as defined by eq. (16) is plotted in Fig. 3, along with the corresponding values of u_0 and R. Thus, a maximum thickness of 300 μm of Li or H₂O may be stably flowed at $\theta = 5^\circ$, but less than 100 μm at $\theta = 30^\circ$. On the other hand, an examination of eq. (13) shows that vertical flows ($\theta = 90^\circ$) are always unstable. Also note that inverted flows ($\theta > 90^\circ$) are possible but always unstable. Unfortunately, as Fig. 3 shows, the (stable) flow rates are rather low, unless h is quite large. However, since $R \sim h^3$ one rapidly approaches turbulent flow. Given a LFR repetition rate of 10/sec., it is apparent that only very modest flow rates can be stably achieved between pulses.

At fixed θ , the curves of γ vs λ have the general appearance depicted in Fig. 4. When $h < h^*$, all modes are damped, and when $h > h^*$, all wavelengths greater than λ^* are amplified, the growth rate reaching a maximum of

$$\gamma_{\max} = \frac{gh^2}{6\Gamma \sin\theta} \left(\frac{6}{5} R \sin\theta - \cos\theta \right)^2 \quad (18)$$

when

$$\alpha_{\max}^2 = \frac{1}{2} \alpha^{*2} \quad (18a)$$

Using eq. (8), h may be eliminated from eq. (14), yielding an expression for the stability limit solely in terms of the physical properties of the liquid, R and θ ;

$$\alpha^{*2} = \frac{1}{\Gamma} \left(\frac{3g^{1/2} v^2 R}{\sin\theta} \right)^{2/3} \left(\frac{6}{5} R \sin\theta - \cos\theta \right). \quad (19)$$

However, it is more convenient to work with the dimensional wave number $k^* = \alpha^*/h$, which takes the simpler form

$$\boxed{k^{*2} = \frac{g}{\Gamma} \left(\frac{6}{5} R \sin\theta - \cos\theta \right).} \quad (20)$$

The dimensional growth rate of the most unstable mode is

$$\frac{u_0}{h} \gamma_{\max} = \frac{\Gamma h^3 k_{\max}^4}{3\nu} \text{ sec}^{-1}. \quad (21)$$

Equations (18) and (19) reduce to Benjamin's results for vertical flows.

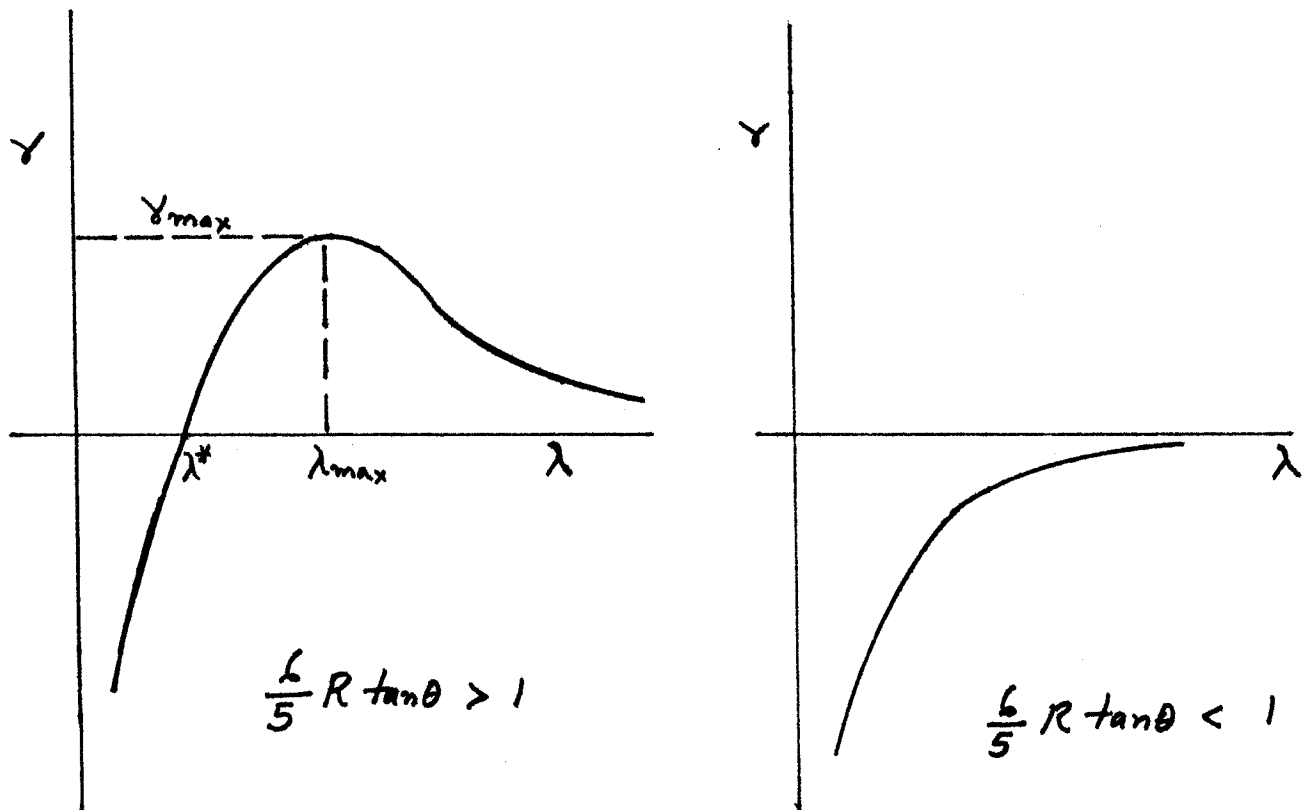


Figure 4

IV. NUMERICAL RESULTS

Consider a one-meter length of lithium, for which $\Gamma = 766 \text{ cm}^3 \text{ sec}^{-2}$. The growth exponent in traversing this distance is

$$G = \frac{u_0}{h} \gamma t = \frac{u_0}{h} \gamma \left(\frac{100}{2u_0} \right), \quad (22)$$

since the surface wave propagates at $2u_0$. The exponent G is plotted vs wavelength in Figs. 5-6 for $\theta = 5^\circ$ and 30° , resp., and a few different thicknesses in each case. Note the tendency to stability at longer and longer wavelengths as θ is decreased, as predicted by eq. (14). The maximum growth, given by using eq. (18) in eq. (22), is plotted vs h in Fig. 7 for various angles of inclination. In general, it is seen that thicker films will flow stably at small angles of inclination, all flows being unstable at $\theta = 90^\circ$.

As was pointed out previously, the stability limit is independent of surface tension. However, the growth rate is strongly affected, as may be seen by comparing Fig. 8, where G is shown vs λ for water at 20°C ($\Gamma = 72.9 \text{ cm}^3 \text{ sec}^{-2}$ and $\nu = 0.01$) with Fig. 6 for lithium. The maximum growth rate is seen to be about 10 times larger and the range of stable modes decreased. More importantly, in the stable regime a given disturbance is much more strongly damped in lithium.

An interesting sidelight emerged in the process of generating the data for Figs. 5-6. Namely, as one continues to reduce h in the stable regime, the damping $|G|$ in a fixed distance reaches a maximum over a range of wavelengths. For example, in Fig. 5 the curve for $h = 100 \text{ } \mu\text{m}$ is sandwiched between the curves for $h = 200 \text{ } \mu\text{m}$ and $300 \text{ } \mu\text{m}$. Thus, there exists an optimum thickness for maximum stability at fixed wavelengths, given by setting

$$\left(\frac{\partial G}{\partial h} \right)_\lambda = 0, \quad (23)$$

which yields

$$\tilde{h}^3 = \frac{5\nu^2(\cos\theta + \frac{4\pi^2\Gamma}{g\lambda^2})}{8g\sin^2\theta} . \quad (24)$$

For $\theta = 5^\circ$ and $\lambda = 16$ cm we find $\tilde{h} = 198 \mu\text{m}$, in good agreement with Fig. 5.

V. SATURATION OF WAVY LAMINAR FLOW

Both theory and experiment verify the existence of finite-amplitude equilibrium waves for moderate Reynolds numbers. Lee's theory⁽⁹⁾ is a rigorous treatment of a nonlinear analysis first carried out by Kapitsa. The result is that the most unstable mode given by eq. (18a) tends to an equilibrium amplitude

$$A_{\text{eq}} = \sqrt{3 N_{\text{We}}} h, \quad (25)$$

where the Weber number is given by

$$N_{\text{We}} = \frac{h\bar{u}^2}{\Gamma}. \quad (26)$$

The wave speed becomes

$$C_r = \frac{2u_0}{1 + \frac{3}{2} A_{\text{eq}}} . \quad (27)$$

In the case of 1mm of Li flowing at 30° , the Reynolds number is too large for the theory to apply (1800) and we find $A_{\text{eq}}/h = 3$. When $h = 100 \mu\text{m}$, however, we find $N_{\text{We}} = 3.9 \times 10^{-5}$ and $A_{\text{eq}} = 0.01 h = 1 \mu\text{m}$, a very small amplitude.

VI. DISCUSSION

We have calculated stability limits and growth rates for travelling wave disturbances on thin films flowing down a smooth plane surface. The results are relevant to lithium flow on the inside of a laser fusion reactor vessel in the laminar flow regime ($R \approx 300$), provided that the ratio of film thickness to cavity radius is small enough that the flow is effectively planar over one growth period. It should be noted that the Reynolds number of 250,000 given in Reference 3 is incorrect.⁽¹³⁾ For $\bar{u} = 15$ cm/sec and $h = 1-2$ mm Eq. (7) gives $R = 150-300$, still in the laminar regime. Thus, the one-dimensional treatment of Reference 3 may not be sufficient to characterize the primary flow.

On the other hand, a fully rigorous calculation of two-dimensional Navier-Stokes flow on a sphere with a free surface is a difficult unsolved analytic or numerical problem. Thus we must rely on approximate treatments, at least for the time being.

With these caveats in mind, the results of our analysis show that inverted flow ($\theta < 90^\circ$) is possible but always unstable. Further, Fig. 7 suggests that lithium films thicker than about 200 μm will be unstable in the range $90^\circ < \theta < 180^\circ$. (Actually $\theta_{\text{max}} < 180^\circ$ to allow room for an exit port). This rather discouraging prognosis may be alleviated by two physical stabilizing mechanisms. First, a flow that is locally unstable in the upper half-sphere may be stable in the lower half. Thus, a wave born in the upper half travelling (necessarily) downward will grow only until it reaches a stable region, where it is damped away. Whether

such a wave fatally disrupts the flow depends on the amount of amplification taking place before damping can act. This convective damping may be aided by nonlinear saturation of the wave amplitude. Indeed, the results of section V suggest that wave growth may saturate at a moderate value, due to the very large surface tension of liquid lithium. A satisfactory treatment of these competing effects awaits a (probably numerical) description of laminar viscous flow on a spherical surface.

ACKNOWLEDGEMENTS

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Figure Captions

- Fig. 1. Primary flow geometry.
- Fig. 2. Perturbation of free surface.
- Fig. 3. Neutral stability curve, along with Reynolds number and surface velocity (cm/sec) for water at 20°C or lithium at 260°C.
- Fig. 4. Schematic of growth rate as a function of wavelength.
- Fig. 5. Nondimensional growth (exponent) versus wavelength for Li flowing down a 5° slope.
- Fig. 6. Nondimensional growth (exponent) versus wavelength for Li flowing down a 30° slope.
- Fig. 7. Maximum growth versus film thickness for Li, various slopes.
- Fig. 8. Nondimensional growth versus wavelength for water flowing down a 30° slope.

Table 1. Flow Parameters for Different Film Thicknesses

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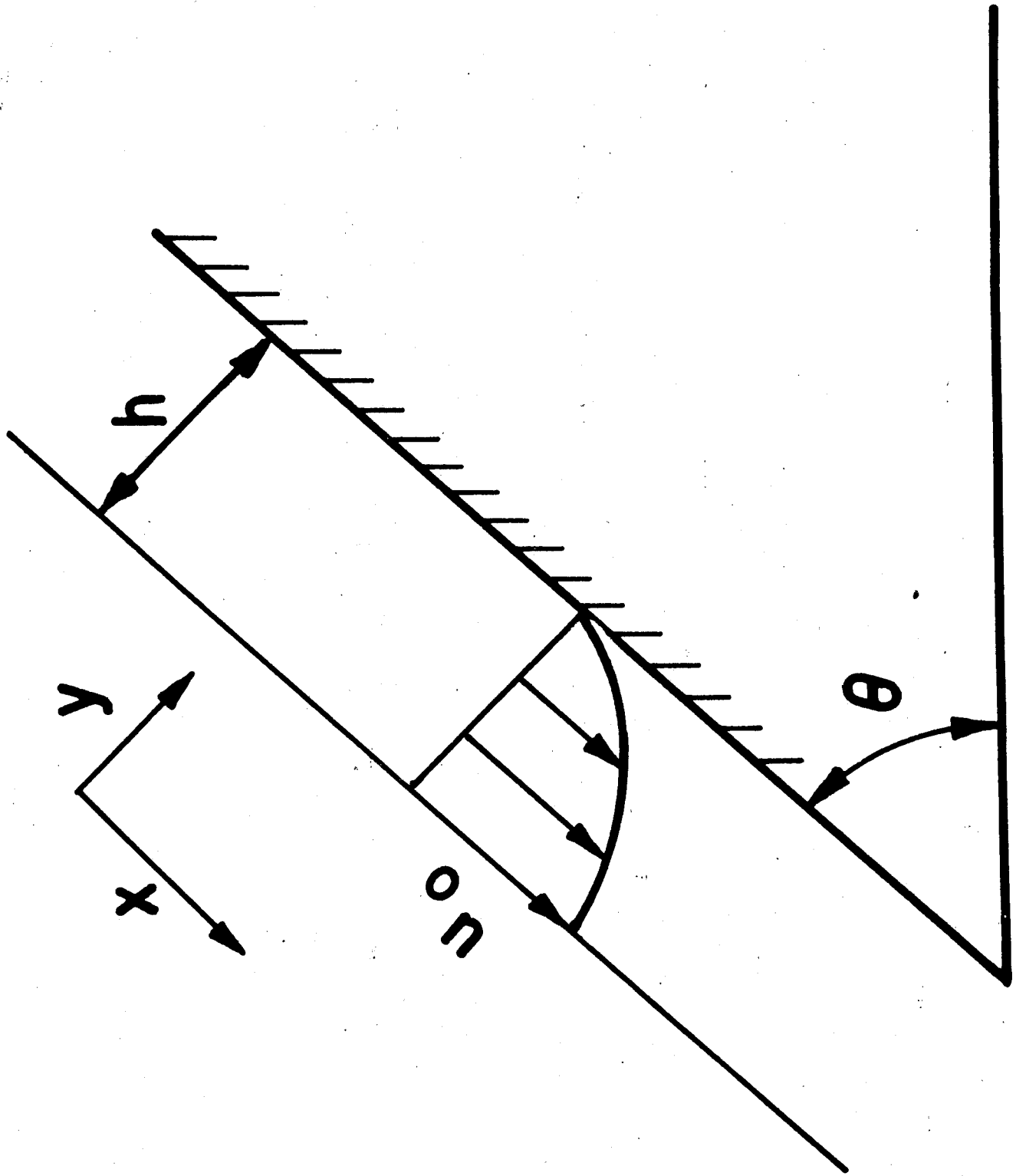


Figure 1

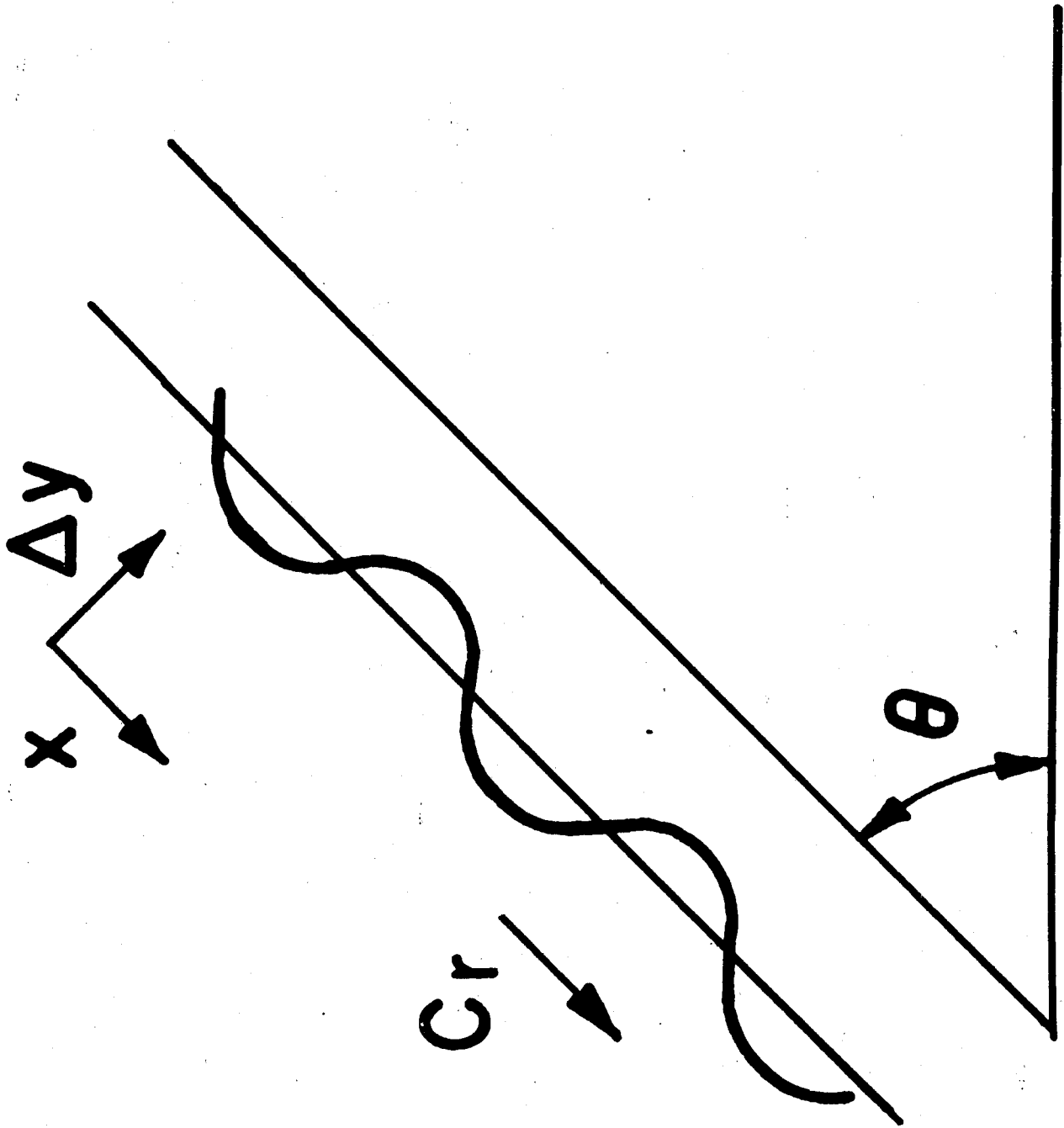


Figure 2

STABILITY LIMIT

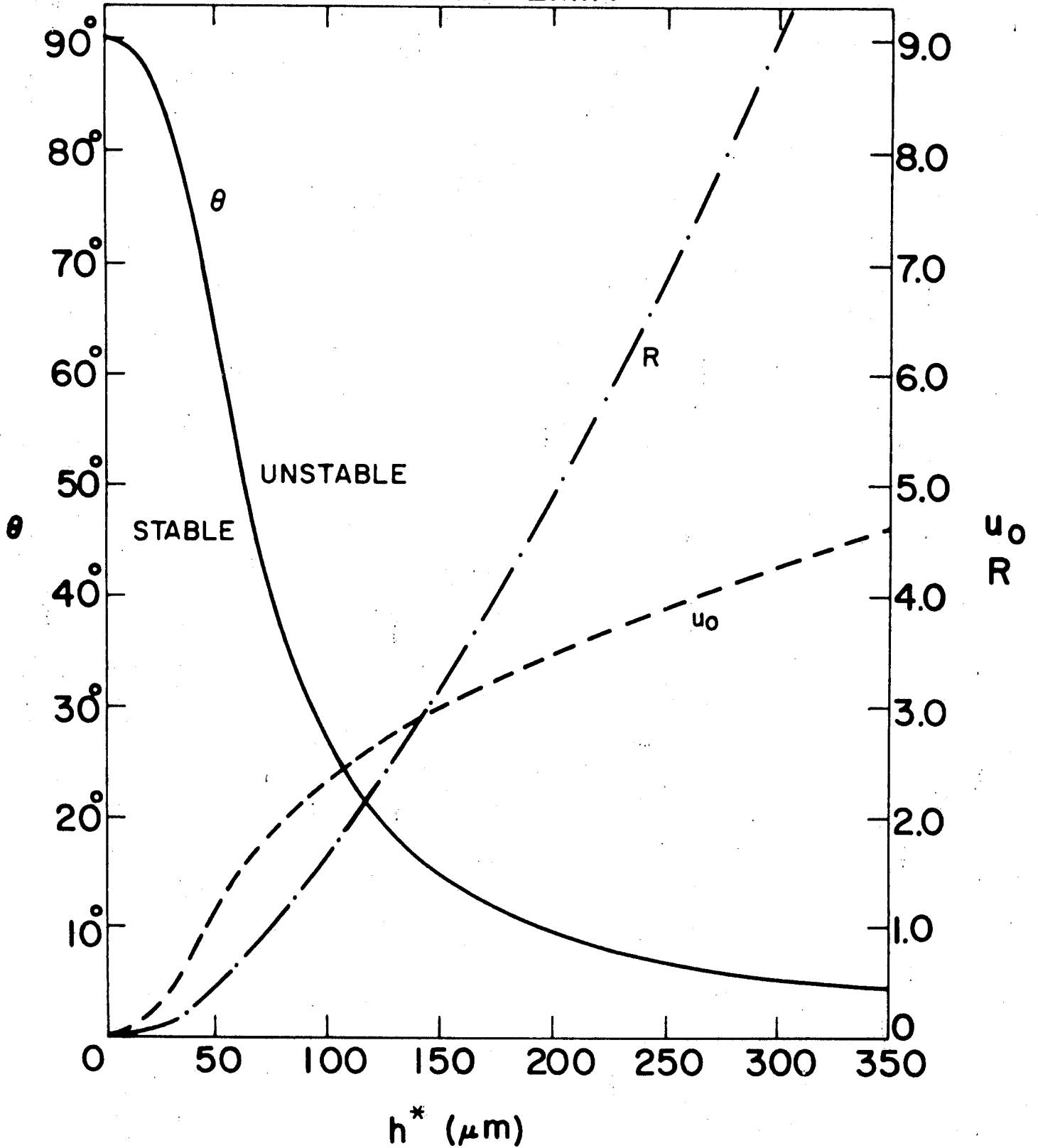


Figure 3

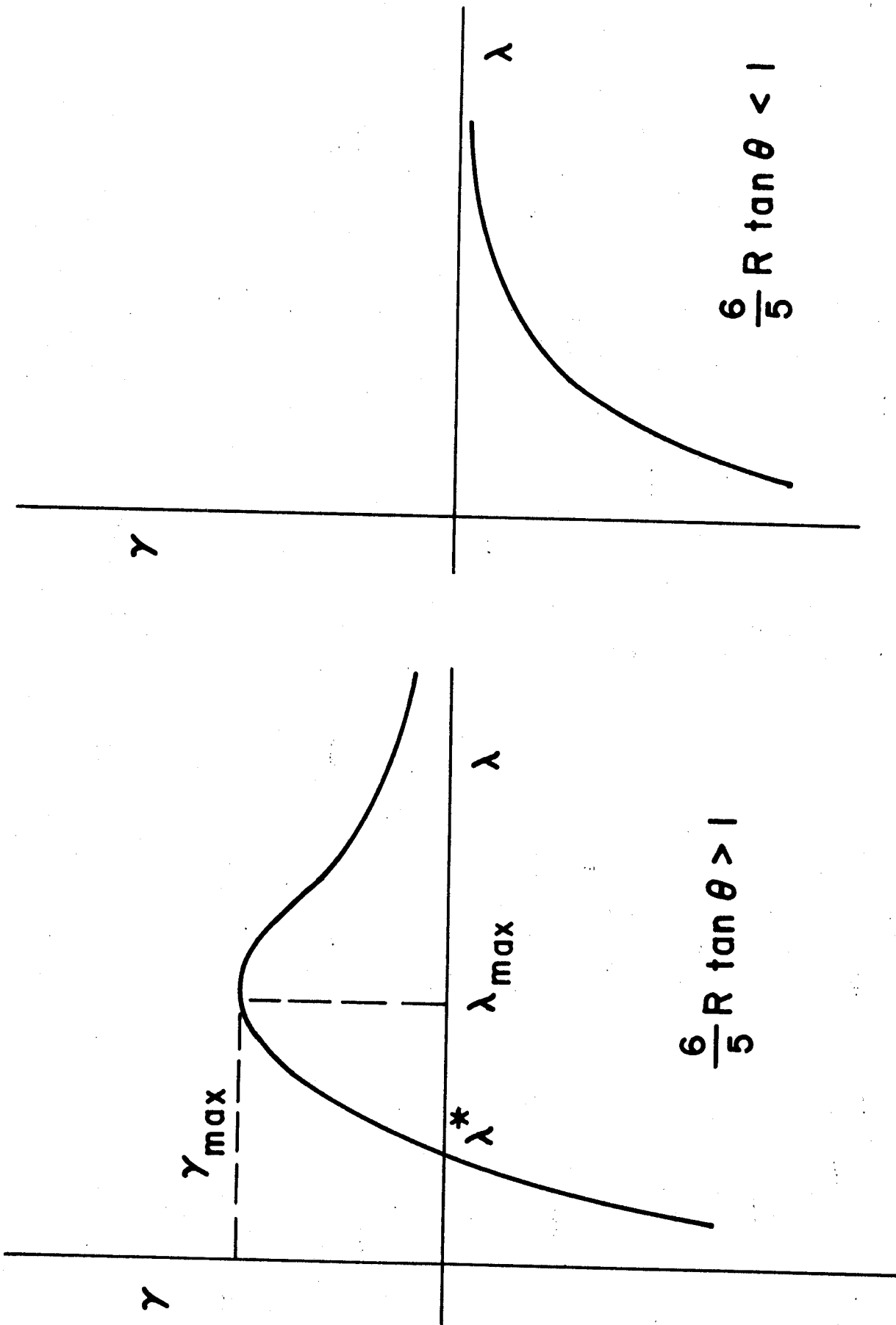


Figure 4

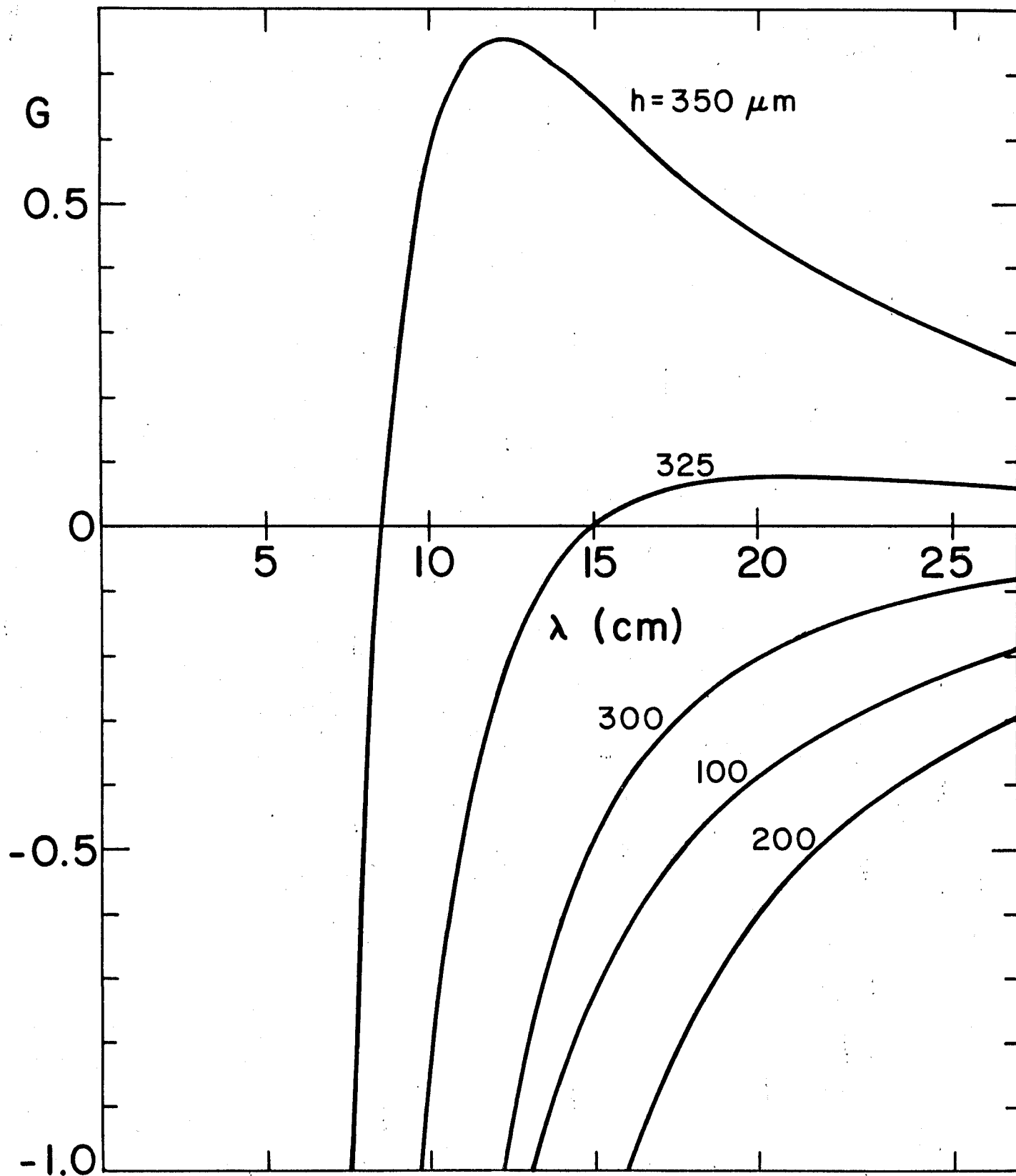
GROWTH vs. WAVELENGTH $\theta=5^\circ$ LITHIUM

Figure 5

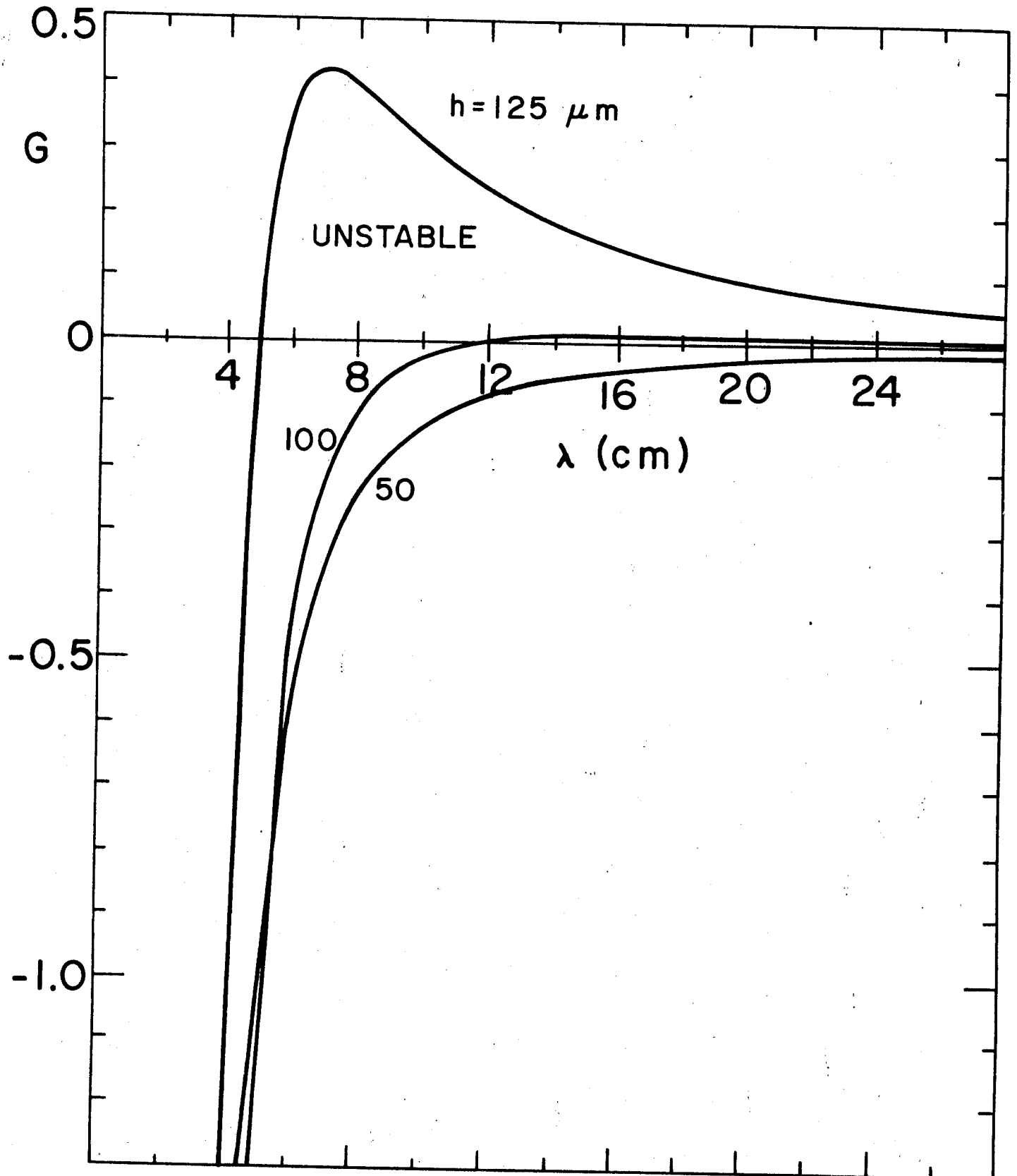
GROWTH vs. WAVELENGTH $\theta=30^\circ$ LITHIUM

Figure 6

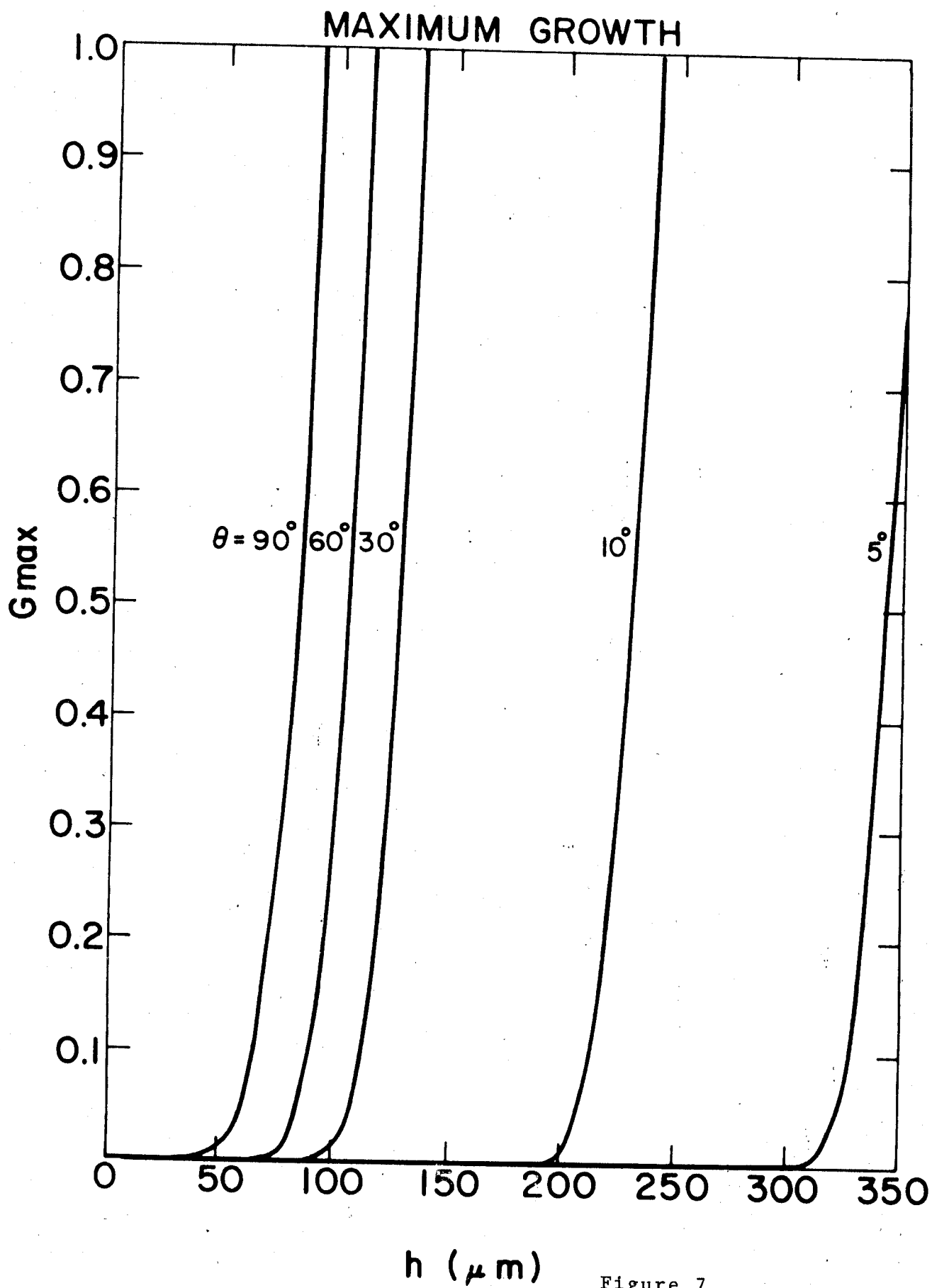


Figure 7

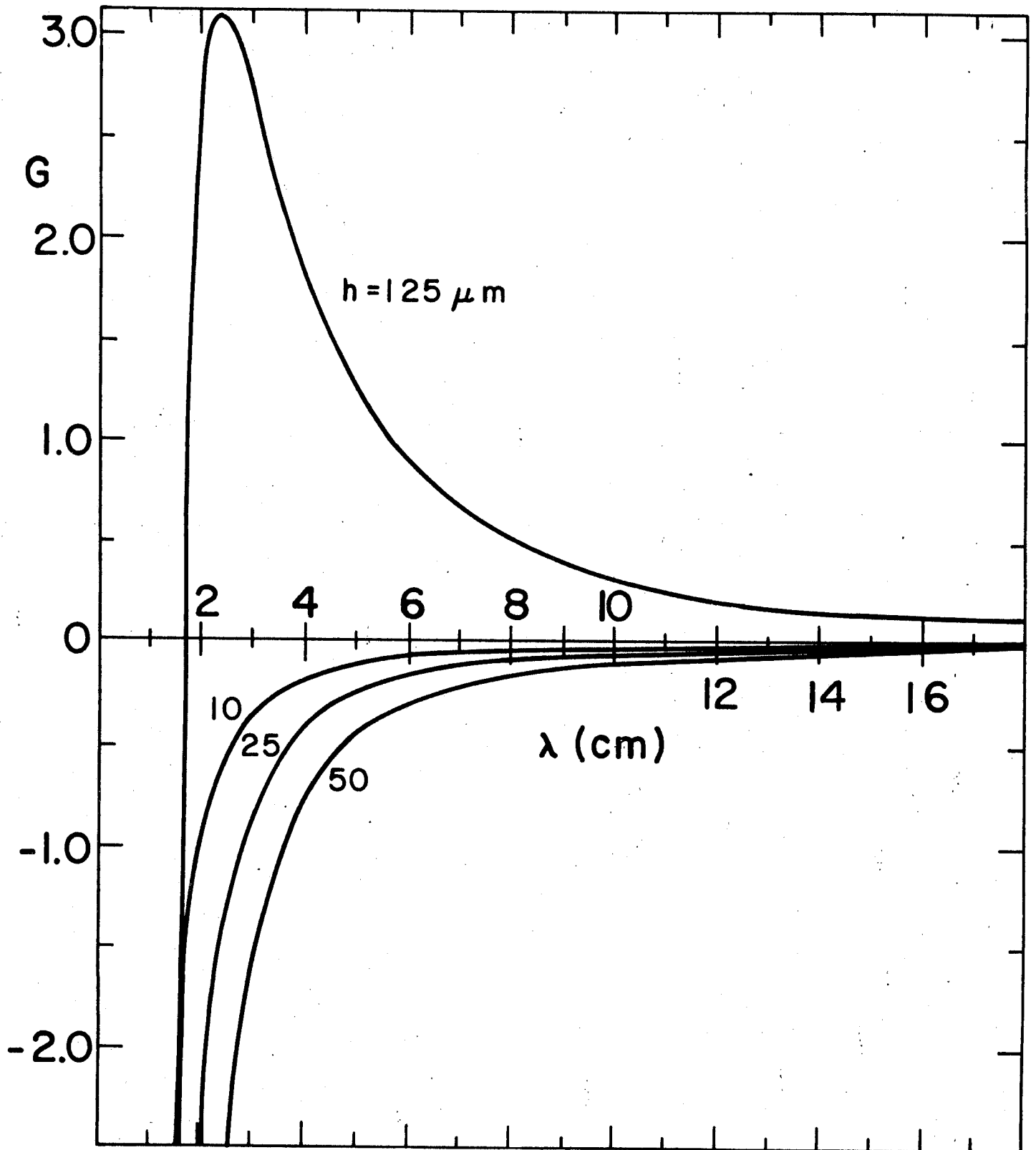
GROWTH vs. WAVELENGTH $\theta=30^\circ$ H₂O

Figure 8