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TRANSITION AND RECOVERY OF CRYOGENICALLY STABLE CONDUCTORS

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Abstract

Film cooling stability is described for a composite conductor of superconductor filaments in a normal metal matrix. It is shown by an electromagnetic and thermal transient solution for current density, temperature and power generated that the following occurs after a filament has gone normal: (1) the temperature rise $\Delta T \approx 50$ K for 0.04 cm radius filaments and is less for smaller filaments, (2) the current spread and temperature rise is fast, \approx milliseconds, and (3) the recovery to $T = T_c$ is relatively slow, \approx seconds. Recovery is assured if the heat flux generated at any temperature is less than the boiling heat flux in helium, in particular less than 0.15 W/cm^2 , the minimum film boiling heat flux at 9 K.

1.0 Introduction

This is the third paper in a series by the authors on cryogenic stability for composite conductors of superconductor filaments in a normal metal matrix. In the first paper [1] we considered the electrostatics of the migration of current from the normal metal back into a superconductor filament during magnet charging when current and field must soak through a normal metal to reach a filament. The conclusions involved certain limitations on I and B so that excessive losses are avoided. The second paper [2] dealt with the dc conditions of current sharing below T_c and found temperature profiles within both the superconductor filament and normal matrix. By means of such temperature profiles the current capacity of a filament was found in order to determine if recovery would proceed. An example showed that for UWMAK conductors it would be necessary to use at least enough TiNb to carry full current at 4.3 K to ensure recovery in a 4.2 K bath, otherwise a stable operating point with 0.9 I in TiNb and 0.1 I in Cu could obtain. In this third paper we consider the transient conditions initiated by a filament going normal. Local temperatures become high, typically $T > T_c$, and then temperatures drop due to current spread throughout the low resistance matrix. If the helium film boiling curve is intercepted en route then cooling exceeds heating and recovery is possible. Recovery can become complete if $Q < 0.15 \text{ W/cm}^2$ at 9 K since then power generated is less than the film boiling minimum, which is a more stringent requirement than for ordinary nucleate cooling stability at 0.4 W/cm^2 .

2.0 Film Boiling Cryogenic Stability

In previous nucleate cooling cryogenic stability studies it was assumed that the current is initially carried uniformly by the normal metal at $T < T_c$. In this section we assume $T > T_c$ and compare q_g , the heat flux generated, with q_b , the film boiling heat flux removed, as a function of T .

Assume that a composite conductor carrying a fixed constant uniform current in the normal metal is designed so that q_g is equal to 0.15 W/cm^2 at 9 K, the film boiling minimum heat flux. Then q_g at any other temperature is determined solely by the temperature dependent resistivity ratio $\rho(T)/\rho_0$ (9 k):

$$q_g = (0.15 \text{ W/cm}^2) \frac{\rho(T)}{\rho_0} \quad (1)$$

Figure 1 is a plot of q_g and q_b , the boiling heat flux, versus T for copper of residual resistivities 7.75×10^{-12} , 7.75×10^{-11} and $1.55 \times 10^{-10} \Omega\text{-m}$ respectively; the last value corresponds to OFHC copper in zero field. Temperatures T_s at which the curves for q_g and q_b intersect are 28 and 79 K. As may be expected, low residual resistivity implies a low temperature at which the two curves intersect. Figure 2 is a plot of T_s versus the residual resistivity. It is often suggested that low residual resistivity stabilizers may be used to minimize the quantity of stabilizer required. However, if large T_s is required, then the amount of stabilizer required will be about the same since the resistivity at high temperature is almost independent of the residual resistivity.

It is, of course, desirable to have the generated power always less than the surface heat removal, which we have called unlimited stability. That means no intersections in Fig. 1 and $\rho > 1.55 \times 10^{-10} \Omega\text{-m}$ in Fig. 2. Recall that a uniform current density in copper has been assumed.

3.0 Current Spread

When a superconductor filament becomes normal the transport current will start to migrate to the stabilizer. To study the current migration or spread to the stabilizer we make the following assumptions: (1) The resistivities of normal metal ρ_0 and superconductor ρ_s are independent of temperature, and (2) the total current carried by the conductor is constant.

The diffusion equation is used to find the magnetic field and current density as a function of time. We consider a superconducting filament surrounded by a stabilizer as shown in Fig. 3 where one filament and its equivalent copper matrix and cooled surface are shown. The ratio of copper to superconductor is taken so that the heat generated in the stabilizer does not exceed the minimum film boiling heat flux. Making use of the above assumptions the magnetic diffusion equation in cylindrical coordinates is

$$\frac{\partial}{\partial r} \frac{\rho_s}{r} \frac{\partial}{\partial r} r H_s = \mu \frac{\partial H_s}{\partial t} \quad \text{and} \quad \frac{\partial}{\partial r} \frac{\rho_n}{r} \frac{\partial}{\partial r} r H_n = \mu \frac{\partial H_n}{\partial t} \quad (2)$$

where the subscripts s and n refer to the superconductor and the normal metal respectively. The boundary conditions are that H and E are continuous at the interface $r = a$ for any time t and that the self field $H_n(R, t) = I/2\pi R = a$ constant, see Fig. 3.

The separation of variables techniques is used to solve this problem. The use of this technique for composite media as discussed in reference 3 will not be detailed here. The solution is given by

$$H_s(r, t) = \sum_{\lambda} A_{\lambda} J_1(\lambda_{\lambda} r) e^{-\frac{\lambda_{\lambda}^2}{\alpha_s} t} + H_s(r, t) \quad (3)$$

and

$$H_n(r, t) = \sum_{\lambda} A_{\lambda} [B_{\lambda} J_1(\eta_{\lambda} r) + C_{\lambda} Y_1(\eta_{\lambda} r)] e^{-\frac{\lambda_{\lambda}^2}{\alpha_s} t} + H_n(r, t) \quad (4)$$

where J_0 , J_1 , Y_0 , and Y_1 are Bessel functions, $\alpha_s = \mu/\rho_s$ is the diffusion coefficient for the superconducting material, and η_{λ} is related to the λ_{λ} by the following equation $\eta_{\lambda} = \sqrt{\rho_n/\rho_s} \lambda_{\lambda}$. The eigen value λ_{λ} is calculated from the following eigen value equation

$$\sqrt{\rho_s/\rho_n} J_0(\lambda_{\lambda} a) = \frac{Y_0(\eta_{\lambda} a) J_1(\lambda_{\lambda} a) J_1(\eta_{\lambda} R) - J_0(\eta_{\lambda} a) Y_1(\eta_{\lambda} R) J_1(\lambda_{\lambda} a)}{J_1(\eta_{\lambda} R) Y_1(\eta_{\lambda} a) - Y_1(\eta_{\lambda} R) J_1(\eta_{\lambda} a)} \quad (5)$$

and the coefficients B_ℓ and C_ℓ are given by

$$B_\ell = \frac{Y_1(\eta_\ell R)J_1(\lambda_\ell a)}{Y_1(\eta_\ell R)J_1(\eta_\ell a) - Y_1(\eta_\ell a)J_1(\eta_\ell R)} \quad \text{and} \quad C_\ell = \frac{J_1(\lambda_\ell a)J_1(\eta_\ell R)}{J_1(\eta_\ell R)Y_1(\eta_\ell a) - Y_1(\eta_\ell R)J_1(\eta_\ell a)} \quad (6)$$

The coefficient A_ℓ is determined by constructing orthogonal functions from eigen solutions and using the initial conditions and the orthogonality property to determine A_ℓ . Once the coefficients B_ℓ , C_ℓ , and A_ℓ are determined, it is possible to calculate the current density and hence the power loss. It may be mentioned that the series representing the solution, Eqs. (3) and (4), converges very slowly for small t so that many terms have to be considered.

To calculate the temperature due to current spread it is assumed that the composite temperature is always uniform. In this case we have

$$[C_s d_s + C_n d_n] \frac{dT}{dt} + f S q_g = S q_b \quad (7)$$

where C and d are the specific heat and density respectively, f is a factor to account for the change of resistivity with temperature, q_g is the instantaneous heat flux generated during current spread calculated for constant resistivity and S is the surface area. The factor f is taken as the ratio of resistivity at temperature T to the constant resistivity used in solving the magnetic diffusion equations above. Using this procedure it is possible to solve for the magnetic and thermal spread of current and temperature separately and not for coupled nonlinear equations, which would be more difficult to solve.

Figure 4 is a plot of q_b , the boiling heat flux, and $f q_g$, the instantaneous heat generated by a filament and its copper, sized for film boiling stability. The temperature, time and power generated are plotted for three representative sized filaments. The sequence of heating is as follows: (1) first the filament goes normal and $f q_g$ is very large since the expelled current is in a thin shell of copper around the filament, (2) the current spreads out into more of the copper causing $f q_g$ to decrease, (3) the temperature continues to rise until $f q_g$ crosses the film boiling curve after which $f q_g$ continues to drop and finally (4) the conductor reaches T_c below which the superconductor again starts to carry current. The times involved for the above steps are shown on the curve in Fig. 4. The maximum temperature which occurs when $f q_g$ intersects the q_b curve is reached rather quickly within milliseconds while it takes seconds to cool down to T_c . Several curves are plotted for different size filaments; note that small filaments experience small temperature rises. Figure 5 is a plot of current in the filament and skin depth in the copper vs. time after the 0.0384 cm filament goes normal. Figure 6 is a plot of maximum temperature vs. filament radius which shows that small filaments are best.

4.0 Current Recovery, $T < T_c$

Following a temperature rise, the composite should be designed to recover by automatically cooling down to lower temperatures. When the critical temperature T_c is reached, current will transfer from the stabilizer to the superconducting filament. Such recovery processes for $T < T_c$ are studied in this section.

Let us assume that the temperature of the composite is equal to or slightly lower than T_c . The current starts migrating to the superconducting filaments which have very small

resistance as long as their ability to carry current is not exceeded. If the current rejected from the stabilizer should exceed the superconductor capacity, then current sharing will take place.

The average filament current density \bar{J} is given in reference [2] as:

$$\bar{J} = 2J(T) + \frac{2J(T)}{\phi} \frac{I_1(\phi)}{I_0(\phi)} \quad \text{and} \quad \phi^2 = -v \frac{\partial J}{\partial T} \frac{a^2}{k_s} \quad (8)$$

where I_0 and I_1 are modified Bessel functions, k_s is the superconductor thermal conductivity and v is the voltage gradient along a filament. The current density J and the decrease in current density ΔJ , $\Delta J = \bar{J} - J(T)$, due to current sharing is plotted against T for different filament radii in Fig. 7.

Consider a superconducting filament surrounded by a stabilizer as before, Fig. 3. The field and the current density in the normal metal are governed by the diffusion equation

$$\frac{\partial}{\partial r} \frac{\rho}{r} \frac{\partial}{\partial r} rH = \mu \frac{\partial H}{\partial t}, \quad r \geq a \quad (9)$$

The diffusion equation cannot be applied, however, inside the superconducting filament. It was shown [1] that an approximate solution to Eq. (9) can be obtained by assuming that the presence of the superconductor imposes the following boundary condition

$$\frac{1}{r} \frac{\partial}{\partial r} rH = h H \quad (10)$$

where h is a parameter related to the velocity of the flux lines at the interface between the normal metal and the superconductor. This was based on the equation of motion of flux lines in the superconductor as introduced by Irie [4].

The procedure for solving Eq. (9) is similar to the procedure used in the current spread case and we have

$$H = \sum_{\lambda} A_{\lambda} \left[Y_1(\lambda_{\lambda} r) - Y_1(\lambda_{\lambda} R) \frac{J_1(\lambda_{\lambda} r)}{Y_1(\lambda_{\lambda} R)} \right] e^{-\frac{\lambda_{\lambda}^2}{\alpha} t} \quad (11)$$

and the eigen value equation is

$$Y_1(\lambda_{\lambda} R) \{ \lambda_{\lambda} \rho J_0(\lambda_{\lambda} a) - h J_1(\lambda_{\lambda} a) \} - \{ \rho \lambda_{\lambda} Y_0(\lambda_{\lambda} a) - h Y_1(\lambda_{\lambda} a) \} J_1(\lambda_{\lambda} R) = 0 \quad (12)$$

The above solution is valid as long as the current in the superconducting filament does not exceed the capacity of the filament; otherwise the problem is much more complicated. The above solution is used to estimate the recovery time and to see how the current transfers from the normal metal to the superconductor. Figure 8 is a plot of the superconductor current and composite temperature as a function of time for one of the 3.84×10^{-4} m radius filaments in a cryogenically stable conductor carrying 10,000 Amp.

5.0 Conclusion

Film cooling stability in liquid helium based on this study is achieved for a reasonably proportioned and cooled composite by: (1) choosing the conductor size, cooling surface and resistivity so that $I^2 R / \text{cm}^2 < 0.15 \text{ W/cm}^2$ at 9 K with the superconductor normal and (2) choosing a superconductor filament size so that the maximum temperature rise following the filament normal transition is less than T_s which is the temperature at which

$I^2R/cm^2 = q_b$, the boiling heat flux. Filaments of TiNb less than 10μ in diameter stabilized in OFHC copper will not exceed 11 K on going normal and will recover to the superconducting state within 100 milliseconds. Unlimited stability can be achieved if $I^2R/cm^2 < q_b$ at any temperature, which is the case for $\rho > 1.55 \times 10^{-10} \Omega\text{-m}$, a typical conductor copper matrix resistivity. If the amount of superconductor is such that the total transport current can be carried at an elevated temperature, e.g. 5.2 K, then a composite is more difficult to drive normal and, if driven normal, can recover more quickly and completely. Designs for short sample 4.2 K quantities are shown to be marginal and unreliable.

Acknowledgement

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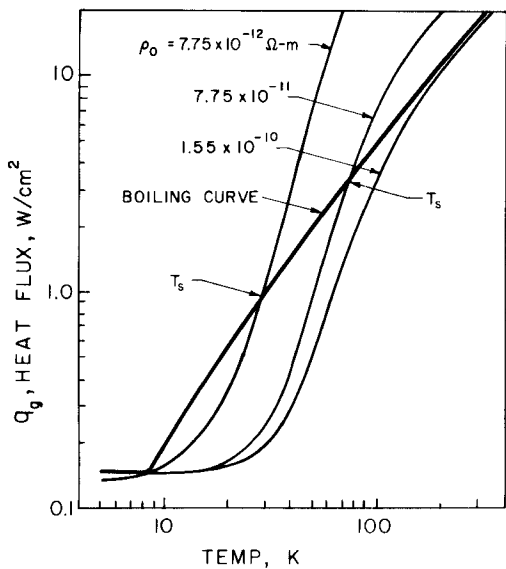


Fig. 1. Heat flux q_g generated by copper conductors sized for $q_g = 0.15 \text{ W/cm}^2$ at 9 K and $q_g(T) = (0.15)\rho(T)/\rho_0(9 \text{ K})$ where ρ is resistivity. q_g curve intersects boiling curve at T_s .

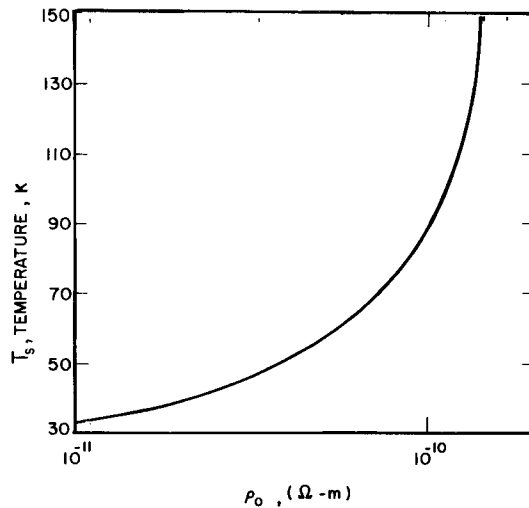


Fig. 2. Intersection temperature from Fig. 1 conductors.

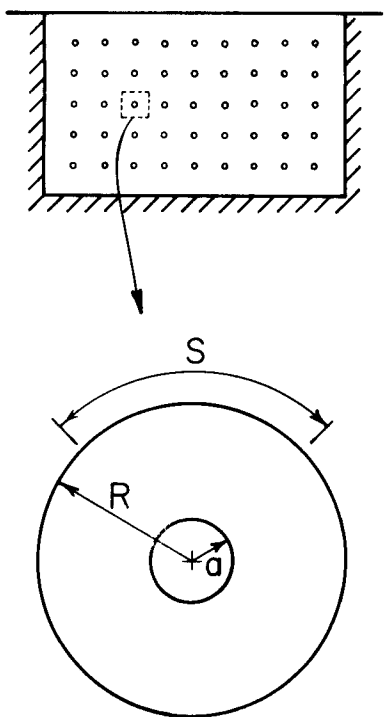


Fig. 3. Equivalent conductor per filament radius a , in normal metal matrix conductor radius R with prorated cooling surface S from large multifilament composite conductor.

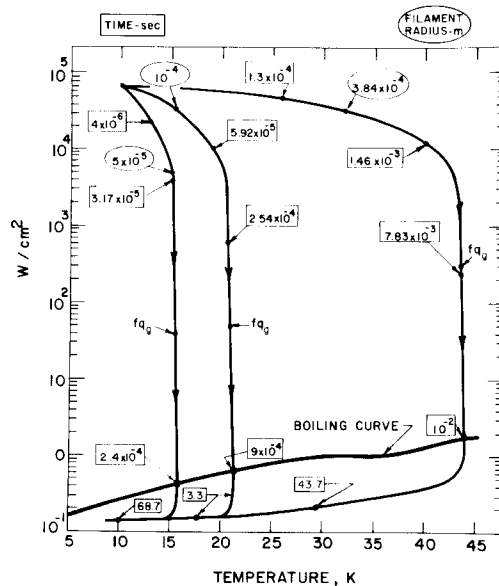


Fig. 4. Heat flux after filaments go normal, arrows toward increasing time. Magnetic solution for $\rho = \rho_0$ and thermal solution for $\rho = \rho_0 f(T)$. Normalized conductor design at 0.15 W/cm^2 at 9 K.

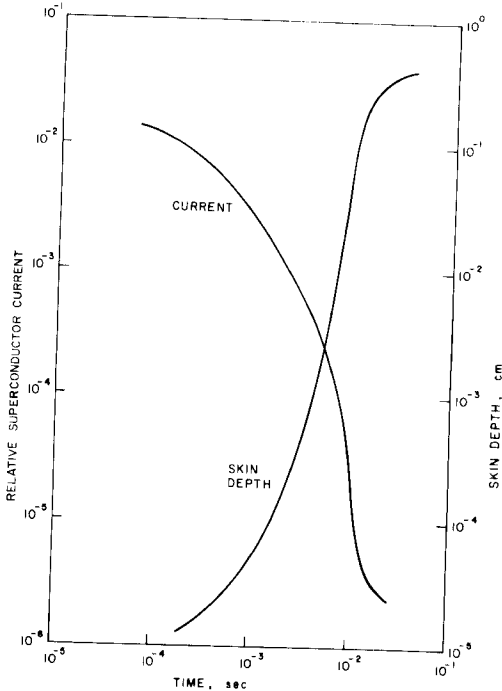


Fig. 5. Filament current and copper skin depth after filament goes normal. Copper current in "skin depth" cylinder produces correct copper power. Filament size 0.0384 cm radius for conductor normalized at 0.15 W/cm^2 at 9 K.

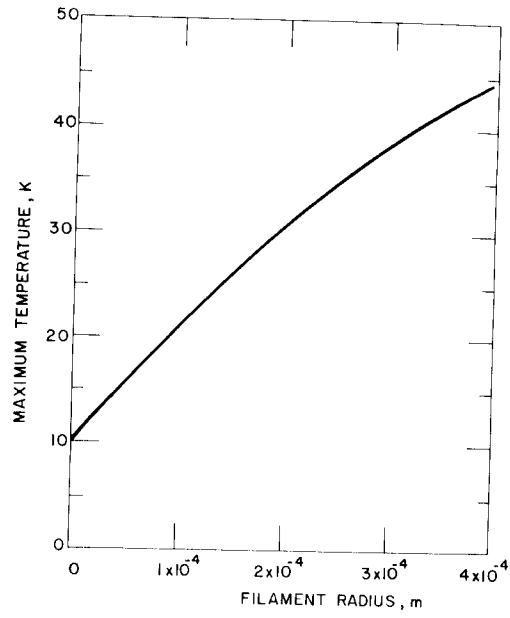


Fig. 6. Maximum temperature after a filament goes normal vs. filament size. Normalized conductor design for $q_g = 0.15 \text{ W/cm}^2$ at 9 K.

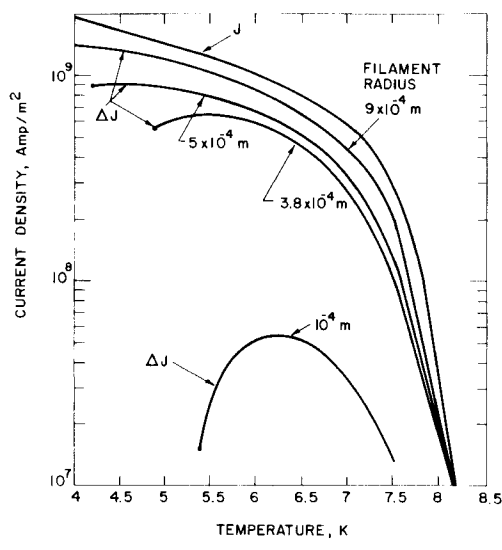


Fig. 7. Current recovery for $T < T_c$. Current density J and current density reduction ΔJ due to current sharing. Normalized conductor design for $q_g = 0.15 \text{ W/cm}^2$ at 9 K.

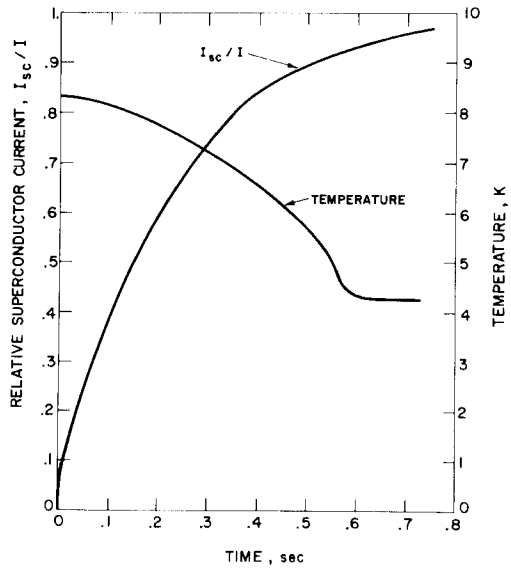


Fig. 8. Recovery from T_c to 4.2 K of current for a "small" filament. Normalized conductor design for $q_g = 0.15 \text{ W/cm}^2$ at 9 K.