The Design of a Noncircular Plasma for the Argonne Tokamak Experimental Power Reactor

T.F. Yang and R.W. Conn

July 1976

UWFDM-164

FUSION TECHNOLOGY INSTITUTE

UNIVERSITY OF WISCONSIN

MADISON WISCONSIN

The Design of a Noncircular Plasma for the Argonne Tokamak Experimental Power Reactor

T.F. Yang and R.W. Conn

Fusion Technology Institute
University of Wisconsin
1500 Engineering Drive
Madison, WI 53706

http://fti.neep.wisc.edu

July 1976

UWFDM-164

The Design of a Noncircular Plasma for the Argonne Tokamak Experimental Power Reactor

T. F. Yang

R. W. Conn

July 1976

UWFDM-164

Fusion Technology Program
Nuclear Engineering Department
University of Wisconsin
Madison, Wisconsin 53706

I. Introduction

In comparison with circular cross section tokamaks having similar values of poloidal Beta, β_θ, and stability factor q, non-circular cross section tokamaks have the following advantages: (1) non-circular tokamaks offer the potential for achieving higher values of total β;[1] (2) a non-circular plasma with a "D" shape tends to fill more optimally the available volume within a set of toroidal field (TF) coils designed with a constant tension shape.[2] The purpose here is to investigate the feasibility of designing such a non-circular plasma for a Tokamak Experimental Power Reactor that will fit within the TF coils designed for the Argonne TEPR.[3] The Argonne machine is designed with equilibrium field (EF) coils outside the TF magnets and this constraint was retained. Stability against vertical displacements combined with this restriction on the plasma of the EF coils limited the maximum height to width ratio of the plasma cross section to 1.3. The calculational method and the stability criteria employed are discussed in section II. Results are given in section III.

II. The Description of the MHD Code

The MHD equilibrium code developed by a group at the Princeton Plasma Physics Laboratory[4] was used for the equilibrium and local stability calculations. The original code has been modified in several ways, including an addition to calculate the general criteria for stability with respect to rigid motions. The MHD equilibrium for an axisymmetric system is determined by solving the usual equation for the flux function Ψ which satisfies

$$X \frac{\partial}{\partial X} \left(\frac{1}{X} \frac{\partial \Psi}{\partial X}\right) + \frac{\partial^2 \Psi}{\partial Z^2} = -XJ_\phi \ , \qquad (1)$$

where

$$J_\phi = X \frac{dp}{d\Psi} + \frac{R_o^2 B_o^2}{X} g \frac{dg}{d\Psi} . \quad (2)$$

The magnetic field has been decomposed into poloidal and toroidal components using

$$\vec{B} = B_o \; [f(\psi)\vec{\nabla}\phi X \vec{\nabla}\psi + R_o g(\psi)\vec{\nabla}\phi]. \quad (3)$$

The coordinates R, ϕ and Z are the axisymmetric coordinates, ϕ is the ignorable variable and X is the distance from the symmetric axis. ψ represents an arbitrary surface label and Ψ is the poloidal flux inside a magnetic surface. The non-linear equation (1) is solved by prescribing the functions $p(\Psi)$ and $g(\Psi)$ in equation (2). The form employed for the functions $p(\Psi)$ and $g(\Psi)$, used in solving eqn. (1), can have a significant effect on the equilibrium and stability of the plasma. If we choose

$$P(\Psi) = P_o \left(\frac{\Psi_* - \Psi}{\Psi_* - \Psi_o}\right)^{\alpha_1} \quad (4)$$

and

$$g(\Psi) = [1 - g_p \left(\frac{\Psi_* - \Psi}{\Psi_* - \Psi_o}\right)^{\alpha_2}] , \quad (5)$$

the plasma current, $j_\phi(\Psi)$, has the form

$$J_\phi(\Psi) = -\frac{XP_o\alpha_1}{\Delta\Psi} \left(\frac{\Psi_* - \Psi}{\Delta\Psi}\right)^{\alpha_1 - 1} + \frac{\alpha_2 R_o^2 B_o^2}{\Delta\Psi X} g_p \left(\frac{\Psi_* - \Psi}{\Delta\Psi}\right)^{\alpha_2 - 1} + \frac{\alpha_2 R_o^2 B_o^2}{\Delta\Psi X} g_p^2 \left(\frac{\Psi_* - \Psi\Psi}{\Delta\Psi}\right)^{2\alpha_2 - 1} \quad (6)$$

where $\Delta\Psi = \Psi - \Psi_m$ and Ψ_*, Ψ_m are the flux at the plasma surface and magnetic axis. The constant g_p is varied until Ψ converges.[5] The plasma boundary is determined self-consistently with the location of external coils. A square of arbitrary size is used as an outer boundary for the problem and the boundary conditions on this border are fixed by specifying the current in the external coils. As such, the procedure does not require the use of a conducting external boundary. The plasma boundary can be fixed by a limiter or a separatrix. The program evaluates $q(\Psi)$, $\beta_\theta(\Psi)$ and $\overline{\beta}_\theta$ from the formulae:[5,6]

$$q(\psi) = \frac{R_o g}{2\pi} \oint \frac{dl}{|\nabla\Psi| X} \quad , \tag{7}$$

$$\beta_\theta(\psi) = \frac{2\pi P}{\left\langle \frac{|\nabla\Psi|^2}{R^2} \right\rangle} \quad , \tag{8}$$

and

$$\overline{\beta_\theta} = \frac{\int P \frac{dv}{d\Psi} d\Psi}{\int^{\Psi} P \frac{dv}{d\Psi} d\Psi - R_o^2 B_o^2 \int^{\Psi} Vg \frac{dg}{d\Psi} \left\langle \frac{1}{X^2} \right\rangle d\Psi} \tag{9}$$

where V and $\frac{dV}{d\Psi}$ are the plasma volume and its derivative with respect to Ψ. The total β and average pressure are defined respectively as follows.

$$\beta_{total} \cong \frac{\int p dv}{\int \frac{B_t^2}{2\mu_o} dv} \tag{10}$$

and

$$\bar{p} = \frac{\int p dv}{\int dv} \tag{11}$$

We have written a new routine that is now included in the general code to compute the general stability criteria related to quasi-rigid motions (vertical and radial displacements and flipping). Details on this can be found in reference 5.

III. Results

The philosophy of this design is to obtain the largest elongation of the plasma and make the best use of the space inside the toroidal field coils consistent with reasonable pressure and current profiles. The exponents α_1 and α_2 in equation (4) and (5) are chosen to be 1.4.[5] The arrangement of the equilibrium field coils is shown in figure 1. The height to width ratio of 1.3 is the largest elongation which can be obtained for this sytem. The shape factor K, the ratio of the circumference of the plasma boundary to that of the largest inscribed circle is 1.2. Fig. 2 shows the detail plot of the flux surface. The major plasma parameters are listed in Table I and the external equilibrium field coils are listed in Table II.

The local instability criteria are all negative for this system while the general stability criteria with respect to quasi-rigid motions are all positive. Therefore, these modes are all stabilized. Also, we have required $q > 1$ over the entire plasma.[7,8]

The pressure and current density profiles plotted as a function of major radius on the midplane are shown in Fig. 3. The magnetic axis is at $R_m = 7m$ and it is shifted from the geometrical center by $\Delta m = 0.7m$. The shift of current axis is $\Delta j = 1.2m$. There is a small amount of reverse current near the inner edge of the plasma due to the diamagnetic effect at the high β. Perspective views of the flux surfaces, the pressure and current density profiles are shown by figures 4, 5 and 6 respectively.

Fig. 1 also shows a comparison of the non-circular and circular plasma (indicated by the broken line.) The volume of the non-circular plasma is about 20% larger. The β value can be made larger by choosing α_1 and $\alpha_2 < 1.4$ because more plasma current can be used. However, this maximization has not been done. Rather the incentive was to keep the current density peaked as close to the magnetic axis as possible.

For comparison, a circular plasma was calculated for the same forms of the $p(\Psi)$ and $g(\Psi)$ functions. The flux plot and the current and pressure profiles are shown in Figs. 7 and 8 respectively. The plasma parameters and the coil arrangement are listed in Tables III and IV respectively. The comparison of Tables I and III shows that, for similar β_θ and q, the total β and average pressure is larger for non-circular plasma. This non-circular design can, therefore, be used in the Argonne TEPR[3] with minimal perturbation on the rest of the design.

References

1. T. Ohkawa, T. H. Jensen, Plasma Physics 12, 789 (1971).

2. J. File, R. G. Mills, J. Sheffield, IEEE Transaction on Nuclear Science NS-18, 277 (1971).

3. W. M. Stacey, et al., "Tokamak Experimental Power Reactor Studies," ANL/CTR-75-2 (1975).

4. M. S. Chance, R. L. Dewar, A. H. Glasser, J. M. Greene, R. C. Grimm, S. C. Jardin, J. L. Johnson, B. Rosen, G. W. Sheffield, and K. E. Weimer, Proc. 5th IAEA Conf. on Plasma Physics and Cont. Nucl. Fus. Research, 1974, (IAEA, Vienna, 1975) Vol. I p. 463.

5. T.F. Yang, R.W. Conn, "Computational Study of MHD Equilibrium of Non-Circular Tokamak Reactor Plasmas", IEEE Trans. on Plasma Science 4, 249 (1976).

6. L. E. Zakharov and V. D. Shafranov, Sov. Phys. Tech. Phys. 18, 151 (1973).

7. M. D. Kruskal, J. L. Johnson, M. B. Gottlieb, and L. M. Goldman, Phys. Fluids 1, 421 (1958).

8. V. D. Shafranov, Plasma Physics and the Problem of Controlled Thermonuclear Reactors (Pergamon, Oxford, 1969), Vol. 2, p. 197.

Table I

Plasma Parameters for the Non-Circular Plasma

I_p = 8.4 MA	a = 2.15 m
B_o = 43.2 kgauss	h = 2.8 m
B_{max} = 100.0 kgauss	h/a = 1.3
R_o = 6.30 m	K = 1.2
β_θ = 2.35	$A = \frac{R_o}{a} = 2.94$
β_{total} = 0.06	R_{in} = 4.15
$\bar{p} = .444 \times 10^6$ Joules/m^3	R_{out} = 8.45
q_o = 1.0	$V_p = 740\ m^3$
q_{max} = 3.1	

Table II

Equilibrium Field Coils for TEPR Non-Circular Plasmas

#	Z(m)	R(m)	I(MA)
V1	0.0	2.0	-0.230
V2	1.0	2.0	-0.230
V3	4.0	2.0	+1.110
V4	5.0	2.2	-0.294
V5	7.5	5.0	+10.102
V6	7.8	6.5	-1.200
V7	7.5	6.5	-0.920
V8	7.0	6.5	-1.473
V9	6.9	7.4	-1.473
V10	6.3	8.5	-1.473
V11	5.3	9.7	-1.473
V12	3.8	11.0	-1.110
V13	1.8	11.8	-1.110
V14	0.0	11.8	-1.110

Table III

Plasma Parameters for Circular Plasma

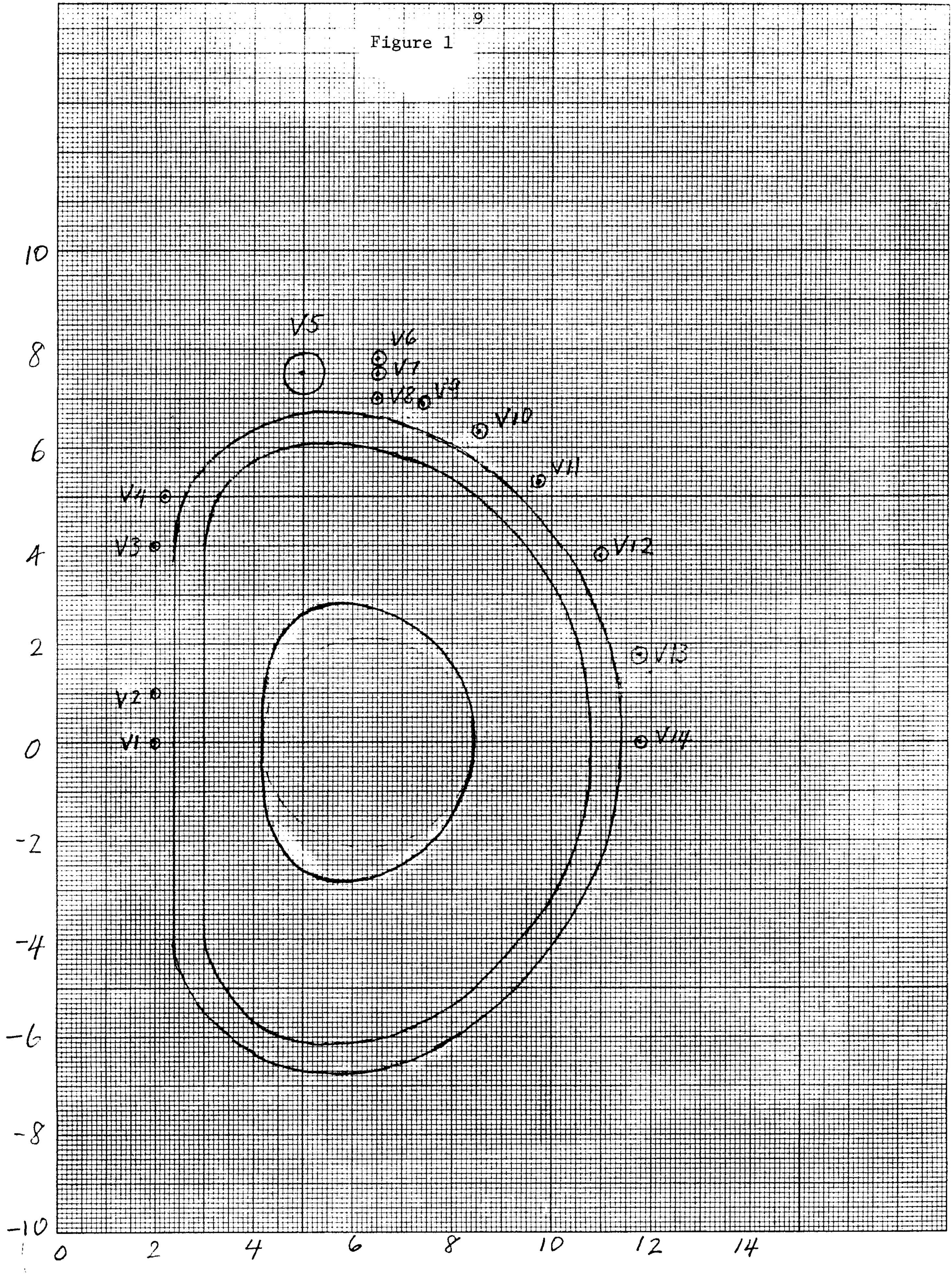
I_p = 6.5 MA	a = 2.15 m
B_o = 43.2 kgauss	h = 2.18 m
B_{max} = 100.0 kgauss	$\frac{h}{a} \simeq 1$
R_o = 6.30 m	$K \simeq 1$
β_θ = 2.3	$A = \frac{R_o}{a} = 2.94$
β_{total} = 0.04	R_{in} = 4.15
$\bar{p}$ = .32 x 10^6 Joules/m^3	R_{out} = 8.45
q_o = 1.06	V(plasma volume) = 615 m^3
q_{max} = 3.2	

Table IV

Equilibrium Coils for TEPR Circular Plasma

#	Z(m)	R(m)	I(MA)
1	3.35	12.05	-1.362
2	4.20	11.38	-1.730
3	7.15	8.88	-1.300
4	7.95	7.13	-1.300
5	8.05	5.63	-1.300
6	7.38	2.75	+3.640
7	7.05	2.40	+1.380
8	6.70	2.20	+1.380

Figure 1


10
8
6
4
2
0
-2
-4
-6
-8
-10
0
2
4
6
8
10
12
14
V1
V2
V3
V4
V5
V6
V7
V8
V9
V10
V11
V12
V13
V14

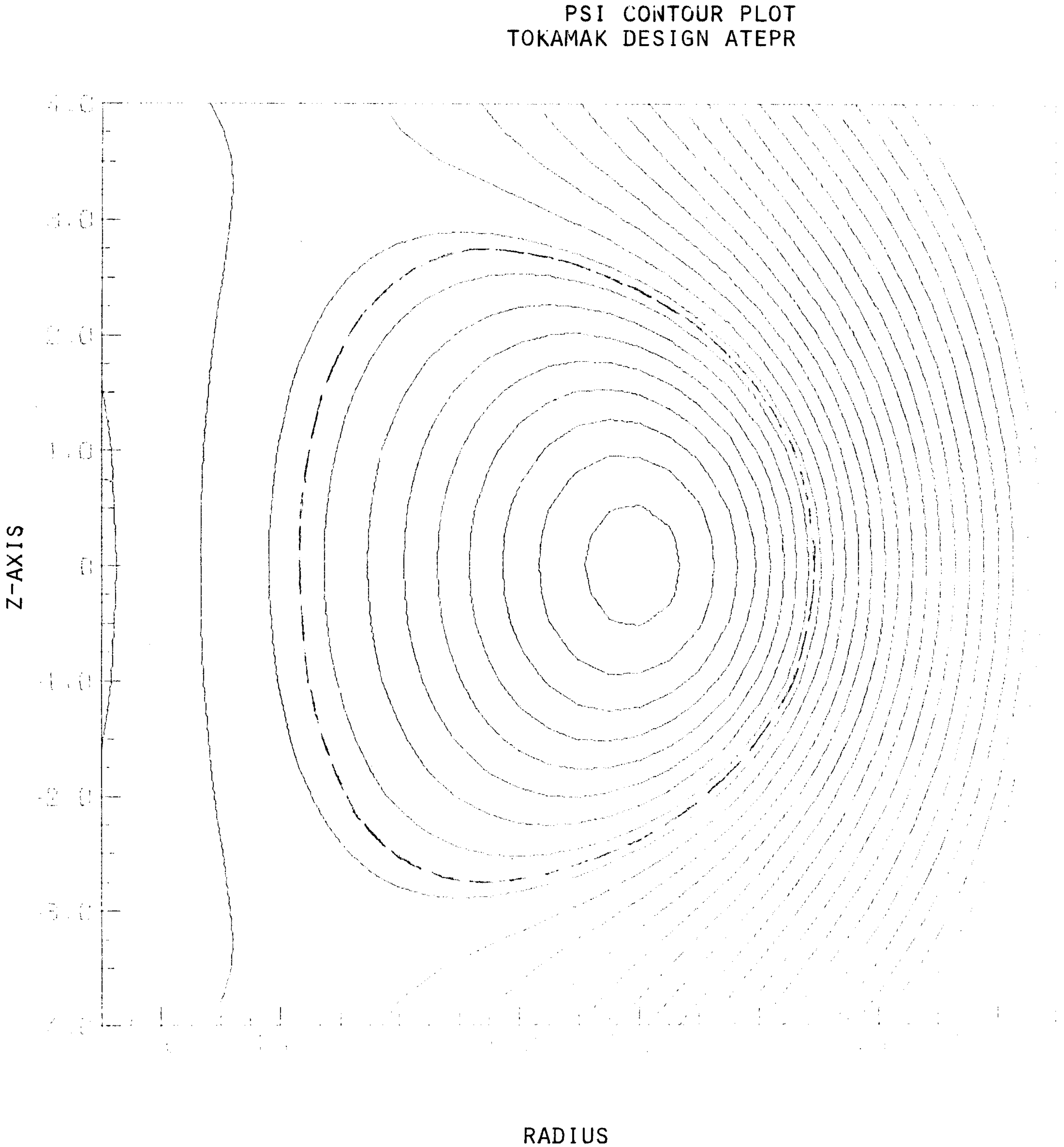


Figure 2

Figure 3

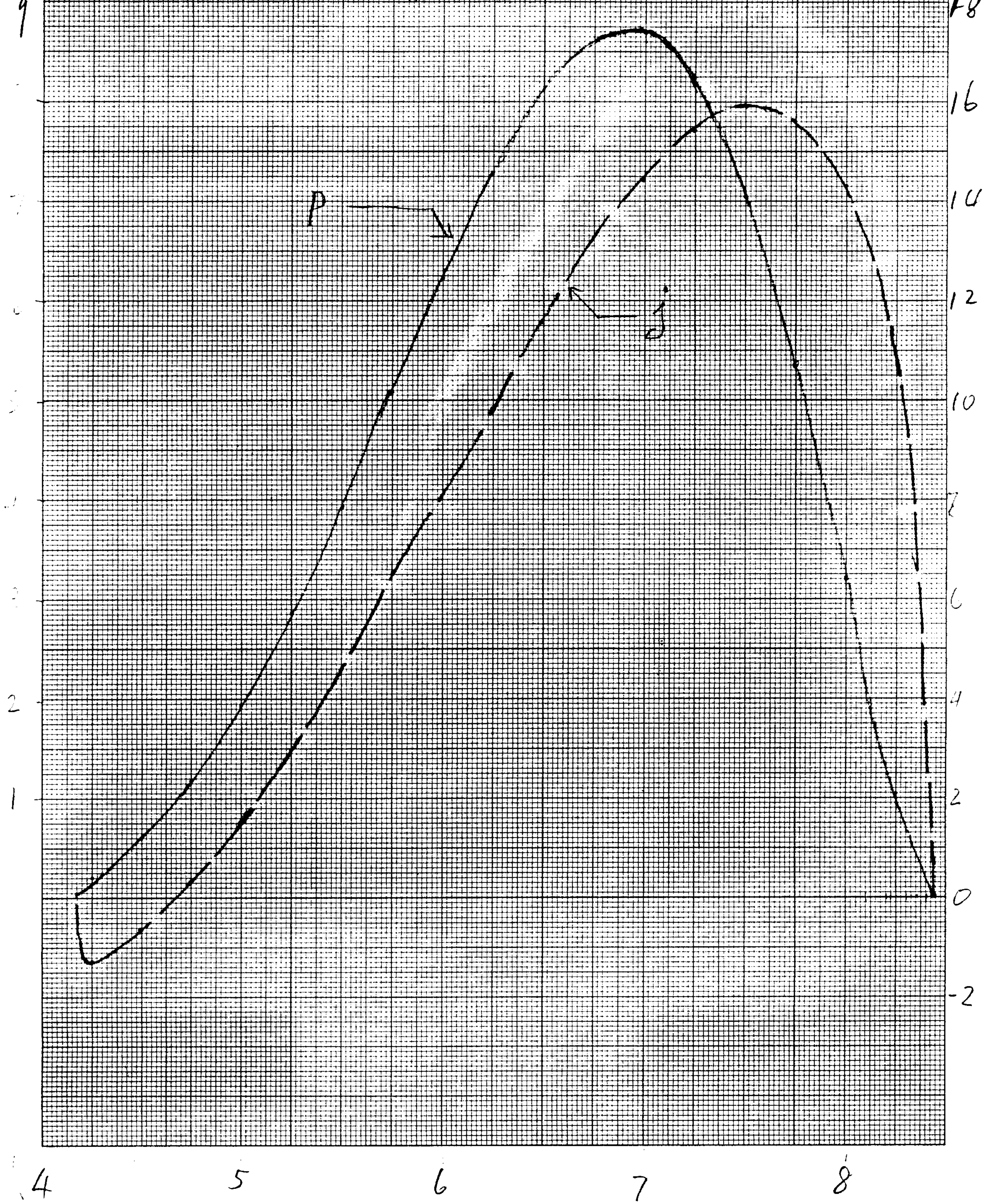
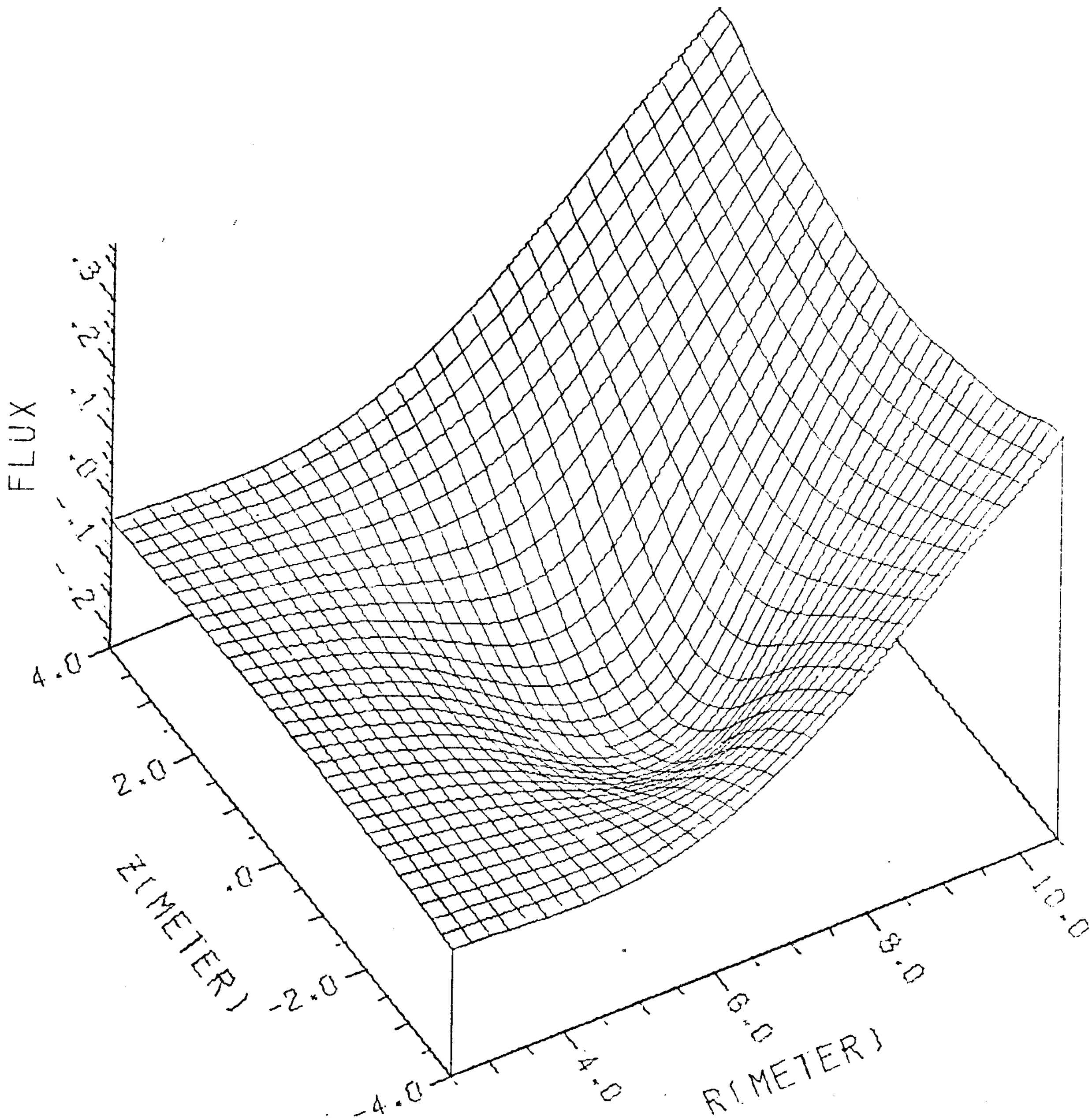
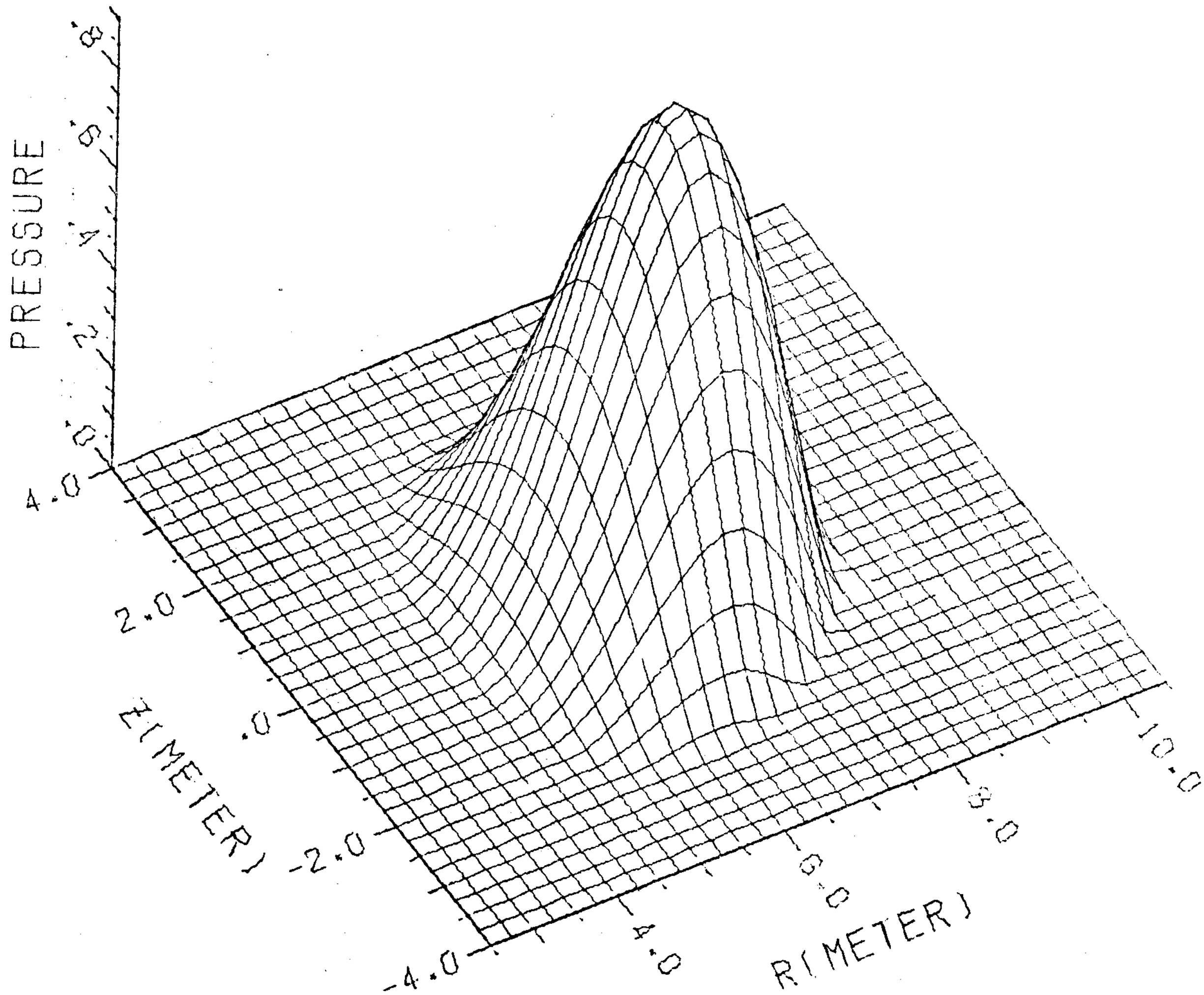


Figure 4



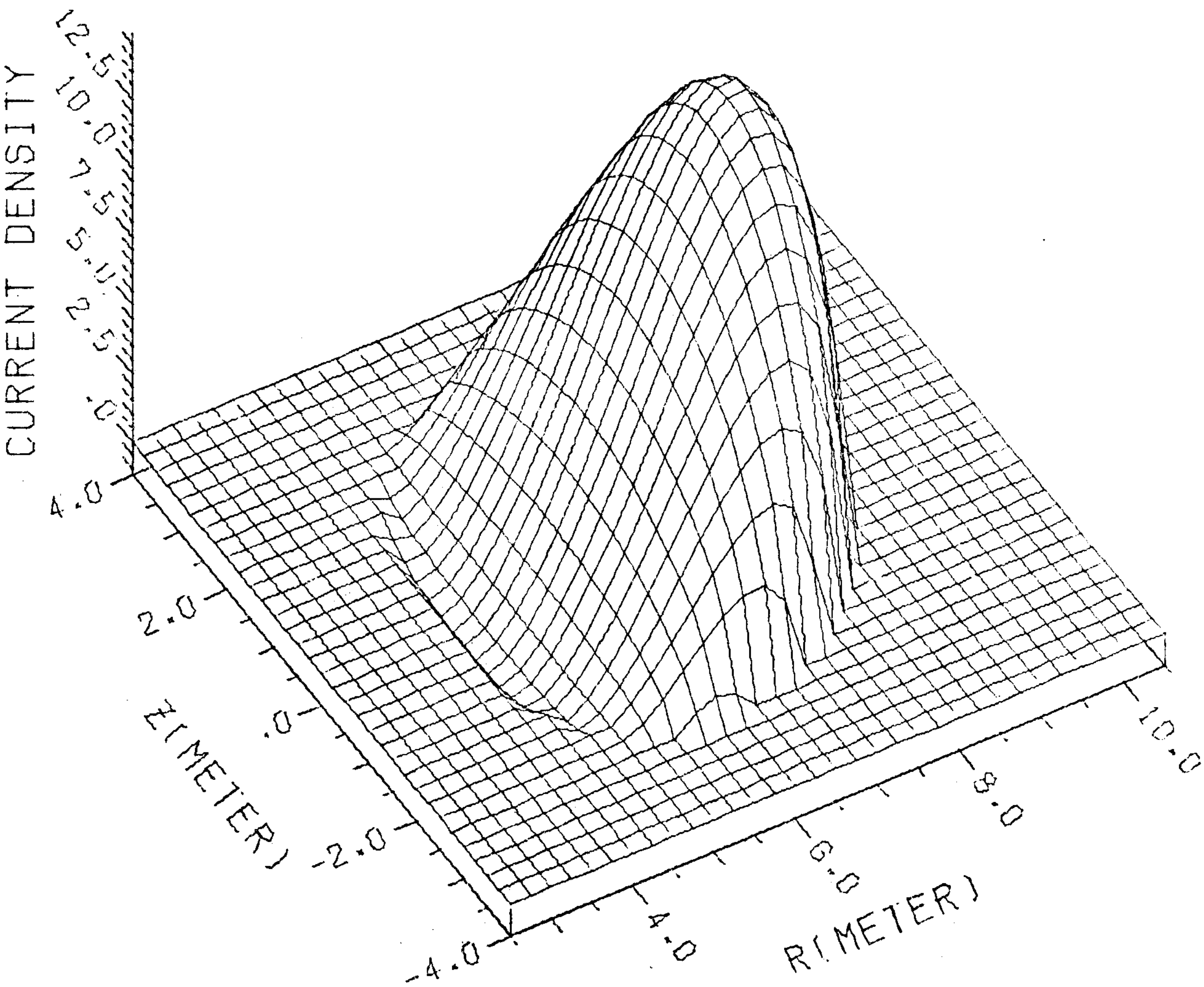
FLUX PLOT

Figure 5



PRESSURE PROFILE

Figure 6



CURRENT DENSITY PROFILE

Figure 7

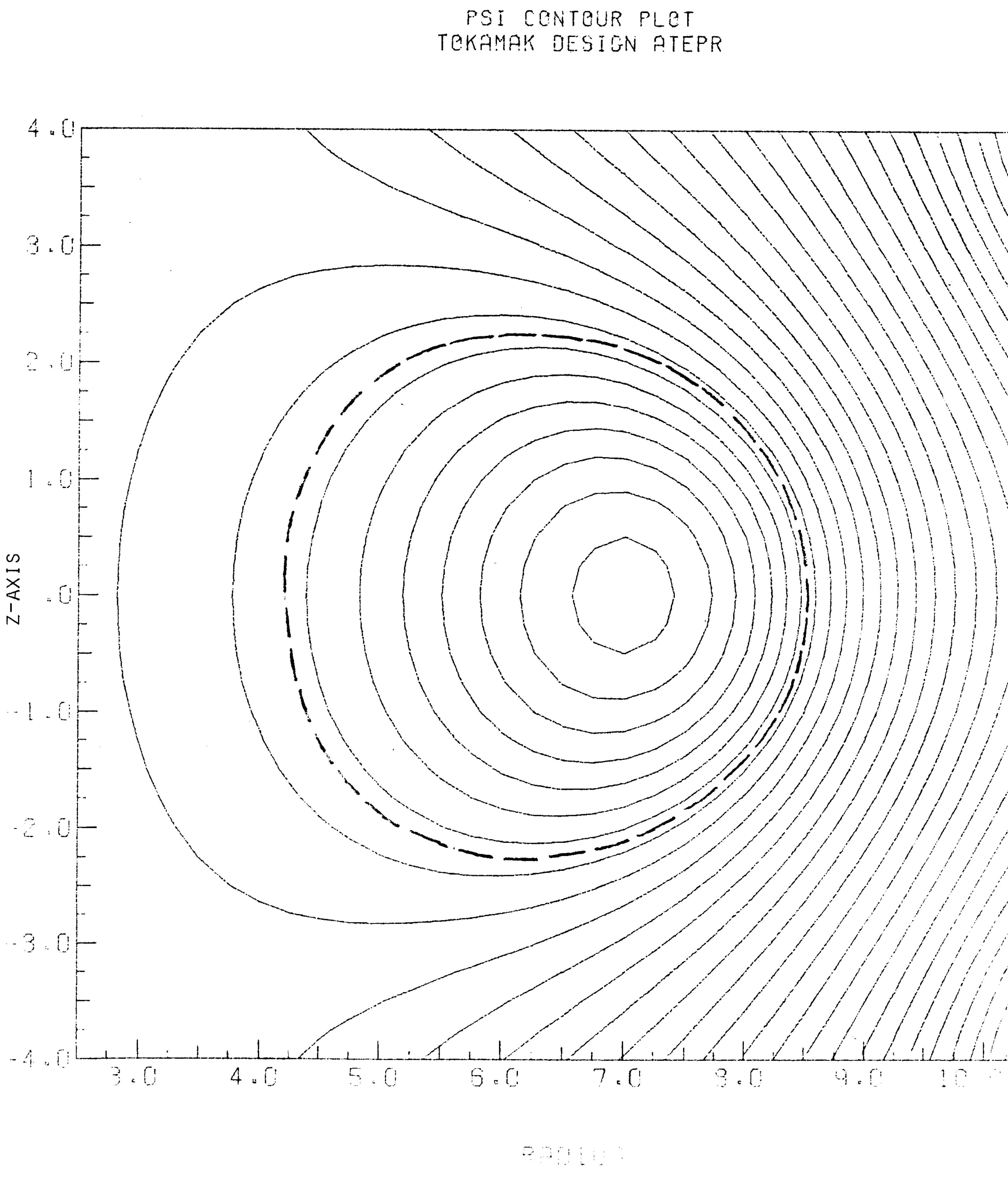
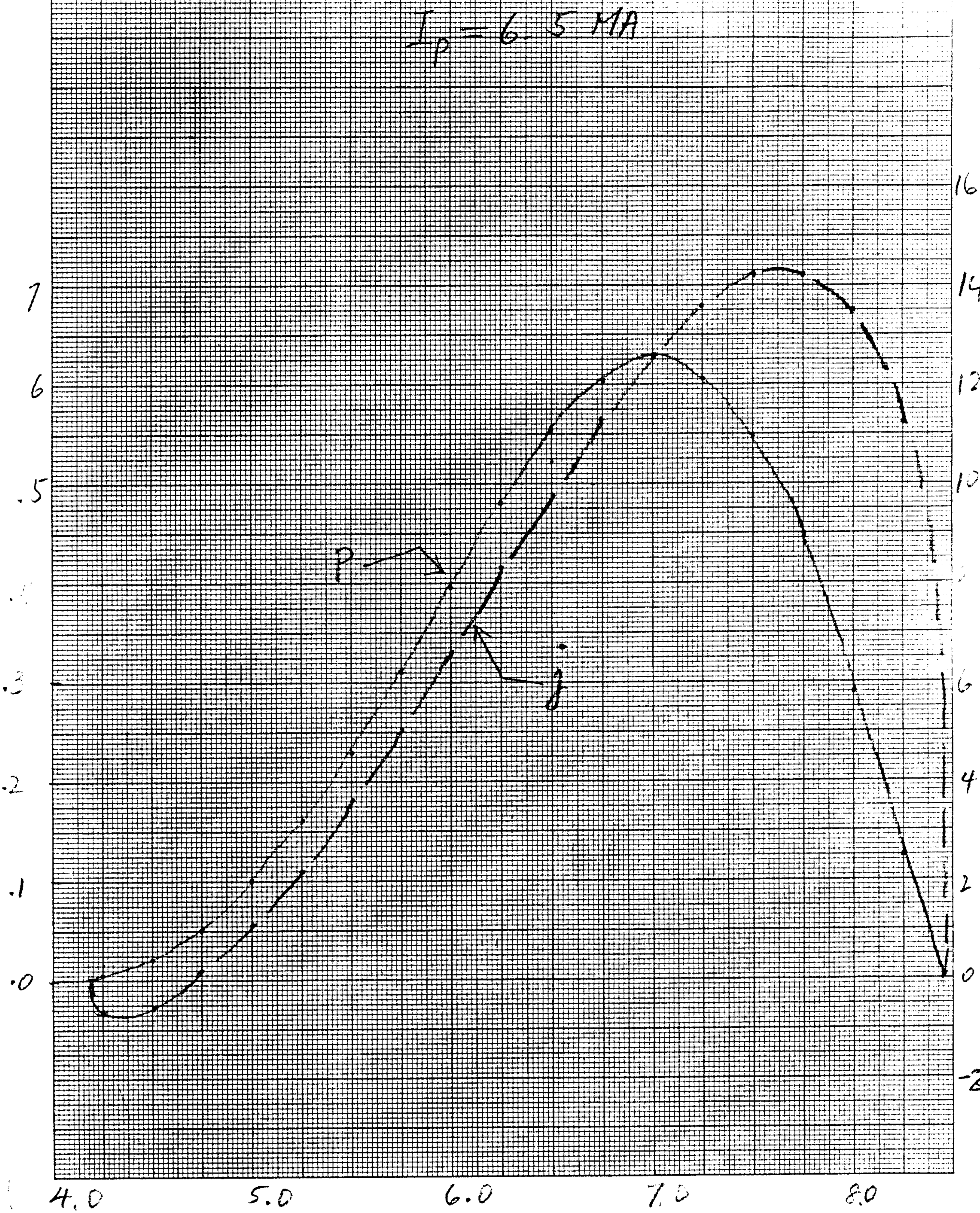

PSI CONTOUR PLOT
TOKAMAK DESIGN ATEPR
4.0
3.0
2.0
1.0
.0
-1.0
-2.0
-3.0
-4.0
Z-AXIS
3.0
4.0
5.0
6.0
7.0
8.0
9.0

Figure 8



46 1512

K+E 10 X 10 TO THE CENTIMETER 18 X 25 CM. KEUFFEL & ESSER CO. MADE IN U.S.A.