Tokamak Fusion Reactor: Plasma Equations and Energy Equilibria and Stability

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May 1972
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FUSION TECHNOLOGY INSTITUTE

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I. Introduction

An important part of any detailed examination of the feasibility of controlled fusion is to establish, as best possible, the plasma operating conditions. A number of recent studies[1,2,3,4] have attempted to calculate possible steady-state plasma operating conditions in CTR systems. The most complete study on closed systems is that of Rose,[2] who examined in detail the dependence of the steady-state conditions on the levels of bremsstrahlung and synchrotron radiation, alpha particle heating, convective particle and energy losses, and details such as electron-ion rethermalization and energetic particle injection. However, it is difficult to apply these results to a specific closed system, such as a Tokamak[5] or Stellerator,[6] because of the schematic used to obtain the solutions. The steady-state conditions (such as the temperature of various plasma components) are derived from particle number and energy conservation equations, including an equation for the particle confinement time, τ_c. In general, τ_c depends on density, temperature, and system parameters. In the work of Rose,[2] however, the solution schematic is to choose a value of the electron temperature, solve for the ion temperature using one of two energy balance equations, and determine a confinement time, τ_c, such that a second energy balance equation is satisfied. It is therefore difficult to uncover the dependence of the steady-state solutions on the functional variation of τ_c with temperature and density.

In this report, we derive a set of steady-state equations that will be used to study steady-state solutions for Tokamak-like CTR systems and to examine the dependence of these solutions on system parameters.[12] The parameters of interest include β_p, the ratio of plasma pressure to poloidal magnetic field

pressure, the stability factor, q, the aspect ratio, A, and the toroidal magnetic field strength, B_T. The steady-state solutions are determined self-consistently from a set of particle and energy conservation equations together with confinement time formulas that have functional forms of particular applicability to Tokamak and/or Stellerator systems.[7,8,9,10] The solutions include restrictions on q and β_p.[9,11] In the next section, a derivation is given of the particle and energy balance equations that are used to obtain the numerical results to be presented in another report[12] and to discuss the stability of steady-state solutions in section III.

II. Basic Equations

The basic particle and energy conservation equations for an axisymmetric Tokamak plasma operating in the banana regime of neoclassical diffusion[7,8] as derived by Rosenbluth, Hazeltine and Hinton[9] (RHH) are:

a. Particle Conservation

$$\frac{\partial n}{\partial t} = -\frac{1}{r}\frac{\partial}{\partial r}(r\Gamma) \tag{1}$$

$$\Gamma = \Gamma_i = \Gamma_e = n\left(\frac{r}{R}\right)^{1/2}\left(\frac{B_T}{B_p}\right)^2 (D^e_{class})\left[-1.12\left(1+\frac{T_i}{T_e}\right)\frac{1}{n}\frac{\partial n}{\partial r} + \frac{0.43}{T_e}\frac{\partial T_e}{\partial r} + \frac{.19}{T_i}\frac{\partial T_i}{\partial r}\right]$$

$$- 2.44\,\frac{E_z n}{B_\theta}\left(\frac{r}{R}\right)^{1/2} \tag{2}$$

b. Electron energy conservation

$$\frac{3}{2}\frac{\partial(n_e T_e)}{\partial t} = -\frac{1}{r}\frac{\partial}{\partial r}\left[r\left(Q_e + \frac{5}{2}\Gamma T_e\right)\right] - \frac{\Gamma}{n}\left(T_i\frac{\partial n}{\partial r} - .17n\frac{\partial T_i}{\partial r}\right) + \frac{cE_z}{4\pi}\frac{1}{r}\frac{\partial(rB_p)}{\partial r}$$

$$- \frac{3m_e}{m_i}\frac{n}{\langle\nu_e\rangle}(T_e - T_i) \tag{3}$$

$$Q_e = \text{electron heat flow} = nT_e\left(\frac{r}{R}\right)^{1/2} D^e_{class} \left(\frac{B_T}{B_\theta}\right)^2\left[-\frac{1.81}{T_e}\frac{\partial T_e}{\partial r} - \frac{.27}{T_i}\frac{\partial T_i}{\partial r}\right.$$

$$\left. + 1.53\left(1 + \frac{T_i}{T_e}\right)\frac{1}{n}\frac{\partial n}{\partial r}\right] + 1.75\,\frac{nT_eE_z}{B_p}\left(\frac{r}{R}\right)^{1/2} \qquad (4)$$

c. Ion energy conservation

$$\frac{3}{2}\frac{\partial(n_iT_i)}{\partial r} = -\frac{1}{r}\frac{\partial}{\partial r}\left[r\left(Q_i + \frac{5}{2}\Gamma T_i\right)\right] - \frac{\Gamma}{n}\left(T_i\frac{\partial n}{\partial r} - .17n\frac{\partial T_i}{\partial r}\right) + \frac{3m_e}{m_i}\frac{n}{\langle\nu_e\rangle}(T_e - T_i)$$

$$(5)$$

$$Q_i = \text{ion heat flow} = -.68\ n\ D^e_{class}\left(\frac{m_iT_e}{m_eT_i}\right)^{1/2}\frac{\partial T_i}{\partial r} \qquad (6)$$

d. Maxwell equations

$$E_z = \frac{n}{\left[1 - 1.9\left(\frac{r}{R}\right)^{1/2}\right]}\left\{\frac{c}{4\pi}\frac{1}{r}\frac{\partial(rB_p)}{\partial r} + \frac{c}{B_p}\left(\frac{r}{R}\right)^{1/2}\left[2.44(T_i + T_e)\frac{\partial n}{\partial r} + .69\ n\frac{\partial T_e}{\partial r}\right.\right.$$

$$\left.\left. - .42\ n\frac{\partial T_i}{\partial r}\right]\right\} \qquad (7)$$

$$\frac{\partial B_\theta}{\partial t} = c\frac{\partial E_z}{\partial r} \qquad (8)$$

The notation is:

Γ = particle flux

n = particle density

r = radial coordinate measured from center of plasma

R = major radius of torus

$D^e_{class} = \langle\nu_e\rangle \rho_e^2$

B_θ = poloidal magnetic field

B_T = toroidal magnetic field

ρ_e = electron gyroradius

η = Spitzer resistivity

subscripts i,e label ions and electrons.

The assumptions that are important for the studies in this paper are that $n_i = n_e$, the plasma is hydrogenic, no alpha particles are present, there are no radiation losses, and the diffusion is in the banana regime[7,8] of neoclassical theory. The quantities $\langle\nu_e\rangle$ and ρ_e are defined as:

$$\rho_e = \frac{(2m_eT_e)^{1/2}}{eB_T} \tag{9}$$

$$\langle\nu_e\rangle^{-1} = \frac{3m_e^{1/2}\ T_e^{3/2}}{4(2\pi)^{1/2}e^4 n \ln\Lambda} \tag{10}$$

The last term in equations (3) and (5) is the electron-ion rethermalization term.

To derive equations for the study of plasma particle and energy steady-state conditions, we need specific limits of the general equations. To simplify the derivation, consider a CTR plasma system operating with a diverter. (In the systems proposed by the Wisconsin[13] and Princeton groups,[14] a poloidal field divertor is contemplated.) Regardless of type, however, when a divertor is in operation, there is a bounding field line such that any particle stepping across that field line is diverted out of the system. Thus, one expects convective energy losses to be important

but the temperature distribution to exhibit a sharp boundary at the plasma edge so that conductive energy losses are minimal. In this case, the steady-state form of equations (3) and (5) simplify to:

$$-\frac{5}{2}\frac{T_i}{r}\frac{\partial}{\partial r}(r\Gamma_i) + Q_{ei} + \frac{T_i\Gamma_i}{n_i}\frac{\partial n_i}{\partial r} = 0 \tag{11}$$

$$-\frac{1}{r}\frac{\partial}{\partial r}[r(\tilde{Q}_e + \frac{5}{2}\Gamma_e T_e)] - Q_{ei} - T_i\Gamma_e(\frac{1}{n_e}\frac{\partial n_e}{\partial r}) + E_z J_z = 0 \tag{12}$$

where

$$\tilde{Q}_e = (\frac{r}{R})^{1/2} D^e_{class}(\frac{B_T}{B_p})^2\ 1.53(1 + \frac{T_i}{T_e})\frac{T_e\partial n_e}{\partial r} + 1.75\frac{nT_eE_z}{B_p}(\frac{r}{R})^{1/2} \tag{13}$$

and

$$Q_{ei} = \frac{3m_e}{m_i}\frac{n}{\langle\nu_e\rangle}(T_e - T_i)\ . \tag{14}$$

To obtain a set of space-independent equations, we make the following heuristic identifications:

$$\frac{\partial n}{\partial r} \rightarrow \frac{n}{r_p} \tag{15}$$

$$(\frac{r}{R})^{1/2} \rightarrow (\frac{r_p}{R})^{1/2} = \frac{1}{\sqrt{A}} \tag{16}$$

$$\frac{1}{r}\frac{\partial}{\partial r} rD^e_{cl}\ \frac{\partial n}{\partial r} \rightarrow D^e_{cl}\ (\frac{n}{r_p^2}) \tag{17}$$

where r_p is the plasma radius and $A = R/r_p$ is the aspect ratio. Using

(15)-(17), equation (11) becomes

$$-\frac{3}{2}\left(1.12(1+\frac{T_i}{T_e})\frac{1}{A}(\frac{B_T}{B_p})^2 D^e_{cl}\right)\frac{n_i}{r_p^2}+Q_{ei}=0\ . \tag{18}$$

It is convenient to define

$$D^e_{ncl} = 1.12(1+\frac{T_i}{T_e})\, q^2 A^{3/2}\, D^e_{class}\cdot S \tag{19}$$

$$q = \frac{1}{A}\frac{B_T}{B_p} \tag{20}$$

where D^e_{ncl} is the neoclassical diffusion coefficient in the banana regime[7,8,9] and q is the stability factor.[10] S is a "spoiling factor" which is included so that one can investigate confinement times other than the theoretical value, S = 1. The particle confinement time is defined as

$$\tau_c = \frac{r_p^2}{D^e_{ncl}} \tag{21}$$

so that (18) becomes

$$-\frac{3}{2}\frac{n_i T_i}{\tau_c}+Q_{ei}=0 \tag{22}$$

In a similar way, one can derive the electron energy conservation equation:

$$-(\frac{5}{2}-\frac{1.53}{1.12}+\frac{T_i}{T_e})\frac{n_e T_e}{\tau_c}-Q_{ei}+E_z J_z=0\ . \tag{23}$$

Equations (20) and (21) were derived assuming no conduction losses (i.e.,

that a diverter is operative) and that the identifications (15) - (17) are reasonable. In this same spirit, we can include conductive losses using an energy confinement time, τ_E, derived as follows: Identifying the standard conduction term in equations (3) and (5) gives

$$\frac{\partial W_i}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}\left(r\kappa_i \frac{\partial T_i}{\partial r}\right) \tag{24}$$

where W_i is the ion energy flow per unit volume per unit time due to conduction. κ_i denotes the ion thermal conductivity. An equivalent equation exists for electrons. Using identifications equivalent to (15) - (17), one can derive:

$$\kappa_e = \frac{1.81 n_e}{1.12\left(1 + \frac{T_i}{T_e}\right)} D^e_{ncl} \tag{25}$$

$$\kappa_i = \frac{.68 n_i \left(\frac{m_i T_e}{m_e T_i}\right)^{1/2}}{1.12\left(1 + \frac{T_i}{T_e}\right)} D^e_{ncl} \equiv \left(\frac{.68 n_i}{1.81 n_e}\right)\left(\frac{m_i T_e}{m_e T_i}\right)^{1/2} \kappa_e \,. \tag{26}$$

Writing (24) as

$$\frac{\partial W_i}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}\left(r\kappa_i \frac{\partial T_i}{\partial r}\right) \sim \frac{\kappa_i T_i}{r_p^2} = \frac{2\kappa_i}{3 n_i r_p^2}\left(\frac{3}{2} n_i T_i\right) \tag{27}$$

we define the ion energy containment time as

$$\tau_E^i = \frac{3 n_i r_p^2}{2\kappa_i} \tag{28}$$

so that

$$\frac{\partial W_i^{\text{conduction}}}{\partial t} \sim \frac{3}{2}\frac{n_i T_i}{\tau_E^i} . \tag{29}$$

Similarly, τ_E^e is defined as

$$\tau_E^e = \frac{3n_e r_p^2}{2\kappa_e} \quad \alpha \sqrt{\frac{m_i}{m_e}}\, \tau_E^i \tag{30}$$

The proportionality in (30) is the well-known theoretical result that ions are expected to preferentially conduct energy out of the system.[15] For $n_e = n_i$, we have

$$\tau_c^i = \tau_c^e \tag{31}$$

$$\tau_e^i \quad \alpha \sqrt{\frac{m_e}{m_i}}\, \tau_E^e \tag{32}$$

and for $T_i = T_e$,

$$\tau_E^i \quad \alpha \quad \sqrt{\frac{m_e}{m_i}}\, \tau_c^i \tag{33}$$

$$\tau_E^e \simeq \tau_c^e . \tag{34}$$

Note that (28) and (30) are neoclassical definitions of the energy containment time. It will be useful to also use the pseudoclassical definition[16] of τ_E in numerical work. Using (29) and (30) in (22) and (23) yields

$$-\frac{3}{2}\frac{n_i T_i}{\tau_c} - \frac{3}{2}\frac{n_i T_i}{\tau_E^i} + Q_{ei} = 0 \tag{35}$$

$$-\left(\frac{5}{2} - \frac{1.53}{1.12} + \frac{T_i}{T_e}\right)\frac{n_e T_e}{\tau_c} - \frac{3}{2}\frac{n_e T_e}{\tau_E^e} - Q_{ei} + E_z J_z = 0 \ . \tag{36}$$

To finally derive the equations we seek, fusion reactions, alpha particles, fuel injection, and radiative losses must be introduced. Consider each in turn. Alpha particles satisfy the particle conservation equation

$$\frac{n_i^2 \langle\sigma v\rangle}{4} = \frac{n_\alpha}{\tau_c^\alpha} \ . \tag{37}$$

Let $U_{\alpha i}(T_e, n_i, n_e)$ and $U_{\alpha e}(T_e, n_i, n_e)$ be the fractions of the initial alpha energy, E_α (3.52 MeV for D-T system) deposited in the ions and electrons, respectively, as the alpha slows down.[19] Then the alpha heating terms to be included in equations (35) and (36) are:

$$\frac{n_i^2}{4} \langle\sigma v\rangle \ E_\alpha \ U_{\alpha i} \tag{38}$$

$$\frac{n_i^2}{4} \langle\sigma v\rangle \ E_\alpha \ U_{\alpha e} \tag{39}$$

$\langle\sigma v\rangle$ is the Maxwellian averaged fusion rate.[2] Injection and heating adds the term

$$\left(\frac{n_i}{\tau_c^i} + \frac{n_i^2 \langle\sigma v\rangle}{2}\right) E_0^i \ U_{ii} + \frac{n_e}{\tau_c^e} E_0^e \ U_{ei} \tag{40}$$

to the ion equation and

$$\frac{n_e}{\tau_c^e} E_0^e U_{ee} + \left(\frac{n_i}{\tau_c^i} + \frac{n_i^2 \langle\sigma v\rangle}{2}\right) E_0^i \, U_{ie} \tag{41}$$

to the electron equation. Here, E_0^i and E_0^e are the injection energies of the ions and electrons (usually $E_0^e = 0$) and

U_{ii} = fraction of ion injected energy going to ions

U_{ie} = fraction of ion injected energy going to electrons

U_{ei} = fraction of electron injected energy going to ions

U_{ee} = fraction of electron injected energy going to electrons.

Finally, radiation losses must be included. Bremsstrahlung losses from electrons are

$$W_x = A_x(n_i + z_\alpha^2 \, n_\alpha + \sum_{im} z_{im}^2 \, n_{im}) n_e T_e^{1/2} \; . \tag{42}$$

Synchrotron losses in low-β systems have only recently been given more attention[17] and the definitive study remains to be done. Indications are that for T_e <20KeV, synchrotron losses are small,but this point is under investigation.[18] We will simply include these losses symbolically as W_c. Each of these terms, i.e., alpha heating, bremsstrahlung losses, and synchrotron radiation are discussed by Rose.[2]

In CTR Tokamaks operating on a D-T fuel cycle, the particle confinement time is expected to be greater than one second and perhaps as many as 50-60 seconds. For these conditions, the alpha slowing down time is short compared to the confinement time and the alpha particles can be expected to fully thermalize.[19] It can be shown that this also implies $T_\alpha \simeq T_i$ to

a very good approximation. With this, the final set of particle and energy conservation equations are:

a. Particle conservation

$$n_e = n_i + 2n_\alpha + \sum_{im} Z_{im}\, n_{im} \tag{43}$$

$$n_\alpha = \left(\frac{\tau_c^\alpha}{\tau_c^i}\right) \frac{n_i X_i \langle\sigma v\rangle}{4} \tag{44}$$

$$X_i = n_i\, \tau_i \tag{45}$$

b. Ion energy conservation

$$\frac{n_i^2\langle\sigma v\rangle}{4} E_\alpha\, U_{\alpha i}(T_e;n_i;n_e) + \left(\frac{n_i}{\tau_c^i} + \frac{n_i^2\langle\sigma v\rangle}{2}\right)E_o^i\, U_{ii} + \frac{n_e}{\tau_c^e} E_o^e\, U_{ei}$$

$$- \left(\frac{1}{\tau_c^i} + \frac{1}{\tau_E^i}\right)\left(\frac{3n_i T_i}{2}\right) + Q_{ei} = 0 \tag{46}$$

c. Electron energy conservation

$$\frac{n_i^2\langle\sigma v\rangle}{4} E_\alpha\, U_{\alpha e} + \frac{n_e}{\tau_e} E_o^e\, U_{ee} + \left(\frac{n_i}{\tau_c^i} + \frac{n_i^2\langle\sigma v\rangle}{2}\right)E_o^i\, U_{ie} + E_z J_z - \left(\frac{5}{2} - \frac{1.53}{1.12} + \frac{T_i}{T_e}\right)\frac{n_e T_e}{\tau_c^e}$$

$$- \frac{3}{2}\frac{n_e T_e}{\tau_E^e} - Q_{ei} - A_x\left(n_i + 4n_\alpha + \sum_{im} Z_{im}^2\, n_{im}\right) n_e\, T_e^{1/2} - W_c = 0\,. \tag{47}$$

III. Stability

Equations (46) and (47) can be used to determine the stability classification of the steady-state operating point. Mills[20] has argued from simpler equations that Bohm diffusion will lead to a stable operating point when T is

between 7 and 28 KeV. However, in recent studies Mills indicates, using an analysis based on a generalized Lawson criterion, that all temperatures below approximately 30 KeV correspond to unstable operating conditions.[21] Ohta, Yamato, and Mori[22] have analyzed plasma energy stability for various temperature dependencies of τ_c and τ_E, and conclude that neoclassical diffusion in the banana regime will lead to unstable operating conditions. Our conclusions, in general, confirm those of Ohta, Yamato, and Mori.

The simplest way to uncover the governing physics is to add equations (46) and (47), and assume $T_i = T_e$, zero injection energy, and negligible ohmic heating. Then we find

$$\frac{n_i^2 \langle\sigma v\rangle}{4} E_\alpha - \left(\frac{1}{\tau_c^i} + \frac{1}{\tau_E^i}\right)\left(\frac{3n_i T}{2}\right) - \left\{\frac{\left(\frac{7}{2} - \frac{1.53}{1.12}\right)}{\tau_c^e} + \frac{1}{\tau_E^e}\right\} \frac{3n_e T}{2}$$

$$-A_x(n_i + 4n_\alpha + \sum Z_{im}^2 n_{im})\, n_e \; T^{1/2} - W_c = 0 \; . \qquad (48)$$

The temperature dependence of the gain term is that of $\langle\sigma v\rangle$ which is shown in figure 1 for the D-T reaction. In the temperature range of greatest interest for CTR Tokamaks (8-30 KeV), the fusion reaction rate is linear in T_i. On the other hand, in the banana regime of neoclassical diffusion,[7,8] regardless of the spoiling constants, S, the temperature dependence of τ_c and τ_E is $T^{1/2}$. Therefore, energy loss by particle convection, conduction, and bremsstrahlung all vary as $T^{1/2}$ and, neglecting synchrotron radiation, the operating point is unstable. A typical plot of gains and losses illustrating this is shown in figure 2. The conclusions are readily verified analytically by performing a standard stability analysis on the equilibrium point. Clearly, since

$\langle\sigma v\rangle_{D-T}$ eventually peaks and begins decreasing, there is a second stable point at higher temperatures (>50KeV). However, the neglect of W_c can no longer be justified in this temperature regime.

If particle losses are Bohm-like or if $D_{ncl} \alpha T^{+3/2}$, as in the plateau regime,[10] or if we have a bumpy torus,[10,23] both stable and unstable points at low temperatures can be obtained. The details depend on the magnitude of these loss terms compared to bremsstrahlung losses and the various possibilities are illustrated schematically in figures 3 and 4. Importantly, a transition from banana to plateau, or maintaining the machine in the plateau regime, can result in a stable operating point at lower temperatures. Furthermore, synchrotron losses have not been included in the above discussion. However, since these losses are generally predicted to have a strong temperature dependence,[2,17] synchrotron radiation can help produce a stable operating temperature below approximately 30KeV <u>if</u> the radiation gets out at these "low" temperatures in low β systems.

The implication of all these reamrks is that <u>if</u> particle and energy confinement times vary as $T^{1/2}$ (banana regime of neoclassical diffusion), low temperature (<30DeV) steady-state operating points will be energetically unstable. Operation at such a point will therefore require a system for feedback stabilization.

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Figure Captions

Figure 1 The fusion rate versus temperature for the D-T reaction.[2] From 7KeV to approximately 30KeV, $\langle\sigma v\rangle_{D-T}$ is linear in T.

Figure 2 Plasma energy gains and losses versus T. The losses include bremsstrahlung radiation, and conduction and convection energy losses. In the banana regime,[7,8] each loss term varies as $T^{1/2}$. The equilibrium point is unstable.

Figure 3 a. Variation of the electron diffusion coefficient with collision frequency for an axisymmetric torus.[7,8,10]

b. Variation of the electron diffusion coefficient versus collision frequency in a bumpy torus[23,10] (radially uniform corrigation in the toroidal magnetic field).

Figure 4 Schematic illustration of the effects of adding to the bremstrahlung loss other loss mechanisms such as synchrotron radiation, Bohm losses, or losses related to τ_C and τ_E when these are for the plateau regime[7,8] or a bumpy torus.[23,10] The upper equilibrium point is stable in each of these cases.

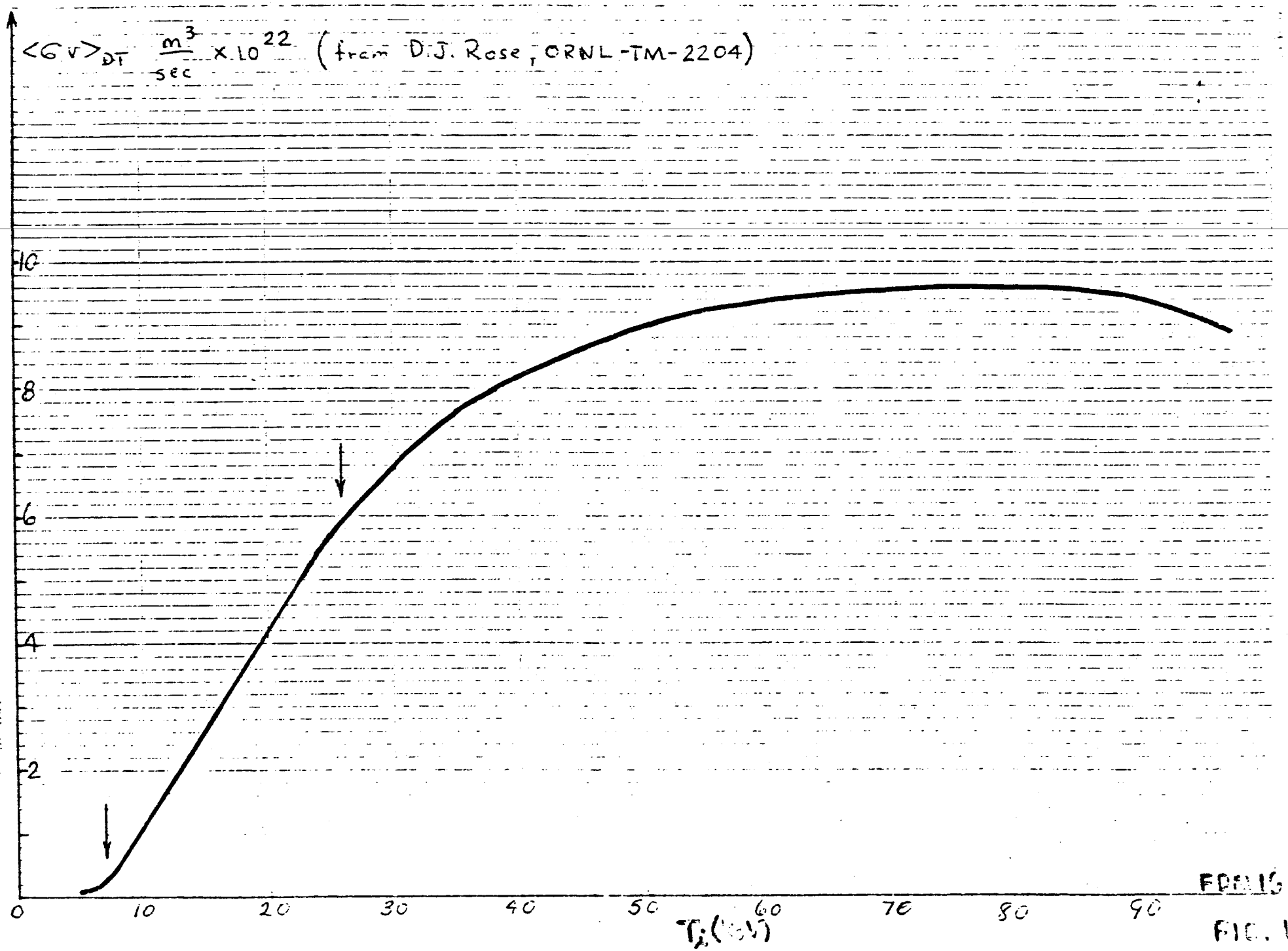
$\langle \sigma v \rangle_{DT}$ $\frac{m^3}{sec} \times 10^{22}$ (from D.J. Rose, ORNL-TM-2204)
10
8
6
4
2
0
10
20
30
40
50
60
70
80
90
T_i (keV)

FIG. 1

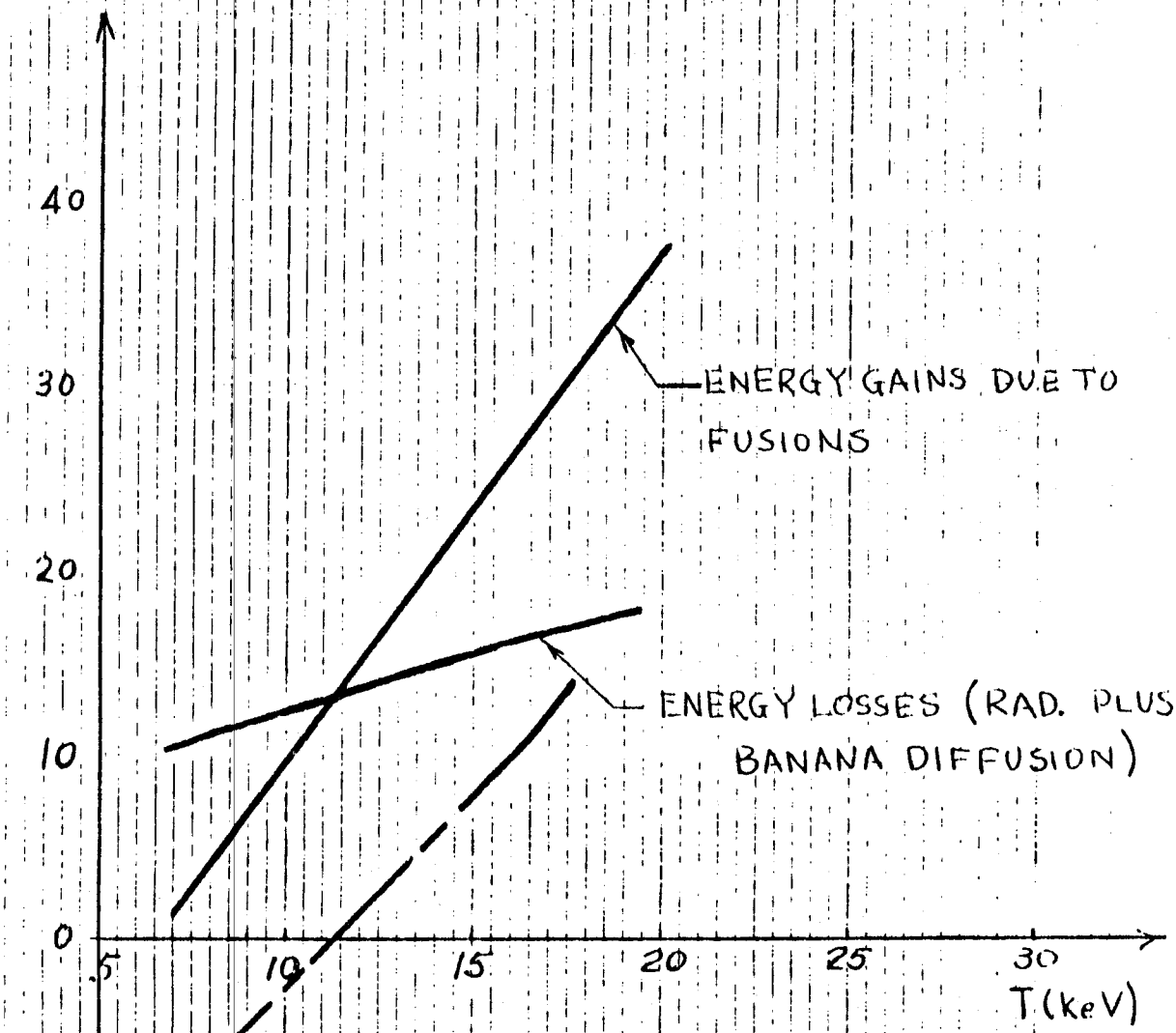
40
30
20
10
0
-10
-20
5
10
15
20
25
30
T (keV)
ENERGY GAINS DUE TO FUSIONS
ENERGY LOSSES (RAD. PLUS BANANA DIFFUSION)
NET ENERGY BALANCE ON PLASMA

FIG. 2

FDM 16

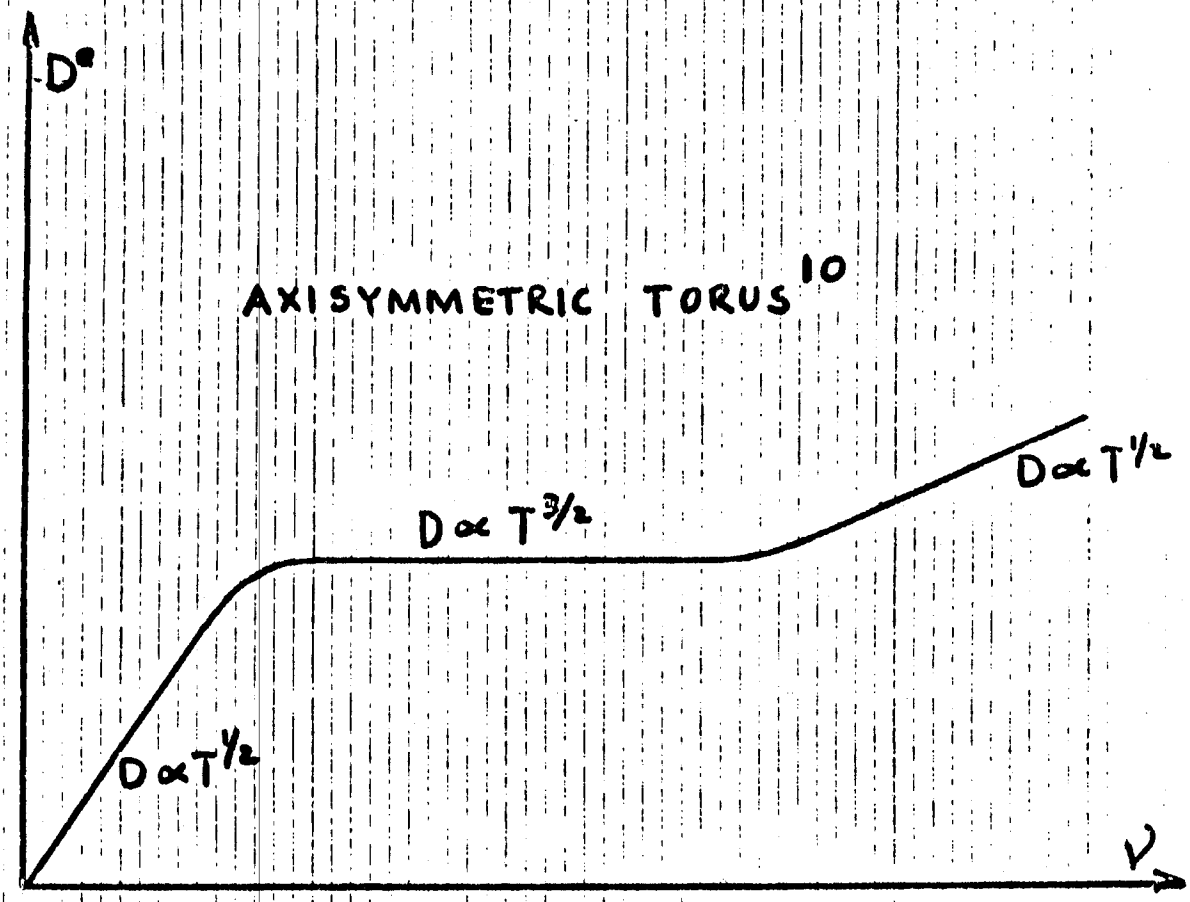
D*
AXISYMMETRIC TORUS 10
D ∝ T^{3/2}
D ∝ T^{1/2}
D ∝ T^{1/2}
ν

FIG. 3a

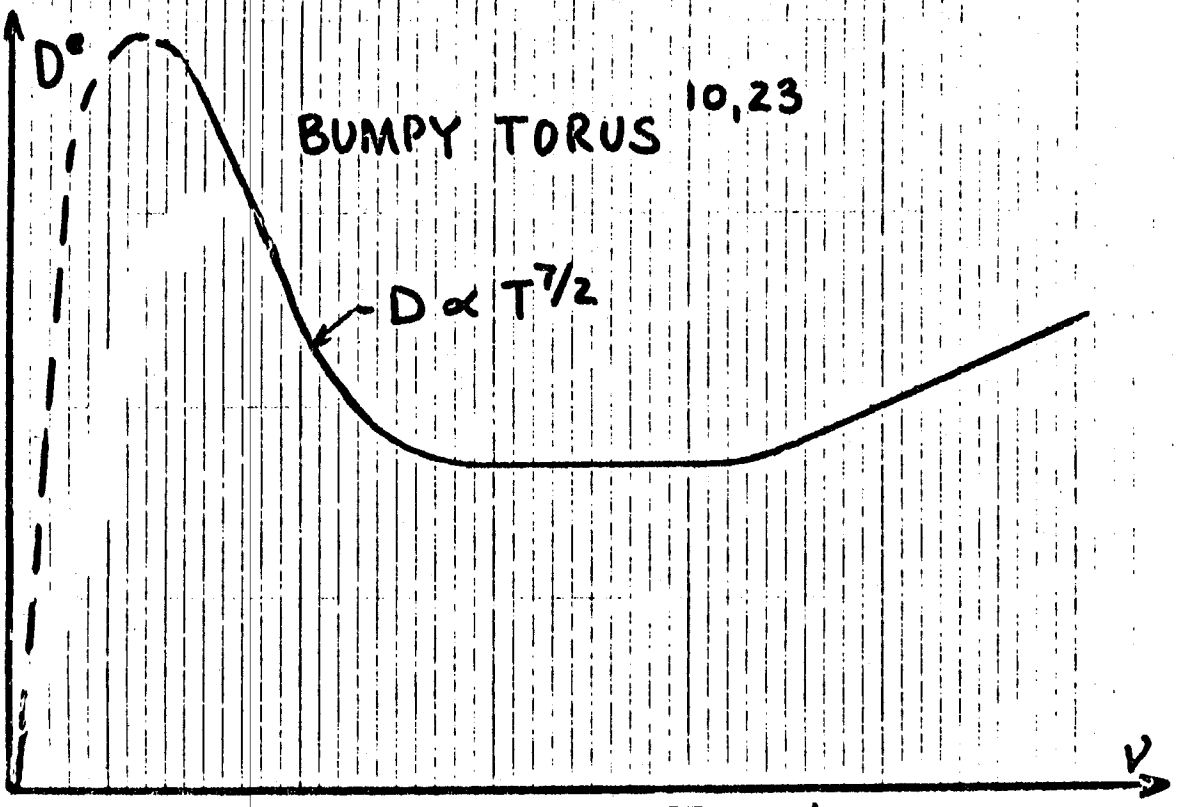
D*
BUMPY TORUS 10,23
D ∝ T^{7/2}
ν

FIG. 3b

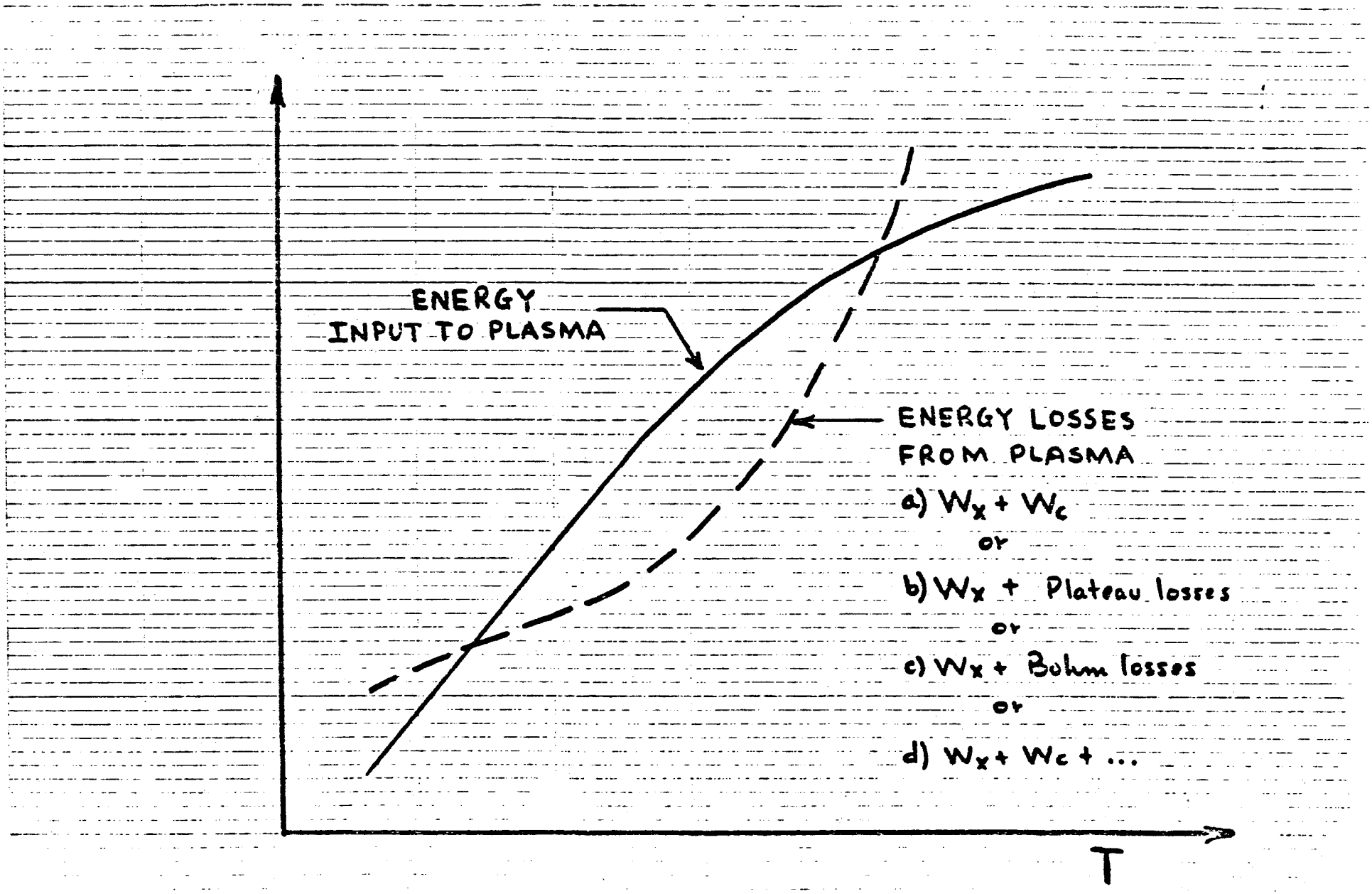
ENERGY
INPUT TO PLASMA
ENERGY LOSSES
FROM PLASMA
a) $W_x + W_c$
or
b) W_x + Plateau losses
or
c) W_x + Bohm losses
or
d) $W_x + W_c + \ldots$
T

FIG. 4. FDM 16