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UWFDM-147

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Summary

Stability and temperature maxima are determined for exposed, uncooled lengths of cryogenically stable composite superconductor-copper magnet turns. This problem corresponds for example, to the accidental depression of the liquid helium level in a TF, OH or VH fusion reactor magnet which is cooled by pool boiling, so that the upper turns are above the liquid helium level and cooled only by helium vapor. It is shown that a temperature excursion results in the following final states as a function of the exposed length ℓ: (1) for $\ell \leq .9\ell_c$, where ℓ_c is a critical length, a stable solution with a high maximum temperature, T_M, can exist via end cooling, (2) for $\ell > .9\ell_c$ unstable conditions would exist and $T_M \to \infty$ and (3) complete recovery takes place if $T_M < T_s$. The saturation current T_s is defined here as the temperature below which the superconductor can carry the total transport current. End cooling and current sharing are the two basic physical phenomena considered.

Introduction

Pool cooling with liquid helium is usually suggested for cooling superconducting Tokamak fusion reactor magnets. To be an effective method the liquid helium must maintain good thermal contact with the conductor. For various reasons a length of conductor may lose contact with liquid helium. We have examined the effect this will have on the stability of the magnet. The analysis that follows is for the UWMAK-II[1] design, but most of the results are applicable to other cryogenically stable magnets.

A length of conductor may lose contact with liquid helium bath under a variety of conditions. For instance the helium level may drop exposing some of the winding as illustrated in Fig. 1(a), or a vapor lock may occur in one of the cooling channels. A pressure increase due to the flow of helium gas can possibly depress the helium level in a poloidal field coil as illustrated in Fig. 1(b). This last phenomenon is discussed in more detail elsewhere.[2,3]

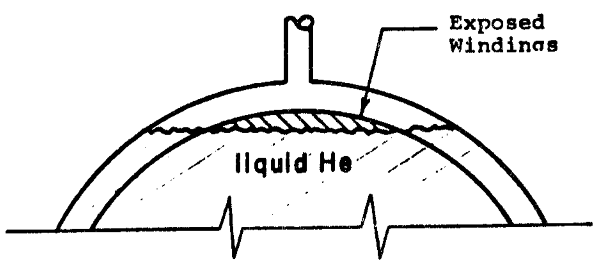


Fig. 1(a). Top of a toroidal field coil.

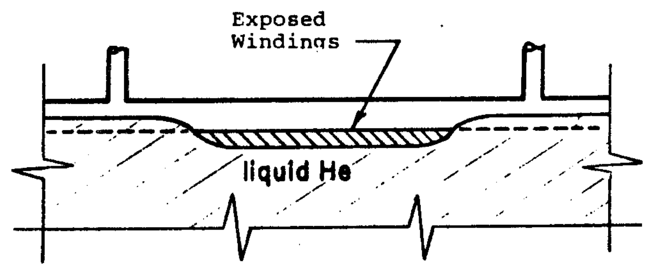


Fig. 1(b). Top of a poloidal field coil.

When a length of conductor loses contact with liquid helium the total current can still be carried in the superconductor at bath temperature. This is true as long as heat losses to the conductor are negligible. In the following sections we study the other possible steady states of the conductor. Our major concern is to examine the stability of these states.

Definition of the Problem

A schematic diagram of an exposed length of the conductor, ℓ, is shown in Fig. 2. The rest of the conductor is submerged in liquid helium.

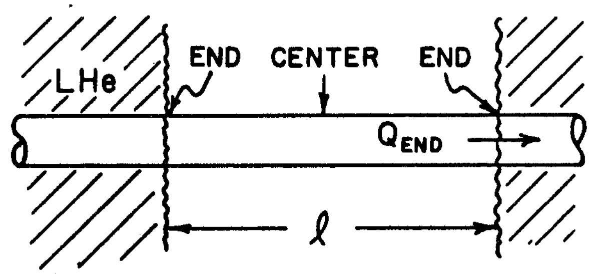


Fig. 2. Schematic drawing of the exposed part of the conductor.

The conductor is a composite consisting of many fine superconducting filaments embedded in copper. The cross sectional dimensions of the composite are assumed to be small compared to the exposed length so that the temperature gradient across the cross section can be neglected. This assumption reduces the problem to one dimension.

The one-dimensional steady state heat diffusion in the conductor is governed by the following equations:

$$\frac{dQ}{dx} = \frac{I_o^2 \rho}{A} \left(\frac{I}{I_o}\right) - qS \qquad (1)$$

and

$$Q = -kA \frac{dT}{dx} \qquad (2)$$

where

- Q = rate of heat transfer by conduction across A in watts
- x = distance measured along the length of the conductor, measured from the center in cm
- I_o = total current carried by conductor in amperes
- ρ = electrical resistivity of copper stabilizer in Ω cm
- I = current carried by copper stabilizer in amperes
- A = cross sectional area of copper stabilizer in cm^2
- q = surface heat flux in W/cm^2
- S = lateral surface area cooled per unit length, cm^2/cm
- k = thermal conductivity of copper in W/cm K
- T = temperature of conductor at position x in °K.

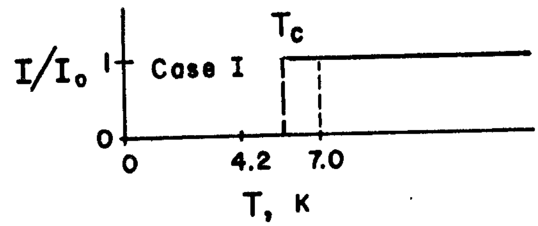
The first term on the right side of Eq. (1) is the rate of ohmic heating per unit length. This term includes the ohmic heating due to the resistance of the copper and the superconductor. The second term is the rate of surface cooling per unit length.

The heat diffusion equation is solved for two cases. Case I is the non-physical example for a fixed end temperature T_E = 4.2 K while Case II is the real case obtained by matching heat transferred from the submerged conductor to the liquid coolant with the incoming heat from the exposed length after accounting for local power generated in the matrix and resistive superconductor filaments. Case I is included because it leads to an analytical solution. Case II is a good representation of the real situation, but requires a numerical solution.

The current in the copper as a function of temperature is plotted in Fig. 3 for both cases. We assume that the exposed length of the conductor is in a low magnetic field. In the real situation, Case II, all the current is carried by the superconductor for temperatures below a certain limiting value which is defined as the saturation temperature, T_s. Above the saturation temperature, but below the critical temperature, the superconductor is unable to carry all the current because of its elevated temperature. For Case II we assume that the amount of superconductor is chosen; as in the UWMAK-II design, so that the superconductor saturates at 5.2 K. The critical temperature chosen, 9 K, is that of NbTi in a low magnetic field.

In general I_o, A, S, ρ and k are specified. For cryogenically stable conductors the following equation is satisfied:

$$\frac{I_o^2 \rho_o}{A} = q_o S \qquad (3)$$



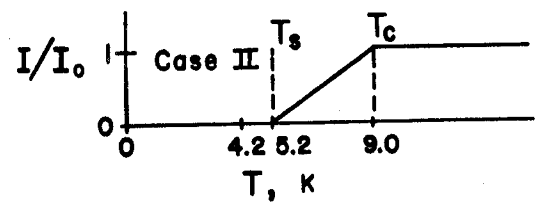


Fig. 3. Current carried in copper stabilizer vs. temperature of conductor.

The design value of heat flux, q_o, is taken as 0.4 W/cm^2, the same value used in the UWMAK-II design. Furthermore, OFHC copper of resistivity $\rho_o = 10^{-8}$ Ωcm is used. The thermal conductivity is determined from the Wiedmann-Franz law:

$$\frac{\rho k}{T} = L, \qquad (4)$$

where L is the Lorenz number.

<u>Solution for Case I</u>

This case is described by the following assumptions:

1. The end temperature of the exposed length, T_E, is held fixed and equal to 4.2 K.
2. Surface cooling, qS, in the exposed region is taken as zero.
3. The supercurrent is taken as $I_{sc} = I_o$ for $T < T_c$ and $I_{sc} = 0$ for $T \geq T_c$, see Fig. 3(a). There is no current sharing. The temperature T_c = 7K is used for numerical examples.

Let the origin of the x-coordinate be located at the center of the exposed conductor. Using Eq. (4) and eliminating the variable x from Eqs. (1) and (2) the following equation for $|x| < \ell/2$ is obtained,

$$-Q \frac{dQ}{dT} = I_o^2 L \left(\frac{I}{I_o}\right) T \qquad (5)$$

At the center of the exposed length Q is zero and the temperature is maximum. Hence, by integrating Eq. (5) we obtain

$$Q^2 = \begin{cases} I_o^2 L(T_M^2 - T^2), & T_M \geq T \geq T_c \\ I_o^2 L(T_M^2 - T_c^2), & T_c \geq T \geq T_E \end{cases} \quad (6)$$

where T_M is the maximum temperature. Note that Q is zero for $T_M < T_c$. The rate of heat transfer at the end of the exposed conductor, Q_{END}, is a relevant quantity to be determined and is given by

$$Q_{END} = I_o \sqrt{L\ (T_M^2 - T_c^2)} \quad (7)$$

If we substitute Eq. (6) into Eq. (2) and integrate assuming k is constant we obtain

$$\frac{\ell}{2} = \frac{\bar{k}A}{I_o\sqrt{L}} \left[\cos^{-1}\left(\frac{T_c}{T_M}\right) + \frac{T_c - T_E}{\sqrt{T_M^2 - T_c^2}}\right] . \quad (8)$$

Equations (7) and (8) enable us to find Q_{END} as a function of the exposed length ℓ by treating T_M as an independent variable. The results are plotted in Fig. 4 using the following:

$$\frac{Q_{END}}{Q_o} = \sqrt{\left(\frac{T_M}{T_B}\right)^2 - \left(\frac{T_c}{T_B}\right)^2} \quad (9)$$

$$\frac{\ell}{\ell_c} = \frac{2}{\pi} \left[\cos^{-1}\left(\frac{T_c}{T_M}\right) + \frac{T_c - T_E}{\sqrt{T_M^2 - T_c^2}}\right] \quad (10)$$

where, for copper at B = 0 tesla,

$\rho = \rho_o = 10^{-8} \Omega$ cm

$L = \rho\bar{k}/T_B = 2.4 \times 10^{-8}$ WΩ/K^2

$\bar{k}$ = 10 W/cmK (constant)

T_B = 4.2 K, bath temperature

and

$$Q_o = I_o T_B \sqrt{L} = 6.5 \times 10^{-4}\ I_o, \text{ watts} \quad (11)$$

$$\ell_c = \frac{\pi\bar{k}}{\sqrt{L}} \frac{A}{I_o} = 2 \times 10^5 \frac{A}{I_o}, \text{ cm.} \quad (12)$$

The constant ℓ_c is of particular importance since no stable steady state solution exists for an exposed length ℓ greater than or equal to ℓ_c. For this reason ℓ_c is called the "critical length."

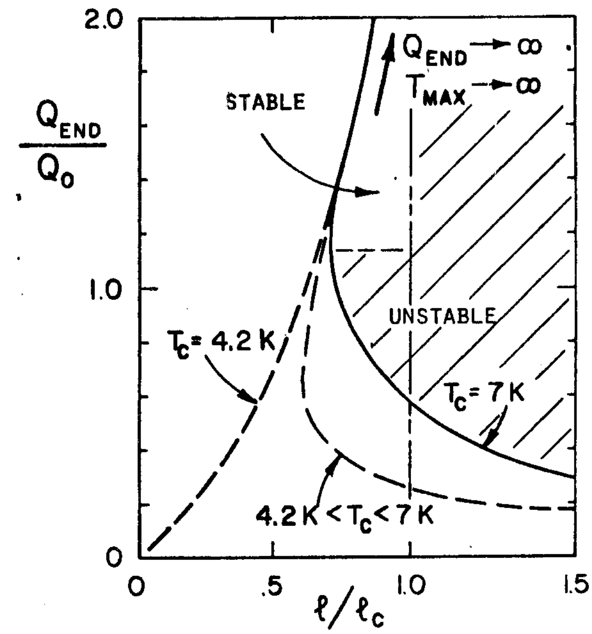


Fig. 4. Stability plot of conduction heat flux vs. exposed length for T_E=4.2K.

In general there are three kinds of steady state solutions to Eq. (1).

1. If the conductor were to completely recover after a transient heat load, we would have the trivial solution T=4.2 K everywhere.
2. The conductor may not recover but instead achieve a stable temperature profile with a maximum temperature greater than 4.2 K. This second steady state solution would correspond to a situation where there is sufficient additional end cooling available to return the conductor to its steady state whenever it is perturbed by a small heat load.
3. If an unstable steady state temperature profile were obtained, this would correspond to a situation where no additional end cooling is available.

For T_c = 7 K each of the three kinds of steady state solutions are possible. For an exposed length less than about .7 ℓ_c complete recovery to 4.2 K occurs. Between .7 ℓ_c and ℓ_c a stable and an unstable steady state solution exist. For $\ell > \ell_c$ there are unstable solutions only. Note that for $T_B = T_E = T_c$ = 4.2 K we obtain

$$Q_{END} = Q_o \tan\left(\frac{\pi}{2}\ \ell/\ell_c\right)$$

and

$$T_{MAX} = \frac{T_{END}}{\cos\left(\frac{\pi}{2}\,\ell/\ell_c\right)} .$$

These two formulas indicate how the end cooling and the maximum temperature of the conductor vary with the exposed length. Both Q_{END} and T_{MAX} approach infinity as ℓ approaches ℓ_c. In the UWMAK-II design the conductor current density I_o/A is approximately 5×10^3 Amp/cm^2 for the toroidal field coils and 2×10^3 Amp/cm^2 for the poloidal field coils. These values yield 40 cm and 100 cm respectively for the critical lengths.

Solution for Case II

From Eqs. (1), (2), (3) and (4) we obtain

$$-Q\,\frac{dQ}{dT} = I_o^2 LT\left[\left(\frac{I}{I_o}\right) - \frac{\rho_o}{\rho}\left(\frac{q}{q_o}\right)\right] \qquad (13)$$

where $q_o = 0.4$ W/cm^2. Equation (13) is valid for the submerged conductor as well as the exposed part. For the submerged part of the conductor q is determined by the heat transfer characteristics of liquid helium. For the exposed part of the conductor q is determined by the heat transfer characteristics of helium vapor.

Figure 5 indicates the empirical correlations which are assumed for pool boiling and free-convection vapor cooling.[4,5] These correlations are for vertical metal surfaces and helium at one atmosphere pressure. Two possible values of the heat transfer flux in the film boiling region, q_F, are considered as shown in Fig. 5.

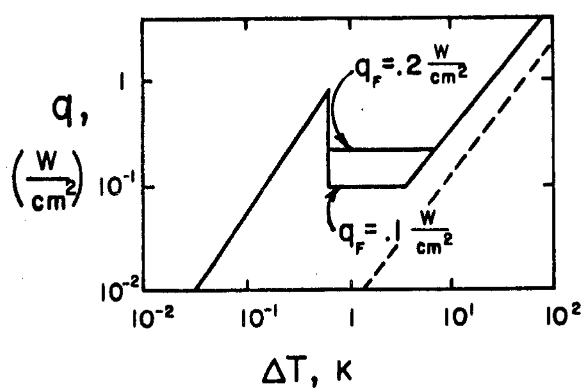


Fig. 5. Heat transfer flux vs. temperature difference between metal surface and 4.2 K helium. Legend: —— pool boiling and --- free-convection vapor cooling.

Equation (13) was solved with $\rho = \rho_o$ for each of the two regions separately and the steady state solutions obtained by requiring that Q be continuous across the boundary between the two regions. The heat transfer rate by conduction out the end of the exposed length is determined by solving Eq. (13) for the submerged conductor. The submerged conductor is assumed to be infinitely long. Thus, the total cooling capacity provided by the submerged conductor, for one end of the exposed length, is found by integrating dQ/dT from T = 4.2 K, the temperature at an infinite distance from the exposed length, to $T = T_{END}$, the temperature at the end of the exposed length. The results of this integration are shown in Fig. 6. The two curves correspond to different assumed values for the heat transfer flux in the film boiling region. Each curve rises to maximum value and falls to zero as the temperature at the end of the exposed conductor increases. This is due to the fact that the normal zone of the superconductor moves into the submerged part of the conductor as the end temperature increases. Thus, ohmic heating occurring in the submerged part of the conductor increases until $Q_{END} = 0$. This mechanism limits the amount of end cooling that can be provided by the liquid helium.

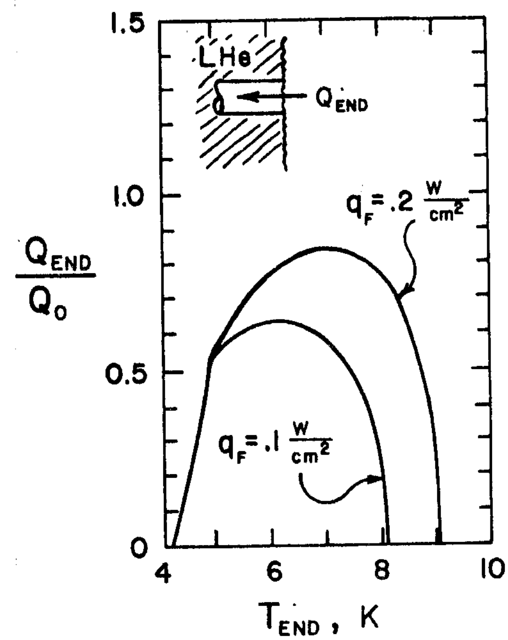


Fig. 6. Heat removed at each end of the exposed length vs. exposed length as limited by the boiling characteristics of helium.

The normalized heat removed at the end of the exposed length is plotted as a function of the normalized length for Case II in Fig. 7. The heat removed at the end is maximum if $\ell = .9\ell_c$. If the length of the exposed part is longer than $.9\ell_c$ then the steady state solution is unstable. A thermal disturbance for lengths > $.9\ell_c$ leads to higher heat generation which results in a temperature T_M that will continue to rise, and eventually exceed the maximum heat removal capability at the ends. As expected the maximum heat removal at the end occurs at

$\ell < \ell_c$. In Fig. 7, we also show the normalized heat removal at the end for Case I as a function of the normalized length, for $T_c = 4.2$, fixed end temperature $T_E = 4.2$ K and zero surface cooling. In this Case I example it can be seen that the temperature goes to infinity at $\ell/\ell_c \Rightarrow 1$. This may be explained physically as follows: the heat generated in the exposed length requires a certain temperature difference in order to transfer heat at the end. Beyond a critical exposed length the total heat generated cannot be transferred which results in a temperature rise to infinity.

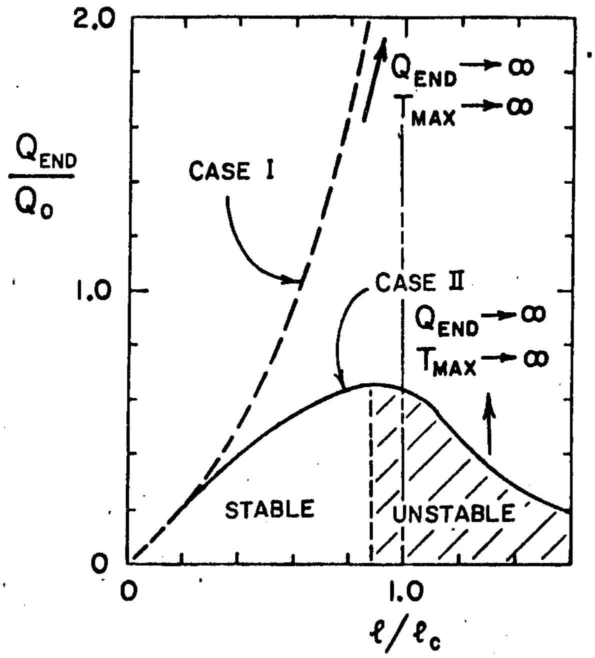


Fig. 7. End heat removal vs. temperature at the end of the exposed length.

Conclusion

For an exposed cryogenically stable conductor of length ℓ the following can be concluded: (1) If a transient heat load is sufficiently small and diffuse so as not to raise the maximum temperature of the superconductor above its saturation temperature, T_s, the highest temperature below which the superconductor can carry all the current, then the temperature of the conductor will return to the ambient temperature of the helium. (2) We predict that a cryogenically stable conductor will not recover from a transient heat load occurring in a length of conductor ℓ which has lost contact with the liquid helium if T_{MAX} exceeds T_s. (3) Moreover, if that length is greater than a critical length ℓ_c, the maximum temperature can increase indefinitely.

Acknowledgements

The authors wish to thank Professor R.W. Boom and Dr. R.W. Moses for their helpful criticisms. In particular, we wish to thank Professor R. W. Boom for his aid and encouragement. This work was supported by the Wisconsin Alumni Research Foundation as part of the Wisconsin Superconductive Energy Storage Project.

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