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November 1975

UWFDM-146

Presented at the Sixth Symposium on Engineering Problems of Fusion Research, San Diego CA, 18–21 November 1975.

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APPARENT DIFFICULTIES ASSOCIATED WITH HIGH MAGNETIC FIELD
LARGE TOKAMAK POWER REACTOR DESIGNS[†]

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Summary

Optimum fusion reactor sizes and magnetic fields have been found for neutron wall loadings between 0.625 and 5 MW/m² for reactor power levels between 1000 and 20,000 MW thermal. It is shown that higher neutron wall loading requires higher magnetic fields, and that there is little incentive to exceed 10 tesla maximum in large, circular plasma Tokamaks. The designs are scaled and normalized from UWMAK-I and II using $\beta_p = A$ and $q = 2.45$. Only circular plasmas are considered. The conclusions are: that the overall cost of a complete power reactor station is almost independent of the magnetic field, that the optimum use of magnetic fields above 10 T requires wall loading above 2.5 MW/m², and that the highest fields are needed for the lowest power reactors. Attempts to design 1000 and 2000 MW_{th} units show that these must be limited to low wall loading.

Introduction

The fact that the fusion power density of a Tokamak reactor plasma varies with the magnetic field as B_T^4 often leads one to believe that economic power reactors require high field superconducting magnets. However, when constraints on factors such as neutron wall loading, transformer core space, magnet winding space and blanket-shield thickness, are taken into account one finds that a "high field" does not necessarily provide an economic reactor.

It has been shown earlier in the UWMAK-I studies¹ that the transformer core space constraint will increase the cost of a reactor and introduce an aspect ratio dependence on magnetic field strength. The latter characteristic is even more pronounced since the magnet winding space increases with increasing magnetic field. The present study uses UWMAK-II, a University of Wisconsin designed Tokamak power reactor,² as the

reference design for analyzing the effects of changing the maximum toroidal magnetic field. It emphasizes the neutron wall loading limitation by designing reactors for a given wall loading that are consistent with other constraints.

In order to produce valid comparisons it is required that the best possible parameters be selected for each case. We have attempted to achieve this impartiality by the following sequential procedure: (1) the total thermal power and neutron wall loading are selected, (2) a consistent set of major and minor plasma radii are found, (3) the only magnetic field which fills the plasma space at the proper power level is determined, (4) the toroidal field (TF) coil inner radius and maximum field is found by assuming 1.5 meters for the blanket and shield thickness and 110% the plasma radius for the first wall radius, (5) the radial thickness of the TF coils at the inner perimeter is determined by stress considerations scaled from UWMAK-II and (6) the minimum core radius of the ohmic heating (OH) coils is found as determined by the plasma inductance and current. The (OH) coils are assumed to be 0.64 m thick. The economic choice in each case is simply to select the smallest reactor which possesses adequate core space for the OH coils.

Scaling With a Given Wall Loading

The fusion thermal power of a Tokamak reactor, P_{th} , depends on the toroidal magnetic field, B_T , at the plasma axis according to:

$$P_{th} \sim \frac{a^2 R_T}{A^4} \frac{\beta_p^2}{q} B_T^4, \quad (1)$$

where a is the plasma minor radius, R_T is the major radius, and A is the aspect ratio R_T/a , see Fig. 1. The plasma poloidal beta and stability factor are taken as $\beta_p = A$ and $q = 2.45$. With these values and normalizing to UWMAK-II, the expression for thermal power becomes:

$$P_{th} = C_1 \frac{a^4}{R_T} B_T^4. \quad (2)$$

Although C_1 depends on β_p , q , and plasma temperature, we assume C_1 is determined at the optimum temperature and take it to be a constant.

The first wall neutron loading p_{nw} is related to P_{th} as

$$p_{nw} = C_2 \frac{P_{th}}{a_w R_T}, \quad (3)$$

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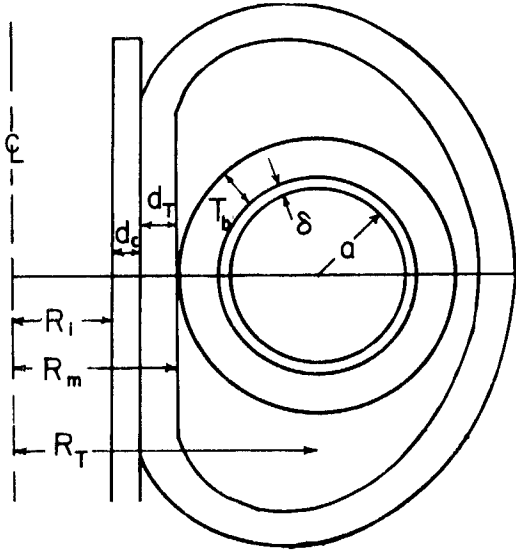


Fig. 1. Tokamak sketch.

where C_2 is a geometric constant, and a_w is the first wall minor radius which is

$$a_w = a + \delta, \quad (4)$$

where δ is the plasma scrape-off zone. It is seen from Fig. 1 that

$$R_m = R_T - (a + \delta + \tau_b), \quad (5)$$

and

$$B_m R_m = B_T R_T, \quad (6)$$

where τ_b is the blanket-shield thickness and B_m the maximum TF field at radius R_m .

Combining Eqs. (1-6) one gets the following two equations which are an aid in making the calculations:

$$a R_T = \frac{C_2}{1.1} \frac{P_{th}}{P_{nw}} \quad (7)$$

$$B_T = \frac{1.1}{C_2 (C_1)^{.25}} \frac{P_{nw}}{P_{th}^{.25}} R_T^{1.25} \quad (8)$$

where it is assumed that $a + \delta = 1.1a$. Using UWMAK-II parameters converted to the above conditions, we use for the normalization of this study: $P_{th} = 5000 \text{ MW}_{th}$, $B_T = 3.67 \text{ T}$, $R_T = 13 \text{ m}$, $a = 5 \text{ m}$, $B_m = 8.30 \text{ T}$, $R_m = 5.75 \text{ m}$, $\delta = 0.1a$, $\tau_b = 1.5 \text{ m}$ and $P_{nw} = 1.25 \text{ MW/m}^2$ from which $C_1 = 0.573$ and $C_2 = 0.01788$. Constants C_1 and C_2 , $\delta = 0.1a$ and $\tau_b = 1.5 \text{ m}$ are used throughout this study.

Constraints

A Tokamak reactor requires a primary coil to generate and sustain the ohmic

heating loss of the plasma current. Core space must be provided for the transformer winding. For an air-core superconducting winding the minimum inner radius, R_i of the core, can be estimated from

$$\Delta\Phi = \Delta B \pi R_i^2 = 2L_p I_p, \quad (9)$$

where:

$2L_p I_p \equiv$ flux change, half for startup and half for burn

$L_p = \mu_o R_T (\ln 8A-1.75)$, plasma ring inductance

$I_p = B_T \frac{2\pi a}{\mu_o q A}$, plasma current

$\Delta B = 2B_{oh}$, positive to negative field change

$B_{oh} = B_{max}$ in core, OH coils.

The core field B_{oh} is taken as 4 T, which is a practical value for superconducting ac pulsed field coils.

The structural support plays a vital limiting role on the design of magnet systems for Tokamak power reactors. The total magnet winding cross section is usually designed to be large enough to keep the hoop stress below a certain value and the bending stresses near zero. The forerunner of such constant tension systems was introduced by File, Mills and Sheffield.³ A critical dimension, the toroidal winding thickness, d_T , is determined by the total tensile load, by the stress level, and by "crowding" at R_m . By means of the virial theorem, which relates structural mass to electromagnetic stored energy E_s , we get an approximate relation:

$$d_T = C_3 \frac{E_s}{R_m (R - R_m)}, \quad (10)$$

where $C_3 = 5.835 \times 10^{-10}$ in UWMAK-II, a tightly packed "crowded" design. On the other hand the core transformer winding thickness, d_c , will be assumed to remain constant at 0.64 m to produce 4 T. The OH coil current density is taken as 1000 A/cm^2 for one half the available space or 500 A/cm^2 average to allow sufficient area for a load bearing solid contact with the TF coil structure.

Referring to Fig. 1,

$$R_i = R_T - (a + \delta + \tau_b + d_c). \quad (11)$$

In reference (1) it was assumed that the cost of a basic reactor is proportional to the magnet cost which is proportional to the energy stored in the magnetic field. Further cost estimates for total systems have shown that the nuclear island represents about one third of a power generator system cost and that the entire magnet

system plus refrigeration and power supplies costs about 50% of the nuclear island cost. The TF coils account for 75% of the magnet cost. Therefore, approximately, the TF coils account for about 12% of a 5000 MW_{th} station cost. The major overall scaling law is that many of the non-magnet system components favor larger power levels, often scaling as Pⁿ, with n < 1.

Within this framework we can still determine optimum selection of the TF coil size and field by determining relative cost/power values which are normalized to UWMAK-II. Those costs were: TF coils 162 M, OH + Diverter 20.8 M, refrigeration 19.3 M and power supplies 32.7 M equal to 235 M total for 5000 MW_{th}. We assume the TF coil cost is proportional to:

$$E_s = \frac{2\pi^2}{\mu_0} B_m^2 R_T^3 (1-\alpha)^2 (1-\sqrt{1-\alpha^2}) \text{ joules} \quad (12)$$

where: $\alpha = \frac{R_T - R_m}{R_T}$, approximate toroidal magnet aspect ratio

$E_s \sim$ stored energy for uniformly wound toroid, moderately equivalent to the energy in a "D" sectored TF system.

Results and Discussion

In Table I are listed some of the key parameters for each design which is characterized by a prior choice of P_{th} and p_{nw}. Each listing is an optimum choice. Recall

the background for choosing these numbers: a series of (R_T, a) matched sets were chosen for given (P_{th}, p_{nw}) sets and then the smallest R_T was chosen which still allowed core space for a 4 T OH coil. The stored energy $E_s \sim B_m^2 R_T^3$, is proportional to the structural mass which in turn is proportional to 75% of the UWMAK-II toroidal field costs while the coil ampere-meter associated costs, $\sim B_m R_T^2$, account for 25% of the TF coil cost. For larger reactors, the primary consideration of this study, structure costs are even more dominant according to the scaling laws above. Therefore we make the simplifying assumption that TF coil costs are proportional to the value of E_s listed in Table I.

These data are also presented in four figures to emphasize the p_{nw} and B_m dependence. In Fig. 2 are plotted B_m vs. p_{nw} for five selected constant power levels. It is seen that the B_m required increases with p_{nw} and that, in fact, high fields would not be well utilized without high wall loading. It is also seen that for a given wall loading, smaller, lower power reactors require higher fields than larger, higher power reactors. Finally it is difficult to justify a design for fields above 10 T, if the initial scaling laws we have used for circular plasmas are correct.

On Fig. 2 is a cross-hatched band between 8.5 and 11 T. It is often assumed that TiNb composite conductors could be successfully operated in 4.2 K liquid helium

TABLE I
Minimum Size Reactors

P _{th} (MW _{th})	p _{nw} (MW/m ²)	B _m (T)	R _T (m)	E _s (10 ¹⁰ J)	R _m (m)	d _T (m)	R _i (m)	a	B _T (T)	A (R _T /A)	E _s /P _{th} (Normalized)
1000	0.625	8.04	9.10	2.55	4.45	.720	3.09	2.86	3.93	3.18	1.71
	1.25	11.3	7.80	2.98	4.46	1.17	2.65	1.67	6.48	4.67	2.00
	2.50	19.4	8.20	7.17	5.83	3.03	2.16	.793	13.8	10.3	4.81
2000	0.625	7.00	11.8	4.16	5.45	.702	4.11	4.41	3.23	2.68	1.59
	1.25	9.38	9.60	4.01	5.12	1.02	3.46	2.71	5.00	3.54	1.54
	2.50	13.7	8.50	5.10	5.32	1.76	2.92	1.53	8.58	5.56	1.71
5000	0.625	6.11	17.1	9.53	7.24	.779	5.82	7.60	2.59	2.25	1.27
	1.25	7.71	13.4	7.46	6.57	.970	4.96	4.85	3.81	2.76	1.00
	2.50	10.5	11.0	7.19	6.26	1.41	4.21	2.95	5.96	3.75	.97
	5.00	15.8	10.0	9.92	6.72	2.63	3.45	1.62	10.6	6.17	1.33
10,000	0.625	5.61	23.1	19.5	9.23	.889	7.70	11.3	2.24	2.05	1.31
	1.25	6.95	17.8	14.2	8.27	1.05	6.58	7.30	3.23	2.44	.95
	2.50	9.04	14.1	11.8	7.53	1.39	5.50	4.61	4.83	3.06	.79
	5.00	12.5	11.8	11.6	7.28	2.05	4.59	2.75	7.74	4.29	.78
20,000	0.625	5.19	31.6	41.8	12.00	1.04	10.3	16.5	1.97	1.92	1.40
	1.25	6.37	23.9	28.4	10.43	1.18	8.61	10.9	2.78	2.20	.95
	2.50	8.03	18.6	21.6	9.41	1.46	7.31	6.99	4.06	2.66	.72
	5.00	10.66	15.0	18.7	8.74	1.99	6.11	4.33	6.21	3.46	.63

up to 8.5 T. Between 8.5-11 T it is suggested that TiNb might be used at 1.8 K cooled with superfluid helium. Above 11 T a composite conductor of Nb₃Sn is envisioned.

The last two possibilities are as yet unproven for large coils. However, one conclusion to be drawn is that TiNb could probably be used for most of the region of interest.

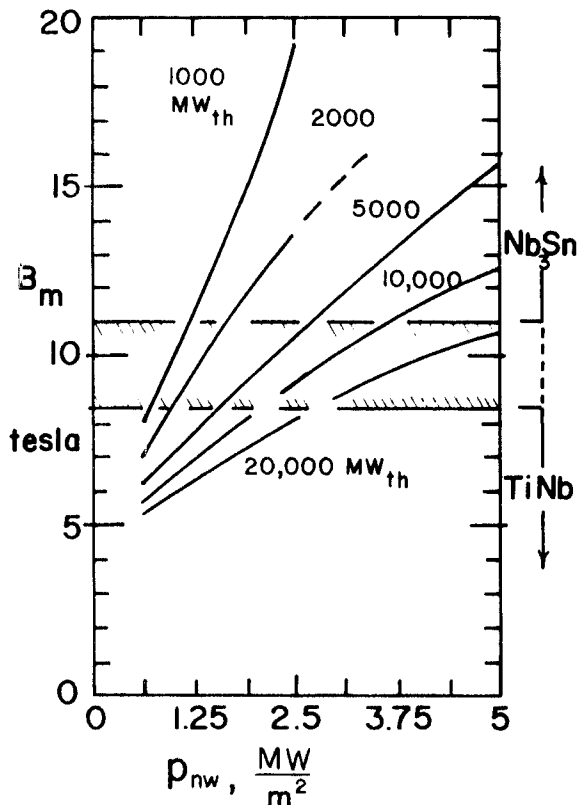


Fig. 2. Maximum magnetic field vs. wall loading for five reactor sizes.

The 1000 and 2000 MW_{th} curves in Fig. 2 deserve mention. First, such low power reactors require the highest fields at wall loadings less than 2.5 MW/m². Note that higher wall loading-higher field units cannot compatibly be designed. The cost effectiveness of these smaller units are at least 50% worse than for the larger units, see Figs. 4 and 5. In addition, a 1000 MW_{th} reactor delivers only 400 MW_c, which even today is smaller than an average size utility company generator. To be significant fusion reactors should produce large amounts of power, not small amounts. The conclusion is that 1000 and 2000 MW_{th} units are probably not viable size choices. Therefore our main discussion continues with emphasis on 5000 MW_{th} and larger.

The design major radii are plotted in Fig. 3 vs. wall loading for the five different reactors. The three larger units exhibit an obvious regularity, with R_T increasing with thermal power and decreasing

with wall loading. Recall that R_T was selected as the smallest radius which still allows an adequate core size. The design process, represented partially by Eqs. (10) and (11), automatically selects values of d_T, the TF coil thickness, in the range of 1-3 m for the sizes studied, see Table I.

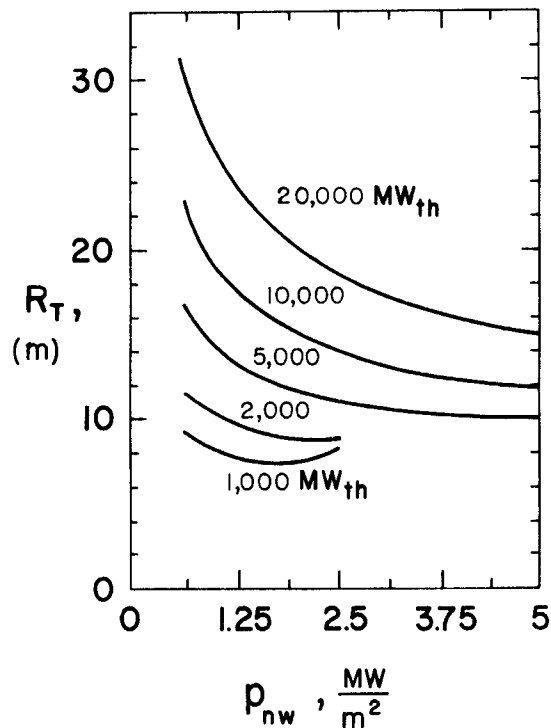


Fig. 3. Toroidal major radius vs. wall loadings for minimum size reactors.

In Fig. 4 are presented cost/power graphs vs. wall loading, again normalized to one for the UWMAK-II case. The small units are inefficient, the 5000 MW_{th} unit is optimum at about 2.0 MW/m², the 10,000 MW_{th} unit is broadly optimized in the range 2-5 MW/m², and the 20,000 MW_{th} unit improves monotonically with higher wall loading. At 1.25 MW/m² all three larger units are equally cost effective regarding TF coils. In fact, if 1.25 MW/m² were an absolute limit, then one would consider building a cluster of four 5000 MW_{th} reactors rather than one 20,000 MW_{th} unit. At 5 MW/m² the 20,000 MW_{th} TF coil per unit cost is about two thirds its cost at 1.25 MW/m².

The same cost/power values are re-plotted in Fig. 5 but this time vs. B_m, the maximum field in the TF coils on the superconducting windings. The 2000 and 5000 MW_{th} units optimize between 8 and 10 T and the 10,000 MW_{th} unit between 9-12.5 T. The 20,000 MW_{th} reactor does not optimize vs. field but appears to still be improving with field at 10.7 T; however, higher fields

would imply wall loadings above 5 MW/m^2 . Three of the curves in Fig. 5 terminate at high field. The wall loading at this termination is 5 MW/m^2 ; higher wall loading, and therefore higher magnetic fields, were not considered in this study. It would appear that $B_m = 8$ to 10 T is the best choice of magnetic field.

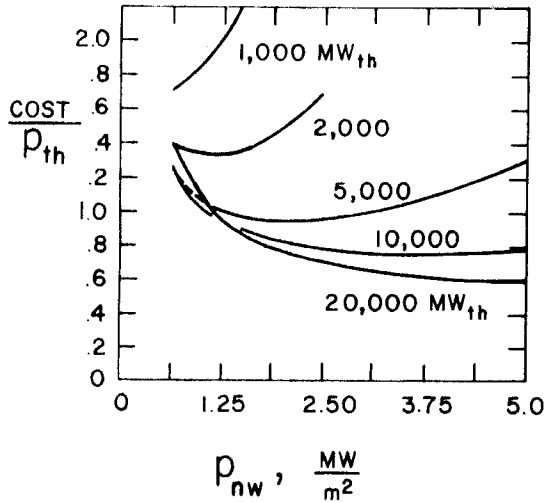


Fig. 4. Cost effectiveness vs. wall loading.

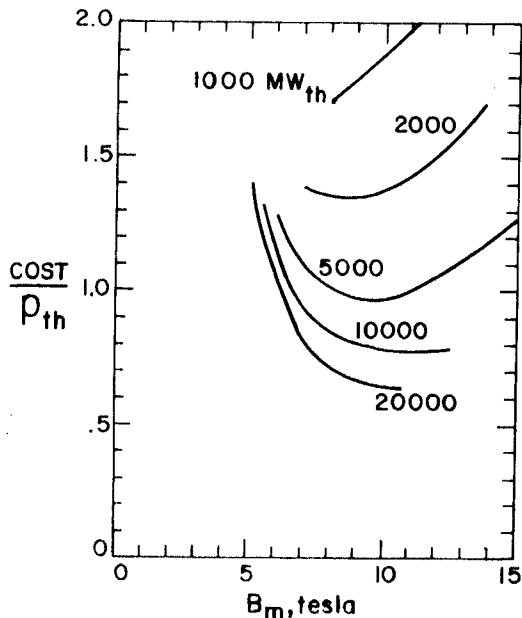


Fig. 5. Cost effectiveness vs. maximum magnetic field.

The reader should be reminded that higher wall loading is of reduced interest because of the more frequent stoppages needed to change the first wall. Kulcinski⁴ estimates that at 1.25 MW/m^2 replacement would occur every two years, with other wall loadings proportionately different. The production energy costs estimated from the UWMAK-I and II studies are 21.0 mills/kWh for a two year wall life. These are esti-

mated to increase 24.3 mills/kWh for a one year wall life and to 33.3 mills/kWh for 0.5 year wall life. The penalty, of course, is the down time for the whole plant, not just replacement costs. One advantage of higher wall loading is that the reactor and some parts of the power station are smaller, which is a saving. These compromises are too complicated to consider here, but we caution that care should be exercised when pushing towards higher wall loading and higher magnetic fields.

Conclusion

This study presents a straightforward case which strongly suggests that circular Tokamak plasma reactors will not benefit much from magnetic fields above 10 T . Other conditions apply to non-circular plasmas or to different reactor designs. One such design by Mills et al.,⁵ for example, requires $B_m = 16 \text{ T}$ to get $B_T = 6 \text{ T}$; obviously a high field is needed in that case. Small experimental reactors of course need the highest field possible, which does not refute the above study for large reactors.

Finally, since the TF coil system represents only 12% of the total plant cost it is reasonable to assume that plant capital cost is almost independent of choice of B_m . However, considering the down time to replace the first wall, it seems apparent that wall loading of 2.5 MW/m^2 or less and maximum magnetic field values of 10 T or less are desirable design goals.

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