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Summary

The optimization of large cryogenically stable TiNb-Cu composite conductors used in toroidal confinement magnets for fusion reactors is presented. Both copper and superconductor amounts are minimized subject to stability and current sharing conditions which are dependent on local filament temperatures. The new feature here is the determination of these local temperatures as a function of the fractional current in the normal metal matrix. The result is a steady state (not transient) assessment of the ability of a composite conductor in the resistive state to achieve complete superconductive recovery. For a UWMAK-I example magnet, a constant tension D magnet operated in a 4.2 K helium bath, it is shown that the minimum amount of superconductor is that required to carry the total current at 4.3 K.

Introduction

The design of large superconductive magnets for fusion reactors has led to renewed interest in the study of cryogenically stable magnets. Cryogenically stable conductors are composite conductors of superconductor and normal metal arranged so that the total transport current can be carried in the normal metal for a significantly long period of time without excessive temperature rises. This is often accomplished by using a sufficiently large cross section of copper with adequate cooling surface exposed to liquid helium in a helium bath. The basic premise is that the transport current which has been diverted to the copper will return to the superconductor provided the temperatures are maintained less than T_c .

In contrast, cryogenically unstable magnets are wound with composite conductors with inadequate amounts of copper and surface cooling for steady state stability. The ideal expectation for cryogenically unstable conductors is that transient recoveries following a disturbance will and can occur before $T > T_c$, due to a variety of processes.

Major work with the development of cryogenically stable conductors has been restricted mostly to a few large coils such as the ANL¹ and NAL² bubble chamber coils. Large fusion reactor magnets such as the D magnet TF (toroidal field) coils seem to require the reliability inherently obtained with cryogenically stable conductors. There is little need to employ high current density cryogenically unstable conductors since the major space requirement is for massive structure. In addition, the amount of copper used is not a major cost item; eliminating copper to obtain a smaller unstable conductor would be a minor achievement. Unexpected super-to-normal transitions typical of cryogenically unstable coils are unacceptable in terms of potential magnet damage and lost time for a utility company.

The present paper concerns two points: (1) assuming current sharing with all the current in the copper and assuming a fixed total width available for a D coil disc cooling surface (the total cooling surface width of all turns per disc is typically one meter), then what is the minimum amount of copper for cryogenic stability and (2) what is the minimum amount of superconductor required for total supercurrent recovery,

again assuming current sharing and the temperature-supercurrent capacity variations resulting from current sharing.

Cryogenic stability may be divided into a nucleate boiling limited stability and film cooling unlimited stability. In the case of nucleate boiling limited stability the design heat flux is assumed not to exceed a certain value, which is a fraction of the maximum nucleate boiling heat flux. Usually a design value of 0.3 to 0.4 W/cm² is taken in case the peak value is 0.8 to 1.0 W/cm² for a given cooling situation. This extra allowance of cooling ability amounts to a reliability derating by one half of the maximum conditions for heat transfer. If the 1.0 W/cm² peak heat flux is exceeded the conductor may or may not recover depending on other factors. In the case of unlimited stability the conductor will recover even if the nucleate heat flux is exceeded since stability is designed for the lower heat fluxes, ~ 0.1 W/cm², associated with film boiling heat transfer. However, only limited cryogenic stability is considered in this paper. We intend to discuss unlimited cryogenic stability in a future publication.

As an example for this paper consider the UWMAK-I conductor³ which is a rectangular copper composite conductor embedded in grooves in an insulated support disc D magnet. The conductor cross section depends on the resistivity variation with magnetic field at the inner radius, R_m in Fig. 1, and was not varied along each turn. The present optimization in this paper not only considers the copper optimization at R_m but also varies the cross section of each turn around the D as a function of R. Even if it is not practical to vary the width and height of a conductor along each turn in manufacture it is still of interest to know the absolute minimum amount of copper which could be used as a gauge for comparison of any design.

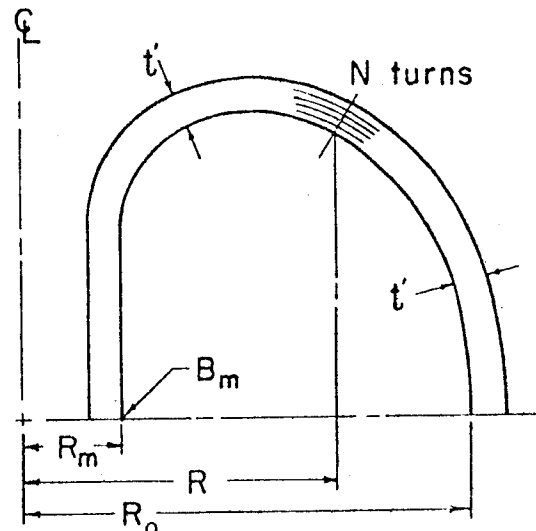


Fig. 1. Toroidal field D magnet disc with N embedded turns.

Copper Optimization

Consider the cross section at R, Fig. 1. Assume that the field falls linearly across the N turns. For any turn the following gross stability equation is satisfied

$$\frac{I^2 \rho_o C_i}{w_i h_i} = q w_i \quad (1)$$

where

- i is a subscript for the ith turn which can be any turn from 1 to N
- I is the current in amperes
- ρ_o is the resistivity in zero field
- C_i is the ratio of resistivity of the ith turn to the zero field resistivity, ρ_i/ρ_o
- w_i is the width of the ith turn
- h_i is the height of the ith turn, and
- q is the surface heat transfer flux.

The volume of copper per unit length, V, required at R is given by

$$V = \sum_{i=1}^N w_i h_i = \frac{I^2 \rho_o}{q} \sum C_i / w_i, \quad (2)$$

where $I^2 \rho_o / q$ is a characteristic volume. The total width of the copper turns $\sum w_i = t$ depends on the width of the disc t' and on δ , the total width of the space between grooves, as:

$$\sum w_i + t' - \delta = t \quad (3)$$

We wish to minimize the amount of copper given by Eq. (2) subjected to the constraint given by Eq. (3). Using the Lagrange multiplier technique the function to minimize is

$$F = I^2 \rho_o / q \sum C_i / w_i + \lambda (\sum w_i - t) \quad (4)$$

where λ is the Lagrange multiplier.

The optimum widths are given by

$$\frac{\partial F}{\partial w_i} = \frac{\partial F}{\partial \lambda} = 0. \quad (5)$$

The optimized values for the ith turn at radius R are:

$$w_i = t \sqrt{C_i} / \sum \sqrt{C_i} \quad (6)$$

and

$$h_i = \frac{I^2 \rho_o / q}{t^2} (\sum \sqrt{C_i})^2 = \text{constant}. \quad (7)$$

The minimum volume of copper required at radius R is given by

$$V = \frac{I^2 \rho_o / q}{t} (\sum \sqrt{C_i})^2. \quad (8)$$

It can be seen that the minimum amount of copper required is inversely proportional to the thickness t, and that h_i is a constant depending only on R.

To continue with the UWMK-I example let us take $I = 10,200$ A, $N = 30$ turns, $q = 0.4$ W/cm², $\sum w_i = t = 0.54$ m, $R_m = 5.75$ m, $R_o = 20.5$ m, $B_m = 8.66$ T and $\rho_o = 10^{-8}$ Ω cm for a disc of 5.44 cm thickness.

Figure 2 is a plot of h vs. R with values noted at R_m , the high field point. Figure 3 is a plot of w_i at R_m which indicates a linear decrease in w_i from 2.35 cm to 1.11 cm for B changing from B_m to zero. In the actual UWMK-I design the w_i variation at R_m is similar to that in Fig. 3; however w_i was unchanged vs R. The height h in an equivalent UWMK-I design varies from 2 cm to 1.6 cm at turn seven and is constant at 1.6 cm to turn 30, while here h varies with R as in Eq. (7) and Fig. 2. The present optimization in comparison to UWMK-I saves about 50% of the copper.

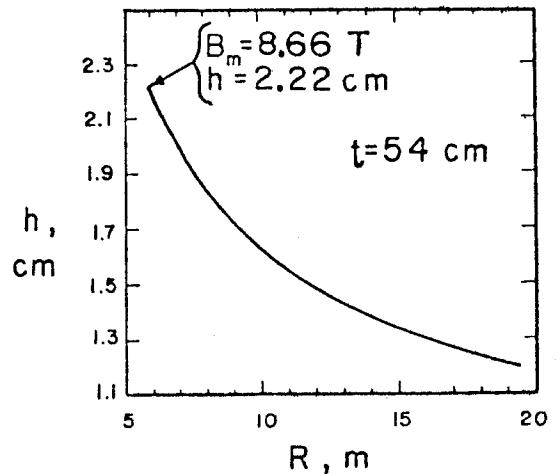


Fig. 2. Height of the turns vs. radius.

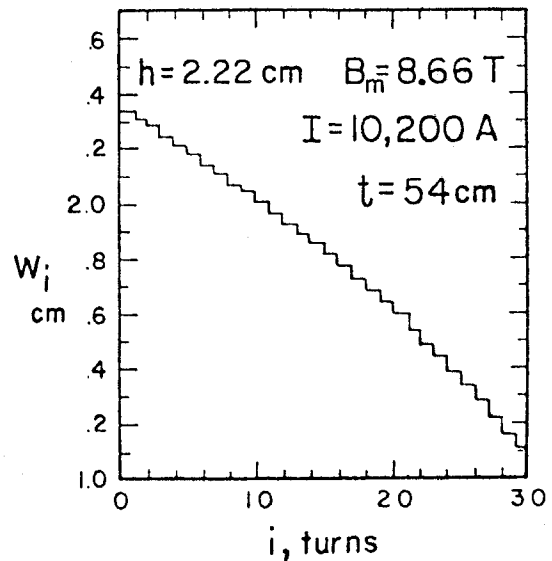


Fig. 3. Width of turns at R_m .

Cryogenic Stability and the
Amount of Superconductor Required

Most large superconducting magnets in operation at this time are cryogenically stable. A good example is the NAL bubble chamber solenoid built by Purcell.² It is stabilized by selecting 0.4 W/cm² and 0.15 W/cm² for edge and restricted face cooling of rectangular turns in a jelly-roll winding. The filament temperature is limited by using enough filaments so that the average temperature of each filament will not exceed T_c; the criteria is based on the fact that the maximum heat which can be removed from any filament is independent of its diameter.

The UWMAC-I design uses the same filament number criteria. Extreme temperatures at a filament are calculated for two cases: (1) one half the current is carried by resistive filaments for which the maximum power is dissipated in the filaments and (2) the total current is carried by the copper causing a temperature rise from the cooled surface to the filament location. Three temperature differences were considered: (1) ΔT at the copper-helium surface, (2) ΔT across the copper from the surface to a filament and (3) ΔT due to resistive power dissipated in a filament. After inspecting the above two extreme conditions it was determined that a total ΔT = 1 K was more than adequate for either case and the amount of superconductor was chosen to carry the total current at 5.2 K for use in a 4.2 K helium bath. The reliability justification used was as follows: if the worst imagined combination of temperature rises will not cause T to exceed 5.2 K and if there is enough superconductor to carry all the current at 5.2 K then extra reliability has been obtained.

Cryogenic stability was introduced by Stekley in 1965.⁴ In his first several papers he considered the problem for the following case: (1) the only temperature difference considered is that between the conductor and helium bath, (2) the surface heat transfer coefficient h was taken constant and (3) the superconductor current capacity was assumed to vary linearly with temperature from zero to T_c. An important aspect of Stekley's study was the introduction of current sharing effects.

A variation on this stability criteria by Whetstone⁵ introduced the real nucleate boiling-film boiling heat transfer coefficient as an improvement over the first assumption that h is constant. In the surface heat transfer expression q = hΔT, where ΔT is the surface to bath temperature difference, Whetstone measured ΔT = 0.4 to 1 K as the stability limit, which corresponds to helium "burn out," the transition from nucleate to film boiling. He also showed that for a wide class of conductor shapes and winding configurations in bath cooling that q = 0.3 to 0.4 watts/cm², the nucleate boiling heat flux, was the limiting and determining factor for cryogenic stability.

In order to further expand on the stability criteria above and, in particular, to extend and improve on the UWMAC-I design example, we wish to determine all of the temperature differences for a continuous set of simultaneous recovery conditions. The new area of concern involves multifilamentary conductors which were not in use during the early developmental phase of cryogenic stability theories. Our new procedure is to determine the local temperature and supercurrent capacity of each filament and then, via current sharing, find if recovery can take place for

any fractional division of current between the copper matrix and superconductor filaments.

We first assume that for some reason the current is carried by the normal metal. If the temperature of the superconductor is less than the field dependent critical temperature, some of the current will transfer to the superconductor. If the superconductor is still able to carry more current then the current in the stabilizer will decrease further. If at all times the superconductor is able to carry more current than required, considering the superconductor temperature and the current in the copper, then total recovery can occur. Assume that the stabilizer carries a certain fraction of the total current α_n. The surface of a given superconductor filament in a bath of temperature T_b will be at a temperature T_s given by

$$T_s = \Delta T_1 + \Delta T_2 + \Delta T_3 + T_b \quad (9)$$

where

ΔT₁ is the temperature difference between the conductor surface and bath temperature,

ΔT₂ is the temperature difference between the filament location and the cooled surface of the conductor and

ΔT₃ is the temperature difference due to the flow of the heat generated in the filament from the filament surface to the cooled surface of the conductor.

The temperature difference ΔT₁ is calculated from the heat flux expression⁶

$$q = \frac{\alpha_n I^2 \rho}{w h} = 1.34 (\Delta T_1)^{1.4} \quad (10)$$

where I is the total current,

ρ is the resistivity (magneto-resistance is included),

w and h are the width and the height of the conductor respectively and

1.34(ΔT₁)^{1.4} is an empirical expression for nucleate boiling heat flux.⁶

The temperature difference ΔT₂ is calculated from the following equation, which is essentially the same as the simplified one dimensional calculation used for UWMAC-I:³

$$\Delta T_2 = \sqrt{\frac{\rho^2 \alpha_n^2 I^2}{L w^2} \left[1 - \left(\frac{x}{w}\right)^2 \right] + T_b^2} - T_b \quad (11)$$

where x is the location of the filament as measured from the cooled surface. The constant L = k_c T, where k is the thermal conductivity of copper, in the above equation is taken to be 2 x 10⁸ ΩW/K² for OFHC copper.⁷

For a filament located at y-b, see Fig. 4, the temperature difference, ΔT, between a point (x,y) and the surface is given by

$$\Delta T = \frac{q'}{4nk} \left\{ \ell_n \frac{\cosh\left(\frac{\pi x}{2a}\right) + \cos\left(\frac{\pi y}{2a} - \frac{\pi b}{2a}\right)}{\cosh\left(\frac{\pi x}{2a}\right) - \cos\left(\frac{\pi y}{2a} + \frac{\pi b}{2a}\right)} + \ell_n \frac{\cosh\left(\frac{\pi x}{2a}\right) + \cos\left(\frac{\pi y}{2a} - \frac{\pi b}{2a}\right)}{\cosh\left(\frac{\pi x}{2a}\right) - \cos\left(\frac{\pi y}{2a} + \frac{\pi b}{2a}\right)} \right\} \quad (12)$$

where q' is the heat generated in the filament per unit length and k is the thermal conductivity of copper.⁶

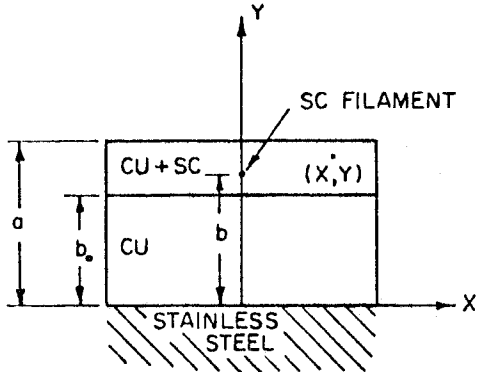


Fig. 4. Conductor cross section with one filament at $(0, b)$ and a general point at (x, y) .

If there are n filaments per conductor a filament at $y = b_0$, Fig. 4, will have the highest temperature difference. The temperature difference ΔT_3 at b_0 will be given by

$$\Delta T_3 = \sum_{i=1}^n \delta T(x_i, y_i) \quad (13)$$

where x_i and y_i are the coordinates of the i th filament.

If the surface temperature at b_0 of n transposed filaments is known, the maximum supercurrent capacity, I_{sc} , is given by

$$I_{sc} = nr^2 \left\{ J(T_s) + \frac{BC}{A} - \frac{2B}{A^{3/2}} C \frac{I_1(\sqrt{A})}{I_0(\sqrt{A})} \right\} \quad (14)$$

where r is the radius of the filament,

$J(T_s)$ is the critical current density corresponding to the surface temperature of the filament and

C is the temperature coefficient of the current density, $\partial J / \partial T$.

A and B are given by

$$A = -vCr^2/k_s$$

$$B = vJ(T_s)r^2/k_s$$

where v is the voltage/meter along the filament and depends on the fraction of current flowing in the stabilizer and k_s is the thermal conductivity of the superconductor.

To determine the amount of superconductor required at temperature T_s the following procedure is followed:

a. Assume that there is a certain amount of superconductor. The amount of superconductor should be equal to or larger than the amount required to carry the current at bath temperature.

b. Equations (10), (11), and (13) are used to determine ΔT_1 , ΔT_2 and ΔT_3 and hence the surface temperature of the filament vs. fraction of current in the normal metal α_n .

c. Determine the maximum current which a filament can carry for a known T_s , Eq. (14). This provides the maximum fraction of current which a superconductor current can carry as a function of α_n , provided the filament capacity is properly summed over all filaments. The fraction of the current the total superconductor cross section is required to carry, α_s , is given by

$$\alpha_s = 1 - \alpha_n \quad (15)$$

The fraction of the current the superconductor can carry as calculated in (c) above should always be higher than the fraction given by Eq. (15) to insure recovery. Figure 5 is a plot of both the fraction of the current the superconductor is required to carry and the fraction of the current the superconductor can carry as a function of the fraction of the current in the stabilizer. As shown from Fig. 5 if the amount of the superconductor is taken as the cross section needed at 4.2 K then the conductor will not recover. The point of the intersection shown in Fig. 5 represents conditions in which the conductor will be operating with 10% of the current permanently flowing in the copper. If the amount of superconductor used corresponds to J_c at 4.3 K then the two curves do not intersect and the conductor is cryogenically stable. The conductor used in Fig. 5 is 2.34 cm wide x 2.22 cm high and the field at the conductor is 8.66 T, the UWMAK-I example considered earlier.

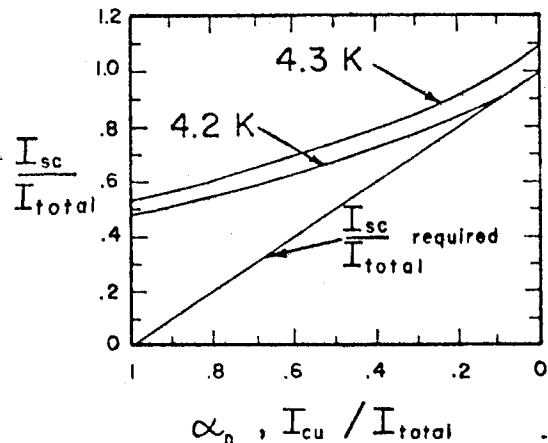


Fig. 5. Supercurrent capacity and requirement vs. copper current. The 4.2 K and 4.3 K curves result from selecting the superconductor cross section to carry the total current at those temperatures.

The amount of superconductor required considering the 4.3 K condition is only 50% of the amount used in UWMAK-I. If we could imagine a practical way to fabricate a conductor in which the superconductor cross section is varied with R then only 25% of the original

UWMAK-I quantity would be needed. Recall that the amount of conductor used in UWMAK-I was determined by insisting that $I_{total} = I_{sc}(5.2K)$ with an argument that this excess in superconductor represented a conservative choice. According to the development above we can now see that the minimum required amount of superconductor is that sized to carry the current at 4.3 K in a 4.2 K bath for copper TiNb composites used at 0.4 watts/cm^2 . The amount of surplus conductor in the UWMAK-I design is now well determined and subject to further consideration in respect to useful surplus quantities, for reliability reasons, which is not the topic of this paper.

Conclusions

An optimization method is presented in order to minimize the copper in a composite cryogenically stable conductor. The copper optimization magnet design would apply in general to any flat pancake type winding which is cooled on one surface, see. Eqs. (6), (7) and (8). This design provides equal stability in terms of heat generation and removal for all locations in a coil for a minimum amount of copper.

The second and perhaps more interesting optimization is the determination of the minimum amount of superconductor which allows complete recovery to the superconducting state following a normal excursion. In a quite general example we show that a composite conductor carefully designed to carry full current in the superconductor at 4.2 K may in fact on occasion carry only 90% of the full current in the superconductor. The amount of superconductor, see Fig. 5, should be sufficient to carry the full current at 4.3 K in order to allow complete recovery. Based on this general example we note that the amount of superconductor should be sized at $T > 4.3 \text{ K}$ for use at 4.2 K. In fact, we predict that short sample 4.2 K currents cannot be reliably achieved at 4.2 K.

Acknowledgement

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