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WIRE DEBRIS MODELING OF THE Z-ACCELERATOR

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The Z-Accelerator is an intense x-ray source located at Sandia National Laboratory. On a typical shot, 20 MA of current passes through a cylindrical array of wires over tens of nanoseconds. The result is the release of 2 MJ of low-energy x-rays at approximately 200 TW. The wires are mostly vaporized in this time, but some wire fragments remain[1]. We have developed a model for the deformation of these wires as they accelerate towards the center of the device. While the shot is generally over 200 nanoseconds, the model only covers times on the order of 1-4 nanoseconds, as it is a continuum model.

The model begins with a 2-D finite element model that determines the forces and magnetic fields the titanium wires experience early in a typical shot. The magnetic field around the wires reaches a maximum of 210 Tesla when the current is a maximum. ANSYS provides a force per unit length that is applied to the wire over time.

The forces that are determined in ANSYS are used in a separate computer code that solves the equations of motion for the wires. The code solves the 1-D wave equation with a periodic forcing function, using only the early portions of a cycle to approximate a monotonically increasing load. As the wire is displaced from its initial position, the tension should increase as the length of the wire increases. An incremental model is used to update the tension as the wire is displaced, effectively linearizing an inherently nonlinear problem[2]. Results will be described that show the wires' behavior as a function of the initial tension applied to the wire.

I. INTRODUCTION

The Z-Accelerator located at Sandia National Laboratory produces over 200 TW and approximately 2 MJ during each shot by driving about 20 MA of current through a cylindrical array of wires. [3] The importance of the wires has been discussed in the paper by Stanford et al and the need for further understanding of the wires' behavior has been suggested by many authors in recent years.[4,5]

A paper by Haines offers a description of the behavior of the wires during the z-pinch process.[5] The wires undergo four steps during the z-pinch process. The first step that Haines describes is the wires carrying current and behaving independently. As the current flows down the wire, the wire is heated up and the wires create a plasma shell around the wires. This shell expands until they merge and become a cylindrical shell. As the wires expand into this cylindrical shell, the individual wires create plasma, completing the second step. As the plasma is formed around the wires, the wires move inward and the initial Raleigh-Taylor instabilities grow in step three. Finally, the fourth step occurs when the plasma stagnates and emits radiation.[5]

The wire model discussed within this paper will deal with the first step described above. The wire will behave independently and will have a large, time-dependent force applied over its length. As the magnetic field increases around the wires, the force will increase, placing a strain on the wire. The wire will fragment under these strains and these fragments will ultimately form a plasma. Our model will only deal with an individual wire undergoing a strain which then leads to the wire fragmenting. No modeling of plasma will be addressed, although a possible procedure for doing so will be discussed later.

Heating effects in the wire model will not be addressed in this paper. In a paper by Chittenden, et al, it was found that at times on the order of tens of nanoseconds, over 99% of the wires' mass remain cold.[6] Hence, for the short times that our model is valid, the wires can be assumed to be solid.

II. BACKGROUND

The description of the Z-Accelerator is covered in many papers.[7,8] The Z-Accelerator is a large device with a diameter of over one hundred feet with the z-pinch occupying the inner ten cm portion. Although the device is large, the model that will be described throughout will be the innermost five to seven centimeters (radially).

The output from the Z-Accelerator has been steadily increasing since it became operational in 1995. As the number of wires has increased, the amount of power radiated and energy emitted have increased to over 200 TW and approximately 2 MJ.[3,4] In determining the amount of power radiated and energy emitted, it has been shown that the instabilities in the wires can have a tremendous impact upon these results [3,9] and the behavior of the wire in the early stages of the shot can have an impact on the instabilities.

Another paper that goes to the heart of developing a model for the wires movement is by Ding.[10] Ding describes a possible outline very similar to that Haines described where the wires are considered individually early in the z-pinch process. After the wires undergo heating, they fragment. It is these fragments that then create the plasma shell, which moves inward towards the center until the plasma shell reaches a zero velocity.[10] Ding states in his paper that he feels that the wires' heating, expansion and fragmentation are very important early in the process, perhaps out to several nanoseconds.[10]

Throughout the model, timing will be critical. The Z-Accelerator shot that will be used for current measurements is number 302. The reason for choosing this particular shot was the availability of data on the current when this investigation was started. Similar comparisons can be made for other shots, once the appropriate parameters are changed from this baseline analysis.

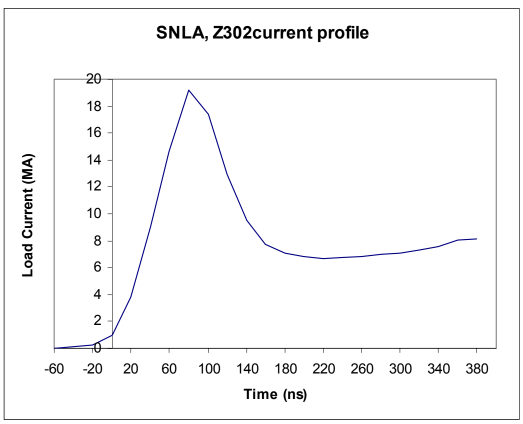

Figure 1
Current profile used in the model.

As has been stated already, the model discussed within will be for a single wire. The wires used on shot 302 were titanium wires of approximately twelve micron diameter.[11] The time for the model has been chosen to be one nanosecond. This will be the time that is used to determine the displacement of the wire. Note that this time is significantly less than the time when the maximum current is reached as shown in figure 1. This time agrees well with the description given by Haines and Ding regarding how long the early stage lasts when the wires act independently.

The heating effects that are described in the papers by Haines and Ding are negligible over the short time during which the results are obtained. Taking the description presented in these two papers along with the paper by Chittenden, the heating effects of the solid core of a wire is minimal over one nanosecond. When fragmentation is incorporated into this model, heating effects will need to be addressed, but they are not yet included in the current model.

III. RESULTS

Modeling was done in two parts before fragmentation occurs. The first part was done using the commercial code ANSYS to determine the forces and magnetic field around the wires for a slotted return can. The second part uses a program written by the primary author as to the displacement of the wire before fragmentation occurs.

III.A. ANSYS Model

The first model constructed within ANSYS incorporated a solid return can versus the slotted return can actually used in Z. Verification within ANSYS was determined using the solid return can as an analytical result could be compared to the results obtained from ANSYS, as shown in figure 2. The results matched well and, following this verification, the ANSYS model was changed to incorporate the slotted return can. The current carrying elements and the vacuum elements used in the ANSYS model are shown in figure 3. The infinite elements in the model are not shown.

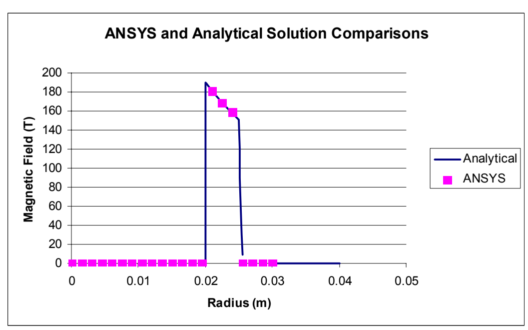

Figure 2
Verification of Magnetic Field.

Once the field's calculations were verified, the ANSYS program was used to model the forces on the wires as a function of time. The results showed little difference between the forces using the solid and slotted cans, despite the large peaking seen in the fields for the slotted can. This is assumed to be a result of the large distance from the can to the wires, as compared to the wire-to-wire distance.

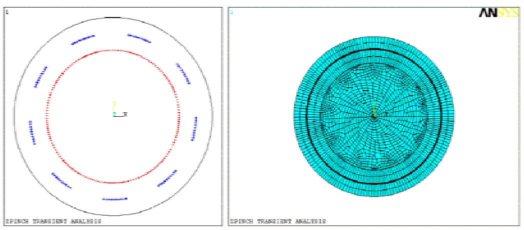

Figure 3
Current carrying elements on the left, the right includes the vacuum elements.

III.B. Wire Model

The wire model is based upon the one-dimensional wave equation with a force applied over its length. Figure 4 shows the wire supported between two ends with a tension applied throughout the wire with a transverse force that varies over its length.

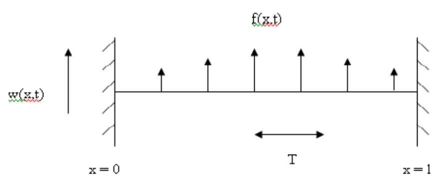

Figure 4
Wire fixed at both ends with an external force f(x,t) acting on it.

The equation of motion for the wire is:

$$w(x,t) = \sin\frac{\pi x}{l}\left(\frac{\omega\tilde{A}cl^3}{\pi(\omega^2 l^2 - \pi^2 c^2)}\sin\frac{\pi c t}{l} + \frac{\tilde{A}c^2 l^2}{\pi^2 c^2 - \omega^2 l^2}\sin\omega t\right) \qquad (1)$$

where x is the position on the wire, l is the length of the wire, ω is the angular frequency, $\tilde{A}$ is the amplitude of the forcing function divided by the tension, and c is the wave speed in the wire. Several assumptions were made to arrive at this solution. The forcing function was assumed to be a quarter wave of the input current. This assumption on the forcing function led the solution to become a finite solution of one term versus an infinite series of terms. Another assumption that was made in the wire's behavior was to model the tension as a function of the wire's length. This required the tension to be determined at the end of each time step before being updated for the next step. One last assumption was that the wire ends will not move significantly over the short time span in which this model is applicable.

The tension used in the model was 5 N. The tension can change significantly depending on how long the model is run. With a time of 1 ns, the tension changes by approximately one percent while at a time of 20 ns, the tension has changed by over eight hundred percent. Also, the initial tension value would affect the rate at which the tension changes as well. If an initial value of 500 N was used instead of 5 N, the percent change at 1 ns and 20 ns would be 0.01% and 11.9% respectively. The initial value of tension will have an affect on the amount the wire displaces and how much the value the tension will change throughout the time the model is ran.

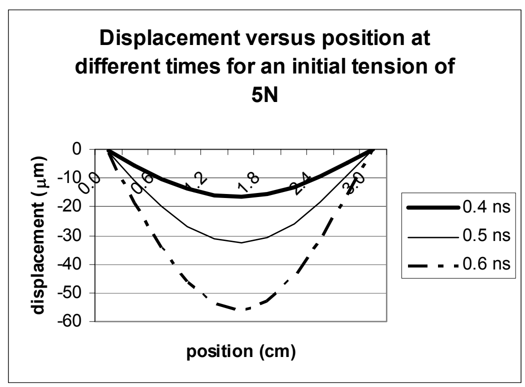

Figure 5
At a tension of 5 N, the displacement is shown.

Figure 5 shows the displacement of the wire at different times for an initial tension of 5 N. The figure shows that the wire bows inward towards the center as the force acts over its length. Note however that the amount of the deflection is not large, but the wire does move under the forces created by neighboring wires. Figure 5 shows the wire horizontally for graphical purposes, but the wire on the actual device is vertical.

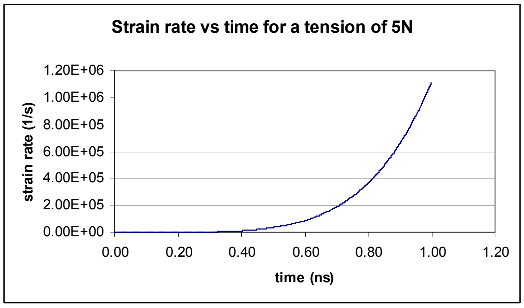

Figure 6
Strain rate versus time.

As the wire is displaced from its initial position, the wire will undergo strain. Figure 6 shows the strain rate of the wire as a function of time. The amount of strain on the wire increases dramatically after approximately 0.6 nanoseconds. The local strain in the wire is not determined in the model at this time.

IV. CONCLUSIONS

The model created thus far simulates the wire's behavior in the early stages of a z-pinch shot. The wire will undergo some displacement before the wire is vaporized but at this point, further work must be done to determine at which point vaporization occurs. This is important because the current model assumes the wire is solid.

The wire undergoes displacement early in the z-pinch process. While the amount of this displacement is not large, it will affect the fragmentation sizes in the wire, depending upon the local strain in the wire. For the wire to move approximately 60 microns, it takes approximately 0.6 nanoseconds. During this time the tension increases slightly.

V. FUTURE WORK

Future work will include modeling the fragmentation of the wire in the early stages of the wire model. After the wire's fragmentation has been modeled, the output of this computer code could be fed into another model that will take position, velocity, current and other necessary information in and give an output that will model the transition from a fragmented solid to plasma.

More work must be done on the wire model so that the motion is more accurately described by a more general forcing function. This will lead to a more complex result for the wires motion, but it can easily be incorporated into the model without significant changes. Also, by incorporating this change, the wire's motion can be more accurately described in the earlier stages.

Changes for the input current must be made as well. Throughout the model, it has been assumed that the current at time zero was zero. Upon inspection in figure 1, the current is not zero and has been increasing for some time. This current must be incorporated into the model to attain more accuracy. Also, this long pre-pulse will have heating effects that need to be addressed to determine what effects are made to the wires elasticity and tension.

The local strain will need to be determined in the future. It will be part of the process in which fragmentation will occur and in order to model the fragmentation process accurately, the local strain will need to be determined. At present, only the strain on the entire wire is found.

REFERENCES

1. GOLUB, T.A., et al, "Multiwire screw-pinch loads for generation of terawatt x-ray radiation," *Applied Physics Letters*, **Vol. 74**, *No. 24*, page 3624, (1999)

2. TOLONEN, T., et al, "Modeling of Tension Modulation Nonlinearity in Plucked Strings," *IEEE Transactions on Speech and Audio Processing*, **Vol. 8**, *No. 3*, page 300-310, (2000)

3. SANFORD, T.L., et al, "Improved Symmetry Greatly Increases X-Ray Power from Wire-Array Z-Pinches," *Physical Review Letters*, **Vol. 77**, *No. 25*, page 5063-5066, (1996)

4. SANFORD, T.L., et al, "Wire Number Doubling in High-Wire-Number Regime Increases Z-Accelerator X-Ray Power," *IEEE Transactions on Plasma Science*, **Vol. 26**, *No. 4*, page 1086-1093, (1998)

5. HAINES, M.G., "A Heuristic Model of the Wire Array Z-Pinch," *IEEE Transactions on Plasma Science*, **Vol. 26**, *No. 4*, page 1275-1281, (1998)

6. CHITTENDEN, J.P., et al, "Plasma Formation and Implosion Structure in Wire Array Z Pinches," *Physical Review Letters*, **Vol. 83**, *No. 1*, page 100-103, (1999).

7. SPIELMAN, R.B, et al, "PBFA II-Z: A 20-MA Driver for Z-Pinch Experiments," *Proceedings of the 10th IEEE Pulsed Power Conference*, page 396-404, (1994)

8. SPIELMAN, R.B., et al, "PBFA Z: A 55 TW/4.5 MJ Electrical Generator," *Proceedings of the 1997 Particle Accelerator Conference*, Volume 1, page 1235-1239, (1997)

9. SANFORD, T.L., et al, "Azimuthal Structure in Wire-Array Z Pinch Experiments," *IEEE Transactions on Plasma Science*, **Vol. 30**, *No. 2*, page 538-546, (2002)

10. DING, N., "The Tentative Opinion of Modeling Plasma Formation in Metallic Wire Z Pinch," *Dense Z-Pinches: 5th International Conference on Z-Pinches*, **Vol. 651**, *No. 1*, page 428-431, (2002)

11. Personal Communication with Chris Deeney at Sandia National Laboratory, Sept. 28, 2003.