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Ripple and Determination of the Number of TF  
Coils**

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***FUSION TECHNOLOGY INSTITUTE  
UNIVERSITY OF WISCONSIN  
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T.F. Yang and G.A. Emmert

Fusion Technology Institute  
University of Wisconsin  
1500 Engineering Drive  
Madison, WI 53706

<http://fti.neep.wisc.edu>

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T. F. Yang

G. A. Emmert

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Nuclear Engineering Department  
University of Wisconsin  
Madison, Wisconsin 53706

Abstract

In conceptual designs of Tokamak reactors, the ripple of the magnetic field produced by the finite number of "D" shape toroidal field coils is found to be both radial and poloidal dependent. This ripple can be modeled by a power function of minor radius and Gaussian function of poloidal angle. The fraction of energetic alpha particles trapped in the magnetic wells between coils is estimated and its minimization is discussed.

The finite number of coils used to generate the toroidal field of a Tokamak produces a bumpiness or ripple of the magnetic field which in turn causes particles to be trapped between coils in the field ripples and to drift vertically out of the plasma. The bumpiness is illustrated by the magnetic field structure in Fig. 1 of the conceptual design UWMAK-I<sup>(1)</sup>, which has 12 constant-tension "D" shaped toroidal magnetic field coils spaced at 30° intervals. The field line at the outer edge of the plasma column shows that the column is corrugated with a deviation of  $\Delta r \approx 0.5$  meter. The amplitude of the ripple is about 20%. Fig. 2 and Fig. 3 show the variation of the ripple with the minor radius  $r$  and poloidal angle  $\theta$ . To evaluate the number of toroidal field coils required on UWMAK-II, we have calculated the leakage of  $\alpha$  particles trapped in these field ripples. We have also investigated the effect of extending the "D" shaped TF coil on the outside such that there is adequate space between coils for removal of blanket segments without moving the TF coils themselves.

Anderson and Furth<sup>(2)</sup> have considered the alpha particle loss for a field ripple which has only a radial variation. To include the poloidal angular dependence shown by Figs. 2 and 3 we write

$$B_{\phi}(r, \theta, \phi) = \frac{B_0}{1 + \frac{r}{R} \cos \theta} [1 + \epsilon(r, \theta) \cos(n\phi)], \quad (1)$$

where  $B_0$  is the true toroidal magnetic field strength,  $B$  is the mean field on the magnetic axis of major radius  $R$ . Here  $(r, \theta, \phi)$  are the usual quasi-toroidal coordinates with  $\theta$  being the poloidal angle. In order to show the variations of ripple with coil number and size, we calculated particle leakage rates when there are 24 TF coils and when the "D" coils have their outer edge extended to larger  $R$ . The results are shown by curves II, III and IV in Figs. 2 and Fig. 3. A fit to the curves in these figures is

$$\epsilon(r, \theta) = \epsilon_a \left(\frac{r}{a}\right)^n \exp\left(-\left(\frac{\alpha\theta}{\pi}\right)^2\right), \quad (2)$$

where  $n, \epsilon_a$  and  $\alpha$  are constants determined by a particular design and  $a$  is the minor radius of the plasma. For low- $\beta$  circular Tokamaks, the trajectory of a field line is  $r = \text{constant}$ ,  $\phi = \phi_0 + q\theta$ , where  $q$  is the MHD stability factor. The field strength along the field line is

$$B_T(r, \theta, \phi) = \frac{RB_0}{R+r \cos \theta} \left[1 + \epsilon_a \left(\frac{r}{a}\right)^n \exp\left(-\left(\frac{\alpha\theta}{\pi}\right)^2\right) \cos(Nq\theta + \phi_0)\right] \quad (3)$$

neglecting the poloidal field strength. Consider the upper half of the  $r$ - $\theta$  plane for fixed  $\phi_0$ . Local magnetic wells are produced by the  $\cos(Nq\theta + N\phi_0)$  term. These are shown in Fig. 4 for the UWMAK-I design (Number of TF coils = 12,  $q = 1.75$ ,  $A = R/a = 2.6$ ,  $\epsilon_a = .20$ ,  $\alpha = 3.2$ ,  $n = 3.0$ ). Particles can be trapped in these wells if they have sufficiently large pitch angle relative to the magnetic field. The lower boundary in  $\theta$  for toroidally trapped particles can be determined from the condition

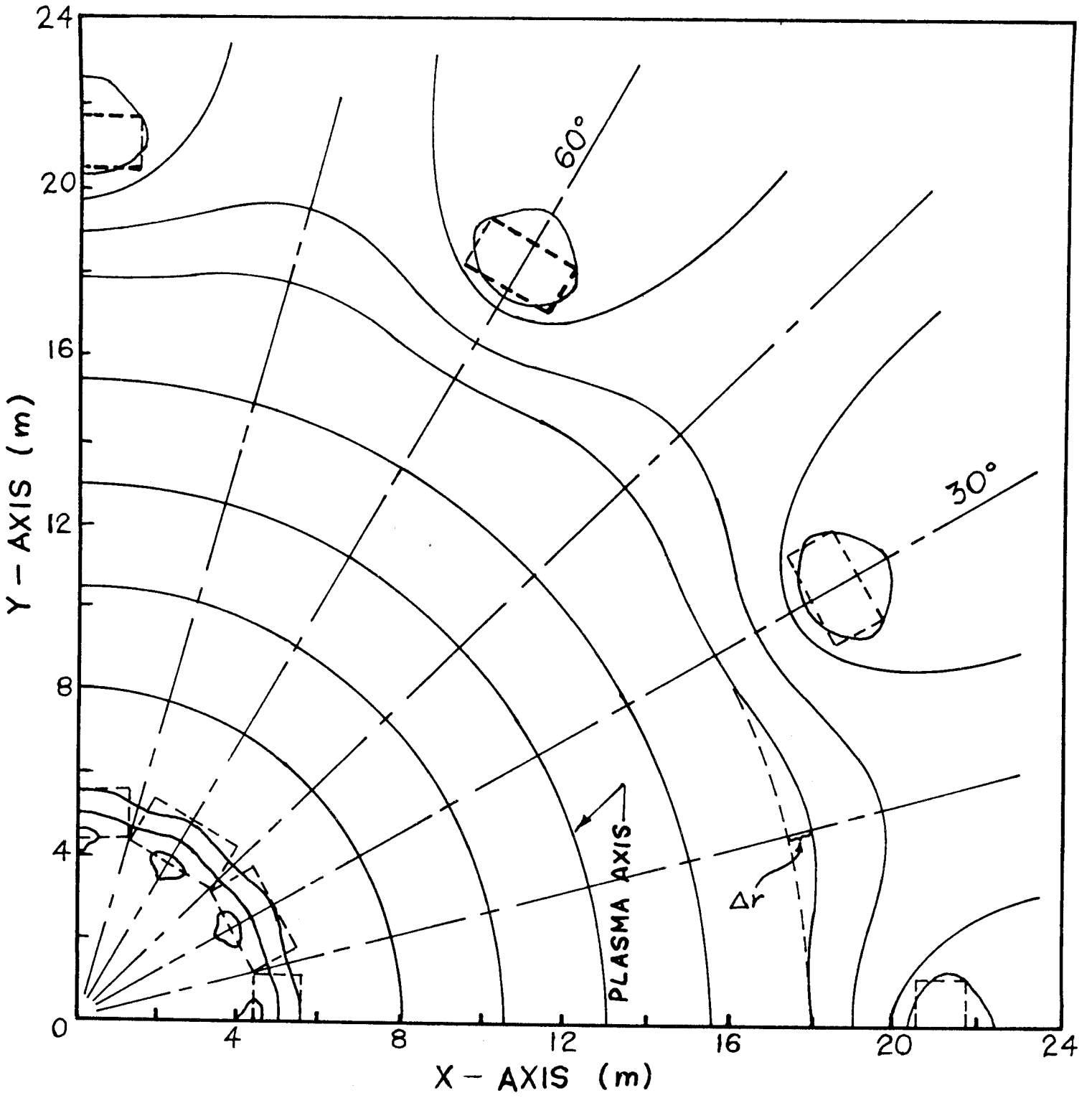


Figure 1

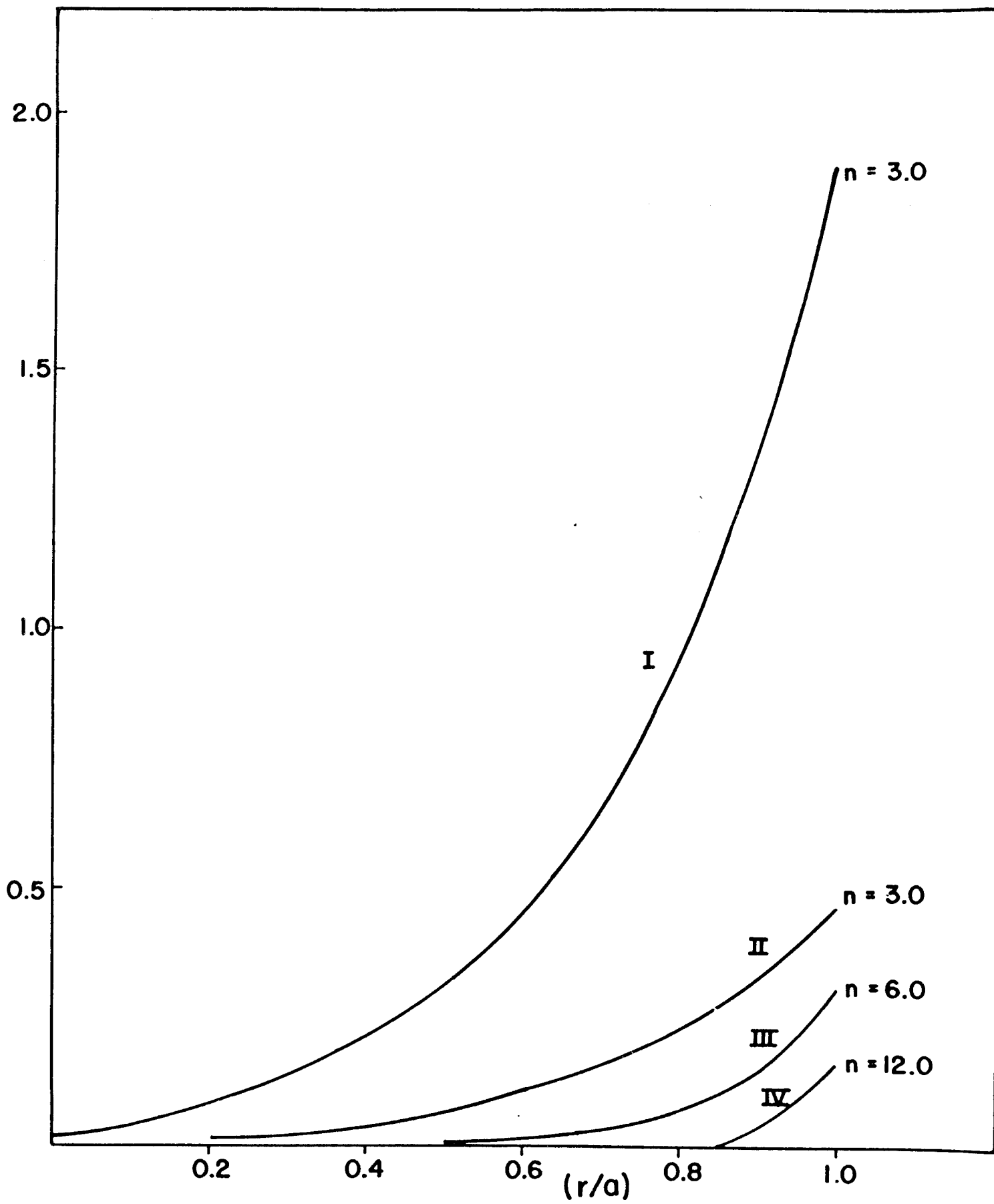


Figure 2

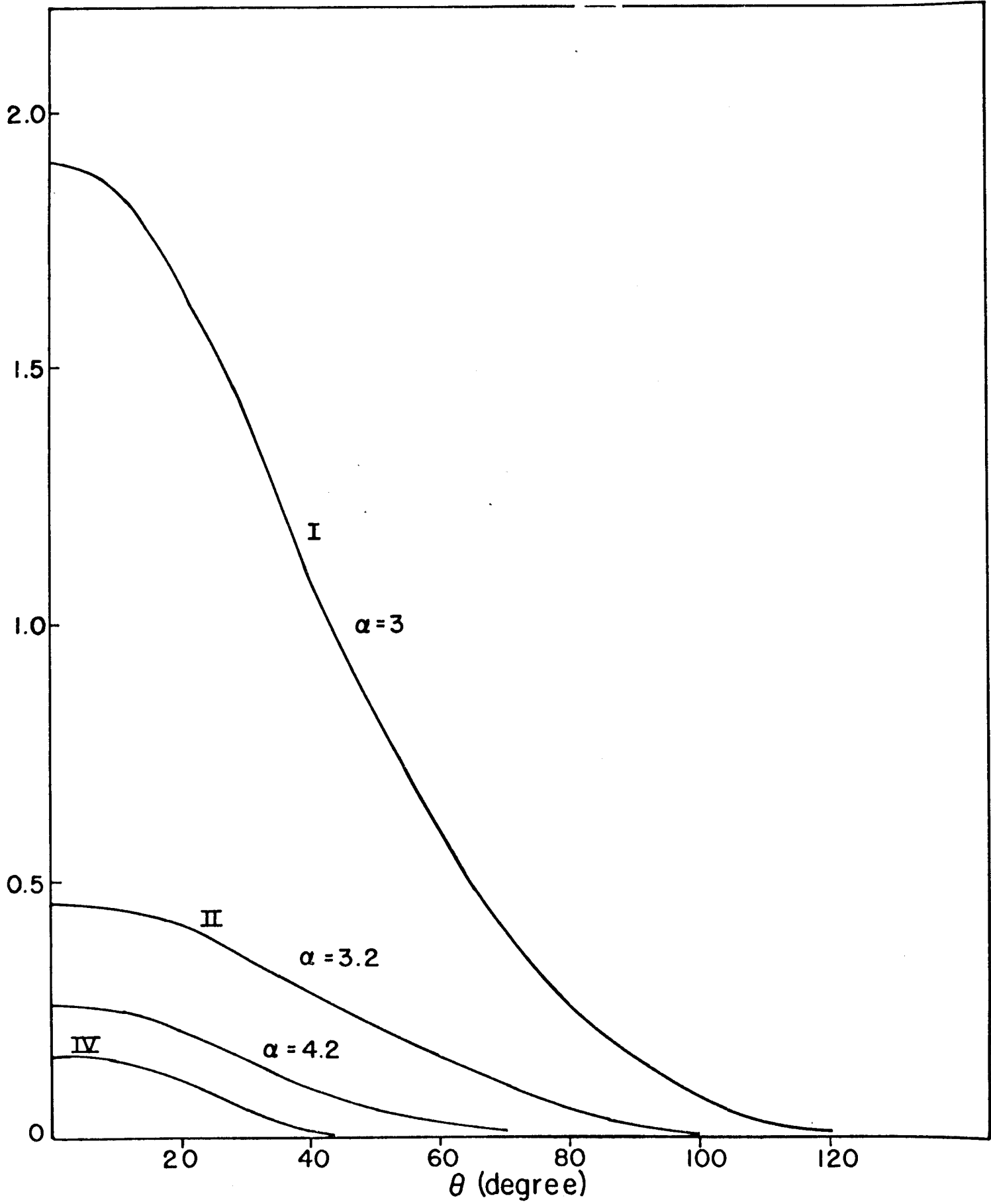


Figure 3



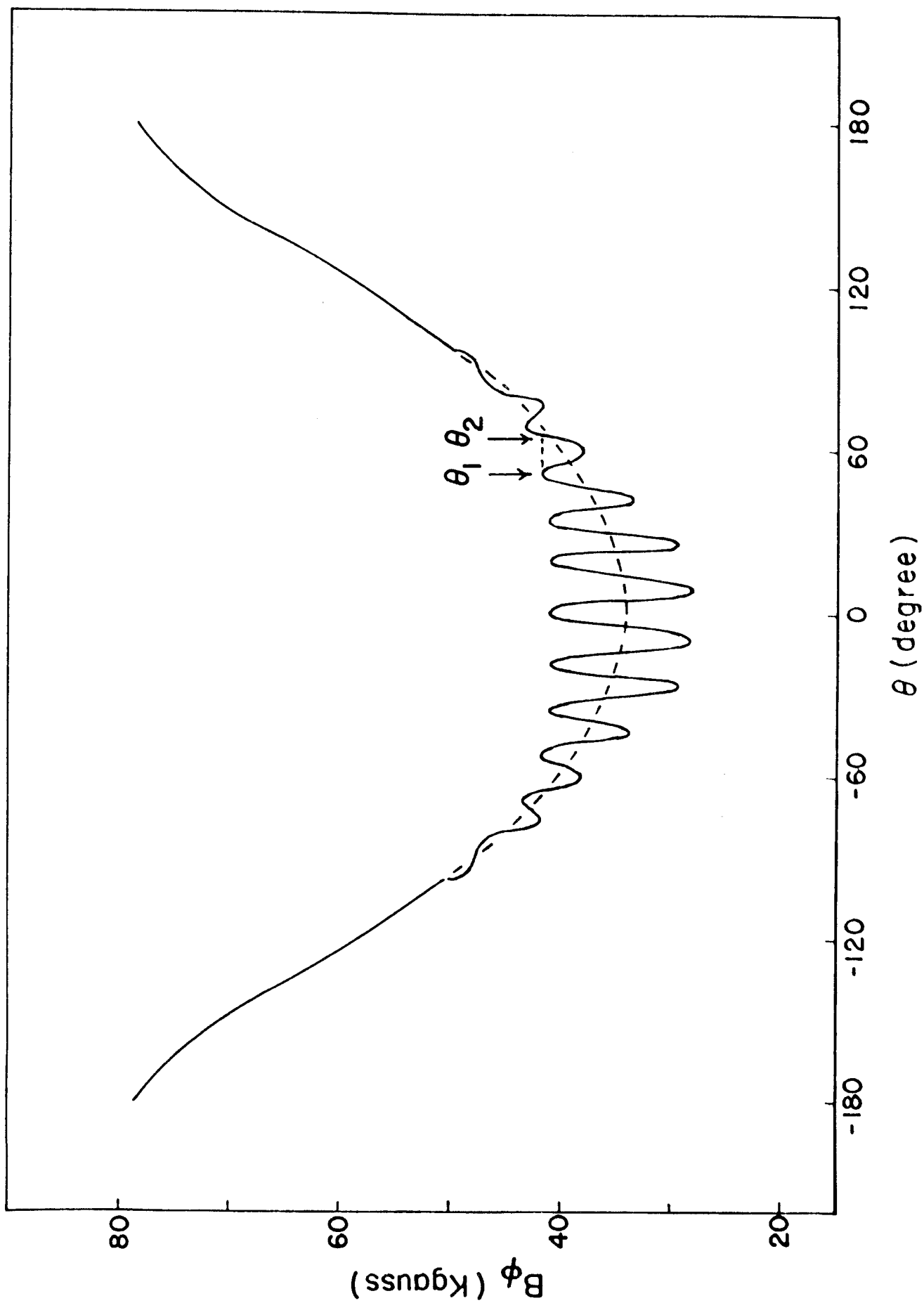


Figure 4

$$\left. \frac{\partial}{\partial \theta} \left( \frac{B_\phi}{B_0} \right) \right| = 0 \quad (4)$$

using (1) we have

$$\frac{\sin \theta_1}{1 + \frac{r}{R} \cos \theta_1} \approx \frac{aK}{\epsilon_a r} \left( \frac{r}{a} \right)^n \exp\left(-\left(\frac{\alpha \theta_1}{\pi}\right)^2\right) \sin(Nq\theta_1 + \phi_0) \quad (5)$$

where K is defined by<sup>(3)</sup>

$$K = \frac{\epsilon NqR}{a} \quad (6)$$

The upper boundary  $\theta_2$  is given by  $B_\phi(\theta_2) = B_\phi(\theta_1)$ . The projection of the trapping regions on the  $r - \theta$  plane is shown in Fig. 6 for constant  $\phi_0$ . Unlike the case of poloidally uniform ripples, the trapping regions are confined to the first and fourth quadrants of the  $r - \theta$  plane even though K is large. These locally trapped particles will drift vertically in the gradient of the toroidal field with a velocity

$$v_D = \frac{mv^2}{2eB_0 R} \quad (7)$$

A 3.5 MeV alpha particle can drift to the wall in about  $10^{-2}$  msec, which is much shorter than the time to slow down or to scatter in pitch angle and become untrapped. Consequently, the fraction of alphas formed in the region of phase space corresponding to being locally trapped in the bumpy field will be lost directly from the plasma and will impinge on the first wall in localized zones.

For an isotropic source of energetic particles, the fractions of particles trapped at  $(r, \theta, \phi)$  is

$$f(r, \theta, \phi) = \sqrt{1 - \frac{B(r, \theta, \phi)}{B(r, \theta_1)}} \quad (8)$$

where  $B(r, \theta_1)$  is the field at the mirror point on the field line passing through  $(r, \theta, \phi)$ . From (2),

$$B(\theta_1) \approx \frac{R B_0}{R+r \cos \theta_1} \left[ 1 + \epsilon_a \left( \frac{r}{a} \right)^n \exp\left(-\left(\frac{\alpha \theta_1}{\pi}\right)^2\right) \right]. \quad (9)$$

Since  $\cos(Nq\theta_1 + N\phi_0) \approx 1$ , we find

$$\frac{B(r, \theta, \phi)}{B(\theta_1)} = \frac{1 + \epsilon_a \left( \frac{r}{a} \right)^n \exp\left(-\left(\frac{\alpha \theta}{\pi}\right)^2\right) \cos N\phi}{1 + \epsilon_a \left( \frac{r}{a} \right)^n \exp\left(-\left(\frac{\alpha \theta_1}{\pi}\right)^2\right)} \quad (10)$$

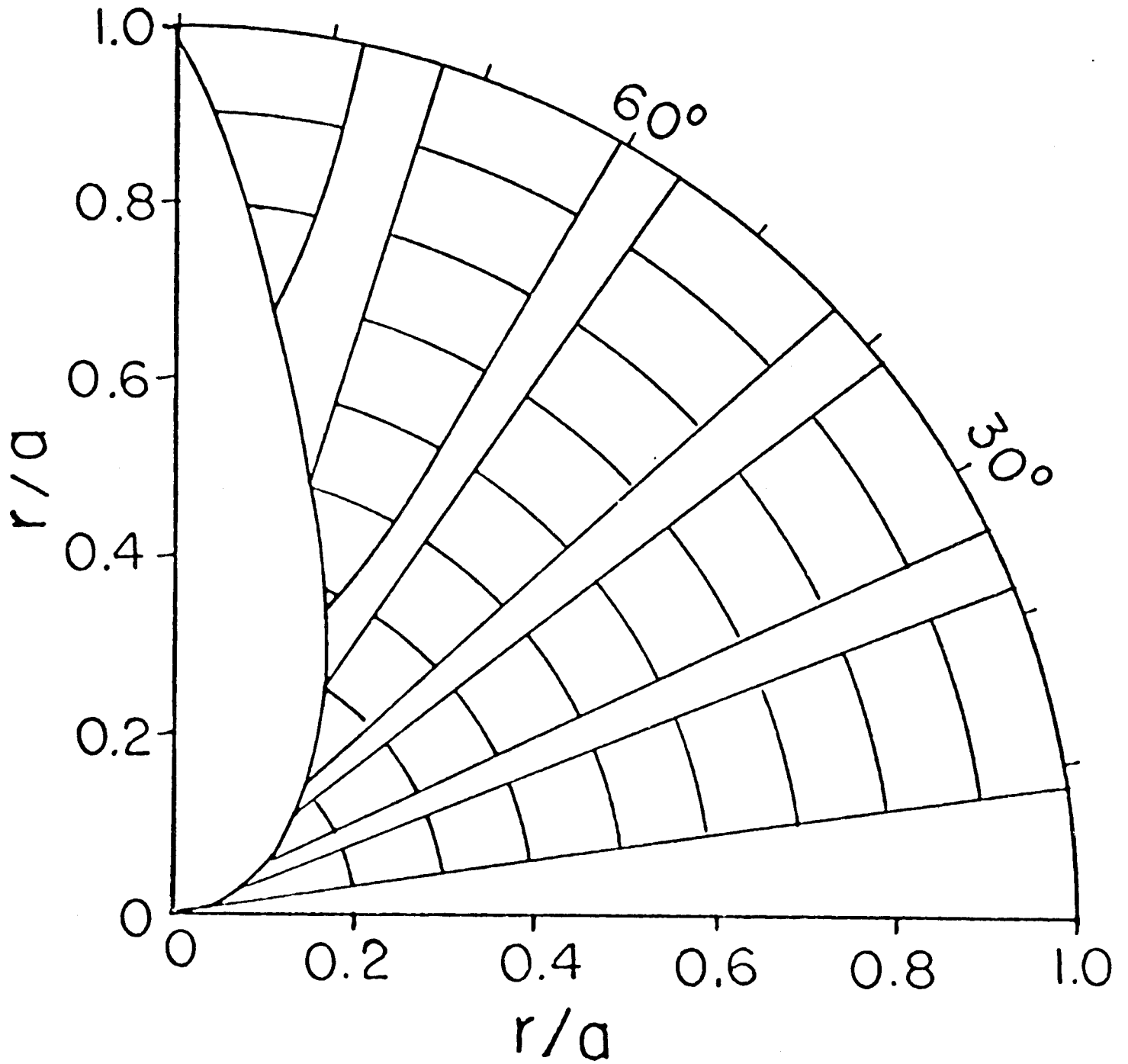


Figure 5

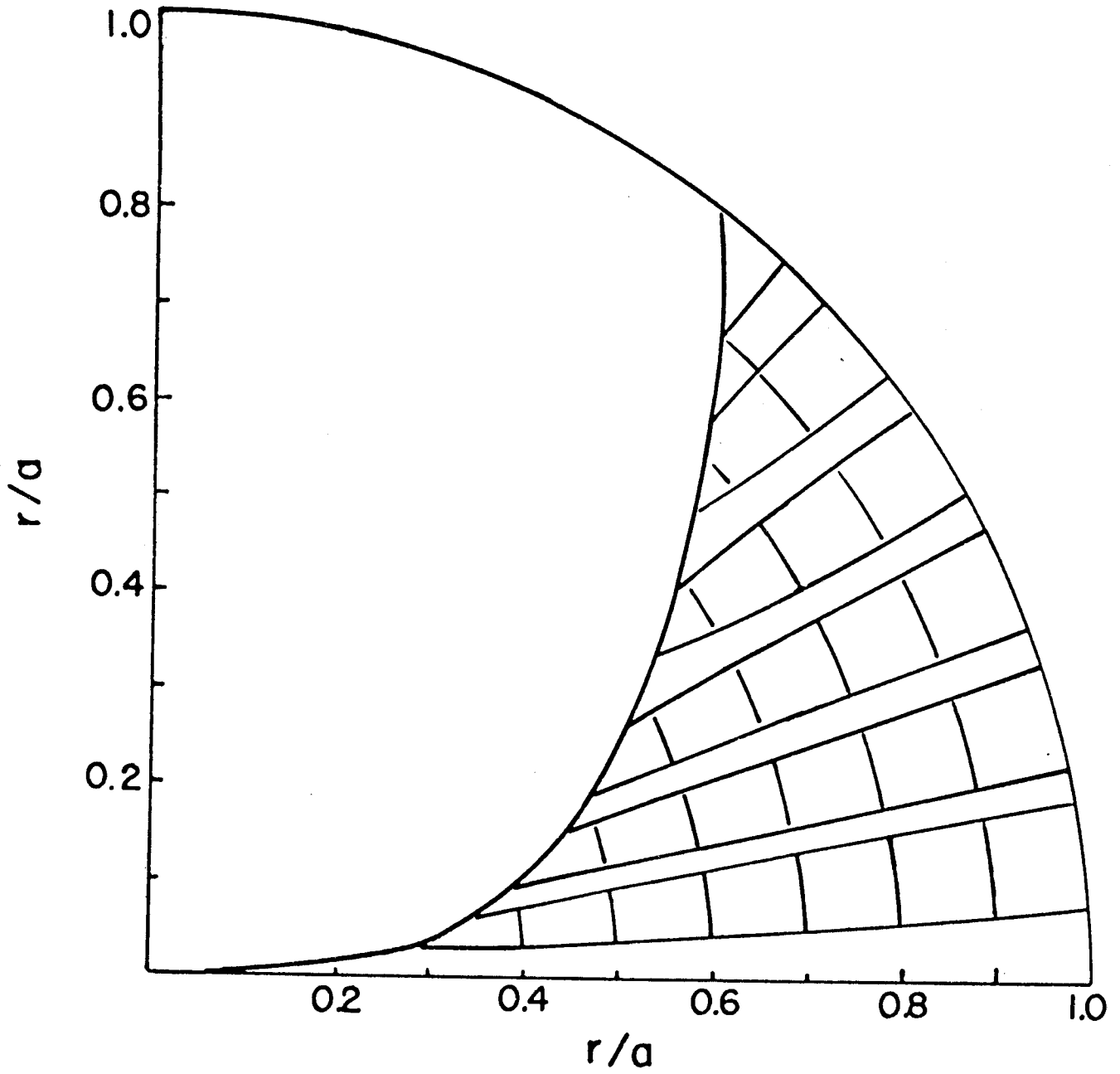


Figure 6

and

$$f(r, \theta, \phi) \approx \sqrt{\epsilon_a} \left(\frac{r}{a}\right)^{\frac{n}{2}} \exp\left(-\frac{1}{2} \left(\frac{\alpha\theta}{\pi}\right)^2\right) \sqrt{1 - \cos N\phi}. \quad (11)$$

The total fraction of particles trapped is

$$f_T = \frac{\int_0^a r dr S(r) \int_0^{\theta_{\max}(r)} d\theta \int_0^{2\pi} d\phi f(r, \theta, \phi)}{2\pi^2 \int_0^a r dr S(r)}, \quad (12)$$

where  $S(r)$  is a source function independent of  $\theta$ .

$$f_T = \frac{\sqrt{\epsilon_a} \int_0^{2\pi} \sqrt{1 - \cos N\phi} d\phi \int_0^a dr r S(r) \left(\frac{r}{a}\right)^{\frac{n}{2}} \int_0^{\theta_{\max}(r)} \exp\left(-\frac{1}{2} \left(\frac{\alpha\theta}{\pi}\right)^2\right) d\theta}{2\pi^2 \int_0^a r dr A(r)} \quad (13)$$

For purposes of estimation, we assume

$$\frac{1}{2\pi} \int_0^{2\pi} \sqrt{1 - \cos N\phi} d\phi \approx 1, \quad (14)$$

and  $\frac{\alpha\theta_{\max}(r)}{\pi} \gg 1$ , and we find

$$f_T = \frac{16 \sqrt{2\pi}}{(n+4)(n+8)\alpha} \sqrt{\epsilon_a} \quad (15)$$

For UWMAK-I conditions, <sup>(1)</sup>  $f_T = 5.6\%$  which means that 5.6% of the 3.5 MeV alpha particles will not be magnetically confined but will strike the wall in localized areas between the coils in the first quadrant of the  $r - \theta$  plane. This may cause additional wall damage and generation of impurities. We would like to minimize this loss by increasing the number of coils and/or extending the coils at the outside of the torus.

The curves marked III in Fig. 2 and Fig. 3. show that the amplitude of the ripple is reduced by a factor of 4 by doubling the number of coils. The amplitude also decreases faster, both radially and poloidally, than when there are only 12 coils. Correspondingly, the trapping regions are also smaller, as shown by Fig. 3. Curves II and IV in Fig. 2 and Fig. 3 are ripples for 12 and 24 "extended" D magnets, respectively. The parameters for modeling these ripples and the fraction of trapped alphas are listed in Table I. We note here that the numerical accuracy in the magnetic field calculation is about 0.01%.

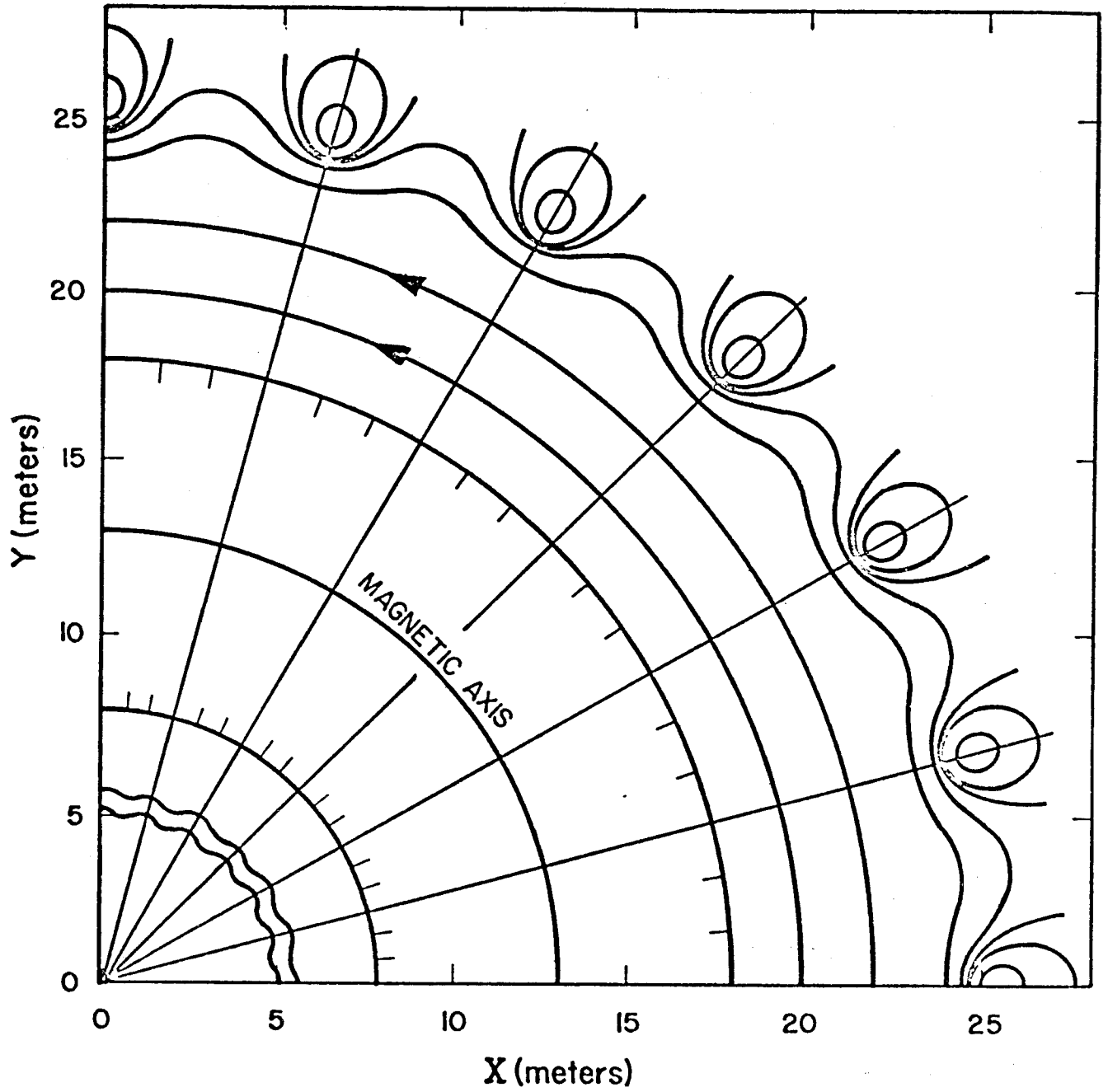


Figure 7

Table I

Parameters of the Magnetic Field Ripples  
for 4 Different Magnetic Field Coil Designs,  
and the Total Fractions of the  $\alpha$ -Particles  
Trapped in Those Ripples

		$\epsilon_a$	n	$\alpha$	$f_T$ (%)
UWMAK-I	12 coils	0.25	3	3	5.6
	12 extended coils	0.046	3	3.2	3.4
	24 coils	0.03	6	4.4	1.1
	24 extended coils	0.016	12	5.0	0.3
UWMAK-II	24 extended coils	<0.01	>12	>5	<0.1

Table I indicates that for the 24 "extended" D-coil magnet design, as chosen for UWMAK-II, the fast alpha loss rates are very low. An auxiliary reason for extending the TF coil to large R is to allow enough space between coils such that blanket segments can be removed without removing the coils themselves.

In conclusion, the UWMAK-II design has 24 "D" shaped toroidal field coils extended on the outer edge. The field ripple is satisfactorily small, less than 0.01%. The magnetic field structure of UWMAK-II is illustrated by some of the field lines shown in Figure 7. The corrugation of the plasma column is now <0.01 meter.

#### References

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