Preliminary Study of Supplementary RF Heating in a Tokamak Fusion Reactor

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FUSION TECHNOLOGY INSTITUTE

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Abstract

A study of the scalability of the fast magnetosonic wave heating from present day tokamaks to UWMAK size reactors is presented. This preliminary work shows that this type of supplementary wave heating in a reactor appears feasible. However, part of the report poses important problems which must be solved in a more detailed manner before further work on a reactor design can be carried out. First a brief review of the wave heating possibilities is presented, concluding that at present the fast magnetosonic mode appears to offer the best scalability to reactor size machines. Next a review of the Vlasov wave theory for this mode is presented, indicating conditions under which the results are valid. Ion cyclotron, ion finite gyroradius, and electron transit time damping rates are presented for the fundamental and first harmonic of the ion cyclotron frequency where account of the variation of the local gyrofrequency across the minor radius of the column has been taken into account. The particular advantages of fast ion cyclotron damping rates for a two component plasma are also presented. Application of the theory to a UWMAK reactor shows that many modes are possible and if modes with sufficiently short perpendicular and parallel wavelengths are excited, plasma absorption will dominate over losses by stainless steel or carbon curtain walls.

Next a discussion of important refinements in the wave theory and design of an appropriate launching structure are presented as problems for future investigation. Finally, adaptation of an rf heating model to a space-time dependent code for reactor startup is posed as the ultimate goal of this work.

I. INTRODUCTION

In order to reach fusion ignition conditions in a tokamak, it is well recognized that a supplementary heating scheme will have to be employed. The main candidates for this necessary additional heating capability at present are (1) neutral beams, (2) rf heating, and (3) magnetic compression. This report will deal only with the rf heating technique and future reports will attempt to make comparisons of the relative merits of rf heating with respect to other supplementary heating schemes.

Relatively successful experimental work has previously been done by means of rf heating in the cases of ECRH heating in magnetic mirror machines and ICRH[1,2] heating in the axial magnetic beach in the case of the C stellarator. Recently, vigorous theoretical and experimental work has been done to determine the efficiency by which rf heating can be applied to the axisymmetric magnetic field and plasma configuration of a tokamak. The basic problem of rf heating breaks up into power generation, launching or coupling this energy efficiently to the plasma with accessibility to the core region, and absorbing this energy by the plasma ions and electrons in a way which does not deteriorate the confinement properties of the tokamak configuration. If one uses the magnetosonic wave mode, the generation problem is minimal since amplifier tubes exist "on the shelf" which provide power outputs in excess of a megawatt at the contemplated frequencies of operation of 10-100 MHz. The problems of wave launching, accessibility to the plasma core, and absorption are somewhat more complicated and will be discussed in the following sections of the report. Special aspects associated with a reactor environment will be discussed and topics which will be treated in future reports will be posed.

Let us briefly mention the various types of rf heating applicable to a tokamak plasma and why we feel that with the information available at present, the magnetosonic mode offers the best promise of supplementary heating in a tokamak reactor. The wave heating types which have been considered for heating tokamak plasmas are fast and slow magnetosonic wave heating, lower hybrid wave heating, transit time magnetic pumping, and electron cyclotron resonance heating. Many other wave modes have been eliminated because the magnetic field configuration and radial density profile of a tokamak configuration do not allow accessibility of the wave to the axis of the discharge. This is because the modes are evanescent or because of strong reflection of the wave coming from the vacuum region at the edge of the plasma boundary. Electron cyclotron resonance heating is usually eliminated from contention because for reactors with magnetic fields of 50 kG, microwave sources at frequencies of 100-150 GHz are only available at powers much less than the megawatt level required and it does not appear technologically feasible to increase their power output significantly. A second minor problem is that in present day machines, direct ion heating is the goal to be achieved since electron-ion equilibration times are long compared to energy confinement times. In a reactor with much longer confinement and heating times it may be possible to heat the electrons and allow the ions to gain energy via electron-ion collisions, assuming serious instabilities do not disrupt this process.

Transit time magnetic pumping has been tried experimentally at Culham and Grenoble but has not been very successful, resulting in plasma pumpout from the confining region in stellarators. It has not yet been experimentally tried for tokamak systems but theoretical work is continuing and may result in a more promising electron TTMP heating. Coil and wall losses are calculated to exceed that due to plasma ion TTMP absorption in calculations done to date for JET.[3] A final judgment here awaits further theoretical and experimental work but at present it does not appear to be as attractive as magnetosonic wave heating.

Lower hybrid wave heating has been considered theoretically by Puri and Tutter[4,5] and experiments have been done in particular by Hooke and Bernabi.[6] The operating frequencies for such a mode would necessarily be higher than those for the magnetosonic modes but not so high that megawatt sources cannot be made available. The most serious problem here seems to be a proper theoretical treatment of the wave properties at the plasma edge where the wave suffers a region of evanescence, the fact that the wave heats both the ions and electrons at a relatively localized hybrid layer rather than body heating, and the difficulty of extrapolating from present day PLT size tokamaks to a reactor. Work at Princeton is planned for an experiment on lower hybrid heating using a phased array waveguide feed on the ATC plasma at 200 kW (800 MHz) levels. It will be most interesting to compare these results with those of neutral beam experiments on the same machine at comparable power levels.

This brings us to the magnetoacoustic mode which will be considered in greater detail in later sections of the report. Most of the theoretical work on this subject has been done by Adam and Samain[7-9] at Fontenay-aux-Roses, Weynants, Messiaen, and Vandenplas[10-17] at the Ecole Royale Militaire, Belgium, and by Perkins et al.[18-20] at Princeton. The slow wave (torsional) branch is subject to mode conversion near the edge of the plasma according to work by Perkins[18] and is thus capable of only edge heating. The fast wave (compressional) magnetoacoustic mode propagates readily above a critical density which is easily achievable in present day tokamaks and more easily achieved in larger systems such as a reactor. Present theoretical work shows that this mode penetrates readily to the center of plasmas of reactor interest and heats ions over the minor axis of the plasma in a relatively gentle way so that instabilities are less likely to occur. Recent experimental work on the ST tokamak by Adam et al.[21] has shown that these modes can be generated in the plasma at megawatt levels with efficiencies of 90% and that the ion temperature can be doubled by short rf pulses with a corresponding heating efficiency of 20%. Present work is being carried out to design a similar magnetoacoustic wave experiment on the French TFR tokamak where stronger toroidal currents and confining fields may provide better confinement of the hot ions produced by the wave heating.

This report will first review the Vlasov theory for the magnetosonic mode, noting the conditions under which the approximations are valid and obtain the dispersion relation and damping rates from the hot plasma dielectric tensor. Then the special advantages of the magnetosonic mode in regard to a two component TCT reactor will be presented.

Next a sample calculation of operating conditions for UWMAK[22] parameters and special reactor considerations including stainless vs. carbon curtain walls[23] of the torus and wave excitation will be considered. Finally, a discussion of the coupling alternatives, implications of the present heating theory for reactors, and important problems to be looked at in future reports will be posed.

II. REVIEW OF HOT PLASMA VLASOV THEORY APPLIED TO THE FAST MAGNETOSONIC WAVE

As previously discussed, rf heating has several advantages over other forms of supplementary heating. In particular, waves generated about the ion cyclotron frequency seem especially promising because of the linear wave damping processes which can be used in heating the ions in large tokamaks.

First of all, slow ion cyclotron waves, which have produced good results in the C-Stellarator[2,3,24] appear to be more difficult to excite in an axisymmetric toroidal system the size of UWMAK. Ion cyclotron waves are characterized by small wavelengths parallel to the confining magnetic field. At high plasma densities ($n > 10^{13}$ cm^{-3}) there is sufficient charge separation so that the electric field rotating with the ions is shielded out.[25] In addition to this disadvantage to using the slow (torsional) ion cyclotron wave, the electromagnetic energy which is coupled to the plasma is absorbed in a localized region of the plasma volume, thus leading to a high concentration of energy with the accompanying problems of nonlinear effects limiting the coupling and absorption efficiencies.[26]

When the ion cyclotron wave is used in a tokamak configuration, as it approaches a magnetic beach in the perpendicular direction, the wave can undergo mode conversion. For a cold plasma ($\omega_{ci}/k_{\parallel} >> v_{Te}$), the inward propagating ion cyclotron wave is converted into an outward propagating electrostatic ion cyclotron wave with the following dispersion relation[18]

$$\omega^2 = \omega_{ci}^{\ 2} \left(1 - \frac{k_x^{\ 2}}{k_{\parallel}^{\ 2}} \frac{m}{M} \right) \tag{1}$$

which propagates on the low density side of the perpendicular resonance. In a hot plasma, the incoming wave becomes a slow wave with the following dispersion relation

$$\omega^2 = k_x^{\ 2} \ c^2 \ k_{\parallel}^{\ 2} \quad D^2 \ Y \left(\frac{\omega}{k_{\parallel}} \left(\frac{m}{2T_e} \right)^{\frac{1}{2}} \right) \tag{2}$$

with

$$Y(\eta) = \int_{-\infty}^{\infty} \frac{du}{\sqrt{\pi}} \ \frac{ue^{-u^2}}{(u - \eta - i\varepsilon)}$$

Since the ion cyclotron wave can only propagate for[18]

$$\frac{1}{2} \ \frac{|\omega^2 - \omega_{ci}^{\ 2}|}{\omega^2} \ k_{\parallel}^{\ 2} < \frac{\omega_{pi}^{\ 2}}{c^2} < \frac{|\omega^2 - \omega_{ci}^{\ 2}|}{\omega^2} \ k_{\parallel}^{\ 2} \tag{3}$$

it is seen that k must be rather large. At these short wavelengths the wave has a large region of evanescence, greatly reducing the wave amplitude inside the plasma while wall losses are enhanced. In addition, the slow wave which propagates on the high density side of the perpendicular resonance heats electrons rather than ions.[18] Nevertheless, the ion cyclotron resonance can still be utilized effectively in conjunction with the propagation of a different ion wave.

The fast magnetoacoustic or hydromagnetic wave is an extension of the right hand compressional Alfvén wave which, due to finite temperature effects, has some left handed polarization. Perkins[19] has shown that only the fast mode of the ion cyclotron wave can become a normal mode of a tokamak plasma if $\omega = \omega_{ci}$ is to be located within the plasma. Fast magnetosonic wave heating has three advantages over other types of heating: (1) The wave easily penetrates to the plasma core (provided $n > $ a critical density, n_c), (2) A relatively uniform distribution of wave amplitude throughout the torus produces more uniform body heating, and (3) Toroidal eigenmodes increase the loading resistance on the launching structure.[18]

When $\omega = \omega_{ci}$ the magnetoacoustic waves are damped by the following mechanisms:

(1) Since $v_{\phi\parallel} \lesssim v_{Te}$ the electrons undergo transit time damping.

(2) Since $T_i \neq 0$, there exists a finite left hand component which heats the ions by ion cyclotron damping.

(3) When $\omega = 2\omega_{ci}$, there exists a relatively large left hand component and finite gyroradius effects provide the damping.

Because this damping is weak, linear wave theory may be used to compute the normal modes. Then, by using Vlasov theory, the damping decrements may be calculated in order to estimate the Q of the cavity, the relative importance of electron and ion damping, and finally, the propagation properties of the wave.

The magnetoacoustic mode is only slightly affected by variations in the magnetic field strengths and minor cross section and for $\lambda_{\parallel} << R$, curvature effects can be neglected. The effect of the rotational transform is to distribute the power throughout the plasma with little effect on the heating efficiencies.[7]

The magnetoacoustic wave has a density and radius cut-off, below which the wave cannot be excited. This critical density is given by[2,27]

$$n_c = 1.64 \times 10^{12} \left(\frac{M}{Z_i m}\right) (\Omega p)^{-3} \quad (4)$$

with

$$\Omega = \frac{\omega}{\omega_{ci}}$$

and p = limited plasma radius in cm.

Adam and Samain[7] and Perkins[18,19] have obtained damping decrements and wave propagation applicable to the fundamental and first harmonic of the ion cyclotron frequency. However, a complete published derivation of these results listing important approximations and limitations under which these results are valid does not exist to our knowledge.

To overcome this, T. K. Mau[28] has carried out a complete derivation of these results from the hot plasma dielectric tensor which is available as a report. This section of the report closely follows this work in abbreviated form with the approximations and limitations clearly put forth.

Let us assume that there exists a homogeneous, unbounded, collisionless, Maxwellian plasma in a uniform magnetic field in the z direction. In addition, an electromagnetic wave is assumed to propagate in an arbitrary direction. Then, both ions and electrons satisfy the Vlasov equation and the following determinant can be obtained without any further assumptions.

$$\begin{bmatrix} \left(\frac{\omega}{c}\right)^2 K_{xx} - k_{\parallel}^2 & \left(\frac{\omega}{c}\right)^2 K_{xy} & \left(\frac{\omega}{c}\right)^2 K_{xz} + k_{\parallel} k_{\perp} \\ \left(\frac{\omega}{c}\right)^2 K_{yx} & \left(\frac{\omega}{c}\right)^2 K_{yy} - k_{\perp}^2 - k_{\parallel}^2 & \left(\frac{\omega}{c}\right)^2 K_{yz} \\ \left(\frac{\omega}{c}\right)^2 K_{zx} + k_{\parallel} k_{\perp} & \left(\frac{\omega}{c}\right)^2 K_{zy} & \left(\frac{\omega}{c}\right)^2 K_{zz} - k_{\perp}^2 \end{bmatrix} = 0 \qquad (5)$$

In the low frequency limit of $\omega \sim \omega_{ci}$, the displacement current can be neglected. For simplicity, an isotropic distribution function of the form

$$f_{\alpha o}(V_{\alpha\perp}^2, V_{\alpha\parallel}^2) = \frac{1}{\pi^{3/2} V_{T\alpha}^3} \exp\left\{-\frac{V_{\alpha\perp}^2 + V_{\alpha\parallel}^2}{V_{T\alpha}^2}\right\}$$

with

$$V_{T\alpha} = \left(\frac{2\kappa T_\alpha}{M_\alpha}\right)^{1/2}$$

is assumed. Then, as obtained by Perkins[18] and derived by Mau,[28] the following dispersion relation can be found for $\lambda_\perp >> 2.2$ cm (UWMAK parameters).

$$\begin{vmatrix} \frac{\omega_{pi}^2}{c^2} X - k_\parallel^2 & i\frac{\omega_{pi}^2}{c^2}\left[X - \frac{\omega^2}{\omega_{ci}(\omega+\omega_{ci})}\right] & k_\parallel k_\perp \\ -\frac{i\omega_{pi}^2}{c^2}\left[X - \frac{\omega^2}{\omega_{ci}(\omega+\omega_{ci})}\right] & \frac{\omega_{pi}^2}{c^2} X - k_\perp^2 - k_\parallel^2 + 2\beta_e^2 k_\parallel^2 (Y-1) & -i\frac{\omega_{pi}^2\,\omega k_\perp}{\omega_{ci}\, c^2 k_\parallel} Y \\ k_\parallel k_\perp & i\frac{\omega_{pi}^2\,\omega k_\perp Y}{c^2\,\omega_{ci}\, k_\parallel} & \frac{\omega^2}{c^2 k_\parallel^2 \lambda_{De}^2} Y - k_\perp^2 \end{vmatrix} = 0 \qquad (6)$$

where

$$X = \frac{1}{2}\left(\frac{\omega}{k_\parallel V_{Ti}}\, Z\left(\frac{\omega - \omega_{ci}}{k_\parallel V_{Ti}}\right) - \frac{\omega}{\omega + \omega_{ci}}\right),$$

$$Y = Y\left(\frac{\omega}{k_\parallel V_{Te}}\right),$$

$$\beta_e = n_e \kappa T_e/(B_o^2/2\mu_o),$$

and Z denotes the Fried and Conte function.

This dispersion relation does not include finite-gyroradius effects and cyclotron damping is assumed dominant. Equation (6) is only valid when

$$\left(\frac{m}{M}\,\frac{T_i}{T_e}\right) << 1.$$

Neglecting the off diagonal terms in the last row and column and assuming

$$k_\perp^2 << \frac{\omega^2}{c^2\, k_\parallel^2\, \lambda_{De}^2}$$

we can obtain the following equation

$$-X\left[q_\perp^2 + 2q_\parallel^2 - \frac{2\omega^2}{\omega_{ci}(\omega+\omega_{ci})}\right] + \left[q_\parallel^2\,(q_\perp^2 + q_\parallel^2) - \frac{\omega^4}{\omega_{ci}^2\,(\omega+\omega_{ci})^2}\right] = 0 \qquad (7)$$

where

$$q_\parallel = \frac{k_\parallel\, c}{\omega_{pi}} \quad \text{and} \quad q_\perp = \frac{k_\perp\, c}{\omega_{pi}} \quad \text{with} \quad \beta_e << 1.$$

The zeroth order dispersion relation for $\omega = \omega_{ci}$ is

$$q_\perp^2 + 2q_\parallel^2 = 1 \qquad (8)$$

The following damping decrement can be calculated by setting $\omega = \omega_{ci} + i\omega_i$, substituting equation (8) into equation (7), neglecting the $\omega/(\omega + \omega_{ci})$ term in X, and equating coefficients of the first power in ω_{ci}.

$$-\omega_i = \gamma_i = -\frac{1}{3}\, k_\parallel \left(\frac{2\kappa T_i}{M}\right)^{\frac{1}{2}} \mathrm{Im}\left[\frac{1}{Z(z_i)}\right] \qquad (9)$$

where $z_i = (\omega - \omega_{ci})/k_\parallel V_{Ti}$.

Finally, averaging over the minor cross section in an axisymmetric torus with $B_T = B_o(1 - \frac{r}{R}\cos\theta)$ one obtains the following

$$\bar{\gamma}_i = (1.58)\, \frac{k_\parallel^2\, V_{Ti}^2}{3\omega_{ci}} (\frac{R}{a}) \qquad (10)$$

Retaining the electron term in equation (6) and since $|X| >> 1$, we have the following dispersion relation

$$q_\perp^2 + 2q_\parallel^2 - \frac{2\omega^2}{\omega_{ci}(\omega_{ci}+\omega)} - 2\beta_e\, q_\perp^2 (Y-1) = 0 \; . \qquad (11)$$

Then, in order to find the damping decrement due to electron transit time magnetic pumping, we again set $\omega = \omega_{ci} + i\omega_i$, equate real and imaginary parts and find

$$\gamma_e = \frac{8}{3}\, e^{-1}\, \pi^{\frac{1}{2}}\, \frac{k_\perp^2}{M}\, \frac{\kappa T_e}{\omega_{ci}} \qquad (12)$$

One can show that $\gamma_e/\gamma_i \sim 1.24 (k_\perp/k_\parallel)^2$.

At the first harmonic ion cyclotron resonance, the following determinant is obtained.

$$\begin{vmatrix} \frac{\omega_{pi}^2}{c^2}\{X'-Y'-W'\} - k_\parallel^2 & i\,\frac{\omega_{pi}^2}{c^2}\left(X'+Y'+\frac{\omega_{ci}\omega}{\omega_{ci}^2-\omega^2} - \frac{\omega}{\omega_{ci}}\right) & k_\perp\, k_\parallel \\ -\frac{i\omega_{pi}^2}{c^2}\left\{X'+Y'+\frac{\omega_{ci}\omega}{\omega_{ci}^2-\omega^2} - \frac{\omega}{\omega_{ci}}\right\} & \frac{\omega_{pi}^2}{c^2}(X'-Y'-W') - k_\parallel^2 - k_\perp^2 + 2\beta_e k_\perp^2 (Y-1) & -i\frac{\omega_{pi}^2}{c^2}\,\frac{\omega k_\perp}{\omega_{ci}k_\parallel}\,Y \\ k_\perp\, k_\parallel & i\,\frac{\omega_{pi}^2}{c^2}\,\frac{\omega k_\perp}{\omega_{ci}k_\parallel}\,Y & \frac{\omega^2}{c^2 k_\parallel^2 \lambda_{De}^2}\,Y - k_\perp^2 \end{vmatrix} = 0$$

with (13)

$$X' = \left(\frac{k_\perp V_{Ti}}{2\omega_{ci}}\right)^2 \frac{\omega}{k_\parallel V_{Ti}}\, Z\left(\frac{\omega - 2\omega_{ci}}{k_\parallel V_{Ti}}\right) , \quad Y' = \frac{\omega}{\omega+\omega_{ci}} \left(\frac{k_\perp V_{Ti}}{2\omega_{ci}}\right)^2$$

and

$$W' = \frac{1}{1 - \frac{\omega_{ci}^2}{\omega^2}} \; .$$

Following a procedure similar to that for the fundamental damping rate, one finally obtains a finite gyroradius effect damping rate given by

$$\langle\gamma_i\rangle_{2\omega_{ci}} = \frac{1}{4}\,\frac{k_\perp^{\,2}\,kT_i}{M\,\omega_{ci}}\,\left(\frac{R}{a}\right) \tag{14}$$

Since $k_\perp V_{Ti}/2\omega_{ci} \sim \langle r_{ci}\rangle/\lambda_\perp$ where $\langle r_{ci}\rangle$ is the average ion gyroradius, this damping process is a finite gyroradius effect which preferentially heats ions with large perpendicular and small parallel energy, creating a high energy tail, thereby possibly enhancing hot ion losses. First harmonic heating can be seen to be comparable with fundamental heating.

III. DAMPING DECREMENTS FOR THE CASE OF A TWO COMPONENT PLASMA

Assume again an infinite, homogeneous, collisionless, Maxwellian plasma in a uniform magnetic field in the z direction. In addition, assume that there is more than one ionic species of which one, α, is the resonant species. The nonresonant ions form a cold background and by an analogous procedure as before, the following determinant can be derived.

$$\begin{vmatrix} \frac{\omega_{p\alpha}^2}{c^2} X'_\alpha - k_\parallel^2 & i\frac{\omega_{p\alpha}^2}{c^2}\left[X'_\alpha - \frac{\omega^2}{\omega_{c\alpha}(\omega+\omega_{c\alpha})}\right] & k_\parallel k_\perp \\ -i\frac{\omega_{p\alpha}^2}{c^2}\left[X'_\alpha - \frac{\omega^2}{\omega_{c\alpha}(\omega+\omega_{c\alpha})}\right] & \frac{\omega_{p\alpha}^2}{c^2} X'_\alpha - k_\perp^2 - k_\parallel^2 + 2\beta_e^2 k_\parallel^2 (Y-1) & -i\frac{\omega_{p\alpha}^2 \omega k_\perp}{\omega_{c\alpha} c^2 k_\parallel} Y \\ k_\parallel k_\perp & i\frac{\omega_{p\alpha}^2 \omega k_\perp}{c^2 \omega_{c\alpha} k_\parallel} Y & \frac{\omega^2}{c^2 k_\parallel^2 \lambda_{De}^2} Y - k_\perp^2 \end{vmatrix} = 0 \qquad (15)$$

where

$$X'_\alpha = \frac{n_\alpha}{n} X = \eta_\alpha X .$$

η_α is the concentration factor of the resonant species and arises from the fact that the zeroth order distribution function is now of the form

$$f_{\alpha o}(V_{\alpha\perp}^2, V_{\alpha\parallel}^2) = \frac{n_\alpha}{n\pi^{3/2} V_{T\alpha}^3} \exp - \frac{V_{\alpha\perp}^2 + V_{\alpha\parallel}^2}{V_{T\alpha}^2}$$

with

$$V_{T\alpha} = \left(\frac{2\kappa T_\alpha}{M_\alpha}\right)^{\frac{1}{2}}$$

In addition, Equation (15) now has the more restrictive condition that

$$\frac{n}{n_\alpha}\frac{m_e}{M_\alpha}\frac{T_\alpha}{T_e} << 1 \tag{16}$$

Hence, for resonant ion concentrations of approximately 5% or greater, Equ. (15) still holds. Following the procedure stated previously, one finds (dropping the α subscript)

$$-\eta X\left[q_\perp^{\,2} + 2q_\parallel^{\,2} - \frac{2\omega^2}{\omega_{ci}(\omega+\omega_{ci})}\right] + \left[q_\parallel^{\,2}\,(q_\perp^{\,2} + q_\parallel^{\,2}) - \frac{\omega^4}{\omega_{ci}^{\,2}(\omega+\omega_{ci})^2}\right] = 0$$

and finally the following averaged damping rate

$$\bar{\gamma}_i = \frac{1.58}{\eta}\frac{k_\parallel^{\,2} V_{Ti}^{\,2}}{3\omega_{ci}}\left(\frac{R}{a}\right) \tag{17}$$

The electron transit time magnetic pumping damping decrement is still given by

$$\gamma_e = \frac{8}{3}\, e^{-1}\, \pi^{\frac{1}{2}}\, \frac{k_\perp^{\,2}\,\kappa Te}{M\;\omega_{ci}}$$

At the first harmonic ion cyclotron resonance, the determinant is now

$$\begin{bmatrix} \frac{\omega_{pi}^2}{c^2}\{\eta X'-Y'-W'\} - k_\parallel^2 & i\,\frac{\omega_{pi}^2}{c^2}\{\eta X'+Y'+\frac{\omega_{ci}\omega}{\omega_{ci}^2-\omega^2}-\frac{\omega}{\omega_{ci}}\} & k_\perp\, k_\parallel \\ -i\,\frac{\omega_{pi}^2}{c^2}\{\eta X'+Y'+\frac{\omega_{ci}\omega}{\omega_{ci}^2-\omega^2}-\frac{\omega}{\omega_{ci}}\} & \frac{\omega_{pi}^2}{c^2}(\eta X'-Y'-W')-k_\parallel^2-k_\perp^2+2\beta_e k_\perp^2(Y-1) & -i\,\frac{\omega_{pi}^2}{c^2}\frac{\omega k_\perp}{\omega_{ci}k_\parallel}Y \\ k_\perp\, k_\parallel & i\,\frac{\omega_{pi}^2}{c^2}\frac{\omega k_\perp}{\omega_{ci}k_\parallel}Y & \frac{\omega^2\, Y}{c^2 k_\parallel^2\lambda_{De}^2} - k_\perp^2 \end{bmatrix} = 0 \tag{18}$$

where the ith particle is the resonant species. In the same manner one obtains the following averaged damping decrement

$$\langle \gamma_i \rangle_{2\omega_{ci}} = \frac{\eta}{4} \frac{k_\perp^2}{M} \frac{\kappa T_i}{\omega_{ci}} \left(\frac{R}{a}\right). \qquad (19)$$

In order to physically interpret the enhanced damping rate due to the presence of two ion species we present the following argument which closely follows that of Adam.[8] In cold plasma theory no ion cyclotron damping occurs. However, when hot ions are included, the electric field (E) is elliptically polarized at $\omega = \omega_{ci}$. Therefore the amplitude of the E^+ component is finite at the ion cyclotron frequency and cyclotron damping is possible. Neglecting rotational transform effects, when the frequency is adjusted such that $\omega = \omega_{ci}$ on the magnetic axis, cyclotron damping will occur in a cylindrical layer of width

$$\Delta R = \frac{k_\parallel V_{Ti} R}{\omega}$$

Since the electric field is largely polarized in the opposite sense of ion rotation, the heating rate is weak. However, the amplitude of the E^+ field may be enhanced by going to a two component plasma. The density of the non-resonant species must be high enough such that the fast wave can propagate but the density of the resonant species must be low enough so that the E^+ component is not too strongly damped by the plasma.

Adam finds that the power absorbed by cyclotron damping goes as

$$P = \frac{1}{4\pi\omega} \frac{\omega_{pi}^2}{c^2} |E^+|^2 \frac{R}{a} (\text{ergs/sec}) \tag{20}$$

where $|E^+|$ varies as η^{-1}, $\eta \ll 1$. Therefore the power absorbed is proportional to η^{-2}. The power absorbed in the main body of the plasma is then proportional to η^{-1}. η cannot be made arbitrarily small since it is desired that the power of the resonant ions be transferred to the entire plasma via collisions.

Physically, the resonant ions are heated by direct interaction with the wave field. Initially, when the plasma is cold, the power absorbed by the resonant ions is almost immediately transferred to the non-resonant ions by collisions. The heating rate is determined by the rate of dissipation of the wave. As the plasma becomes hotter, the power absorption increases while the rate of transfer of the power from one species to the other decreases; hence the resonant ions are heated faster and may eventually become runaways. To prevent this the wave amplitude must be reduced when the temperature ratio is optimum for collisional transfer.

Though the dielectric properties of the plasma are altered by the presence of resonant particles, this is a minor effect. The wave number is not significantly altered by the presence of resonant ions so that the fast wave can be excited in a large tokamak. In a D-T tokamak reactor, it is possible to have resonance for deuterium on the axis and for tritium near the inside plasma edge. Further, since there are two ion species, the possibility of mode conversion to the Buchsbaum two-ion hybrid resonance exists and its impact on heating rates needs to be investigated.

IV. APPLICATION TO UWMAK REACTOR

It is expected that for a fusion reactor having an electric power output of 5000 megawatts, supplementary power of the order of 100 megawatts will have to be given to the plasma ions in order to obtain ignition. Aware of the pitfalls that are possible when one attempts to extrapolate an approximate wave theory intended for present day machines with large aspect ratio to a reactor size machine, we nevertheless set out to determine the scalability, work out sample calculations, and pose important aspects which will require more detailed investigation in the future for reactor size systems. In Table 1, various parameters for the TFR, PLT, and UWMAK II machines are given as well as their critical densities and damping decrements.

The first criteria is that the rf power is primarily deposited in the plasma and not on the walls of the toroid through skin effect losses. Next the damping rate of the waves should be fast compared to the confinement time in the UWMAK which is the order of seconds. The ion cyclotron damping rates ($\omega = \omega_{ci}$) of the compressional magnetosonic mode has a damping rate given by

$$\langle \gamma_i \rangle_{\omega_{ci}} = 1.6\ k_{\parallel}^{2}\ (\kappa T_i) R / M_i \omega_{cio}$$

where the variation in major radius of the local ion gyro-frequency has been taken into account. The corresponding electron damping rate due to transit time effects has a comparable order of magnitude except that it scales as $k_\perp$ rather than $k_\parallel$ as in the ion cyclotron damping case. Assuming $T_i = 1$ keV, $R/a = 13/5$, and $B_o = 35.7$ kG at a frequency of $f(H) = 54$ MHz, one obtains an ion damping rate for UWMAK parameters (see Table I) which can be written as

$$\langle\gamma_i\rangle_{\omega_{ci}} = 3.0 \times 10^4/\lambda_\parallel^2(\mathrm{m})\ \sec^{-1}$$

where the scaling with parallel wavelength is explicitly noted. Thus for parallel wavelengths less than 0.4 m, the ion cyclotron damping rate would exceed that of either the stainless steel or carbon curtain wall. Since the damping rate scales as ion temperature it can be expected that as the ions are heated, the efficiency with which power is given to the plasma is increased (by a factor of 10 at an ion temperature of 10 keV).

The corresponding ion damping rate due to finite gyroradius effects at the first harmonic ($\omega = 2\omega_{ci}$) of the ion gyrofrequency scales with perpendicular wavelength at an ion temperature of 1 keV as

$$\langle\gamma_i\rangle_{2\omega_{ci}} = 7.4 \times 10^3/\lambda_\perp^2(\mathrm{m})\ \sec^{-1} .$$

For UWMAK parameters it is required that $\lambda_\perp \lesssim 0.4$ m so that plasma absorption dominates losses by either carbon curtain or stainless steel walls. Since the damping rates are only derived for lower order modes, it is necessary to examine the higher order modes to see that comparable damping rates are obtained. The higher order mode might also imply that the damping would be more distributed over the plasma cross section rather than over the central slab, in both cases heating primarily those ions with large $v_\perp/v_\parallel$.

The shorter perpendicular and parallel wavelengths are doubly advantageous for fast magnetosonic wave heating. They increase the damping rates of the wave. In addition, large values of $k_\parallel \simeq 10\ m^{-1}$ increase the radial width over which appreciable damping exists, minimizing the possibility of mode conversion to the ion Bernstein mode as expressed by Weynants.[17]

One can obtain an estimate of the absorption of the rf energy due to the walls by using the definition of Q or quality factor. The frequency of the rf energy is essentially at the ion cyclotron frequency and one defines a wall damping rate due to skin effect, $Q = \omega_{ci}/\gamma_W$. Alternatively one can determine the energy stored in the rf fields and the energy lost to the walls through the skin depth $\delta = (\pi f \mu_o \sigma)^{-\frac{1}{2}}$. By integrating over a unit length along the torus one finds a result of order $Q = a/\delta$. Solving for the damping rate for the walls one obtains $\gamma_W \simeq \omega_{ci}\delta/a \simeq (\omega_{ci}/\sigma\mu_o)^{\frac{1}{2}}/a$, where σ is the conductivity of the wall material. The values used are $\sigma_{ss}(800^oC) = 3 \times 10^6\ (\Omega\text{-m})^{-1}$ and $\sigma_{cc}(500^oC) = 3.1 \times 10^4\ (\Omega\text{-m})^{-1}$. This yields a stainless steel damping rate of $\gamma_{ss} = 1.9 \times 10^3\ sec^{-1}$ and a carbon curtain damping rate of $\gamma_{cc} = 1.8 \times 10^4\ sec^{-1}$. In Table 2, the wall damping decrements for UWMAK II[22] are calculated for stainless steel as well as for the carbon curtain.[23] As Table 2 shows, the carbon curtain does not increase wall losses greatly. However, caution should be used in dealing with the carbon results since the conductivity is used for a uniform slab. The fact that the curtain is a relatively loosely woven structure will modify this due to the contact between the individual strands of the weave. This effect could be very important and warrants further study.

For present day tokamaks such as the ST, TFR, PLT and T-10, it is possible to excite only one or two modes in the device in the fast magnetosonic mode. However, in much larger devices such as the UWMAK, the critical density for excitation of a magnetosonic mode is reduced to $\sim 10^{10}/cm^3$ and many modes can be excited. Indeed, application of the cold plasma dispersion relation as derived by Perkins[18] can be written

$$\omega_{pi}^2 a^2/c^2 = 2/3\ [(n+1)(n+3)(n+4) + k_{\parallel}^2 a^2 (n+4)]$$

where a parabolic density profile has been assumed $\omega_{pi}^2 = \omega_{pio}^2 (1 - r^2/a^2)$ and $n = |m-1|$ $(m = 0, \pm 1, \pm 2, \ldots)$ where m denotes the minor azimuthal mode number for fields varying as $e^{im\theta}$. For a fixed $k_{\parallel}$ such that $k_{\parallel} a \lesssim 5$, a plasma density of $n = 6 \times 10^{13}/cm^3$, and $a = 500$ cm as is the case for UWMAK, one obtains an estimate that about 48 azimuthal modes can be simultaneously excited with different amplitudes and damping rates.

At first glance one can assume that this will tend to provide field peaks for different modes throughout the plasma region and deposit energy more uniformly over the plasma cross section. However, the problem requires a more detailed investigation including such topics as calculation of the relative excitation of the higher order modes for a given launching structure, derivation of damping rates for higher order modes, investigation of the effects of rotational transform on the modes, and the possibility of wave-wave scattering when a large number of modes are present in the machine.

In order to supply a 10 MW/waveguide power requirement, a waveguide of dimension 3 m wide by 1.5 m high could be used between the toroidal field coils. It would operate in a TE_{10} mode with the electric field oriented in the vertical direction and the transverse magnetic field oriented in the toroidal direction. This would provide a peak electric field of about 916 volts/cm with a power density of 220 watts/cm^2. This is well below breakdown conditions and the waveguide could accommodate higher power levels. Future work will have to consider the effect of the finite wave excitation region on mode excitation and coupling to the plasma column.

V. DISCUSSION

This section of the report will deal briefly with topics which will require more detailed investigation in future reports but could have a major impact on the role of rf heating in tokamak fusion reactors. The first point is that of a detailed description of the launching structure used to couple waves to the plasma column. In order to design this for large systems, a precise determination of the desired wavenumbers $k_\perp$ and $k_\parallel$ must be determined for optimum heating. The role of the many higher order modes which can propagate must also be determined more precisely. Once this is determined, one can consider the use of rectangular waveguides inserted in the walls of the torus or waveguide coupling through periodically spaced irises should the periodic structure of the launching system turn out to be an important factor. If a slow wave structure outside the plasma column should be necessary, rippling of the torus walls could be considered in order to effect such a launcher. If the rf energy is added at a frequency that is less than about 100 MHz, insertion loops coupled at some distance from the plasma might be used if the rf feed points were deeply recessed from the plasma region so that arcing would not be a problem. Lastly, the use of the toroidal cavity modes as observed on ST[21] would be quite useful if high $k_\parallel \approx 10\text{m}^{-1}$ could be excited in the device. The eigenmodes would produce a continuum providing good matching of the wave at all times.

Important problems in the area of wave theory include an examination of whether nonlinear wave effects (e.g., parametric instabilities, trapping effects) will play an important role in reactor plasmas or whether a linear wave theory will be sufficient due to the relatively small amount of wave energy to plasma thermal energy density. Further problems in this area include a more precise determination of the effects of wave energy velocity space deposition on the confinement of trapped and untrapped particles in the neo-classical diffusion regime and a more careful study of the effect of wave heating on the ohmic heating plasma resistivity and the effects of rotational transform on the damping rates and generation of possible instabilities. Dei-Cas and Samain[29] have indicated that, under certain conditions, one may be able to use magnetic pumping at the ion gyrofrequency of impurity ions to keep impurities from penetrating to the center of the discharge.

One of the most important problems that we will work on in the future is a space-time model incorporating ohmic heating, neoclassical diffusion losses, and impurity radiation together with an appropriate model for rf heating to properly treat the startup problem of a reactor. At present Kessner, Khelladi, and Conn[30] are working on a fluid code developed by Oak Ridge for ORMAK parameters and adopting this to scaling indicative of a UWMAK size reactor. An important problem here is to properly incorporate the velocity space effects of rf heating in the Vlasov theory to a fluid model and to present the important differences that one would expect when compared with heating by means of neutral beams. Other problems will involve an appropriate choice of initial density and temperature profiles over the plasma cross section and a more rigorous treatment of wave boundary conditions imposed by the plasma and launching structure.

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Table I. Magnetosonic Wave Damping Rates for Sample Tokamak Plasmas.

	PLT	TFR	UWMAK II
r_p (cm)	45	20	500
R_M (cm)	130	98	1300
B_T (kG)	45	60	35.7
I (MA)	1.4	0.4	14.9
T_i (keV)	$\leq$ 1	1 (D), 1.2 (H)	15.2
T_e (keV)	~1	3	13.5
τ	.1 - 1 s	15 ~ 20 ms	~4 s
n (cm^{-3})	5×10^{13}	$3.5\text{-}7 \times 10^{13}$	$\sim 6 \times 10^{13}$
A	2.9	4.9	2.6
n_c (cm^{-3})	1.83×10^{12}	5.21×10^{12}	2.35×10^{10}
f_{ci}	68.5 MHz	91.4 MHz	54.4 MHz
f_{pi}	1488 MHz	~1800 MHz	1723 MHz
$\bar{\gamma}_i$ (sec^{-1})	$2.7 \times 10^4/\lambda_{\parallel}^2$ (m)	$3.4 \times 10^4/\lambda_{\parallel}^2$ (m)	$4.6 \times 10^5/\lambda_{\parallel}^2$ (m)
$\bar{\gamma}_{2\omega ci}$	$6.4 \times 10^3/\lambda_{\perp}^2$ (m)	$8.1 \times 10^3/\lambda_{\perp}^2$ (m)	$1.1 \times 10^5/\lambda_{\perp}^2$ (m)
$\bar{\gamma}_e$	$3.3 \times 10^4/\lambda_{\perp}^2$ (m)	$1.3 \times 10^5/\lambda_{\perp}^2$ (m)	$5.1 \times 10^5/\lambda_{\perp}^2$ (m)

Table II. Comparison of Carbon Curtain vs. Stainless Steel Walls for Various Operating Temperatures at f = 54.4 MHz for UWMAK II.

	Temp (°C)	Resistivity (Ω-m)	Conductivity $(\Omega\text{-m})^{-1}$	γ_{wall} $(sec)^{-1}$	Ratio γ_{cc}/γ_{ss}
Carbon	20	42×10^{-6}	2.4×10^{4}	2.2×10^{4}	7.9
S.S.	20	74×10^{-8}	1.35×10^{6}	2.8×10^{3}	
Carbon	500	32×10^{-6}	3.1×10^{4}	1.9×10^{4}	5.6
S.S.	500	105×10^{-8}	9.5×10^{5}	3.4×10^{3}	
Carbon	1000	20×10^{-6}	5.0×10^{4}	1.5×10^{4}	3.9
S.S.	1000	130×10^{-8}	7.7×10^{5}	3.8×10^{3}	

References

1. Rothman, M.A., Sinclair, R.M., Brown, I.G., Hosea, J.C., "Ion Cyclotron Heating in the Model C Stellarator," Phys. Fluids 12, 2211 (1969).

2. Hosea, J.D., Sinclair, R.M., "Ion Cyclotron Wave Generation in the Model C Stellarator," Phys. Fluids 13, 701 (1970).

3. Messiaen, A.M., Vandenplas, P.E., "Discussion of R.F. Heating of JET," Report 60, Lab. Phys. Plasmas, Ecole Royale Militaire (Brussels, 1973).

4. Puri, S. and Tutter, M., "Coupling to the Lower-Hybrid Resonance," IPP IV/54 (Garching), March, 1973.

5. Puri, S. and Tutter, M., "Slow-Wave Coupling to the Lower-Hybrid Resonance," IPP IV/59 (Garching), July, 1973.

6. Hooke, W.M. and Bernabi, S., Phys. Rev. Lett. 28, 407 (1972).

7. Adam, J. and Samain, A., "Chauffage par Absorption Cyclotronique," Rapport du Groupe d'etude du Chauffage d'un Plasma Confine dans une Configuration Magnetique de Type Tokamak Report EUR-CEA-FC-579.

8. Adam, J., "Ion Heating Mechanisms Based on the Absorption of the Fast Hydromagnetic Wave Around the Ion Cyclotron Frequency.", Symposium on Plasma Heating and Injection, Varenna, Italy, (1972).

9. Adam, J., "Etudes des Possibilities de Chauffage du Plasma de TFR par Absorption de l'onde Hydromagnetique Rapide," Report EUR-CEA-FC-711, Octobre (1973).

10. Messiaen, A.M., Vandenplas, P.E., "Comprehensive Theory of Radio Frequency Absorption by a Magnetized, Hot Electron-Ion Plasma Column," Nuc. Fusion 11, 556 (1971).

11. Messiaen, A.M., Vandenplas, P.E., "Comprehensive Theory of R.F. Energy Absorption by a Hot Ion-Electron Plasma Cylinder Excited by an Arbitrary Electromagnetic Field," Plasma Phys. 15, 505 (1973).

12. Messiaen, A.M., Vandenplas, P.E., "Bounded Plasma Theory of Radio-Frequency Heating of Toroidal Machines," Report 61, Lab. Phys. Plasmas, Ecole Royale Militaire (Brussels, 1974).

13. Messiaen, A.M., "Bounded Plasma Aspects of the R.F. Heating in Toroidal Devices (Varenna, Italy, 1974).

14. Messiaen, A.M., Vandenplas, P.E., "Toroidal Plasma Resonances and Electron Transit Time Damping for Heating Large Thermonuclear Tori," Phys. Letters 49A, 475 (1974).

15. Messiaen, A.M., Vandenplas, P.E., "The Present State of Research into Plasma Heating and Injection Methods," Euratom Advisory Group on Heating and Injection, pp. 62-62A, (1974).

16. Messiaen, A.M., Vandenplas, P.E., "Radio-Frequency Heating of Large Thermonuclear Tori," Fifth Conf. on Plasma Physics and Controlled Nuclear Fusion Research (Tokyo, 1974) Paper IAEA-CN-33/C4-2.

17. Weynants, R.R., "Ion Heating at Twice the Ion Cyclotron Frequency in Reactor Oriented Machines," Phys. Rev. Letters 33. 78 (1974).

18. Perkins, F.W., "Hydromagnetic Wave Heating of Tokamak Plasmas," Symposium on Plasma Heating and Injection (Varenna, 1972).

19. Perkins, F.W., Chance, M., Kindel, J.M., "Hydromagnetic Wave Heating of Tokamak Plasmas," Third International Symposium on Toroidal Confinement (Garching, 1973).

20. Chance, M.S., Perkins, F.W., Sperling, J.L., "Ion Cyclotron Heating of Tokamaks - Theoretical Considerations," Princeton University Plasma Physics Laboratory (unpublished).

21. Adam, J., et al., "Wave Generation and Heating in the ST Tokamak at the Fundamental and Harmonic Ion Cyclotron Frequencies." Proc. 5th International Conf. on Plasma Physics and Controlled Nuclear Fusion Research (Tokyo, 1974) IAEA-CN-33/A3-2.

22. Conn, R.W., et al., "Major Design Features of the Conceptual D-T Tokamak Power Reactor, UWMAK-II," Proc. 5th International Conference on Plasmas and Controlled Fusion Research (Tokyo, 1974) IAEA-CN-33/G1-2.

23. Kulcinski, G.L., Conn, R.W., Lang, G., "Reduction of Plasma Contamination Effects and First Wall Erosion in Fusion Devices," Nuclear Fusion (in press). See also G.L. Kulcinski et al., UWFDM-108, Nucl. Eng. Dept., Univ. of Wisc. (Aug. 1974).

24. Hooke, W.M., Rothman, M.A., "A Survey on Ion Cyclotron Resonances in Plasmas," Nuc. Fusion 4, 33 (1964).

25. Buchsbaum, S.J., "Resonance in a Plasma with Two Ion Species," Phys. Fluids 3, 418 (1960).

26. Cattanei, G., "Collisionless Plasma Heating at the Ion Cyclotron Resonance," Phys. Rev. Letters 27, 980 (1971).

27. Hooke, W.M., Hosea, J.C., "Wave Generation and Heating, Near the Ion Cyclotron Frequency in the ST Tokamak," Vth European Conference on Controlled Fusion and Plasma Physics (Grenoble, 1972).

28. Mau, T.K., "Preliminary Studies on RF Heating of Ions in Tokamak-Type Plasma," Plasma Laboratory Preprint No. 599, University of Wisconsin, (1975).

29. Dei-Cas, R., and Samain, A., "Extraction Des Impuretes d'Une Configuration Tokamak," Paper CN-33/A15-1, IAEA Meeting, Tokyo (1974).

30. Kesner, J., Khelladi, M., Conn R., Private Communication.