Mirror Instabilities in Divertors

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ABSTRACT

The role of loss-cone type microinstabilities in a divertor plasma is discussed. Since many types of loss-cone modes appear to be unstable in this plasma, it is inferred that the confinement time of particles along the field lines is governed by their transit time to the particle collectors and hence that the divertor plasma will be very thin in its radial extent.

As impurity control in tokamaks appear to be one of the most pressing problems in present day plasma research [1], much thought and some calculation has gone into its eventual attainment. One proposed method is through the use of a divertor [2]. There are many unknowns associated with a divertor's presence on a tokamak. Lacking any experimental observations, one may propose a myriad of simplistic (and a few not so simple) models to illustrate different potential problem areas. The problem addressed here is the possibility that the plasma in the divertor region is unstable to mirror microinstabilities and how this may affect plasma transport in this region.

In the operation of a divertor there exists a region (bounded on one side by a magnetic separatrix and on the other side by the liner or first wall of the reactor) in which the magnetic field lines are diverted from the local vicinity of the plasma and are channeled into some type of particle collection chamber. This region of diverted B field lines has been labeled appropriately a scrape-off region [2]. Plasma feeds this zone by diffusion across the field lines from the plasma core and in turn leaves the zone (mostly) by following along the field lines to the collection region. Cross sections of some proposed models are shown in Fig. 1. The mean thickness characterizing the plasma density "drop-off" perpendicular to the field lines is determined by a balance between the cross field diffusion with coefficient $D_\perp$ and the parallel flow to the collectors on a time scale $\tau_{\parallel}$ [3]. For the model of a poloidal divertor which we consider here this thickness is given approximately by

$$\lambda \sim \sqrt{D_\perp \tau_{\parallel}} \quad .$$

For the purposes of this note $D_\perp$ (in the divertor region) will be taken to be some fraction of the Bohm value $[10^8 (T_e/1\ \mathrm{keV})/(16\ B/1\ \mathrm{kgauss})\mathrm{cm}^2/\mathrm{sec}]$. The electron temperature T_e is to be determined self-consistently from energy balance equations. This choice reflects a belief, substantiated by some experimental observations [4], that low frequency turbulence (say drift waves) may be present in the divertor and this causes the Bohm type scaling of $D_\perp$. With this choice for $D_\perp$ one needs to ascertain what $\tau_\parallel$ might be. The low frequency turbulence will not effect $\tau_\parallel$ because all except the short wavelength (compared to the ion gyroradius) forms of these waves cannot change the magnetic moment of the partide. Therefore particle collisions and/or high frequency turbulence ($\omega \gtrsim \Omega_i$ instabilities) will determine parallel transport times.

Since the distance plasma must flow along field lines to reach the divertor collector region is quite long (2 to 10 m. in possible next generation devices, (50 m) in future large scale fusion reactors), a lower limit on $\tau_\parallel$ is the time it takes the plasma to flow there at the ion speed

$$(\tau_\parallel)\text{min} \sim L/v_s$$

where L is the mean distance to be traveled to the collector plates and $v_s \simeq (\max\,(T_e,\ T_i)/m_i)^{1/2} \simeq 3.1 \times 10^7 \left(\frac{\max(T_e, T_i)}{A}\right)^{1/2} \frac{\text{cm}}{\text{sec}}$ for ions of A amu.

For illustrative purposes we consider two numerical examples:

1.) a next generation tokamak with divertor,

and 2.) a fusion power reactor.

Thus,

Table 1

Divertor Characteristics

Case 1	Case 2
Hydrogen	Mass = 2.5 a mu
B_T = 25 kgauss	B_T = 30.8 kgauss
T_e = .3 keV } Divertor	T_e = 7 keV
T_i = .5 keV } Divertor	T_i = 9 keV
L = 1000 cm	L = 6000 cm
q = 3.6	q = 2.3
R_o = 140 cm	R_o = 1300 cm
a = 45 cm	a = 500 cm
$n = 6\times10^{12}\,cm^{-3}$ (@ separatrix)	$n \simeq 10^{11} - 5\times10^{12}\,cm^{-3}$
$D_\perp = \frac{1}{10} D_{Bohm} = 7.5\times10^3\ cm^2/sec.$	$D_\perp = \frac{1}{50} D_{Bohm} = 2.8\times10^4\,cm^2/sec$
$(\tau_{\parallel})$min = 45 μsec	$(\tau_{\parallel})$min = 100 μsec
$(\tau_{\parallel})$max = 2 msec	$(\tau_{\parallel})$max = ≃ .3 to 13 sec.
$\lambda_{min} \simeq$ .5 cm	$\lambda_{min} \simeq$ 2 cm
$\rho \simeq$ .13 cm	$\rho_i \simeq$ .7 cm
$L_{\nabla B} \sim qk_o \sim$ 500cm ~ 3900ρi	$L_{\nabla B} \sim qk_o \sim$ 3000cm ~ 4300ρi
$\omega_{pi} > \Omega_i \Rightarrow n_i > 3.3\times10^{10}\,cm^{-3}$	$\omega_{pi} > \Omega_i \Rightarrow n_i > 2.0\times10^{10}\,cm^{-3}$
$\omega_{pe} < \Omega_e \Rightarrow n_e < 6.1\times10^{13}\,cm^{-3}$	$\omega_{pe} < \Omega_e \Rightarrow n_e < 9.2\times10^{13}\,cm^{-3}$
$\Omega_i = 2.4 \times 10^8$ rad./sec	$\Omega_i = 1.18 \times 10^8$ rad./sec
f = 38.2 MH_z	f = 18.8 MH_z
$\lambda_{De} \simeq 7 \times 10^{-3}$ cm	$\lambda_{De} \simeq$ .2 to .03 cm
Im $k_{\parallel} \simeq .24\ cm^{-1}$	Im $k_{\parallel} \simeq$ .005 to .033 cm^{-1}

One notes that for case 1 $\lambda_{min} \sim \sqrt{D(\tau_{\parallel})_{min}} \sim$ 1/2 cm and for case 2 $\lambda_{min} \sim$ 2 cm.

The poloidal configurations considered in this note with reference to Fig. 1, can be seen to be ones in which the plasma once in the (outer) divertor region must move into a higher magnetic field ($B \sim 1/R$) before it can reach the collectors. In these cases there may be some magnetic confinement as in a mirror machine. The magnetic mirror ratio is less than 2:1 along the field line so one has a fairly weak mirror. One sees that at least half the plasma in the scrape-off region would not be "trapped" by the mirror; it would flow out in a time $(\tau_{\parallel})_{min}$. The maximum time that the remaining mirror-confined plasma could be trapped would be on the order of the 90° scatter time [5] for ions to be scattered into the loss cone:

$$(\tau_{\parallel})_{max} \sim \tau_{ii} \sim \frac{3\text{x}10^{-3} A^{1/2} (T_i/1 \text{ keV})^{3/2}}{[n_e/10^{13}\text{cm}^{-3}]\ Z^3} \text{ sec} \quad .$$

As can be seen from $\lambda \sim \sqrt{D_\perp \tau_{\parallel}}$ the density drop-off thickness can vary by as much as a factor of $\sqrt{(\tau_{\parallel})_{max}/(\tau_{\parallel})_{min}} \sim 10$ (case 1) and ~ 100 (case 2), depending upon what is the appropriate parallel flow time [6].

In addition to the clasical processes discussed above, plasma instabilities can decrease the minimum flow time. Specifically, since the mirror-confined plasma has a loss-cone, it is susceptible to all the usual loss-cone driven microinstabilities [7]. The four most important loss-cone driven instabilities for this case are: 1) the Post-Rosenbluth convective loss-cone instability [8]; 2) the absolute loss-cone instabilities [9]; 3) the negative energy loss-cone modes [10]; and 4) the drift-cone modes [11]. Before discussing these modes in detail, one needs to estimate some parameters of this mirror-confined plasma. First, one needs to know the ratio of the ion gyro-radius, pi, to the plasma inhomogeneity scale lengths. For case 1 $\rho_i \sim .13$ cm and case 2 $\rho_i \sim .70$ cm. Thus it appears that the scrape off thickness is from 4 (case 1) to 3 (case 2) ion gyro radii in width. The characteristic inhomogeniety scale length along a magnetic field line is $L_{\nabla B} \sim q\, R_o$, where $q = r\, B_T/R\, B_p$ is the MHD stability factor and R_o the plasma major radius. For case 1, $L_{\nabla B} \sim 500$ cm and case 2, $L_{\nabla B} \sim 3000$ cm. Therefore, $L_{\nabla B}/\rho_i \sim 3800$ for case 1 and 4300 for case 2. This is essentially an infinite, homogenous plasma for loss-cone instability calculations where $\lambda_{\parallel} \sim \rho_i \sqrt{\frac{m_i}{m_e}} \sim (40 - 60)\, \rho_i$.

The last three of the four loss-cone instabilities listed above are standing wave (or absolute) modes that are typically found to be radially localized within a few gyroradii. The drift-cone mode [11] has been shown to be present and unstable for radial scale lengths as short as one or two ion gyroradii[12]. The negative energy modes are finite medium forms of the absolute loss-cone and drift-cone modes [7], which are most easily derived in infinite medium theory. In general, for $L \gg \lambda_{\parallel}$, as appears to be the case here, these standing wave instabilities are all unstable for $\omega_{pi} \gtrsim \Omega_i$, (i.e. $n_i > 1.3 \times 10^{10} cm^{-3}$ @ B = 25 kG) where ω_{pi} is the ion plasma frequency and Ω_i is the ion gyro frequency. (These modes generally require $T_{\parallel e} < T_{\Delta i}$, which seems reasonable to expect; however n_i may drop below a few $10^{10}\ cm^{-3}$ if the above instabilities begin to appear and affect $\tau_{\parallel}$. Thus these modes may stabilize somewhere between the separatrix and the wall.) When this density threshold is exceeded the mode growth rates range from a few percent (for absolute loss-cone modes) to large fractions (for drift-cone modes) of the ion gyrofrequency. The nonlinear consequences of the modes are apparently to enhance the pitch angle scattering which causes the loss-cone to be filled in, on a time scale of a few growth times [13]. If one presumes a growth rate of 0.01 Ω_i and requires 10 growth times for significant effects, then the loss-cone fill-in time is $\tau_{inst} \sim 10^3\ \Omega_i^{-1} \sim 5$ to 10 μsec. For the divertor scrape off region considered here $\tau_{inst} << (\tau_{\parallel})_{min}$. This indicates that the instability could cause sufficient scattering to keep the loss-cone filled with particles and thus make $\tau_{\parallel}$ only a few times $(\tau_{\parallel})_{min}$, at most.

In addition to the standing wave modes discussed in the preceeding paragraph there are the convective loss-cone instabilities, which can give rise to significantly enhanced ("quasi-classical") scattering [14] into the loss cone by virtue of the long range polarization fields of the ions in the convectively unstable plasma. The polarization fields around individual "test" ions extend only a few Debye lengths ($\lambda_D << \rho_i$) perpendicular to the field line, but they extend all the way to the mirror "throat" region along the field lines, i.e., in essence we have a zero of the parallel dielectric coefficient. For the divertor scrape off region where we expect $\omega_{pe} < \Omega_e$ (i.e., $n_i < 6 \times 10^{13} cm^{-3}$ for case 1; $n_i < 9 \times 10^{13}\ cm^{-3}$ for case 2) and a fairly sharp loss cone, the effective collision frequency for scattering into the loss cone is given approximately by [14]

$$\nu_{eff} \simeq \frac{1}{\tau_{ii}} \left(\frac{T_i}{T_e}\right)^{3/2} \left(\frac{m_e}{m_i}\right)^{1/2} \frac{\exp\ (2\ \mathrm{Im}\ k_{\parallel} L)}{\mathrm{Im}\ k_{\parallel} L\ \ \ln\Lambda}$$

where

$$\mathrm{Im}\, k_{\parallel} \sim \frac{0.1}{\rho_i} \frac{\omega_{pe}}{\Omega_e} \mathrm{cm}^{-1} \sim 9.51 \left(\frac{m_e}{m_i}\right)^{1/2} \left(\frac{n/10^{13}\mathrm{cm}^{-3}}{T_i/1\ \mathrm{keV}}\right)^{1/2} \mathrm{cm}^{-1} \quad .$$

For the parameters used above one finds for case 1 ($\mathrm{Im}k_{\parallel} = .24\ \mathrm{cm}^{-1}$, $\omega_{pe} \sim .3\ \Omega_e$, $L_{pB} \simeq 500$ cm), $\nu_{eff}\,\tau \sim 10^{100}$ and for case 2 ($\mathrm{Im}k_{\parallel} = .01\ \mathrm{cm}^{-1}$, $\omega_{pe} \sim .1\ \Omega_e$, $L_{\nabla e} \simeq 3000$ cm) $\nu_{eff}\,\tau_{\parallel} \sim 10^{21}$.

These are absurdly large because L is long and the parallel growth length is on the order of a few hundreths of $L_{\nabla B}$ in both cases. In such a situation one expects [14] ν_{eff} to be no larger than the ion bounce frequency $\omega_{bi} \sim v_{Ti}/L_{\nabla B}$, since otherwise the instability is filling the loss-cone before the plasma even knows it has one. We thus conclude for the divertor scrape-off region that $\tau_{\parallel} \sim \omega_{bi}^{-1} \sim \frac{L}{v_{Ti}}$ which is equal to $\tau_{\parallel\, min}$ to within a factor of $\sqrt{\max(T_e,T_i)/T_i}$.

The derivations of the loss-cone driven modes in mirror confined plasmas [7,12,14] have not taken shear into account, since it is unimportant in open-ended systems. However, in tokamaks the magnetic field is sheared in the minor radius direction, with a typical scale length

$$\frac{1}{L_s} \sim \frac{1}{q\,R_o} \frac{r}{q} \frac{\partial q}{\partial r} \lesssim \frac{1}{10r}$$

and it is worth considering the effects of shear on these modes. The effective parallel wavelength of a mode in a sheared magnetic field is given by [15]

$$k_{\parallel} \simeq k_z + k_y \frac{x}{L_s}$$

where k_y, k_z are the local y,z components of the wave propogation and x is the distance from the modal surface investigated. For a mode radially localized to a width δx, the effective $k_{\parallel}$ over the region of interest is

$$k_{\parallel} \sim k_z \left[1 + \frac{k_y \delta x}{k_z L_s} \right] \quad .$$

For the microinstabilities discussed above

$$\delta x \gtrsim \rho_i \sqrt{m_i/m_e} > k_y \rho_i > 1 \text{ and } k_z L_s \sim \frac{L_s}{40 \rho_i} \quad .$$

Thus, for a typical shear length of say 1000 cm, the effect of shear on the effective parallel wavelength is quite small. It should be noted that if, in addition to the shear, there was significant fanning of the magnetic field lines as in the minimum-B mirror systems, then these loss-cone driven instabilities might be more significantly affected by the magnetic topology [16]. Shear may also affect $D_{\perp}$ [17].

In summary we have found that while the scrape-off region in a divertor may appear to contain a mirror-confined plasma, the plasma so confined seems to be unstable to a large variety of microinstabilities driven by the free energy associated with the loss-cone distribution. The net result of the instabilities would apparently be to reduce the expected $\tau_{\parallel\, max} \sim \tau_{ii}$ down to very close to the free flow value of L/v_s. Thus the best estimate of the parallel flow time is probably $\tau_{\parallel} \sim \tau_{\parallel\, min}$, which implies a relatively thin density "drop off" thickness of only a few centimeters or less. As such the hopes of having an effective "screening" divertor diminish - i.e. the probability of ionizing an incoming neutral in the scrape-off zone is very small.

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