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for a Field-Reversed Configuration**

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# Energy and Particle Confinement Times for a Field-Reversed Configuration

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## Abstract

The energy and particle lifetime scalings in a field-reversed configuration (FRC) are shown to depend on transport regimes and geometric parameters. The improved formulas have been compared with other empirical and theoretical scaling laws. The result is that confinement time dependence agrees with experimental measurements (for the present status of the experiments  $T_i \leq 1$  keV and  $s > 1$ ) of energy and particle loss both in magnitude and scaling, becoming smaller as the collision frequency decreases. The importance of the anomalous transport theory for a FRC is discussed.

## 1 Introduction

A field reversed configuration (FRC) is a high  $\beta$  ( $\approx 1$ ) magnetic configuration with a purely toroidal magnetic field. The typical experimental values of  $\langle \beta \rangle$  (volume-averaged  $\beta \equiv$  plasma pressure/magnetic-field pressure) range from 0.8 to 0.95, which gives a FRC a big advantage as a fusion reactor,<sup>1-3</sup> compared with other configurations.

The FRC has been studied for the past three decades, mainly in the USA, Russia and Japan, and significant progress has been made recently towards understanding and predicting energy and plasma confinement in FRC plasmas, using various methods and theories.<sup>4,5</sup> The main problems of FRC physics, such as magnetic reconnection, Finite Larmor Radius (FLR) and transport effects at the null-field point, separatrix and open field lines (scrape-off layer), have so far been investigated from both theoretical and experimental points of view. But there are still open questions about the main mechanism of energy and particle losses. Nonetheless, the discrepancy between empirical and classical scaling laws suggests that the FRC transport is anomalous. However, the lower-hybrid drift (LHD), low-frequency drift (LFD) and other theories do not describe FRC transport adequately.<sup>6,7</sup>

The confinement time in a FRC has a strong dependence on  $r_s^2/\rho_i$ , where  $r_s$  is the separatrix radius and  $\rho_i$  is the local ion gyroradius. The finite ion Larmor radius effect

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is characterized by the parameter  $s$  which is defined by

$$s = \int_R^{r_s} \frac{r}{r_s} \frac{dr}{\rho_i} \quad (1)$$

where  $R$  is the field-null radius ( $R = r_s/\sqrt{2}$  in elongated FRCs). The formula which may be used to approximate this number directly is  $s \approx 0.22r_s/r_c \cdot R/\rho_{i0}$ . This formula assumes a rigid-rotor like profile and an average beta condition.

Experimental work on FRCs has been done for a very broad range of parameters. There are three main ratios that define plasma regimes: 1) the  $s$  parameter, which is the ratio of the minor radius to the average internal gyroradius; 2) collisionality (the ratio of the ion collision frequency to the average bounce frequency); 3) the ratio of global confinement time to the end loss time. Present experiments are carried out at collisional plasmas with the global confinement greater than the end loss time. All of these parameters span a significant range of conditions. Such a wide range in parameters requires the use of estimates for various regimes of turbulence rather than global confinement theory. Furthermore, fusion plasmas (weakly collisional case, higher temperature) will operate at different conditions.

The next section discusses the previous experiments and provides useful formulas. Section 3 deals with improved scaling laws and some transport issues. The results and comparison with other scalings are discussed in Sec. 4.

## 2 Scaling laws and useful formulas

Previous experiments have been conducted over a wide spectrum of parameters.<sup>1,8,9</sup> For example, elongations (ratio of the half length to the separatrix radius of the FRC) of 3 – 13 have been used. Typical  $x_s$  values (ratio of the separatrix to coils radii) varied from 0.3 to 0.7 and the  $s$  value ranged from 0.5 to 6. Average FRC temperatures ranged from  $T_i \approx T_e \approx 100$  eV at 20 mtorr to  $T_i = 1.5$  keV,  $T_e = 0.5$  keV at 0.7 mtorr fill pressure. The FRC major radii ranged from 0.02 to 0.17 m with an external magnetic field of 0.4 – 1.4 T.

Early studies of both energy and particle confinement were performed by many authors<sup>6,10-13</sup> and the energy confinement time from these scaling laws goes as  $\tau \sim l_s^a x_s^b r_c^c n^d T^e$ , where  $l_s$  is the separatrix length,  $n$  is the density and  $T$  is the total temperature. Different empirical and theoretical scalings, including Bohm and classical transport, contain various exponents  $a, b, c, d, e$ .

A recent formula<sup>13</sup> based on results from the Large  $s$  Experiment (LSX) is  $1.3 \times 10^{-5} r_s^2 / \rho_{io}^{1.35}$  for the energy confinement time and  $2.1 \times 10^{-5} r_s^2 / \rho_{io}^{1.6}$  for the particle confinement time, where the reference ion gyroradius is based on the external magnetic field and the peak temperature. Confinement laws for FRCs including the present work, have been derived from empirical fits to confinement databases. Typical of these empirical scalings is the one derived for the LSX experiment for high  $s$  and temperature. These experiments have shown that theoretical models of anomalous transport are needed to explain the main loss channel for past experiments.

The LSX data indicate a stronger particle decay scaling with  $R$  and  $\tau_p \sim s^{1.49}$  which we will see is equivalent to Eq. (5). The energy decay time was also observed to scale with  $s$ .

Other experiments found that the energy confinement time has no  $\beta$  dependence, which means that  $E \times B$  transport (electrostatic origin) dominates over magnetic transport (electromagnetic turbulence). The energy confinement time is also found to increase with decreasing coil radius. Finally, the energy confinement time correlates with collision frequency, which explains that these plasmas in the experiments are collisional and indicates that collisional effects are important.

To compare the confinement time and results from different sources, the electron or ion temperature must be known. However, only the total temperature  $T_t = T_e + T_i$  was inferred from the density and external magnetic field by a pressure balance for a large number of experiments. For temperatures with  $T_t < 1$  keV, an approximation for the electron temperature is  $T_e = 1/2T_t[1 + \sqrt{T_t/400eV}]^{-1/2}$ .

### 3 Plasma energy and particle decay times

We have applied dimensional analysis to this problem by looking at the dependence of energy and particle confinement time on the dimensional and dimensionless plasma physics parameters such as the plasma radius normalized to the gyroradius and the ratio of separatrix length to the separatrix radius. The dependence on  $x_s$ ,  $s$ , and collisionality have also been investigated. After the methods of dimensional analysis were used, we obtained energy decay times as follows:

$$\tau_E \propto \frac{l_s}{r_s} \left(\frac{R^2}{\rho_{i0}}\right)^{1.2} r_c^{-\frac{2}{3}} \quad (2)$$

where  $\tau_E$  is in  $\mu\text{s}$ , and all other units are in m. The energy confinement time written in terms of the  $x_s$  and  $R$  parameters is

$$\tau_E \propto l_s x_s^{0.7} R^{0.7} \rho_{i0}^{-1.2}. \quad (3)$$

For the case where  $T \sim T_e \sim T_i$ , because  $\rho \sim T_i^{1/2}/B_e$  we have  $\rho \sim n^{-1/2}$  and thus the equivalent formula in terms of density is

$$\tau_E \propto l_s x_s^{0.7} R^{0.7} n^{0.6}. \quad (4)$$

Finally, the next expression is ready for use in calculating the plasma energy decay time  $\tau_E = 2.1l_s/r_s \cdot (R^2/\rho_{i0})^{1.2} r_c^{-2/3}$ , where  $\tau_E$  is in  $\mu\text{s}$ , and all other units are in m.

The particle lifetime dependence has the following form

$$\tau_p \propto \frac{l_s}{r_s} \left(\frac{R^2}{\rho_{i0}}\right)^{1.4} r_c^{-2/3} \quad (5)$$

where the same units are used as in Eq. (2). Or again, in terms of the internal magnetic field we can write

$$\tau_p \approx l_s x_s^{0.7} R^{1.1} \rho_{i0}^{-1.4}. \quad (6)$$

This scaling acts as a generalized law that includes LSX confinement. It will be shown that this empirical scaling agrees better with experimental results than other scalings for these device regimes and range of parameters.

We have chosen some experiments from the collisional regime from the ‘‘FRC Data Digest’’<sup>8</sup> with directly measured  $T_e$  and  $s > 1$ . Table 1 compares the experimental

Table 1: Comparison of the scaling laws with experimental results

FRC	$B_e$ (kG)	$T_t$ (eV)	$r_c$ (cm)	$r_s$	$l_s$	$\tau_E^{EXP}(\mu s)$	$\tau_E^{LFD}$	$\tau_E^{Hof}$	$\tau_E^{new}$
FRX-B	6.5	310	12.5	5.4	70	52	25	18	42
FRX-C	8	800	25	9	130	60	47	46	67
	7	250	25	10	150	135	170	133	180
TRX-2	13.5	900	12	4.6	29	21	17	15	18
LSM	4.1	516	35	15.1	145	95	109	108	77
	4.5	431	35	16.7	131	88	113	187	102
LSX	7.6	1500	45	14.3	135	63	-	108	64
	7.5	950	45	15	150	100	-	162	96
	5.7	430	45	14	260	155	-	174	192

plasma energy decay time, and the energy confinement time predicted by the empirical formula with LFD theory<sup>6</sup> and Hoffman scaling.<sup>13</sup>

The OCT, PIACE, FIX, NUCTE and TRX-2 devices had  $s$  factors  $\leq 1$ , and some experiments had confinement in much lower-density plasmas than the FRCs presented in Table 1. We must consider these confinement parameters as of a different regime and other scaling will apply for this class of experiments. As shown in Ref. 9, the total power loss at smaller  $s$  is more significant. Larger  $s$  ( $> 20$ ) such as that required for fusion reactors will have a different scaling of geometric parameters and, of course, other working regimes, and this empirical scaling may not be applicable to such systems. The reason for this is that data were obtained from a wide range of FRC equilibrium conditions. Good confinement was observed at all values of  $s$ , but more difficulties appeared for sustaining the plasma at large  $s$ .

One of the main elements in the scaling,  $R^2/\rho_{i0}$ , is a dependence that has been found in previous scaling experiments and is consistent with transport predictions based on microturbulence. The another one is the elongation that may be written as  $1/2 \cdot l_s/r_s$ . What is new is that a strong dependence on  $r_c$  has also been obtained. Many empirical and theoretical scalings have been suggested,<sup>1</sup> but none of them correlate well with all of the experimental results. One explanation of such behavior may be the fact that most of the scalings describe only one set of experiments or one device. The experiments lie in a very wide range of collisionality,  $s$  factor, elongation and other geometric parameters. Note that the lower-hybrid-drift instability does not explain FRC transport and Ref. 14, which derived LFD scaling, was published before the LSX experiments and did not include full information.

In general, depending on temperature and density, it is possible to operate in collisional, intermediate, or weakly collisional regimes. In these cases, either electrostatic or electromagnetic turbulence will be dominant. This means that under some conditions  $\tau_E$  may be the same order of magnitude or larger than  $\tau_p$ .<sup>15</sup>

## 4 Discussion and conclusions

In summary, the experiments have different parameters and the FRC data base<sup>8</sup> includes records from 10 different FRC facilities, all using deuterium fuel. So far the data set contains only results from FRCs formed in  $\theta$ -pinch devices. A large  $s$  value is only achieved in a theta pinch with high density and low temperature. Thus  $s$  FRCs are relatively collisional. This means that the  $s$  parameter correlates to the collisionality parameter by the formation method,<sup>16</sup> but presently it is not known how.

The energy confinement time in  $\mu$ s is plotted as a function of the parameter  $E_s \equiv l_s/r_s \cdot (r_s^2/\rho_i 0)^{1.2} \cdot r_c^{-2/3}$  in Fig. 1. The particle confinement time is plotted as a function of the parameter  $P_s \equiv l_s/r_s \cdot (r_s^2/\rho_i 0)^{1.2} \cdot r_c^{-2/3}$  in Fig. 2.

The scaling predictions agree well with past experiments. However, further experimental and theoretical investigation is required for hot fusion plasmas.

These scaling laws give unfavorable confinement and do not benefit the use of a FRC as a power plant if they remain valid in the collisionless, hot-plasma regime. There is no theory which describes the main mechanisms of energy and power losses in a FRC. Future experiments on TCS (Translation, Confinement, and Sustainment studies) at the University of Washington will also be useful in helping to better understand the nature of transport in a FRC, especially in collisionless plasmas. But the fusion regime (weak collisions because of high temperature) will probably have a different confinement scaling which may lead to a more optimistic prediction for larger FRC devices. Furthermore, the proposed project SPIRIT (Self-organized Plasma with Induction, Reconnection, and Injection Techniques), based on the counter-helicity spheromak merging, will be interesting, and should provide additional transport information and some knowledge concerning spheromak merging based experiments.

We do not have a clear picture of the FRC. What is needed is a comprehensive kinetic analysis and investigation of the transport theory for different regimes and parameters. This must be a critical goal in FRC research. A better understanding of particle orbits inside and outside the separatrix is needed as well as the creation of an anomalous transport theory for the FRC. Our preliminary results using velocity-space particle loss (VSPL)<sup>17</sup> theory have shown that a significant part of trapped particles (fusion products) may go through the separatrix and will still be trapped outside, depending on the constants of motion. Indeed, other aspects such as temperature,

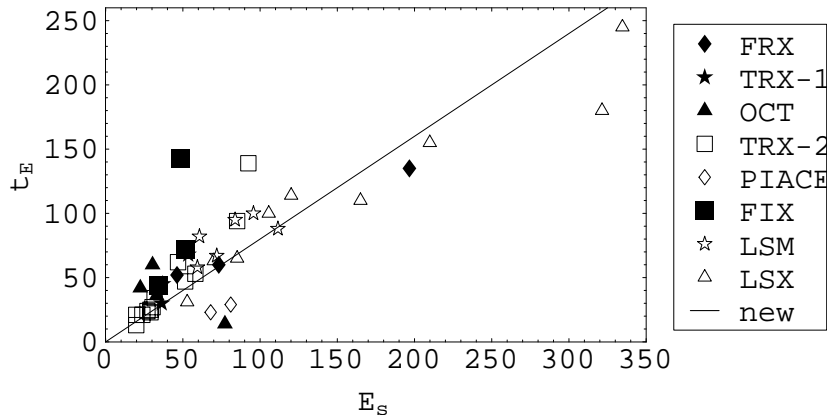


Figure 1: Generalized energy confinement time for different devices.

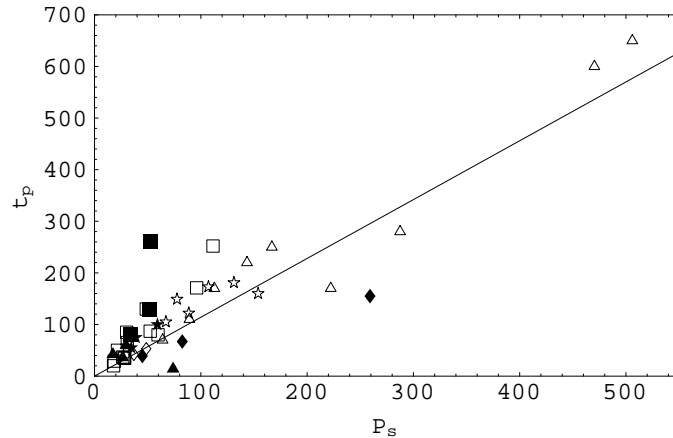


Figure 2: Particle confinement time in microseconds versus parameter  $P_s$  for various FRC experiments where the curve is in generalized scaling.

density and magnetic field profiles play a very important role. However, this theme will be addressed in a future paper.

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