Charging Losses of the Divertors and Transformer Coils

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September 1974

UWFDM-110

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Introduction

The charging losses in the divertor and transformer coils are expected to be significant (because of rapid charging about 10-100 seconds). It is the purpose of this paper to calculate these losses approximately.

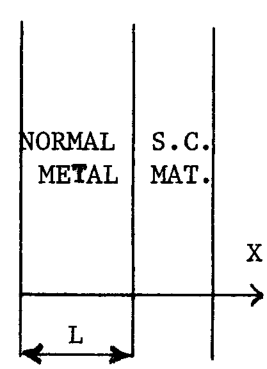
In the first part of this paper, the field and the current distribution in a slab are found. It is assumed that the slab is subjected to varying field on one side and in contact with superconducting slab on the other side. The current distribution is used to calculate power and energy loss. In the second part, the results for the slab were used to approximately derive an equation for calculating the losses in a circular cross section coil. In the third part, the losses for one of the converter coils were calculated. In the last part, we discussed calculating the losses in a rectangular cross section as well as a possible method of reducing them.

Flux Diffusion in a Normal Metal Slab

Consider a slab of thickness L. The magnetic field on one side of the slab is changing with time and the slab is in a perfect contact with superconducting slab on the other side, Figure 1.

The diffusion equation and the boundary conditions are

$$\left.\begin{aligned} &\frac{\partial^2 B}{\partial x^2} = \frac{1}{\alpha}\,\frac{\partial B}{\partial x} \\ &B(o,t) = f(t) \\ &\rho/\mu_o\,\frac{\partial B}{\partial x}\,(L) = v\;B(L,o) \end{aligned}\right\} \quad (1)$$



where

- α is the diffusivity of the normal metal
- ρ is the resistivity of the normal metal
- v is the velocity of flux lines in the superconductor at the interface
- $f(t)$ is any function of time.

As an approximation we take $v=0$ where the superconducting material is assumed to have zero resistivity. The solution of the above problem is given by

$$B(x,t) = \int_{\tau=0}^{t} B^{x}(x,t-\tau)\, \frac{\partial f}{\partial \tau}\, d\tau$$

where f is a continuous function of t.

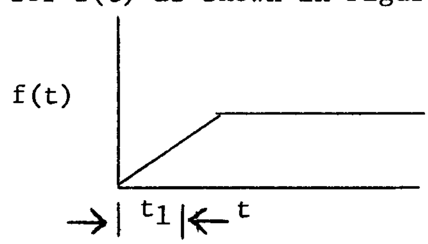
$B^*(x,t)$ is the solution for the above slab problem where the surface is subjected to a unit step function, i.e. where

$$f(t) = 1 \qquad t \geq 0$$
$$\quad = 0 \qquad t < 0$$

$B^*(x,t)$ is given by

$$B(x,t) = 1 - \frac{4}{\pi} \sum_{n=0}^{\infty} \frac{\sin \frac{(2n+1)\pi x}{L}}{(2n+1)} \; e^{\frac{-\alpha\pi^2 (2n+1)^2\, t}{L^2}}$$

We solve the present problem for f(t) as shown in Figure 2. The solution will be



$$B(x,t) = \frac{B_m}{t_1} \left\{ t_1 + \frac{4}{\pi} \sum_{n=0}^{\infty} \frac{\sin \tau_n x}{\alpha\tau_n^{\,2}(2n+1)}\; e^{-\alpha\tau_n^2 t}(1-e^{-\alpha\tau_n^2 t_1}) \right\} \qquad \text{for } t>t_1$$

where

$$\tau_n = \frac{(2n+1)\pi}{L}$$

and

$$B(x,t) = \frac{B_m}{t_1} \left\{ t_1 + \frac{4}{\pi} \sum_{n=0}^{\infty} \frac{\sin \tau_n x}{\alpha\tau_n^2 (2n+1)}\; e^{-\alpha\tau_n^2 t}(1-e^{+\alpha\tau_n^2 t_1}) \right\}$$

We consider the first terms in the above expansion. This is possible because $\alpha\tau_0^2 \simeq 30\ \mathrm{sec}^{-1}$ and after a short period, the other terms are practically zero. The solution, therefore, is

$$B(x,t) = \frac{B_m}{t_1}\left\{ t_1 + \frac{4L^2}{\pi}\,\frac{\sin \pi x/L}{\alpha\pi^2}\; e^{\frac{-\alpha\pi^2}{L^2}t}\left(1 - e^{\frac{+\alpha\pi^2}{L^2}t_1}\right)\right\} \quad \text{for } t>t_1$$

and

$$B(x,t) = \frac{B_m}{t_1}\left\{ t_1 + \frac{4L^2}{\pi}\,\frac{\sin \pi x/L}{\alpha\pi^2}\; e\;\frac{-\alpha\pi^2 t}{L^2}\left(1 - e^{\frac{+\alpha\pi^2}{L^2}t}\right)\right\} \qquad \text{for } t<t_1$$

and the current distributions are

$$J(x,t) = \frac{1}{\mu}\frac{\partial B}{\partial x}$$

$$= \frac{4B_m L^2}{\mu\alpha\pi^2 t_1}\cos\frac{\pi x}{L}\; e^{\frac{-\alpha\pi^2}{L^2}t}\left(1 - e^{\frac{\alpha\pi^2}{L^2}t_1}\right) \quad t>t_1$$

$$= \frac{8B_m L^2}{\alpha\mu\pi^2 t_1}\cos\frac{\pi x}{L}\left(1 - e^{\frac{-\alpha\pi^2 t}{L^2}}\right) \quad t<t_1$$

The power losses are given by

$$P(t) = \frac{8B_m^2 L^3}{\mu^2\alpha^2\pi^4 t_1^2}\,\rho\; e^{\frac{-2\alpha\pi^2 t}{L^2}}\left(1 - e^{\frac{\alpha\pi^2 t}{L^2}1}\right)^2 \quad t>t_1$$

$$= \frac{8B_m^2 L^3\rho}{\alpha^2\mu^2\pi^4 t_1^2}\left(1 - e^{\frac{-\alpha\pi^2}{L^2}t}\right)^2 \quad t<t_1$$

and the total energy losses for charging are given by

$$\Delta E_1 = \frac{4\,B_m^2\,L^5\rho}{\alpha^3\mu^2\pi^4 t_1^2}\left\{ \frac{2t_1\alpha\pi^2}{L^2} + 4e^{\frac{-\alpha\pi^2}{L^2}t_1} - e^{\frac{-2\alpha\pi^2}{L^2}t_1} - 3\right\} \qquad t<t_1$$

and

$$\Delta E_2 = \frac{4\, B_m^2 L^5 \rho}{\mu^2 \alpha^3 \pi^6 t_1^2} \left(1 - e^{\frac{-\alpha\pi^2}{L^2} t_1}\right)^2 \qquad t>t_1$$

As shown $\alpha\pi^2/L^2$ is large and the exponentials in the above expressions can be neglected. In this case, ΔE_1 and ΔE_2 may be written as

$$\Delta E_1 = \frac{4\, B_m^2 L^5 \rho}{\alpha^3 \mu^2 \pi^6 t_1^2} \left(\frac{2 t_1 \alpha\pi^2}{L^2}\right) \text{ for } t<t_1$$

and

$$\Delta E_2 = \frac{4\, B_m^2 L^5 \rho}{\mu^2 \alpha^3 \pi^6 t_1^2} \quad \text{for } t>t_1$$

and

$$\frac{\Delta E_1}{\Delta E_2} = 2\,\frac{t_1}{L^2/\alpha\pi^2} = 2\, t_1/\tau$$

where $\tau = L^2/\alpha\pi^2$

for $t_1 \simeq 10$ sec, $L \simeq .03$m and $\alpha \simeq 10^{-4} m^2/sec$ the above ratio is about 17. It can be seen that it is a good approximation to write the total energy loss, ΔE, as

$$\Delta E = \Delta E_1 + \Delta E_2 \simeq \Delta E_1$$

i.e.

$$\Delta E = \frac{8 B_m^2 L^3 \rho}{\alpha^2 \mu^2 \pi^6 t_1} = \frac{8 B_m^2 L^3}{\rho \pi^4 t_1}$$

<u>Energy Loss in a Donut Shaped Coil</u>

We make the following assumptions before calculating losses in the coil.

1. The coil major radius, R, is large compared to $\sqrt{A}$.
2. The coil has a uniform cross section A.
3. The charging of the coil, I vs t, is shown in Figure 4.

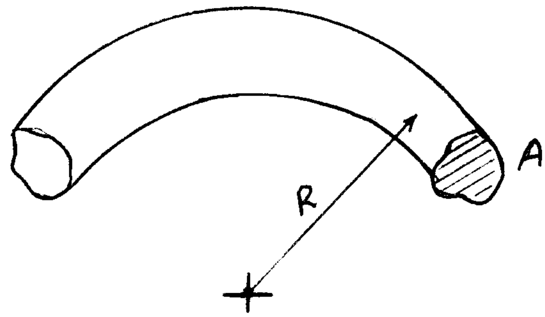


Figure 3

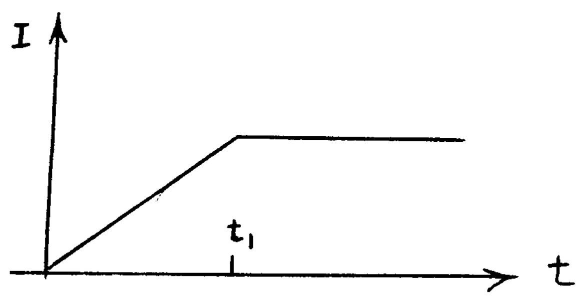


Figure 4

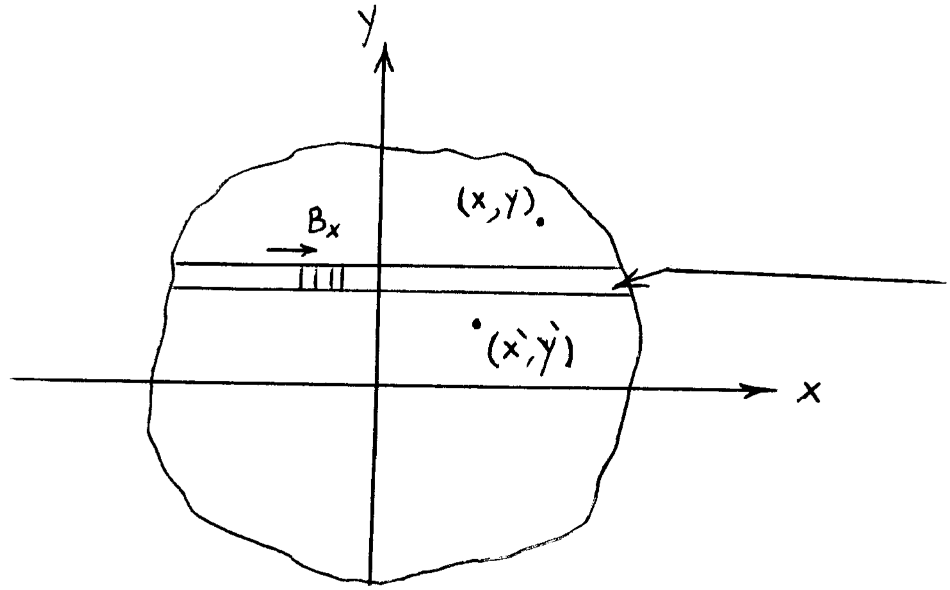


Figure 5

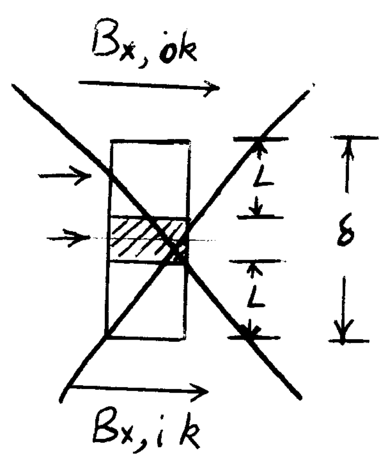


Figure 6

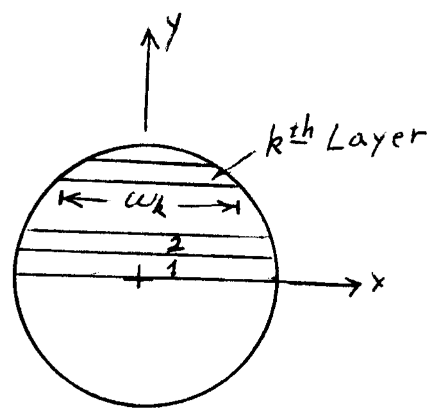


Figure 7

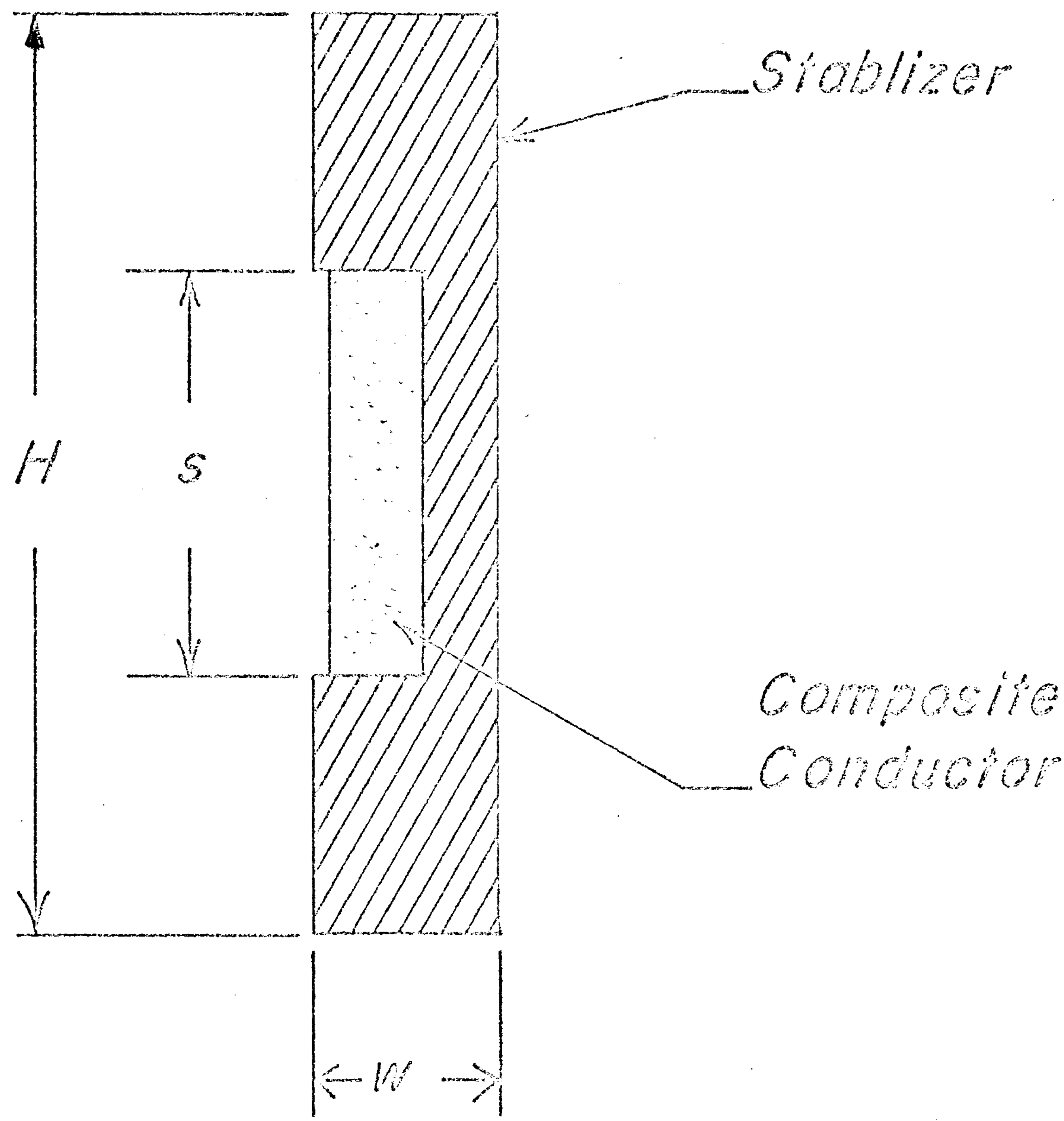


Figure 6 - Composite Conductor Cross Section

Consider a coil with a cross section as shown in Figure 5. For a current density J(x,y), the x-component of the field is given by

$$B_x = \int\int \frac{dx'dy'J(x',y')(y-y')}{(x-y')^2 + (x-x')^2}$$

This equation is approximately valid as long as $R >> \sqrt{A}$.

The magnet is assumed to be wound as shown in Figure 5. The individual conductors are vertical and is assumed to have similar design as the National Accelerator Energy Storage conductor, Figure 6. On the surface of the conductor there are the two components of the field B_x and B_y. It is assumed, because of the small thickness of the conductor, that B_y penetrates to the superconductor very fast with negligible losses. The main losses, therefore, are due to B_x where the conductor width is significant. (More discussion of this point will be included in another FDM).

We assume that the coil consists of n layer of conductors. The x-component of the field, B_x, can be calculated on the upper and lower surface of the k^{th} layer using the above equation. The energy loss formula derived for the slab is used to calculate the energy losses on the surfaces of the conductor. Therefore,

$$(\Delta E)_k = \frac{8L^3}{\rho\pi^4 t_1} \quad W_k(\bar{B}^2_{x,ik} + \bar{B}^2_{x,ok})$$

where

$(\Delta E)_k$ is the energy loss in the layer k

δ is the height of the normal edge, $\frac{H-S}{2}$ Figure 6.

$\bar{B}^2_{x,ik}$ is the average of B_x^2 on the lower side of the k^{th} layer

$\bar{B}^2_{x,ok}$ is the average of B_x^2 on the upper side of the k^{th} layer.

The total losses is given by

$$\Delta E = \frac{8L^3}{\rho\pi^4 t_1} \sum_{k=1}^{n} w_k \left(\bar{B}^2_{x,ik} + \bar{B}^2_{x,ok}\right)$$

As an example, we consider one of the divertor magnets and we assume that it has circular cross section. The x-component of the field will be easy to calculate. On the surface of the k^{th} layer $B_{x,ik}$ and $B_{x,ok}$ are given by (see Figure 7)

$$B_{x,ik} \simeq \mu I \frac{y_{ik}}{2\pi a^2}$$

$$B_{x,ok} = \mu I \frac{y_{ok}}{2\pi a^2}$$

where

- a is the radius of the coil
- y_{ik} is the y-coordinate of the lower surface of the k^{th} layer
- y_{ok} is the y-coordinate of the upper surface of the k^{th} layer

and

$$y_{ik} = \delta(k-1)$$

$$y_{ok} = \delta k$$

where δ is the thickness of the layer.

The average width of the layer is given by

$$w_R \simeq 2\sqrt{a^2-k^2\delta^2}$$

and the energy loss will be given by

$$\Delta E = \frac{8L^3}{\rho\pi^4 t_1} \sum_{k=1}^{n} w_k \left(\bar{B}^2_{x,ik} + \bar{B}^2_{x,ok}\right)$$

$$= \frac{16L^3\mu^2}{\rho\pi^4 t_1} \sum \frac{I^2 \; 2\sqrt{a^2-k^2\delta^2}}{4\pi^2 \; a^4} \left[\delta^2(k-1)^2 + \delta^2 k^2\right]$$

In writing the above formula, we may notice

1. 8 was replaced by 16 to consider both halves of the coil.
2. $\bar{B}^2_{x,ok}$ and $\bar{B}^2_{x,ik}$ for the circular cross section is the same as $(B_{x,ok})^2$ and $(B_{x,ik})^2$.

The above equation may reduce to

$$\Delta E = I^2 \frac{8L^3\delta^2\mu^2}{\rho\pi^6 a^3 t_1} \sum_{1}^{n} \sqrt{1-k^2\left(\frac{\delta}{a}\right)^2} \left[(k-1)^2 + k^2\right]^2$$

If the conductor was taken as NAL conductor we may take

$n \simeq 4$

$L \simeq .03\text{m}$

$\delta \simeq .1\text{m}$

$\rho \simeq 10^{-10}\Omega\cdot\text{m}$

and the divertor coil we are considering has $a \simeq .5\text{m}$ and $I \simeq 7 \times 10^6\text{A}$, therefore

$$\Delta E \simeq \frac{451.95}{t_1} \quad \text{K Joule/meter}$$

$(\Delta E)_{total} \simeq 5.395 \times 10^3$ KJ $\quad t_1 \simeq 10$ sec, $R = 19\text{m}$

$\simeq 1.499$ KWhr/charge at 4.2°K

$= .449$ MWhr/charge at room temperature

Acknowledgement

I wish to acknowledge Dr. Y. Eyssa for reviewing, discussing and helping in the preparation of this paper.