Parametric Studies of Driven Tokamaks

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Parametric Studies of Driven Tokamaks[†]

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Abstract

Parametric studies of driven Tokamaks have been carried out to develop a better understanding of the relationships between two energy component devices, driven tokamaks in general, and ignition machines. It appears that for plasma currents less than 2-3 MA, the largest energy amplification factor is attained by injecting deutrons of greater than 150 keV into pure tritium target plasma at the maximum beam power possible. In higher current machines, the optimum Q occurs when the plasma is a 50-50 deuterium-tritium mixture maintained in energy equilibrium by the minimum amount of injected power. In addition, high thermonuclear amplification factors (greater than 10) appear possible in unignited, driven Tokamaks with large Z_{eff}. Thus, the special technical aspects of a pure two energy component device are of direct relevance to larger, driven Tokamaks, which can be acceptable power systems.

Table of Contents

I. <u>Introduction</u>

The potential for a beam driven plasma to achieve energy breakeven at lower $n\tau$ than is required to meet either Lawson or ignition conditions has spurred wide interest in this concept.[1] Calculations indicate that a high energy deuteron beam injected into a pure tritium plasma can have its energy reproduced in fusion reactions at $n\tau$ values on the order of 10^{13} sec-cm^{-3}. This would amount to an energy amplification factor, Q, of one.

In Tokamaks, this situation may occur in plasmas at the 1 MA level. Importantly, the energy amplification factor depends primarily on the electron temperature (the ions may be cold) and the breakeven condition corresponds to a T_e value of about 4.5 keV. In larger devices, this general two-energy-component approach (now referred to as TCT) can yield Q values as high as six, depending on the specific mode of operation, particularly whether some form of energy clamping is used.[2]

For the ultimate goal of fusion power reactors, one wants as large an energy amplification factor as possible. This does not, of course, mean that economic fusion plasmas must be ignited. By the definition used here, ignition corresponds to an infinite Q. In between the pure TCT and ignition devices lies a class of unignited, high Q, driven machines that can be viable power systems.

To study the relationship between a pure TCT, driven machines in general, and ignition machines, we have carried out parametric studies of driven Tokamaks and investigated the performance characteristics of such devices. In particular, the scaling of these characteristics with plasma current (size), neutral beam energy and power, target plasma composition, magnetic field strength, impurities, and transport scaling

laws has been investigated. The remainder of this report describes the calculational models used and the results that have been attained. In section II, a brief discussion is given of the plasma amplification factor used to present the results. A calculational model has been developed from our earlier work to perform this study and brief details are described in section III. The model includes alpha particle production and heating from beam-plasma and Maxwellian plasma fusions, injection heating, detailed slowing down, energy losses from electrons and ions based primarily on the predicted scaling of the trapped ion mode,[4,5] and energy losses due to radiation, including bremsstrahlung, recombination, excitation, and synchrotron radiation. Limitations on the poloidal beta are considered and detailed calculations are made to account for the pressure of the beam particles and alpha particles as they slow down. The calculation of beam energy deposition profiles is also briefly described. In section IV, we present and discuss the essential results of this study which underlie the summary and condusions contained in section V.

II. Definition of Q

There have been several expressions advanced as candidates for the plasma amplification factors, some involving an imposed external thermal efficiency, η, of 40% and many using different values of the energy per fusion, typically either 17.6 MeV or 22.4 MeV.

In this study, we have adopted the definition of the plasma amplification factor, Q, as

$$Q = \frac{\text{FUSION POWER PRODUCED IN THE PLASMA}}{\text{INJECTED POWER ABSORBED BY THE PLASMA}} \; .$$

The numerator includes all thermonuclear fusion power, either from beam-plasma fusions or from Maxwellian plasma fusions. Each fusion event generates 17.6 MeV of energy. No external energy amplification or thermal efficiency is included. Likewise, the denominator is the injected power absorbed by the plasma. It does not include efficiency factors to account for the neutral beam injection system or for the beam power deposited on the limiter or chamber walls and not absorbed by the plasma. We have considered the neutral beam to be a single atomic species at a specified energy. Also excluded is the power required to generate the steady state plasma, that is, the power to drive the OH coils, the TF coils, and other auxiliary systems. Further, since we are reporting expected Q values from plasmas in a given, quasi-steady-state condition, we have not had to consider averaging Q over a calculated or specified dynamical history of plasma behavior.

III. Calculational Model

The parametric studies reported here required the development of a calculational model which will indicate the expected operating and performance characteristics of large Tokamaks and the scaling of these characteristics with current (size), neutral beam injection energy and power, target plasma composition, impurities, and transport scaling laws. Since many calculations are involved, we have developed a point plasma model involving the space-average particle densities and temperatures. In addition, the model is time independent since our interest is in the expected plasma properties at the end of the current rise and heating phases. The energy conservation equations are:

Ion Energy Conservation:

$$\frac{d}{dt}(\frac{3}{2} n_i k T_i) = W_{inj}^i + W_{BF}^i + W_{PF}^i - W_E^i + W_{ei} = 0 \qquad (1a)$$

Here, W_{inj}^i is the fraction of injected power deposited in the ions, W_{BF}^i is the fraction of alpha particle energy from beam-plasma fusions deposited in the ions, W_{PF}^i is the fraction of alpha energy from thermal plasma fusions deposited in the ions, W_E^i is the energy loss rate due to transport processes, and W_{ei} is the energy exchange rate due to electron-ion rethermalization.

Electron Energy Conservation:

$$\frac{d}{dt}(\frac{3}{2} n_e k T_e) = W_{inj}^e + W_{BF}^e + W_{PF}^e - W_E^e - W_{ei} - W_{RAD} = 0 \qquad (1b)$$

Each term in eqn. (1b) has the same origin as in the ion equation except that W_{RAD} incorporates radiative losses.

These two equations can be written out explicitly, in units where plasma current is in MA, length is in cm, temperature is in keV, magnetic field is in units of 50,000 gauss, and the reduced aspect ratio, A, is $\frac{R}{3a}$:

Ion Energy Conservation:

$$.98 \times 10^{-5} \frac{P_{inj} \, b_T^3 \, U_{bi}}{q^3 \, I^3 \, A^4} + 5.61 \times 10^{-13} X_\alpha n_i^2 \alpha(1-\alpha) <\sigma v>_{D-T} U_{\alpha i}$$

$$+ \frac{0.034 \, X_\alpha P_{inj} b_T^3 \, F_{TCT} U_{\alpha i}}{E_o q^3 \, I^3 \, A^4} - \frac{1.45 \, T_e^{7/2} \, T_i^3 \, b_t^2 \, (n_i/n_e)}{A^{13/2} \, I^4 \, q^4 \, Z_{eff} \, (T_i + T_e)^2}$$

$$- \frac{4.37 \times 10^{-28} \, n_e n_i \, (T_i - T_e)}{A_p \, T_e^{3/2}} = 0 \qquad (2a)$$

Electron Energy Conservation:

$$\frac{.98 \times 10^{-5} \, P_{inj} \, b_t^3 \, (1-U_{bi})}{q^3 \, I^3 \, A^4} + 5.6 \times 10^{-13} \, X_\alpha \, n_i^2 \, \alpha(1-\alpha) <\sigma v> (1-U_{\alpha i})$$

$$+ \frac{.034 \, X_\alpha \, P_{inj} \, b_T^3 \, F_{TCT}(1-U_{\alpha i})}{E_o \, q^3 \, I^3 \, A^4} - \frac{1.45 \, T_e^{9/2} \, T_i^2 \, b_t^2}{A^{13/2} \, I^4 \, q^4 \, Z_{eff}(T_i + T_e)^2}$$

$$+ \frac{4.37 \times 10^{-28} \, n_e \, n_i \, (T_i - T_e)}{A_p \, T_e^{3/2}} - W_{RAD} = 0 \qquad (2b)$$

In both equations, P_{inj} is the beam power injected in W, F_{TCT} is the fraction of beam particles which undergo fusion, b_t is the reduced toroidal field, q is the stability factor, I is the plasma current, A is the reduced aspect ratio, n_i and n_e are the ion and electron

densities, T_i and T_e are the ion and electron temperatures, U_{bi} and $U_{\alpha i}$ are the fractions of beam energy and alpha energy, respectively, going to the ions. X_α is the fraction of alphas that do not have orbits extending beyond the plasma radius, a (that is, the fraction of alphas retained) α is the tritium fraction of the plasma ions, and $A_p = 3\,\alpha + 2(1-\alpha)$ is the effective atomic mass of the plasma ions. The parameter X_α has been computed from a recent study[6] and is shown in Figure 1 as a function of plasma current.

In both equations, the transport scaling law governing the energy containment time has been assumed to be the most pessimistic presently proposed for future Tokamak performance, namely, that predicted for the trapped ion mode[4,5]. Where applicable, we have

$$j(r) = j_o$$
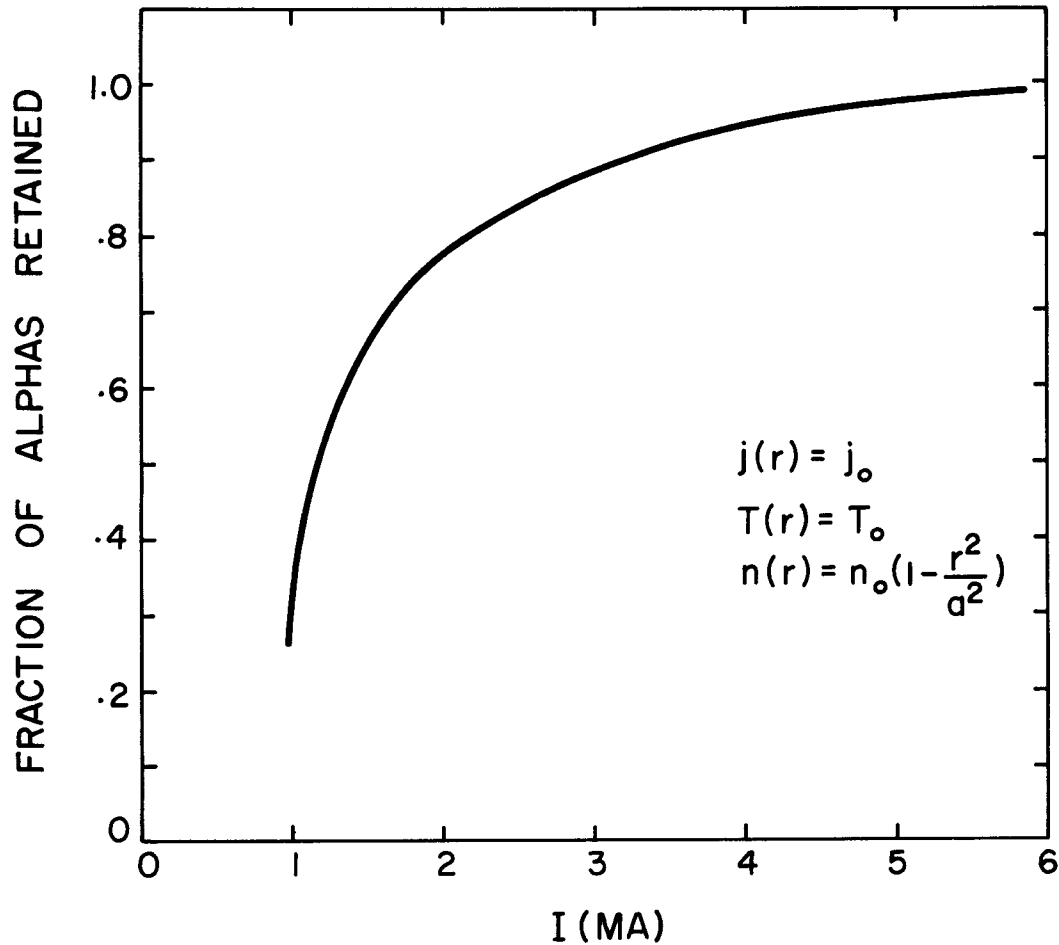$$T(r) = T_o$$
$$n(r) = n_o(1-\frac{r^2}{a^2})$$

Figure 1 – Fraction of Alpha Particles Retained as a Function
of Plasma Current.

9

used the scaling laws appropriate to the other predicted microinstabilities.
The various regimes are shown in Figure 2.

The term, W_{RAD}, includes bremsstrahlung, W_B, as

$$W_B = 4.8 \times 10^{-31} \, n_e^2 \, Z_{eff} \, T_e^{1/2} \tag{3}$$

and synchrotron radiation, W_{Sync}, as[13]

$$W_{Sync} = 8.7 \times 10^{-13} \, (5 + .18 \, (5 - 3A))^3 \sqrt[3]{\frac{n_e b_T}{ab_T}} \, T_e^{2.1} \quad . \tag{4}$$

In addition, terms to account for line and recombination radiation have
been included using formulae presented by Post[7] and by Vasilev, Dolgov-
Savelev, and Kogan.[8] From the paper by Post, recombination radiation is
given approximately by

$$W_{rec} = 0.5 \times 10^{-32} \, Z^4 \, n_e^2 \, \xi_Z \, T_e^{-1/2} \quad , \tag{5}$$

one electron excitation radiation by

$$W_{1e} \simeq 0.18 \times 10^{-26} \, n_e^2 \, \xi_Z \, T_e^{-1/2} \tag{6}$$

and three electron line radiation by

$$W_{3e} \simeq 1.2 \times 10^{-26} \, \xi_Z \, n_e^2 \quad . \tag{7}$$

In these formulae, ξ_Z is $\frac{n_{im}}{n_e}$, the ratio of the impurity density to electron
density. Whether W_{1e} or W_{3e} is the dominant line radiation term depends
on the stripping fractions at a given temperature, also discussed by Post,
and this has been accounted for.

Another fit, developed by Vasilev, Dolgov-Savelev, and Kogan[8]
and summarized recently by Hopkins[9] can also be used to account for exci-
tation and recombination radiation. It has been developed to fit experimental
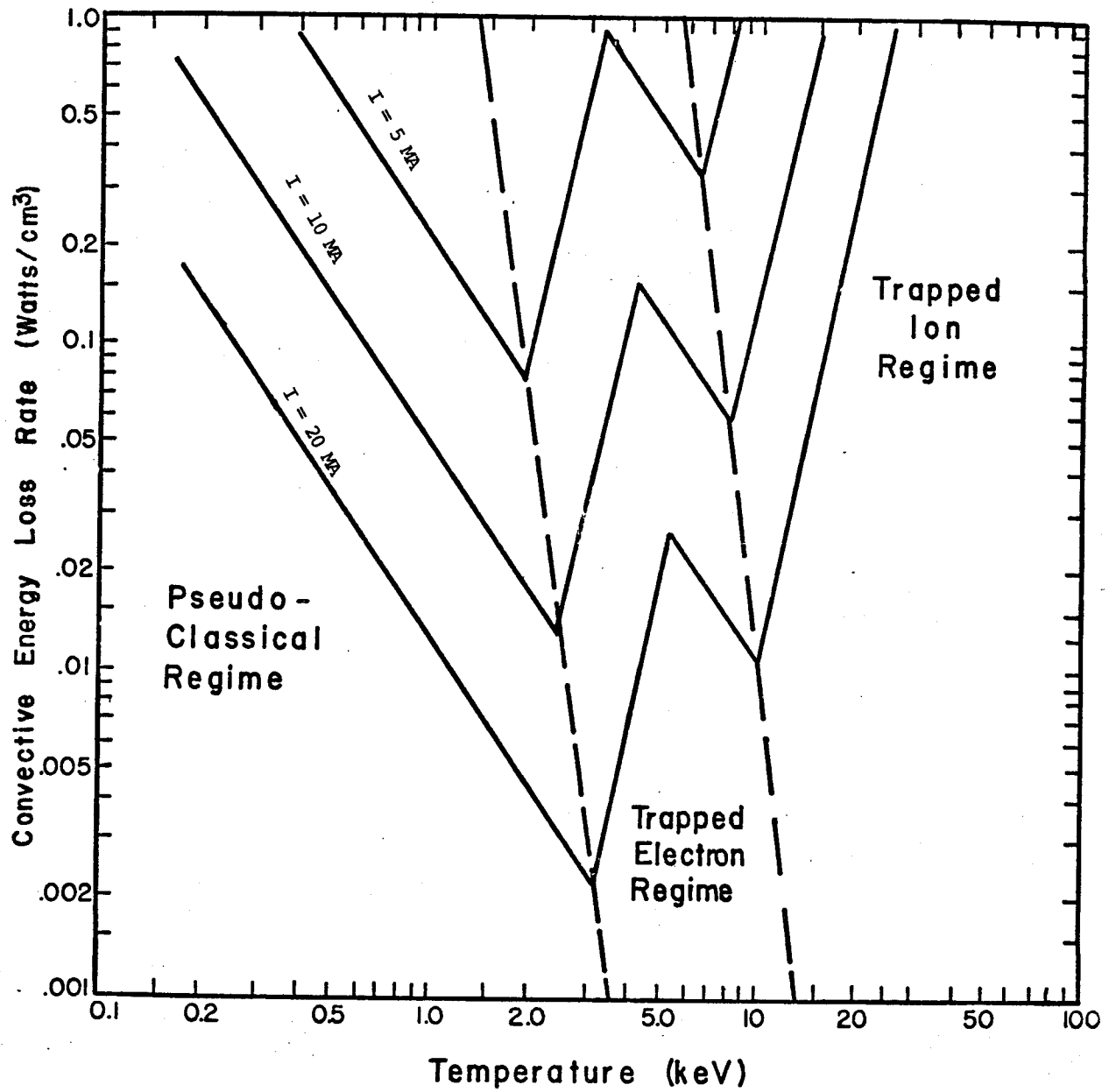
Figure 2 — Convective Energy Loss Rate as a Function of Electron Temperature for Various Predicted Microinstabilities.[4,5]

data for $Z \leq 18$ but has been used for $Z > 18$ since experimental data was not available. This expression for radiation from an impurity with charge Z is given by

$$W = W_B \left(1 + \frac{3.97 \times 10^{-2} \, \xi_Z \, Z^4}{Z_{eff} \, T_e} + \frac{8.6 \times 10^{-4} \, \xi_Z \, Z^6}{Z_{eff} \, T_e^2} \right) . \tag{8}$$

We have found the results using eqn (8) to be similar to those obtained using the formulae of Post. Thus, for W_{RAD}, the formula

$$W_{RAD} = W + W_{Sync} \tag{9}$$

has been used although, as noted the formulae from Post have been used as a cross check.

In addition to the energy conservation equations, the charge neutrality condition

$$n_e = n_i + 2 \, n_\alpha + \sum_j n_j \, Z_j \tag{10}$$

is maintained and a limit on the total poloidal beta, β_θ, has been imposed. The expression for β_θ is

$$\beta_\theta = \frac{1.44 \times 10^{-16} \, q^2 A^2}{b_T^2} \left(n_e \, T_e + n_i \, T_i \right.$$
$$\left. + \frac{2}{3} \left(n_{\alpha TCT} + n_{\alpha Th} \right) \left(\bar{E}_\alpha \, \tau_{SD}^\alpha + T_i \, \tau_p^\alpha \right) + \frac{2}{3} \, n_h \, \bar{E}_b \right) \tag{11}$$

where $n_{\alpha TCT}$ and $n_{\alpha Th}$ are the alpha particle production rates from TCT and Maxwellian fusions, respectively, $\bar{E}_\alpha$ is the mean energy of the slowing down alpha particles, τ_{SD}^α is the time to slow down from 3.52 MeV, τ_p^α is the alpha containment time (taken equal to the ion confinement time) n_h is the density of high energy beam injected particles, and $\bar{E}_b$ is the average

energy of the slowing down beam particles.

The calculations of $\bar{E}_\alpha$ and $\bar{E}_b$ were obtained from recent studies we have completed on the thermalization of an energetic heavy ion in a multi-species plasma. This is based on a Fokker-Planck treatment and extends the earlier work of Butler and Buckingham. [10] Expressions have also been obtained for $U_{\alpha i}$ and U_{bi}, the fractions of alpha and beam energy, respectively, going to the ions. The analysis pertains only to the mean energy slowing down and not to the spread in energy about the mean.

The rate of change of the energy of a test particle, E_T, slowing down in a multispecies plasma is given by

$$\frac{dE_T^{3/2}}{dt} = - 1.5 \times 10^{-19} \frac{Z^2 \, n_e \, \ln \Lambda_e}{A_T \, T_e^{3/2}} \, (T_e^{3/2} \sum_j \gamma_j + E_T^{3/2}) \qquad (12)$$

where Z is the charge of the test particle, A_T is its mass, $\ln\Lambda_e$ is the Coulomb logrithm for electrons, and γ_j is given by

$$\gamma_j = \frac{57 \, A_T^{3/2}}{n_e \, \ln\Lambda_e} \, \frac{Z_j^2 \, n_j \, \ln\Lambda_j}{A_j} \qquad (13)$$

for impurity species j of charge Z_j, mass A_j, and Coulomb logarithm $\ln\Lambda_j$. Let

$$\gamma = \sum_j \gamma_j \, . \qquad (14)$$

The test particle energy at which the rate of energy loss to all ion species is equal to the rate of energy loss to the electrons is defined as E_{crit} and is given by

$$E_{crit} = 14.8 \, A_T \, T_e \left(\frac{1}{n_e \, \ln\Lambda e} \sum_j \frac{n_j \, Z_j^2 \ln\Lambda_j}{A_j} \right)^{2/3} \qquad (15)$$

$$E_{crit} = \gamma^{2/3} T_e \tag{16}$$

The solution for the test particle energy as a function of time is

$$E_T^{3/2} = E_{To}^{3/2} e^{-t/\tau} - E_{crit}^{3/2} (1 - e^{-t/\tau}) \tag{17}$$

where E_{To} is the initial energy and τ is given by

$$\frac{1}{\tau} = 1.5 \times 10^{-19} \frac{Z^2}{A_T} \frac{n_e \ln\Lambda_e}{T_e^{3/2}} \tag{18}$$

The time to slow down is obtained from eqn (17) by setting $E_T = T_i$ to

obtain

$$\tau_{SD} \simeq \tau \ln \left(\frac{E_{To}^{3/2} + E_{crit}^{3/2}}{T_i^{3/2} + E_{crit}^{3/2}} \right) . \tag{19}$$

An analytic expression for the fraction of test particle energy going to

all ion species (which rapidly thermalize together) is

$$U_{Ti} = \frac{1}{3} \frac{E_{crit}}{E_{To}} \left\{ \ln \left[\frac{E_{crit} - E_{crit}^{1/2} E_{To}^{1/2} + E_{To}}{E_{crit} + 2 E_{crit}^{1/2} E_{To}^{1/2} + E_T} \right] \right.$$

$$\left. + 2\sqrt{3} \tan^{-1} \left(\frac{2 E_{To}^{1/2} - E_{crit}^{1/2}}{\sqrt{3} E_{crit}^{1/2}} \right) + \frac{\sqrt{3}}{3}\pi \right\} . \tag{20}$$

This expression is valid for T_e, $T_i \ll E_{crit}$, E_{To} and is used for both the

injected neutral beam and the alpha particles.

The average energy of the slowing down particles, required to compute

β_θ, is obtained by time averaging $E_T(t)$ over the slowing down time from zero

to τ_{SD}. Assuming the final energy of the test particle is E_f, the expression

for the average energy, $\bar{E}_T$, is

$$\bar{E}_T = \frac{3}{2} \frac{(E_{To} - E_f)}{\ln\left(1 + (\frac{E_{To}}{E_{crit}})^{3/2}\right)} (1 - U_{Ti}) \tag{21}$$

and, for $E_{To} \gg E_f$,

$$\bar{E}_T = \frac{3}{2} \frac{E_{To}(1 - U_{Ti})}{\ln\left((1 + (\frac{E_{To}}{E_{crit}})^{3/2}\right)} . \tag{22}$$

This expression is used to calculate $\bar{E}_\alpha$ and $\bar{E}_b$ in the expression for β_θ.

Beam energy deposition profiles for various machines have been computed assuming the beam is a pencil beam and using methods developed in earlier work[11]. The programs were generalized for the present study to include injection both inside and outside the magnetic axis.

IV. Results and Discussion

a. Parametric Study of TCT's and Driven Tokamaks

For an impurity free plasma ($Z_{eff} = 1$), a Q = 1 breakeven experiment can be performed on a Tokamak carrying a plasma current of as low as 1 MA. For larger machines and as ignition conditions are approached, the target composition must be altered from the pure tritium TCT target to a 50-50 deuterium-tritium target plasma to obtain the maximum Q. In this section, we present results on the determination of the maximum Q as a function of plasma current, and thus of machine size. In particular, the effects of target composition, beam power, impurities, and magnetic field strength on Q are discussed. This leads naturally to the analysis of the transition from a pure TCT, which gives the maximum Q when the plasma current is relatively low (< 3 MA), to larger 50-50 D-T driven machines. For the results reported, the following plasma parameters were assumed, unless otherwise noted: safety factor, q(a) = 2.5, total poloidal beta, $\beta_\theta = 2$, aspect ratio, A = 3, toroidal field strength, B_T = 50 KГ and energy of the injected beam, W_o = 180 keV. In addition, we have used I as the figure of merit characterizing the plasma. Thus, many of the results have been put into the form, Q versus I.

The plasma target composition which generates the maximum Q varies from 100% tritium to a 50-50 D-T mixture, depending on the plasma current, as seen in Figures 3 and 4. For I = 2 MA (Fig. 3), the maximum Q is obtained with a 100% tritium target using the maximum injected power consistent with the β limit. The reason for injecting the maximum allowable power is to increase the electron temperature, which in turn causes Q
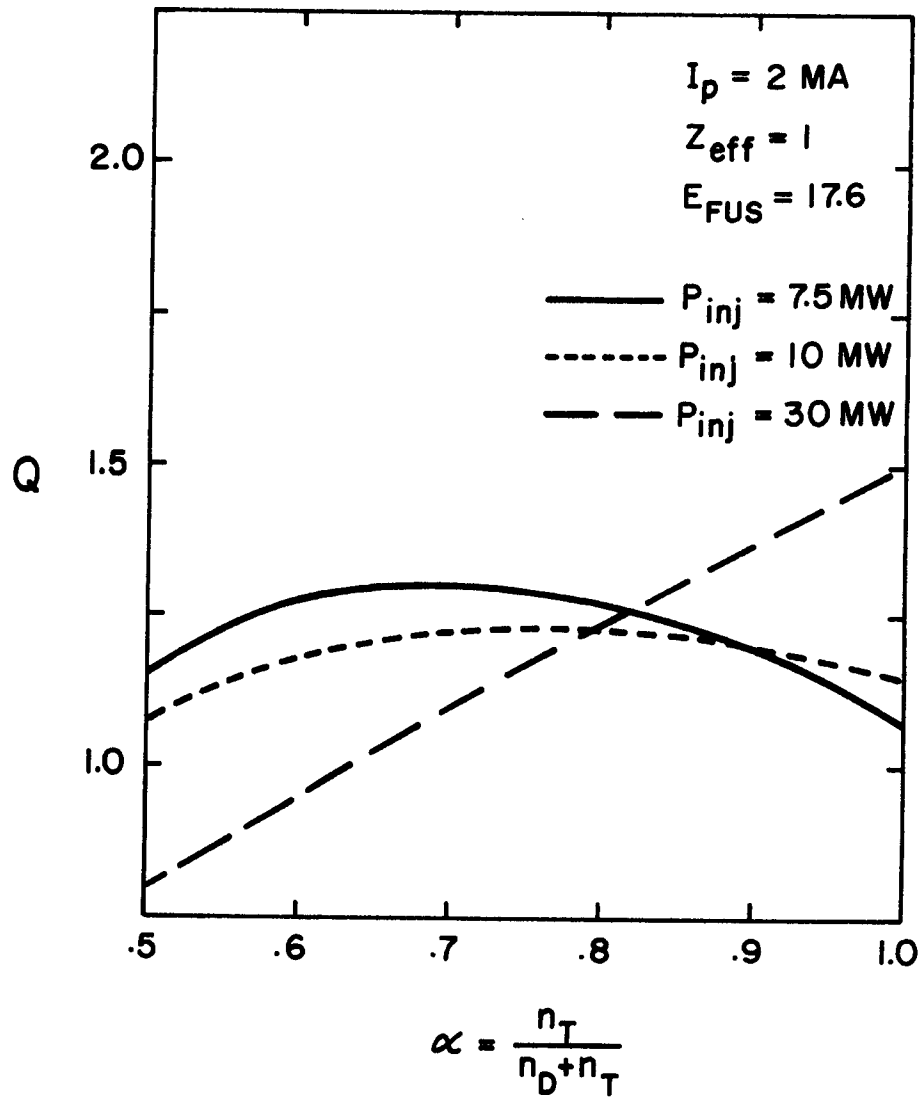
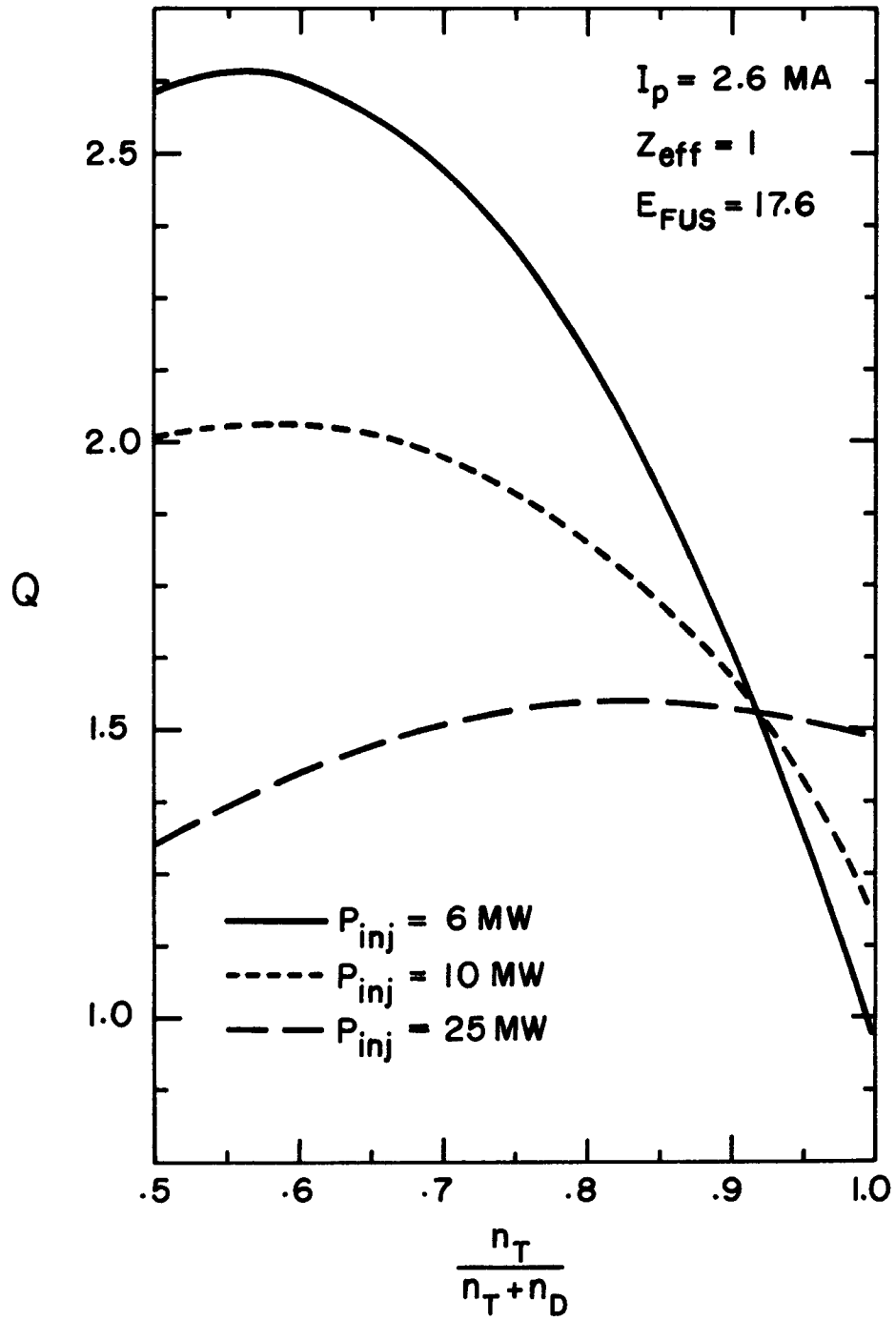Figure 3 - Q as a Function of Tritium Concentration in the Plasma for Various Amounts of Injected Power.

$I_p = 2.6$ MA

$Z_{eff} = 1$

$E_{FUS} = 17.6$

Q

$\dfrac{n_T}{n_T + n_D}$

—— $P_{inj} = 6$ MW

------ $P_{inj} = 10$ MW

— — $P_{inj} = 25$ MW

Figure 4 - Q as a Function of Tritium Concentration in the Plasma for Various Amounts of Injected Power.
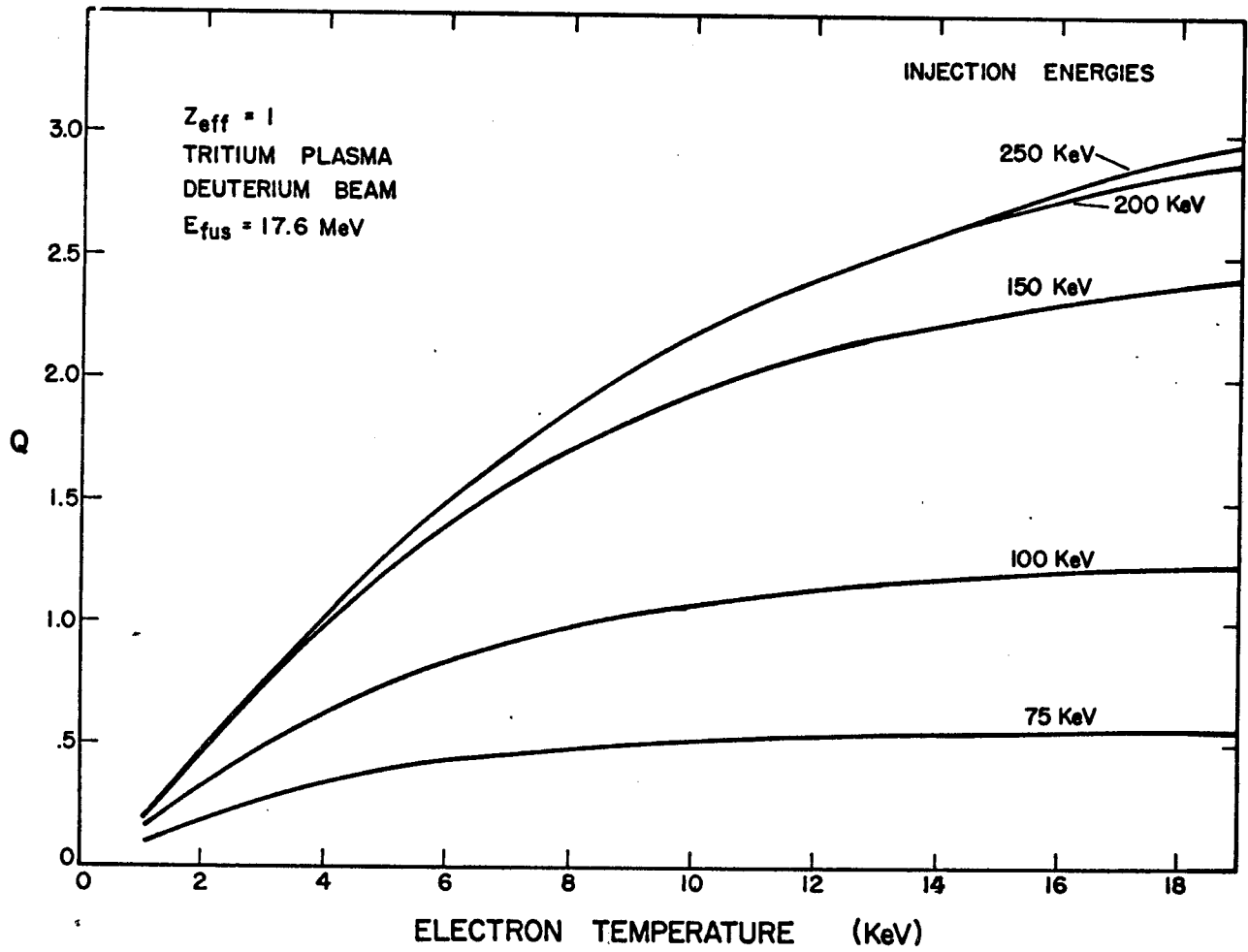
Figure 5 – TCT Amplification Factor, Q, as a Function of Electron
          Temperature for Neutral Deuterium Beams of Various Energies
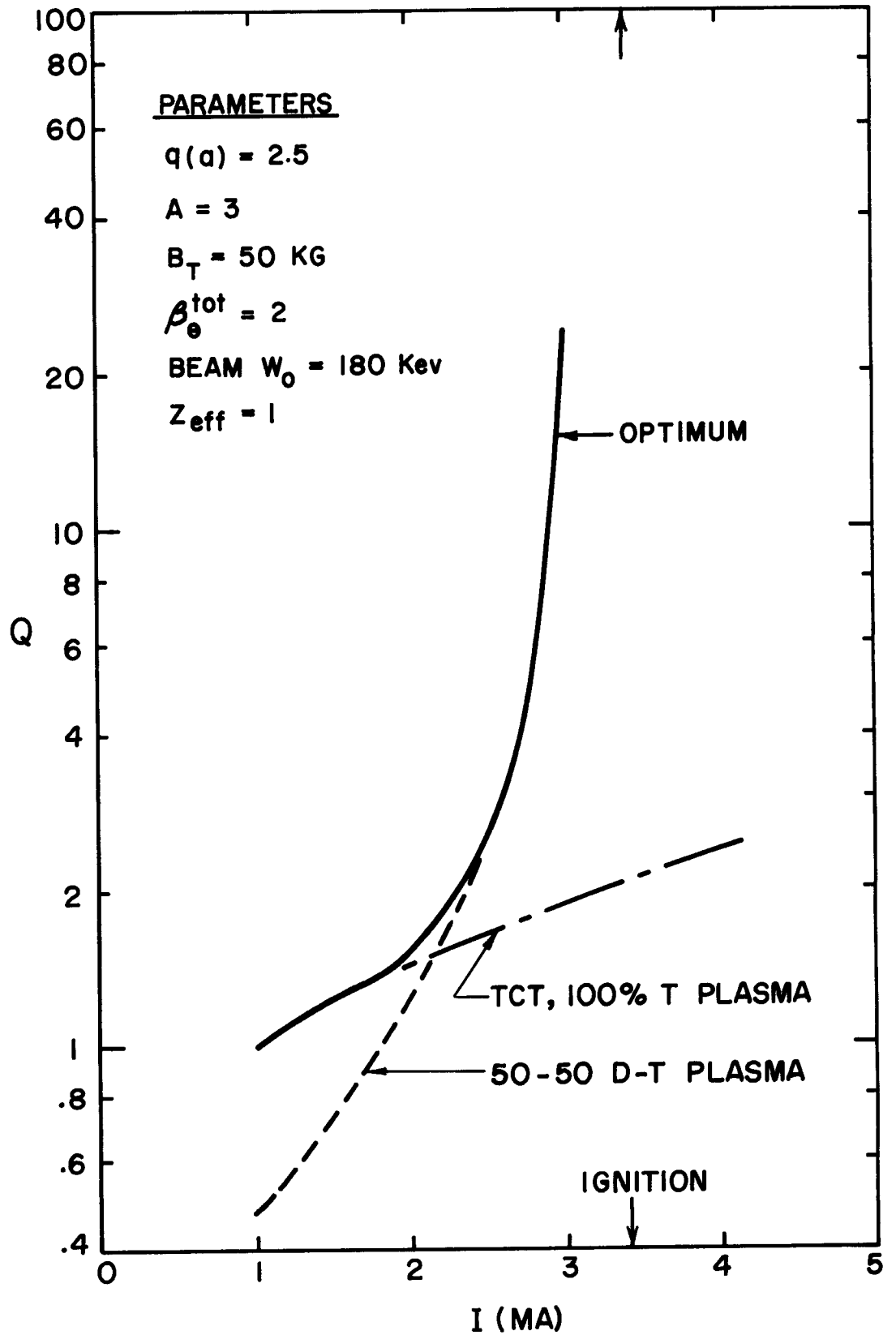   Slowing Down in an Infinite Tritium Plasma.

Figure 6 – Optimum Q as a Function of Plasma Current for $Z_{eff} = 1$.

to increase, as seen in Figure 5. On the other hand, at I = 2.6 MA (Figure 4), the optimum Q is attained for a plasma composition of roughly 60% tritium, 40% deuterium, and the minimum injection power consistent with maintaining the plasma in energy balance. Thus, for Z_{eff} = 1, the transition in maximum Q from a pure TCT to a 50-50 D-T minimum injection system takes place very quickly for I between 2 and 3 mega amperes. This transition is shown most clearly in Figure 6. With the assumed plasma conditions, the current at ignition is approximately 3.4 MA.

It has been noted that the maximum injected power yields the largest Q in a pure TCT only at relatively low plasma currents. This can also be seen by examining the optimum Q as a function of plasma current for various amounts of injected power, as in Figure 7. The curves extend only over a finite range of plasma currents. The low current cutoff is due to the β-limit whereas the cutoff at high currents comes about because the injected power cannot maintain the discharge.

The presence of a small amount of high Z impurity can alter the preceeding results substantially. The main effects are shown in Figures 8, 9, and 10, and table 1. The effect of iron at Z_{eff} = 3 (Fig. 8, 9 and table 1) is to cause the transition in maximum Q from pure TCT's to 50-50 D-T driven machines to take place at somewhat larger plasma currents. On the other hand, the breakeven experimental condition still occurs within the range of 1 to 1.5 MA. Thus, high Z impurities have a smaller impact on the TCT. Low Z impurities can have a big impact on the TCT. Q since the target ion composition is lowered.

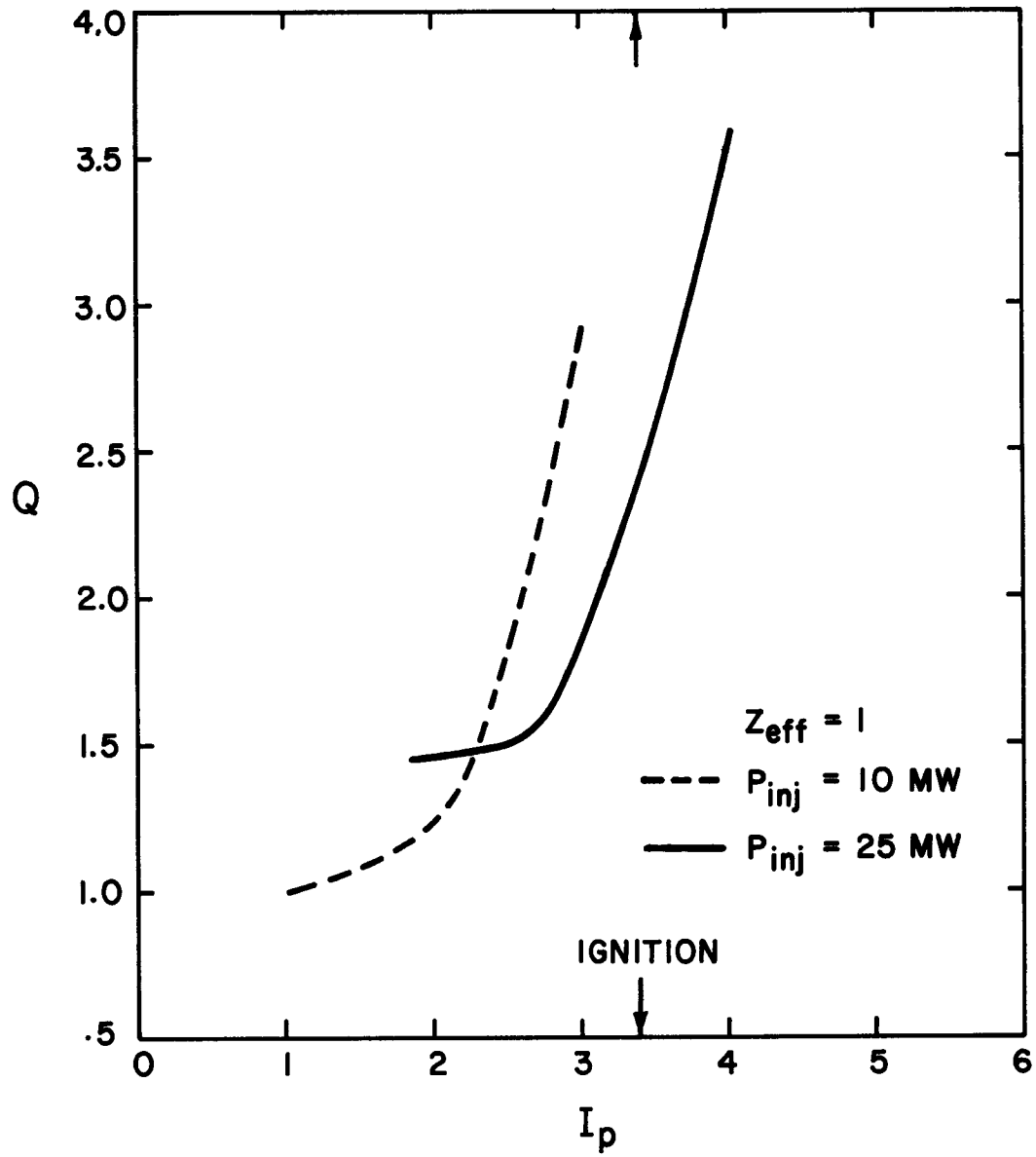Figure 7 - Q Versus Plasma Current for Different Amounts of Injected Power. The Plasma Ignites at 3.4 MA for $Z_{eff} = 1$.
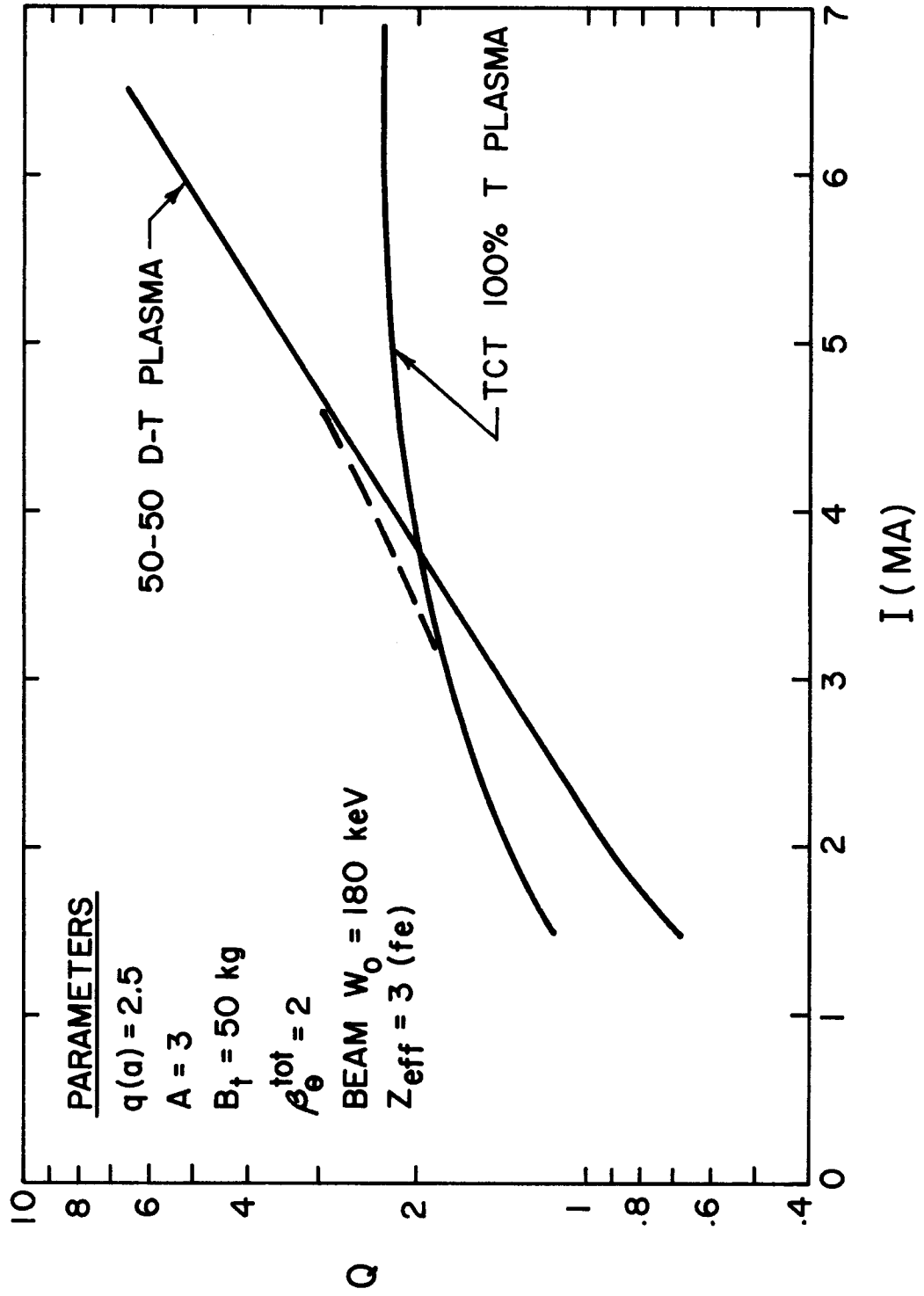
Figure 8 – Q Versus Plasma Current for 50-50 D-T and 100% Tritium Target Plasmas.

Table 1

Amplification Factor, Q, and Plasma Characteristics

for Several Values of Plasma Current

Z_{eff} = 3 (Fe) B_T = 50 kg

$q(a)$ = 2.5 β_θ^{tot} = 2

Beam Wo = 180 keV

I (MA)	n_e (cm^{-3})	T_i (keV)	T_e (keV)	$\dfrac{n_T}{n_D + n_T}$	$\dfrac{n_\alpha}{n_e}$ (%)	max
2	1.3×10^{14}	7.5	6.0	1.0	0.13	1.4
3	9.5×10^{13}	10.3	8.1	1.0	0.17	1.76
4	1.0×10^{14}	11.1	8.8	0.7	0.36	2.4
5	9.0×10^{13}	12.4	10.0	0.6	0.51	3.3
6	9.1×10^{13}	12.5	10.4	0.6	0.87	5.2

Another view of the impact of impurities is given in Figure 9

which shows the ratio of thermal fusion power to total fusion power

and indicates that the changeover from a pure TCT at low currents

to a thermal fusion machine occurs much more slowly for Z_{eff} = 3.

Ignition will not occur in this case until the current is about

12 MA.

The full impact of impurities is shown in Figure 10. Due to

increased radiation losses, the injection requirements are increased.

In high current machines, thermal fusions dominate and, relatively

as such, the Q in these systems tends to vary inversely with the

injected power, P_{inj} In a TCT, Q is less dependent on P_{inj} TCT,

because beam-plasma fusions increase with injected power

increase with inj of power

We noted earlier that impurities have a substantial impact on the

plasma current at ignition. This is shown most clearly in Figure 11

and Table 2, where iron has been assumed as the impurity. For

Z_{eff} = 2 (or $\frac{n_{im}}{n_e}$ = .0015) the current at ignition has about tripled

from the value of 3.4 MA when Z_{eff} equals one. Since $n\tau$ is proportional

to I^4 for the trapped ion modes, the result is that high Z impurities

substantially increase the $n\tau$ requirement for ignition. This same

point has been made by Meade[3].

Such results lead to interest in how these results change when one

assumes that low Z materials, such as silicon carbide, aluminum oxide, or

a metallic carbide, constitute the impurities. This may be possible if

low Z limiters and low Z wall coatings are feasible. Low Z impurities

have a much less detrimental effect on plasma performance since they are

generally fully stripped. As an example, for a Z_{eff} = 2.35 made up of a

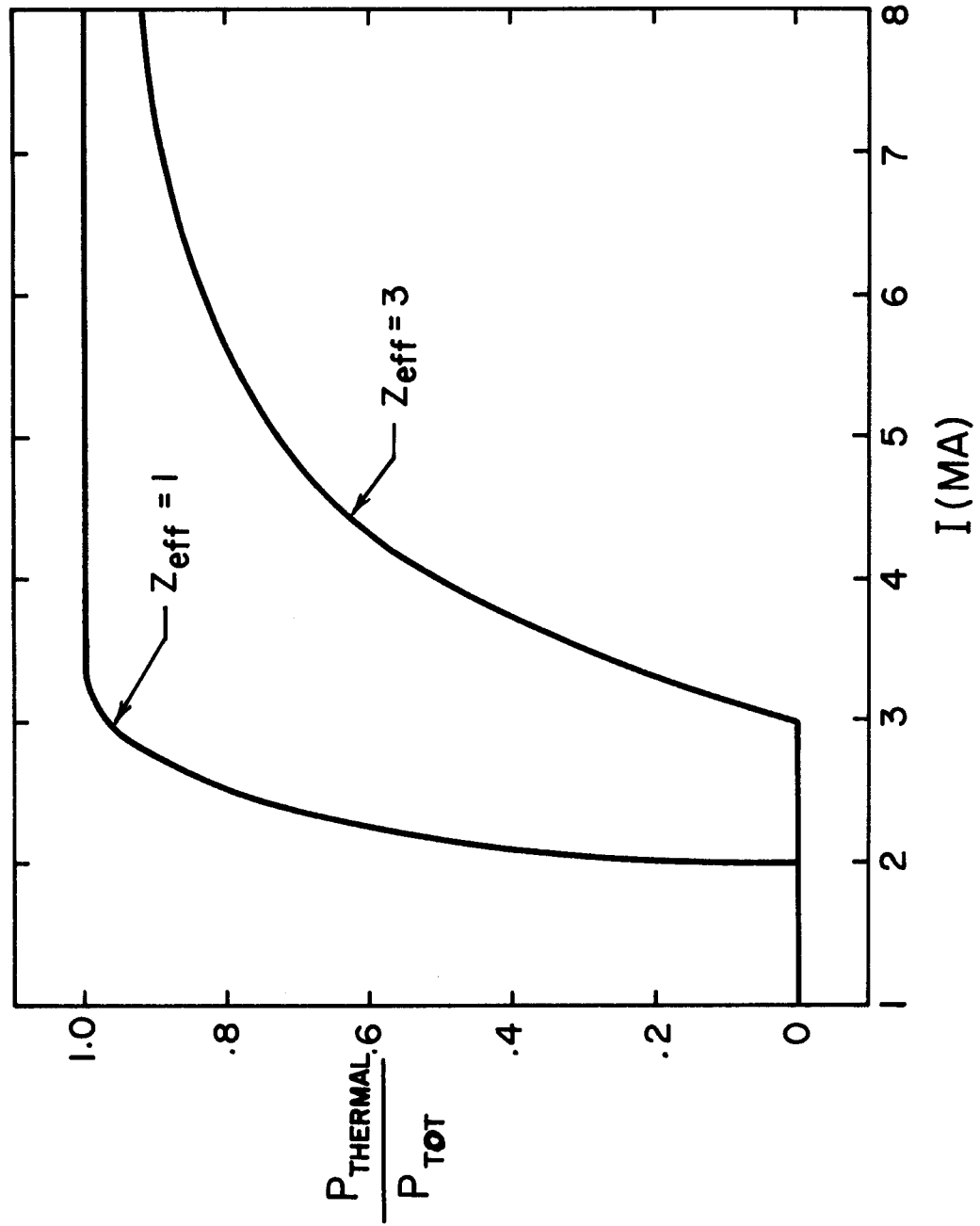low Z materials such as silicon carbide (0.5% Si, 0.5% C, 0.5% O), ignition

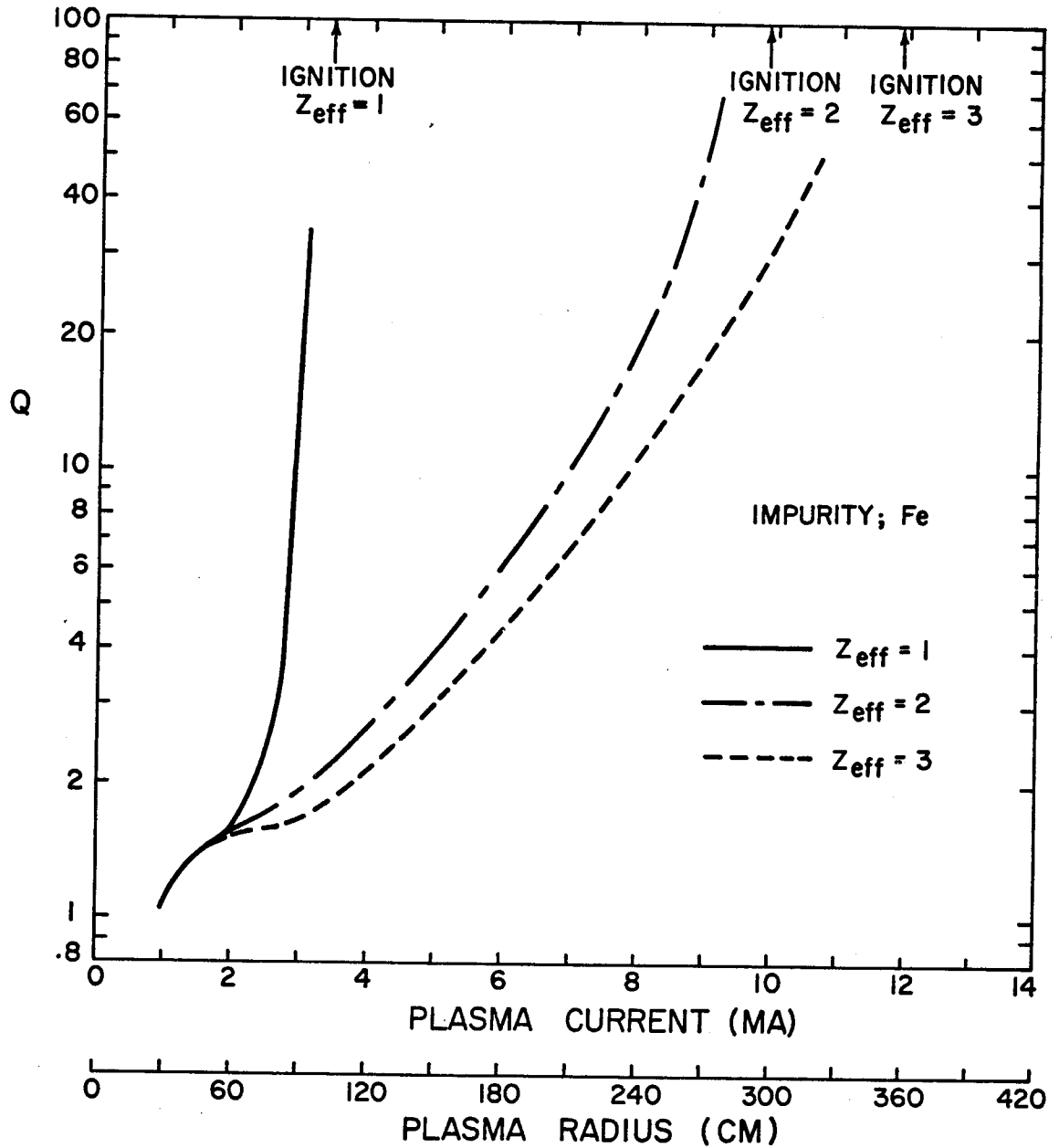Figure 9 – Ratio of Thermal Power Produced to Total Power in Systems which Yield the Highest Q for a Given Current.

Figure 10 – Optimum Q as a Function of Plasma Current for Different Values of Z_{eff}. Plasma Parameters are q(a) = 2.5, B_T = 50 kG, β_θ^{tot} =2.

Table 2

Maximum Z_{eff} (for Iron) Allowable for Ignition

q (a) = 2.5 β_θ^{tot} = 2

B_T = 50 kg   Beam Energy, Wo = 180 keV

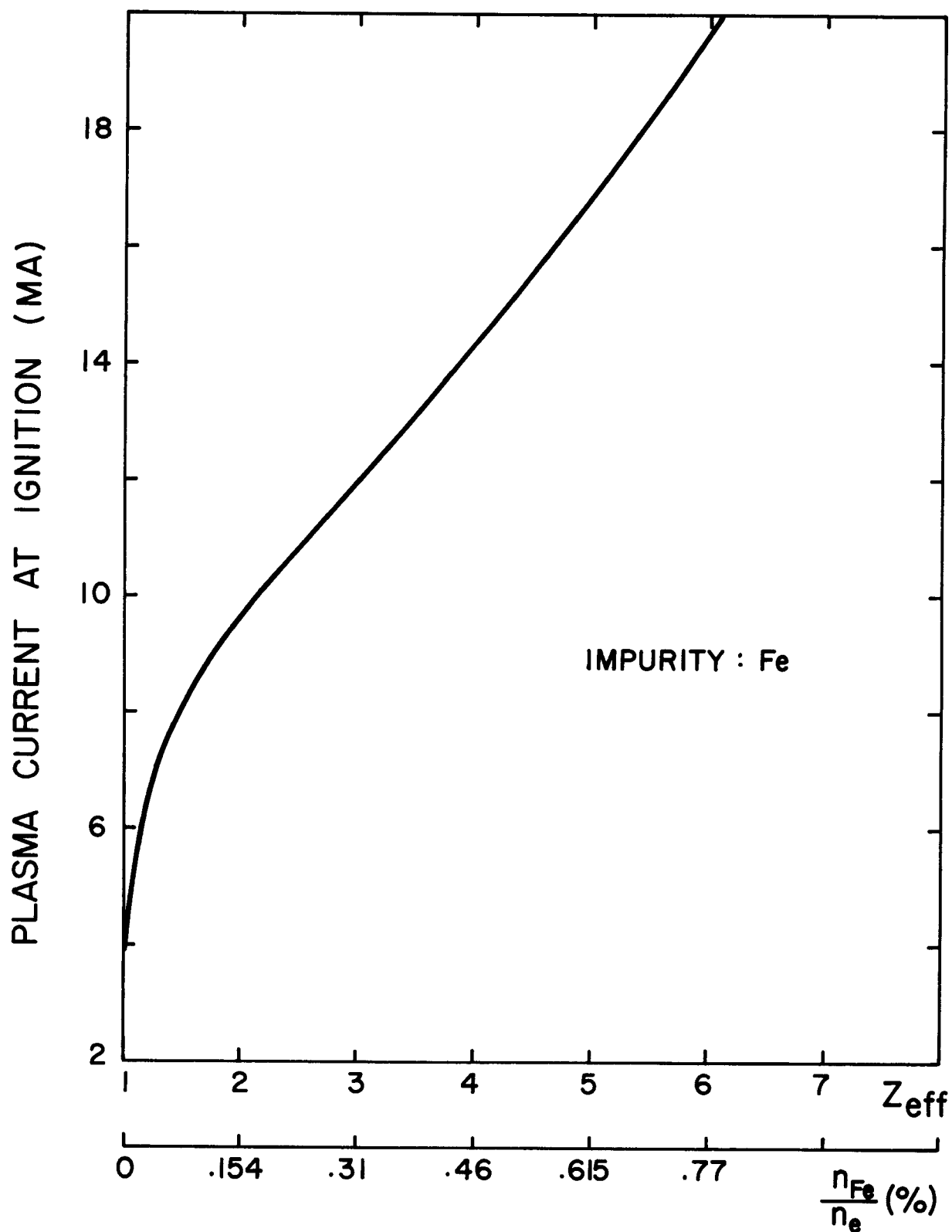| Current (MA)	n_e (cm^{-3})	T_i (keV)	T_e (keV)	$\frac{n_\alpha}{n_e}$ (%)	Maximum Z_{eff} Allowable for Ignition
4	2.3×10^{14}	4.8	4.8	1.98	1.05 ± .05
6	1.75×10^{14}	6.3	6.3	1.44	1.15 ± .05
8	1.19×10^{14}	9.0	9.1	1.96	1.45
10	8.9×10^{13}	12.1	11.9	2.82	2.15
12	7.2×10^{13}	15.1	14.2	4.03	3.05
14	6.1×10^{13}	18.4	16.1	5.32	4.05
16	5.3×10^{13}	21.7	17.8	6.50	4.75
18	4.5×10^{13}	26.0	19.9	6.50	5.4
20	4.1×10^{13}	30.0	21.2	7.65	6.1

Figure 11 – Plasma Current at Ignition as a Function of the Amount
of Impurity in the Plasma.

will occur below 8 MA, as seen on Figure 12. (We have included an additional .5% oxygen in these calculations.) For .15 Si, .15 C, and .5% O, the current at ignition is approximately 6.3 MA. This is less than double the ignition current when $Z_{eff} = 1$. By comparison, .5% Fe increases the ignition current to approximately 15 MA and .5% Mo raises the current required for ignition to greater than 25 MA. It should be noted that for low Z, as compared to high Z, a comparable Z_{eff} indicates a much higher percentage of impurities present. However if the amount of low Z impurities becomes large enough ($Z_{eff} \gtrsim 5$), the Q begins to drop because the impurity now occupies a significant fraction of the allowable β.

The impact of low Z impurities on the optimum Q versus I curve is shown in Figure 13. One notes that below 2 to 3 MA, the curves are similar to those found with high Z impurities. However, for comparable impurity concentrations, the transition to high Q machines is now much more rapid. Thus, low Z wall coatings on chamber walls and low Z limiters will, if feasible, alter the nature and severity of the impurity problem substantially, and for the better. In a sense, low Z coatings can achieve the same effect (namely, lowering the machine size required for ignition) as divertors for the control of high Z impurities.

The effect of magnetic field on the maximum Q is shown in Figures 14 and 15. The optimum Q versus plasma current for three values of the toroidal field on axis is shown in Figure 14. The loss rates due to conduction-convection are increased as B_T is reduced thus increasing the injected power necessary to sustain the plasma. At low currents (<3 MA), where the optimum devices are pure TCT's (see Figure 15), the maximum Q is relatively insensitive to P_{inj}, and therefore to b_T. The opposite is true
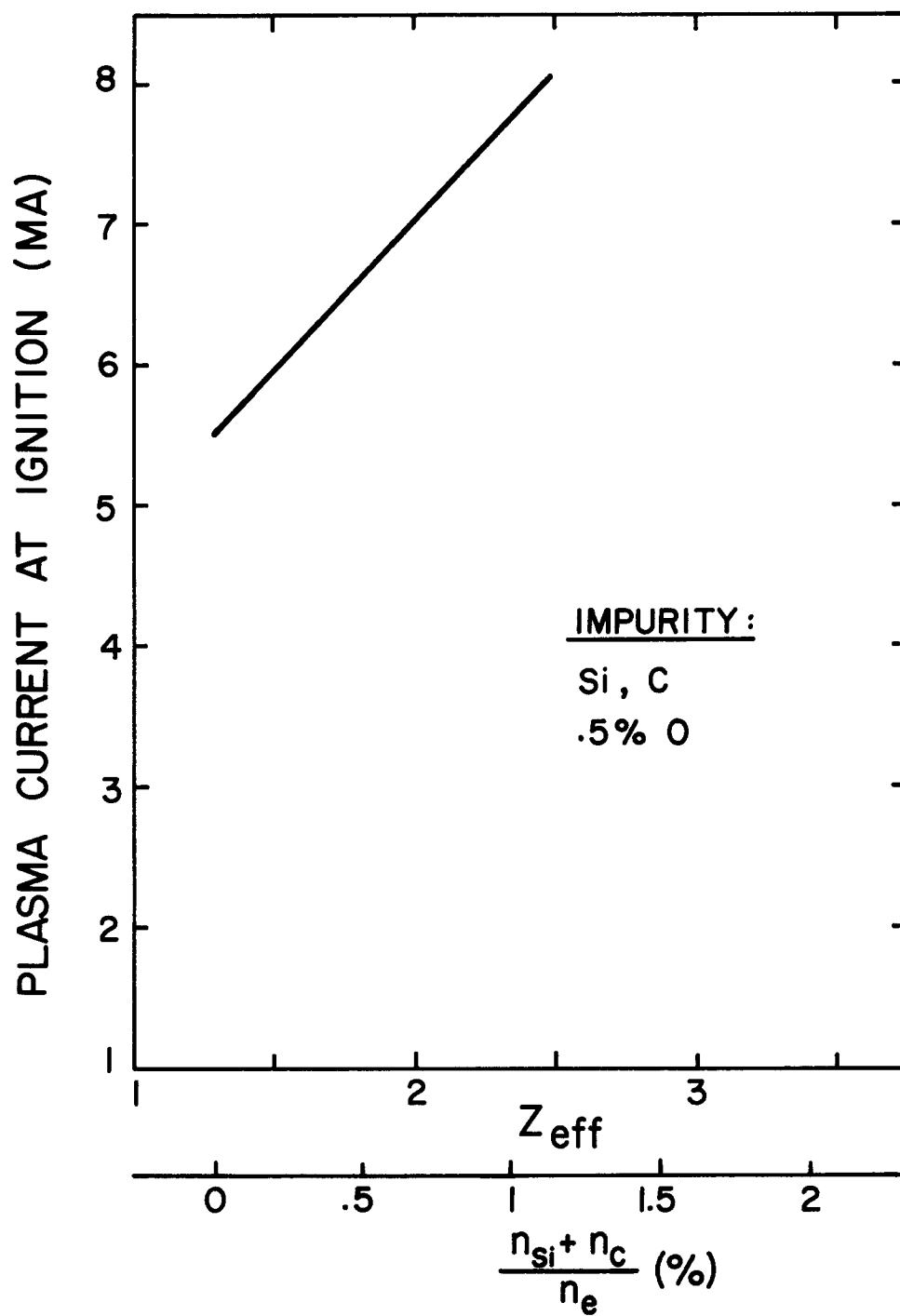
Figure 12 – Plasma Current at Ignition as a Function of the Amount
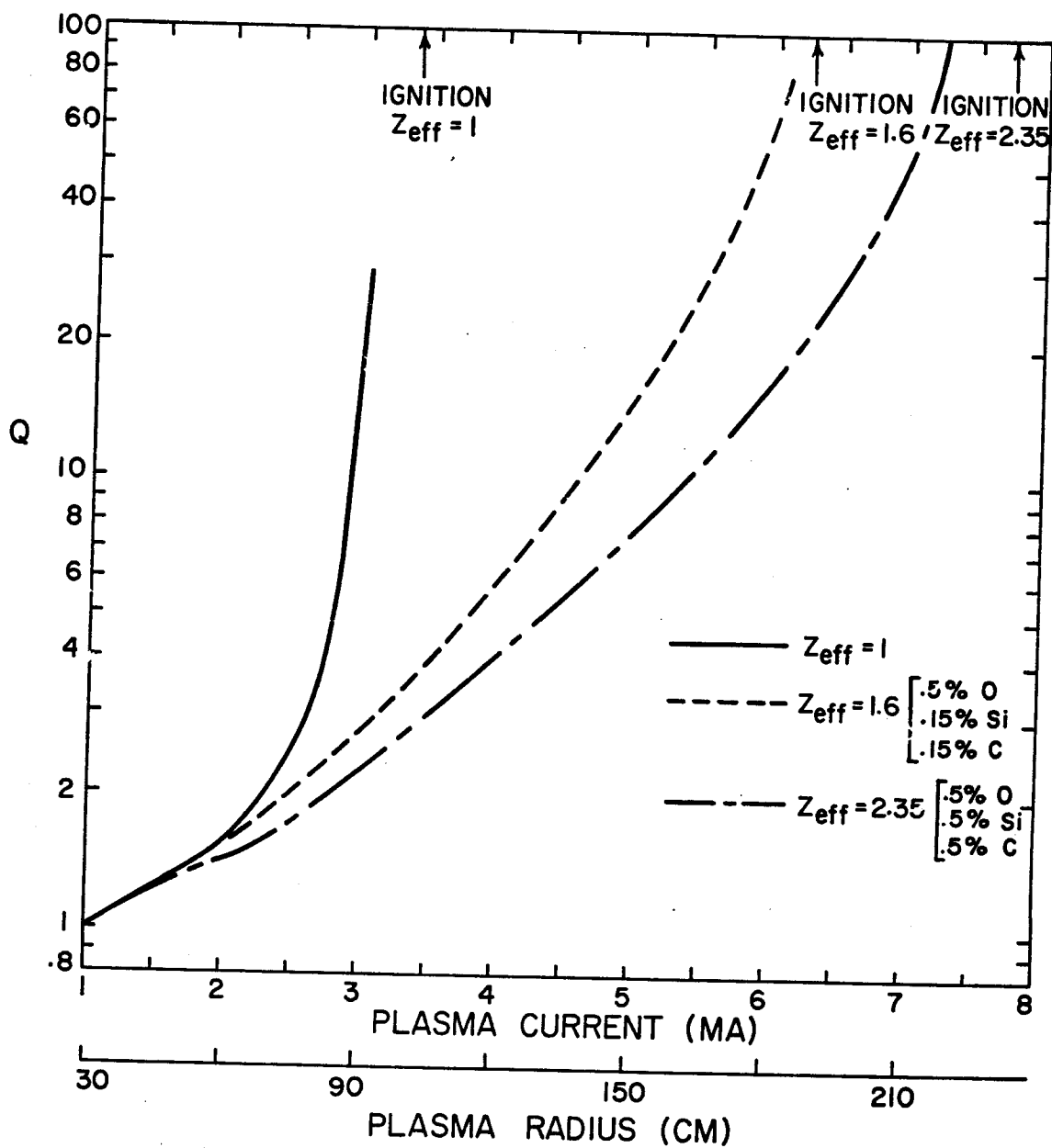of Impurity in the Plasma.

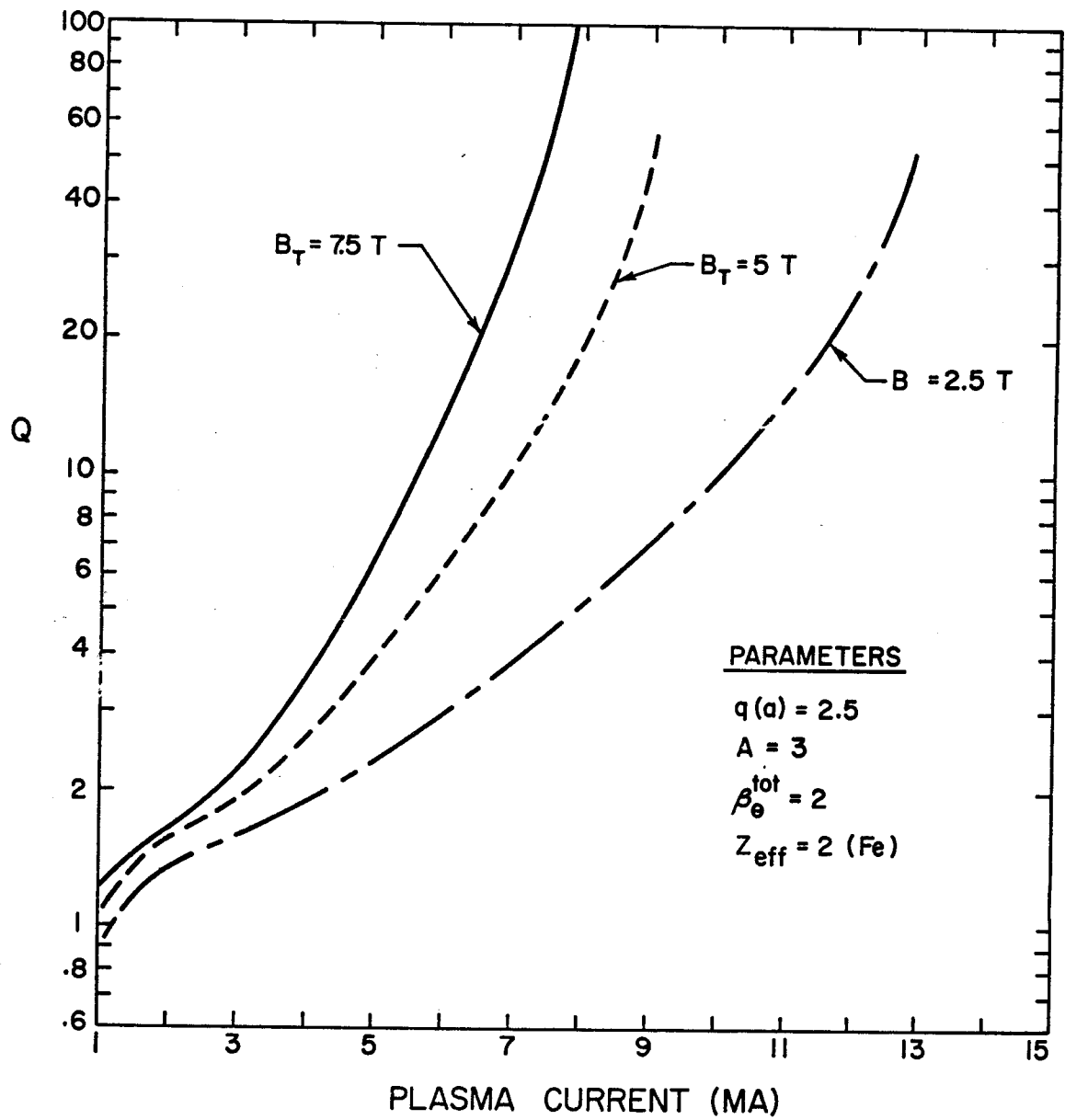Figure 13 — Optimum Q as a Function of Plasma Current for Varying Amounts of Si, C, and O as Impurities.

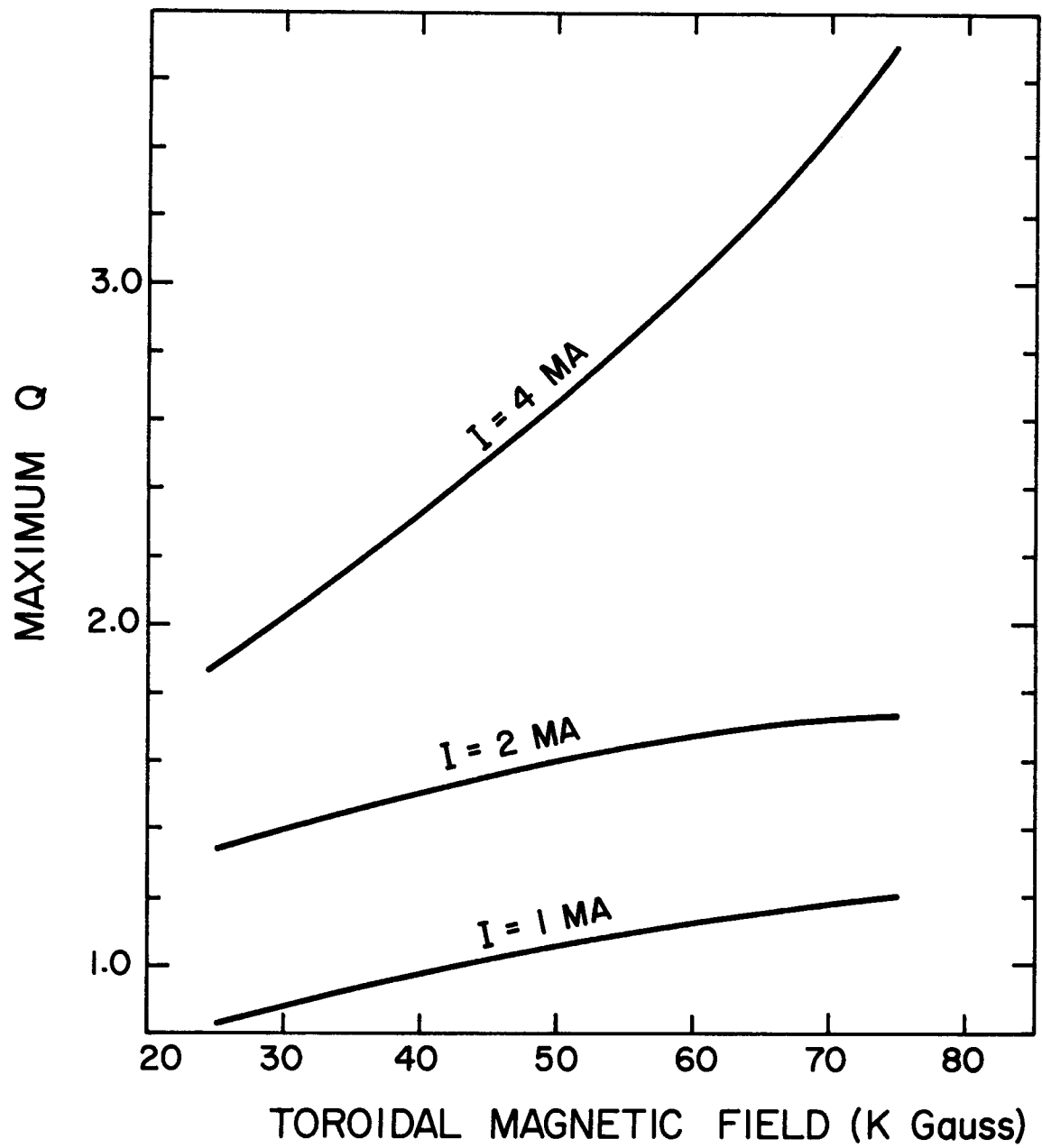Figure 14 – Optimum Q as a Function of Plasma Current for Different Values of the Toroidal Field on Axis.

Figure 15 - Optimum Q as a Function of Toroidal Field Strength.
$Z_{eff} = 2(Fe)$.

for high currents (I > 3 MA), because Q is proportional to $(P_{inj})^{-1}$ in machines dominated by thermal fusions.

b. Burning Dynamics of a TCT

Figure 16 indicates typical power balance curves for a TCT. The solid curve is the power input to the plasma, which is ohmic heating, injection heating, and alpha particle heating. The dashed curves represent the power losses from the plasma to which the main contributions are from bremsstrahlung, line radiation, recombination radiation and the trapped ion convective mode. For low Z_{eff} there is only one thermally stable equilibrium, namely point D on Figure 16. In addition, for a given injection power level and a sufficiently high Z_{eff}, thermal equilibrium will only occur at the low temperature point where ohmic heating is dominanat. For a moderate Z_{eff}, there will, however, exist three equilibrium solutions. The first (point A in Figure 16) represents the temperature at which an ohmically heated plasma will operate. To heat beyond this point requires an additional energy source to drive the plasma to the second, thermally unstable, operating point (point B in Figure 16). This is a "pseudoignition" point and, once there, the plasma will spontaneously heat to a third equilibrium point (point C in Figure 16), which is thermally stable. This spontaneous heating in a TCT means care must be taken to insure that, as the plasma heats, the β limit is not violated (β_θ < A).

Another way of viewing this process is to note that, for a given plasma current and Z_{eff}, there is a limited range of injected power within which the tokamak can operate. If too little power is injected, the plasma
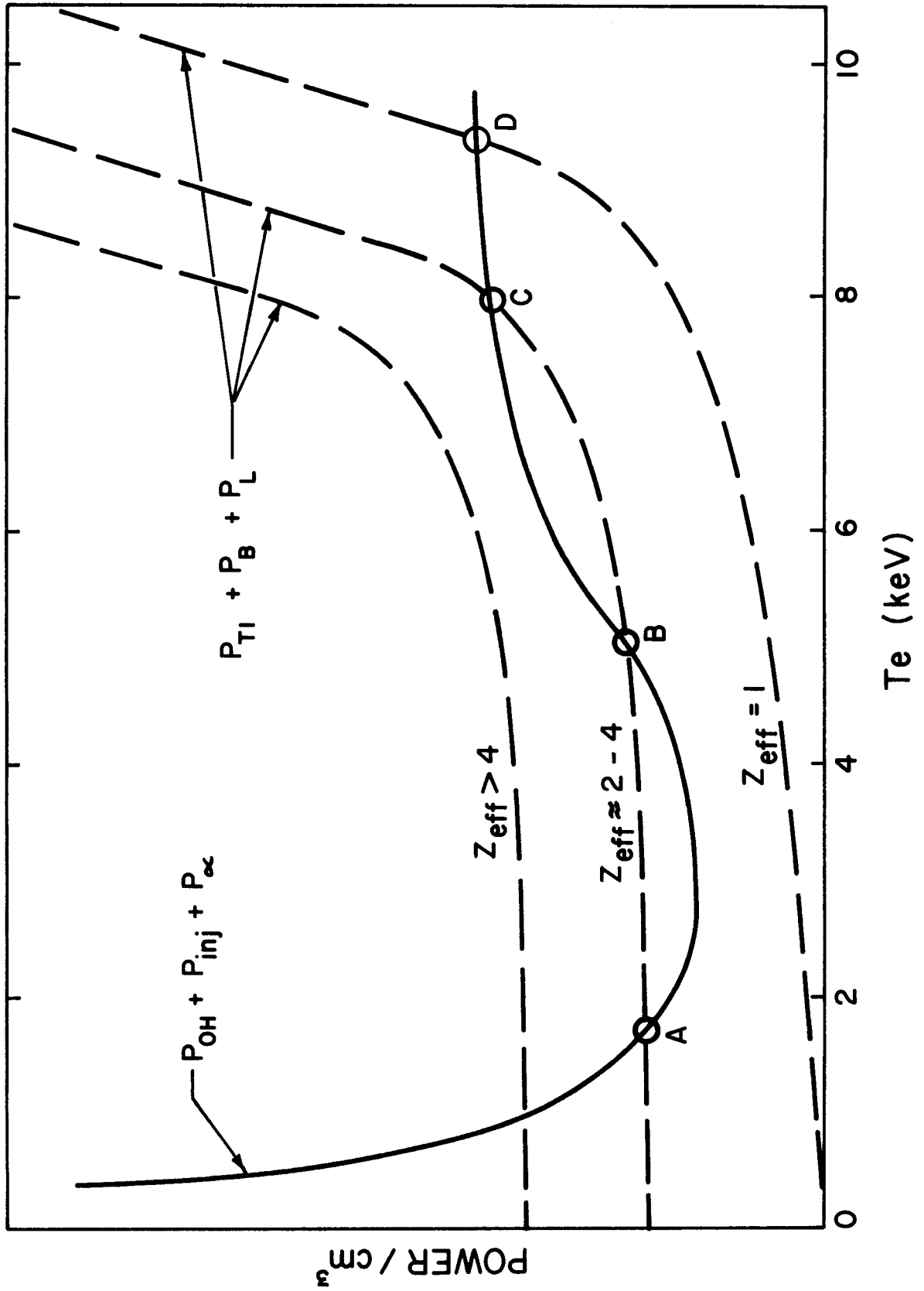
Figure 16 - Schematic to Illustrate the Various Thermal Equilibria Possible in a TCT Plasma.

cannot be sustained. If too much power is injected, the plasma will become too hot and violate the β limit. In the case of a TCT, it is desirable to inject the maximum power possible, consistent with the β constraint on the system, because Q is higher for hotter electron plasmas. This generally leads to a maximum Γ achievable of about one when Γ is defined as the energy in the energetic beam component divided by the thermal plasma energy

$$\Gamma = \frac{n_h W_o}{\frac{3}{2} (n_i T_i + n_e T_e)}$$

Here n_h is the density of high energy beam particles and W_o is the injection energy. In the case of a large driven Tokamak, however, the maximum Q is attained when the minimum energy is injected, as we noted previously, and the values of Γ are much lower.

c. High Currents and Driven Tokamaks

As we have already pointed out, a small increase in Z_{eff} above one causes a sharp increase in the current necessary to achieve ignition. This effect is more pronounced for high Z impurities. Figure 17 shows the increase in the current required to achieve ignition as a function of the impurity concentration for both iron (Z=26) and molybdenum (Z=42) as the impurity. For as little as .3% impurity, the current at ignition is approximately 12 MA for iron impurities and about 20 MA for molybdenum impurities. Thus, a modest contamination of high Z impurities would make ignition for a fusion reactor quite difficult. This, however, is perhaps not the primary consideration. The real question is whether high Q values are possible even in unignited, and therefore driven, Tokamak devices. For this reason, we have considered the potential of unignited driven Tokamaks as potential power producing systems. For convenience, we refer to such unignited systems as driven plasma reactors, or DPR.

The basic points are illustrated in Figure 18 which shows the variation of Q with Z_{eff} in Tokamaks of 6 MA and 10 MA. For the 6 MA case, high Q values are attained only for Z_{eff} quite near one. On the other hand, for the 10 MA discharge, the potential Q values are quite high, exceeding 10 for Z_{eff} as large as 5. As such, the potential of a DPR as a high gain amplifier is substantial. In Figure 19, we show the 10 MA case again along with the amount of auxillary power required to maintain the discharge. For Z_{eff} less than 2.2, the plasma would be ignited. Interestingly, the net electrical power output of such
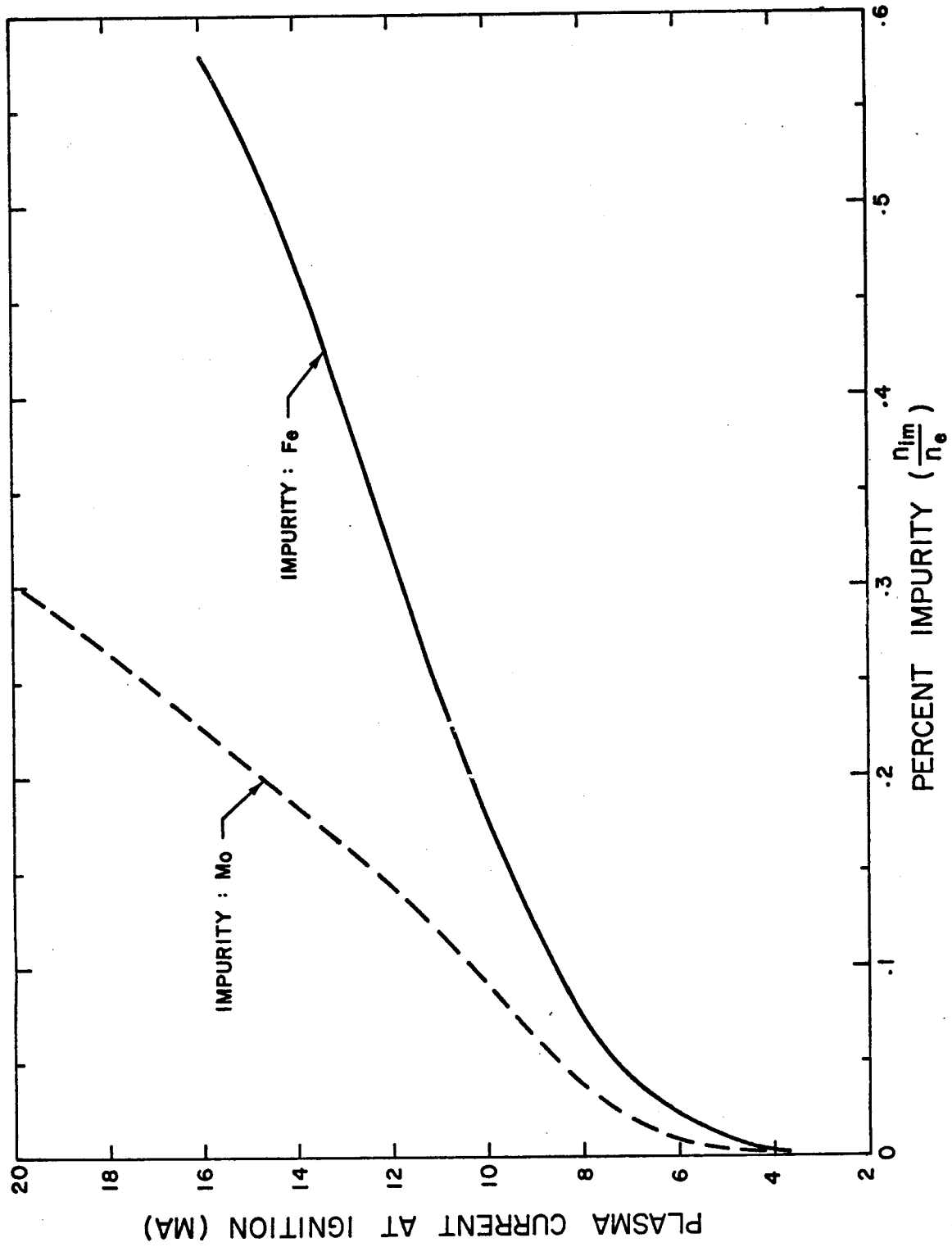
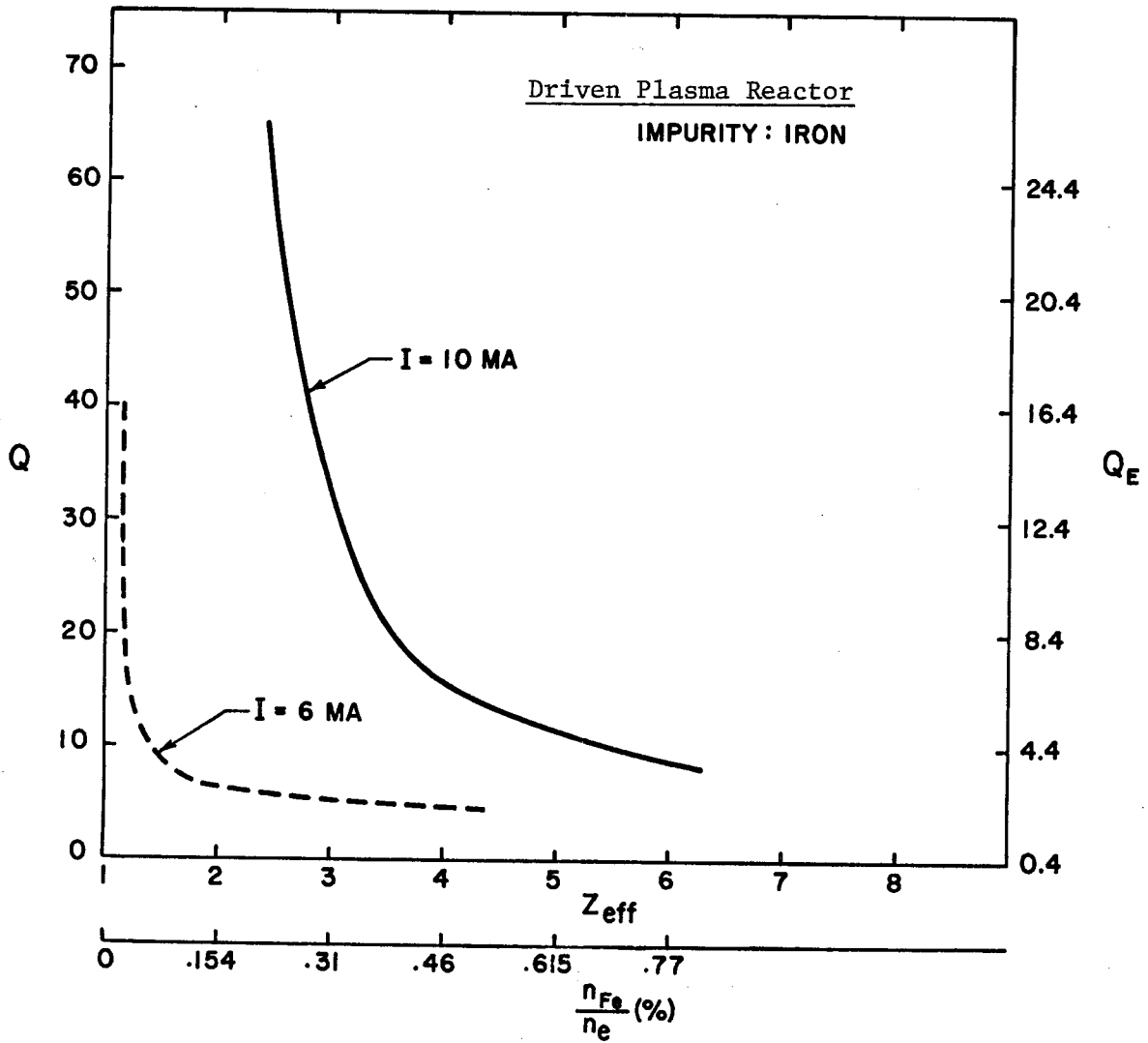Figure 17 – Plasma Current at Ignition as a Function of Impurity Concentration for Fe and Mo as the Impurity.

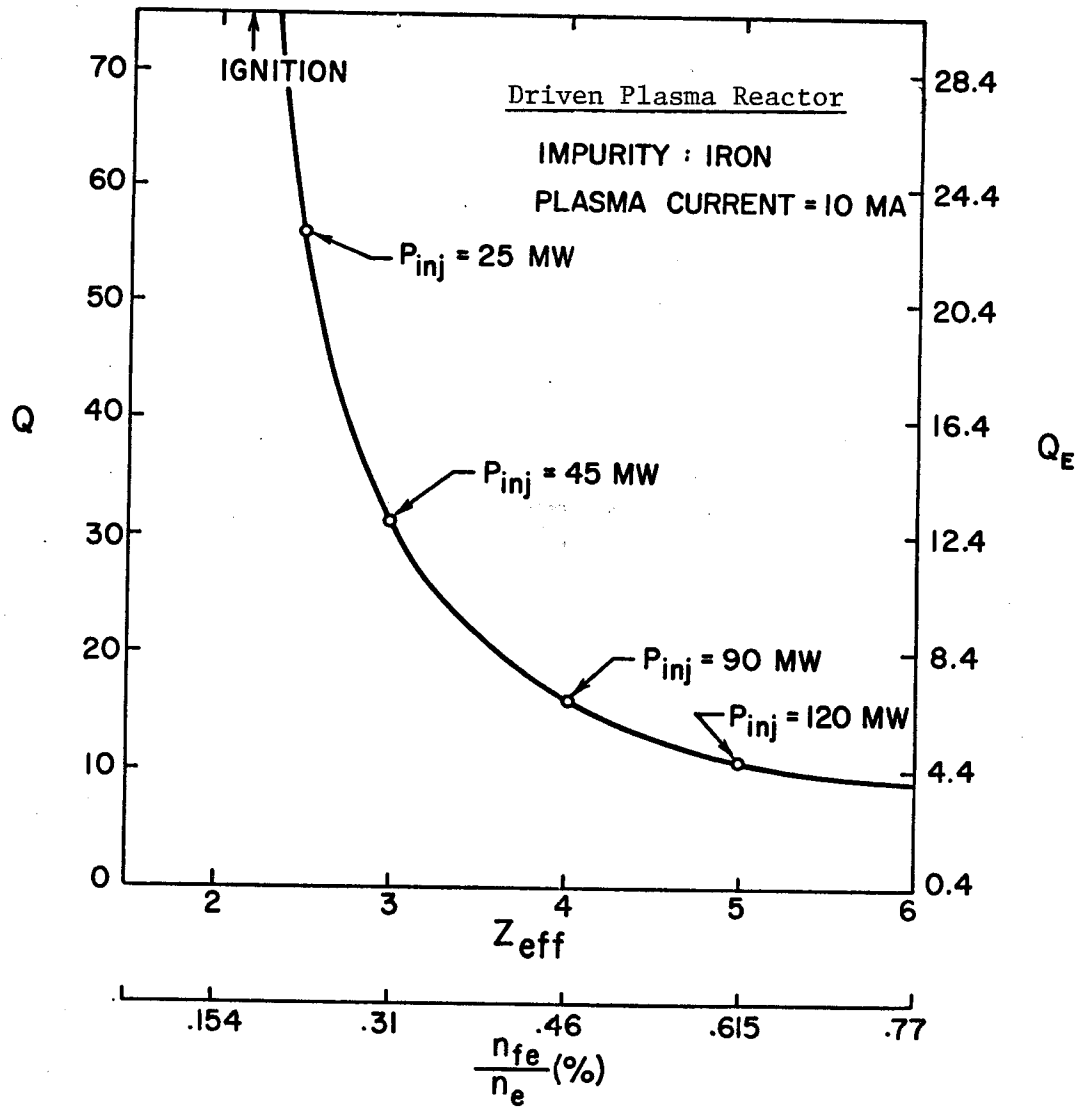Figure 18 - Plasma Amplification Factor, Q, as a Function of Impurity
Concentration.

Figure 19 – Plasma Amplification Factor, Q, as a Function of Impurity
Concentration. Also Indicated is the Amount of Auxiliary
Power Required to Sustain the Discharge.

driven systems varies by only about 10% as Z_{eff} varies from 1 to 5.
This is seen in Figure 20 and also in Table 3, which summarizes the
main results for this 10 MA case. For comparison, we note that the
current required for ignition at $Z_{eff} = 5$ is approximately 17 MA.

Since the injected power couples largely into the ions (~70% or
more) and the radiative energy loss comes from the electrons, the
electrons in large machines will often be up to 50% cooler than the ions.
This resulting temperature difference is beneficial because it will
reduce the trapped ion diffusion coefficient ($D_{\perp} \propto T_e^{9/2}/(1 + \frac{T_e}{T_i})^2$ for a β limited
system). Also, the energy balance for DPR's indicates that the injected power
plus fusion produced alpha power will basically balance the turbulent convective
energy loss and the electron radiative loss. Therefore, an increase in
either the rate of turbulent energy loss or radiative loss requires an
increase in injected power which translates into a decrease in Q. We
noted previously that Q in a TCT is relatively insensitive to field
strength and injected power. For the same reasons, it is also true that
the TCT Q-value is relatively insensitive to the numerical coefficient
in the trapped ion mode expression for the diffusivity. A factor of 10
change in $D_{\perp}$ produces an approximate 15% variation in the TCT Q-value.
By contrast, a DPR is quite sensitive to variations in the loss rate.

The reason is a DPR relies primarily on thermal fusions and not
on amplification of the beam. The maximum Q is attained with the minimum
injection power. and $Q \propto P_{inj}^{-1}$. As such, an increase or decrease in the
trapped ion mode losses translates directly into an increase or decrease
in the minimum injected power required to maintain the discharge in energy
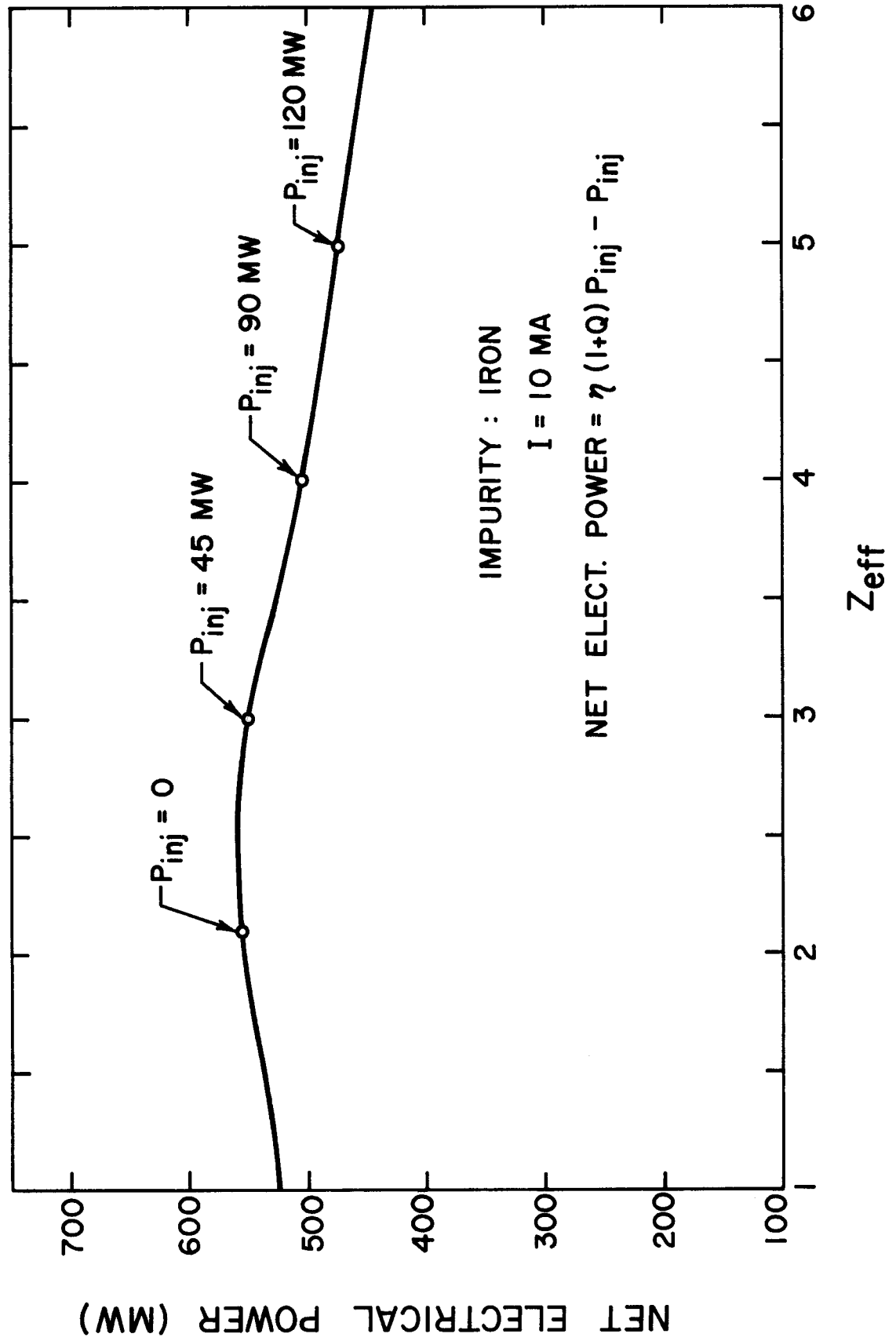balance, and thus to changes in Q.

Figure 20 – Net Electrical Power from a 10 MA Machine as a Function of Z_{eff}. The Thermal Efficiency, η, is .4 and E_{FUS} = 17.6 MeV.

Table 3

10 MA Driven Plasma Reactor

$q(a) = 2.5$
$B_t = 50 \text{ kg}$

$\beta_\theta^{tot} = 2$

Beam Energy = 180 keV

Z_{eff}	P_{inj} (MW)	N_e (cm^{-3})	T_i (keV)	T_e (keV)	$\frac{n_\alpha}{n_e}$ (%)	Q	P (MW$_e$)
2.5	25	8.2×10^{13}	13.2	12.6	2.64	56.4	549
3.0	45	8.1×10^{13}	13.9	12.5	3.18	32.1	553
3.5	70	7.2×10^{13}	15.8	13.6	2.58	20.2	534
4.0	90	6.7×10^{13}	17.3	14.1	2.41	15.6	506
5.0	120	6.2×10^{13}	19.7	14.4	2.47	11.5	480
6.0	150	5.6×10^{13}	27.2	15.1	2.17	8.8	438

This sensitivity does not invalidate the basic idea of the DPR but rather shifts the range of plasma currents in which the DPR concept is of interest. If the $n\tau$ is a factor of 10 smaller than the scaling law used, then the plasma current in a DPR shifts from approximately 10 MA to approximately 18 MA. An 18 MA discharge as a driven reactor is a unit greater than 1000 MWe. If the $n\tau$ is a factor of 10 larger than predicted, then 5-6 MA discharges make interesting DPR's and a 10 MA discharge is an ignited, self-sustaining machine. Another way of stating this result is that if the energy loss rate is 10 times smaller than calculated, a DPR with $Z_{eff} = 5$ could operate in the 6 to 10 MA range, with ignition occuring at 10 MA, whereas if the energy loss rate is 10 times higher, a DPR with the same Z_{eff} would operate in the 18-33 MA region. These results are summarized in Table 4. It therefore appears that early Tokamak fusion reactors might ultimately be driven, high Q, DPR's.

The approach of driven machines can also be used as a viable method for extending the burn time of Tokamaks if impurities accumulate slowing in time. Essentially, as Z_{eff} increases, one increases the injected power to sustain the plasma. Q therefore decreases with time and the burn would terminate when Q falls below some value, but not when the system is no longer self-sustaining. This can mean viable operation until Z_{eff} exceeds 5 or 6. In essence, one rides down along the curve in Figure 18.

Table 4

Sensitivity to Trapped Ion Scaling

For Z_{eff} = 5 (for Iron)

Scaling	Current at Ignition (MA)	Current Range for DPR's having Q >10 (MA)
Trapped Ion Mode Diffusion Rate (Ref. 5)	17	10 - 17
10X Trapped Ion Mode Diffusion Rate	33	18 - 33
0.1X Trapped Ion Mode Diffusion Rate	10	6 - 10

As noted earlier, the net power out as a function of Z_{eff} is relatively constant so that decreasing Q does not mean severely decreasing power output until Z_{eff} gets quite large (>5).

As a final point, we have computed injected energy deposition profiles for neutral beam injection in some of the typical systems considered herein. These are shown in Figures 21 through 24. A parabolic density profile has been used in all these calculations. From Figure 21, which is for injection tangent to the magnetic axis in an I=2MA machine, a 180 KeV beam is adequate for penetration and provides a quite flat deposition profile. In the 4 MA (Figure 22), 6 MA (Figure 23), and 10 MA (Figure 24) cases, injection was along a chord half way between the inside edge of the plasma and the magnetic axis[12]. We have included the 180 KeV case along with the beam energy which gives a relatively flat deposition profile. For the 10 MA case, where the peak density is 1.5×10^{14}, between 500 KeV and 750 KeV beams would be required to deliver relatively flat deposition profiles. We note, however, that the power required for the DPR case (I > 5 MA) are nevertheless relatively low and lower than is required for a TCT. Importantly, neutral beams are not a unique heating source for the DPR concept, and the power levels required are fairly modest (between 25 and 150 MW). This in nominal for RF sources and the main open question is the coupling of the RF power to the ions. If relatively uniform RF heating is possible, it would serve equally well as the power source for the DPR. Thus, the technology of negative ions for energies like 500 KeV could be circumvented.
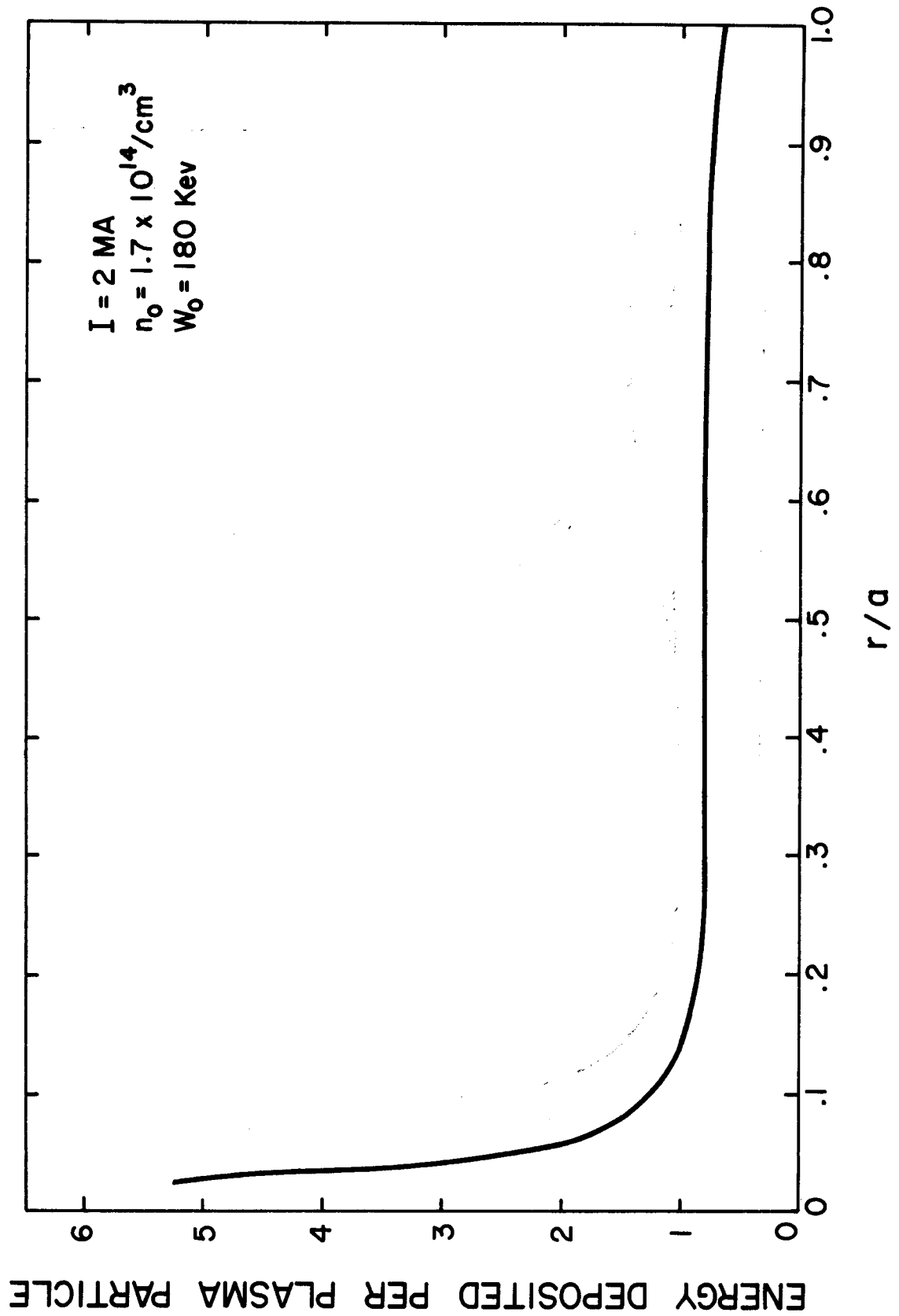
Figure 21 – Beam Energy Deposition Profile in a 2 MA Discharge. The Injection Chord is Tangent to the Magnetic Axis and the Density Profile is Parabolic.
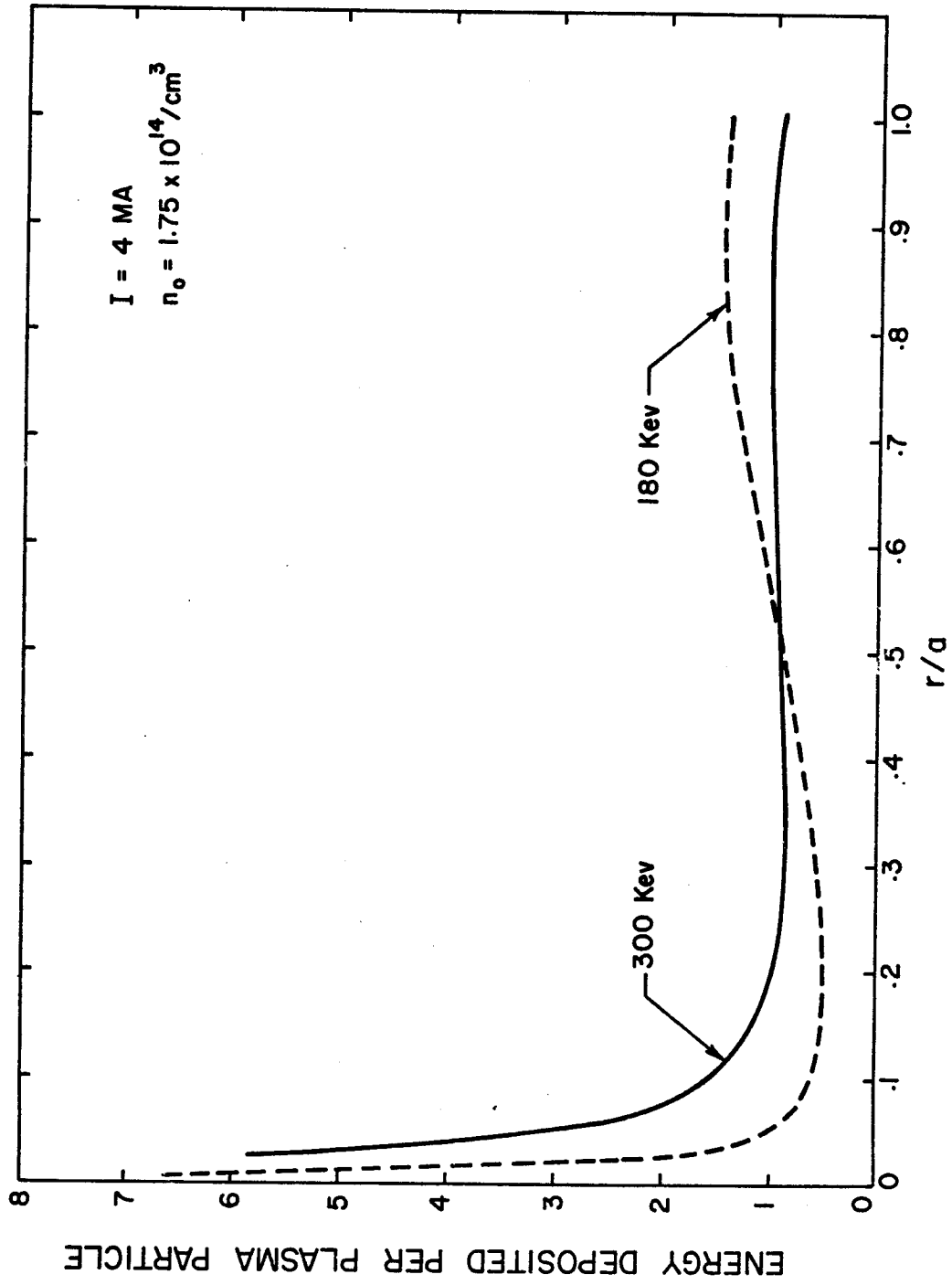
Figure 22 – Beam energy deposition profile in a 4 MA discharge for injection half way between the inner edge of the plasma and the magnetic axis. The density profile is parabolic.
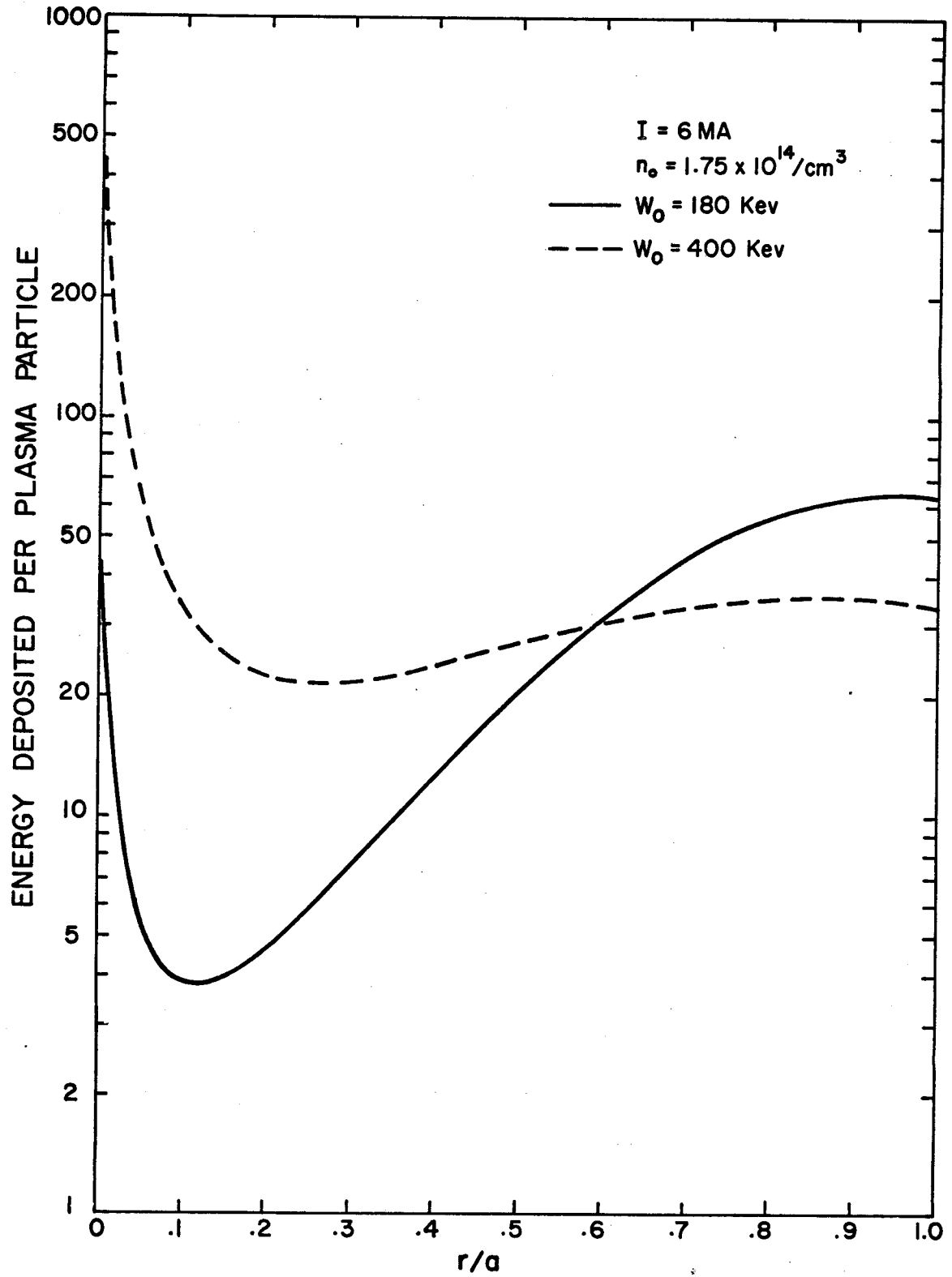
Figure 23 – Beam energy deposition in a 6 MA discharge for injection half way between the inner edge of the plasma and the magnetic axis. The density profile is parabolic.
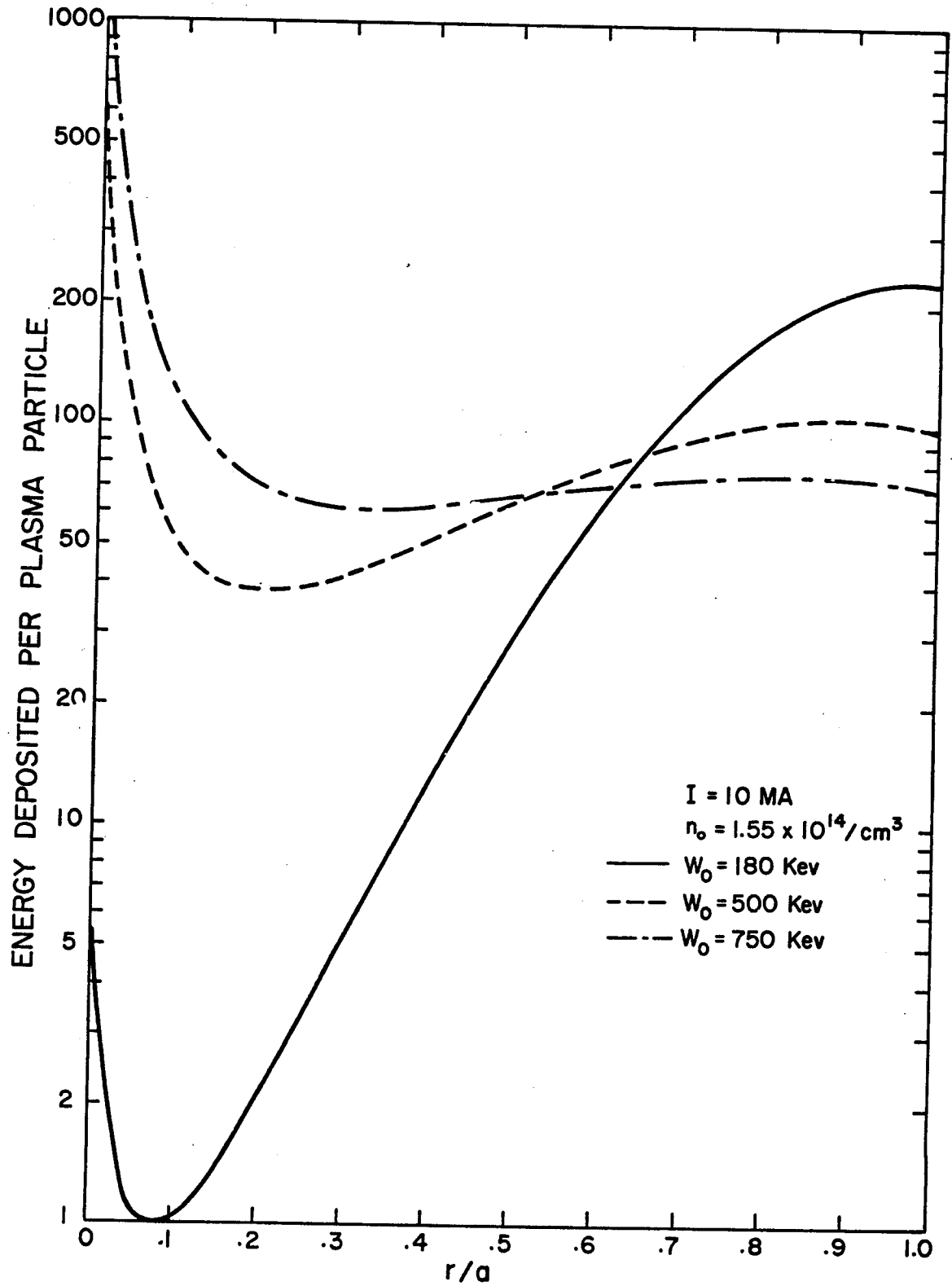
Figure 24 — Beam energy deposition in a 10 MA discharge for injection half
way between the inner edge of the plasma and the magnetic axis.
The density profile is parabolic.

V. Summary and Conclusions

We summarize first the main results of the parametric study and then describe the results for the driven plasma reactor concept.

1. Parametric Study of Driven Tokamaks

The study of the transition from a pure TCT (tritium plasma and deuteron beams) at low plasma currents to 50-50 D-T driven Tokamaks at larger currents has been carried out by examining the variation of the optimum plasma thermonuclear amplification factor, Q, as a function of plasma current, and thus of machine size. The following results have been found:

a. For Tokamaks with plasma current less than 2 MA, a pure TCT with 100% T ions as the target plasma (no deuterium but possibly impurities) gives the optimum Q. For machines with I>4MA, the maximum Q is obtained when the plasma is a 50-50 D-T mixture. The transition in maximum Q versus I from a pure TCT to a driven Tokamak with a 50-50 D-T plasma typically takes place when the plasma current is between 2MA and 4MA, depending on Z_{eff}. For $Z_{eff} = 1$, this transition takes place for I between 2 and 2.5 MA and ignition takes place in a 3.4 MA discharge. These and other results are based on the assumption that the trapped ion mode governs the transport scaling.

b. The effect of injected power on the optimum Q is opposite in low current machines than in high current devices. For I ≤ 2.5, it is best to inject the maximum power permitted by the β-limit to obtain the highest Q. The target should be pure T. For I ≥ 2.5, the minimum power which will sustain the plasma as a 50-50 D-T target gives the optimum Q. Also, for a given injected power, there are no equilibria if I is too small because β is exceeded. Above some maximum I, there are no solutions because the injected power cannot sustain the discharge.

c. The effect of available magnetic field on Q is small for B_T between 2.5 T (tesla) and 5 T when the plasma current is less than 2-3 MA. However, in larger systems, there is a significant gain in Q for higher B_T, essentially because for I > 2.5 MA, the plasma is 50-50 D-T, the injected power is low for optimum Q and thermal fusions are most relevant. Thus, increasing B_T and increasing $n\tau$ has a big effect.

d. The impact of high Z impurities, such as Fe and Mo, on the TCT Q for I between 1 and 2 MA is small. Low Z impurities have the most significant impact because they lower the allowed ion density in the target plasma. This is opposite the effect in larger current machines where high Z impurities have the biggest impact. This is discussed shortly.

e. The maximum obtainable Q in a pure TCT is insensitive to the exact numerical coefficient in the thermal diffusivity predicted for the trapped ion mode. A factor of 10 change either way in $D_\perp$ causes an approximately 15% change in the predicted Q. The reason is that the maximum Q in a TCT is obtained by amplification of the maximum injected beam power consistent with the β limit.

f. The impact of impurities on machines with I > 2.5 MA is very significant. From graphs of the plasma current at ignition (I_{ign}) versus Z_{eff}, we find that I_{ign} increases very sharply, almost as a step function, as Z_{eff} increases above 1. For example, less than .05% Fe or less than .025% Mo causes I_{ign} to more than double from ~3.4 MA to >7 MA. Doubling the plasma current is a factor of eight increase in the volume of the machine.

g. Impurities cause the transition from low Q to high Q to take place much more slowly than for Z_{eff} = 1. We find that Q is approximately constant for I in the range, 2-3 MA, and then begins to increase. For Z_{eff} = 3 and iron as the impurity, the ignition current is 11.9 MA. Thus, the maximum possible Q in the current range of 4-6 MA, the possible range for an Experiment Power Reactor (EPR), is relatively low unless Z_{eff} is very close to 1. However, this does not mean that high Q values are not possible in reactor systems such as I = 10 MA. The question is how high is Q for plasma currents of the order of 10 MA. It is found that Q can be very high and leads to consideration of the driven plasma reactor (DPR).

2. Driven Plasma Reactor

a. From calculations of Q versus Z_{eff} at I = 10 MA with iron as the impurity, we find Q greater than 10 for Z_{eff} as large as 6. For Z_{eff} equal to 3 the Q is approximately 35. The injected power required to sustain the plasma is only 25 MW at Z_{eff} = 3 and 120 MW at Z_{eff} = 5. A 10 MA discharge would be ignited for $Z_{eff} \lesssim 2$. On the other hand, at I equal to 6 MA, the idea of a driven machine does not appear as interesting when dominated by impurities since the attainable Q values are not nearly as high.

These results indicate that impurity control is necessary to keep $Z_{eff} \lesssim 6$ but that it is not necessary to have Z_{eff} approach 1 in systems of reactor interest. Thus, driven Tokamaks can constitute a serious and viable approach to power production for the low $- \beta$ program. This also lends additional virtue to experiments that can add to our knowledge of the behavior of driven machines.

    b. The approach of driven machines can also be used as a viable method for extending the burn time of Tokamaks if impurities accumulate slowly in time. Essentially, as Z_{eff} increases, one increases the injected power to sustain the plasma. Q therefore decreases with time and the burn would terminate when Q falls below some value, but not when the system is no longer self-sustaining. This can mean viable operation until Z_{eff} exceeds 5 or 6. Interestingly, the net power out as a function of Z_{eff} is relatively constant so that decreasing Q does not mean severely decreasing power output until Z_{eff} gets quite large (>6).

    c. High Z impurities, such as Fe or Mo, produce the most severe effects. If limiters or wall coatings can be made feasible with low Z materials, such as metallic carbides, SiC or Al_2O_3, the tolerable level of impurities is significantly altered. For example, .3% Fe impurity increases the current at ignition to be approximately 12 MA. Yet .15% Si, .15% C and .5% O only increases the ignition current from 3.4 MA to 6.25 MA. Further, a plasma with an impurity level as high as .5% Si, .5% C, and .5% O ignites at I = 7.8 MA whereas a plasma with .5% Fe ignites at I approximately 15 MA and .5% Mo raises the current at ignition to much greater than 25 MA. Therefore, low Z coatings on chamber walls and low Z limiters alter the nature and severity of impurities substantially, and for the better.

d. Neutral beam injection at energies of between 150 KeV and 200 KeV are adequate to penetrate plasmas up to I = 4 MA. Beyond this, the profiles become inverted even for injection inside the magnetic axis and higher energies are required for penetration. At 10 MA, where a = 300 cm and $\bar{n}_e = 8x10^{13}$, one requires between 500 KeV and 700 KeV to obtain fairly flat beam energy deposition profiles.

Importantly, beams are not a unique heating source for the DPR concept and the power levels required are fairly modest (between 25 and 150 MW). **These are not outside the technical potential of RF sources** and **the main open question is the coupling of the RF power to the ions.** If relatively **uniform RF heating is possible, it would serve equally well as the power** source for the DPR. **Thus, the technology of negative ions for energies like 500 KeV** could be circumvented.

e. Whereas a TCT is relatively insensitive to the numerical coefficient in the trapped ion mode scaling formula for $D_\perp$, a DPR is sensitive to this coefficient. The reason is a DPR relies primarily on thermal fusions and not on amplification of the beam. The maximum Q is attained with the minimum injection power. As such, an increase or decrease in the trapped ion mode losses translates directly into an increase or decrease in the minimum power required to maintain the discharge in energy balance.

This sensitivity does not invalidate the basic idea of the DPR but rather shifts the range of plasma currents in which the DPR concept is of interest. If the $n\tau$ is a factor of 10 smaller than the scaling law used, the plasma current in an DPR shifts from approximately 10 MA to approximately 18 MA. An 18 MA discharge as a driven reactor is a unit greater than 1000 MWe. If the $n\tau$ is a factor of 10 larger than predicted, then 5-6 MA discharges make interesting DPR's and a 10 MA discharge is an ignited, self-sustaining machine.

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